# Cosmic Neutrinos

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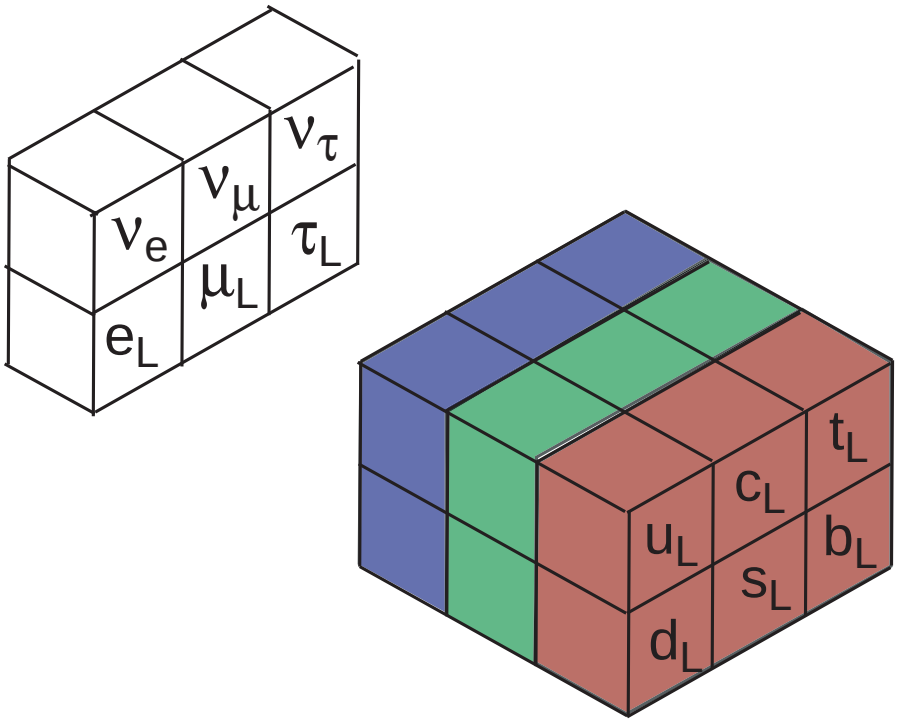
# Neutrinos are abundant!

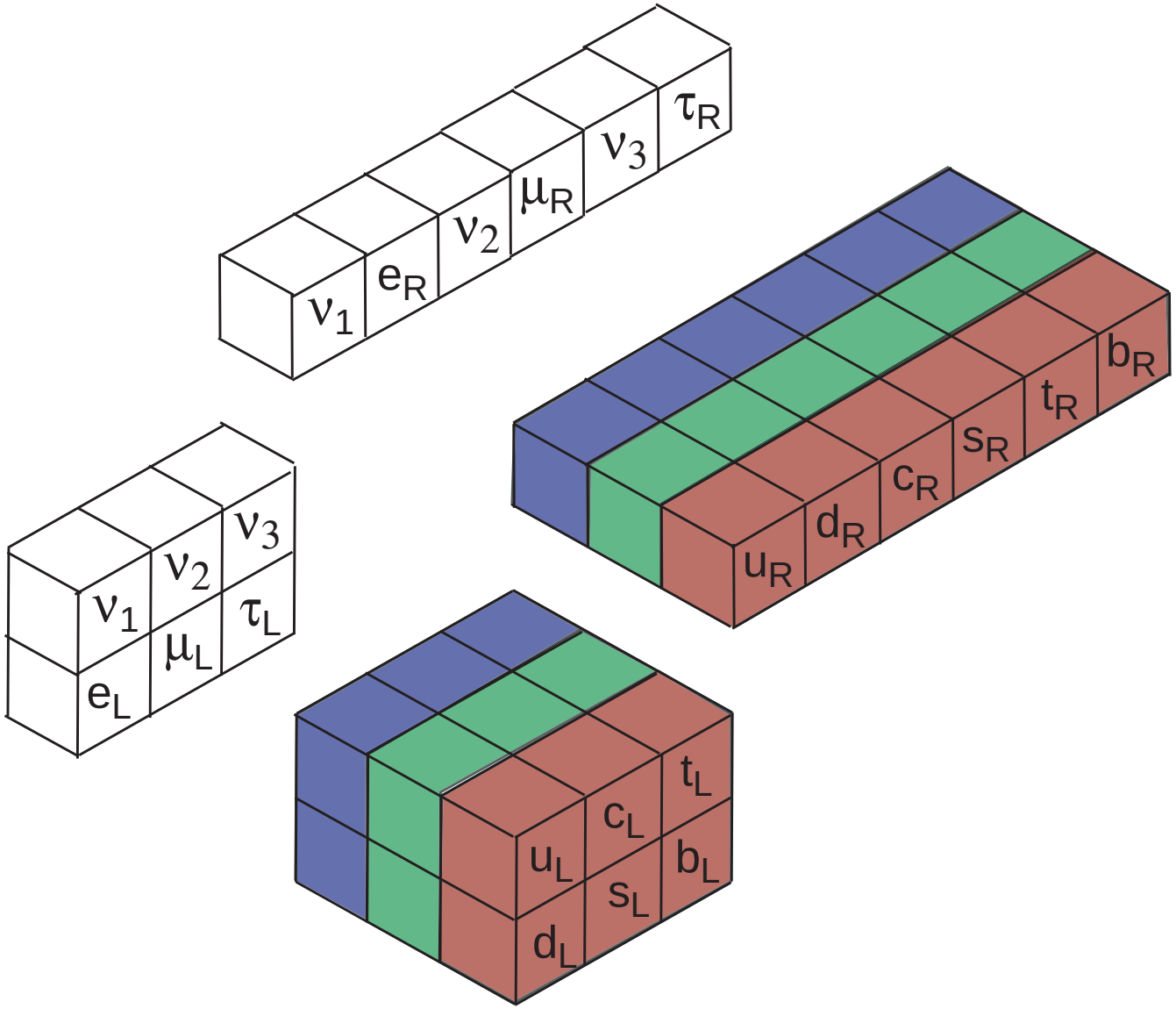
- Each second, some $10^{14}$ neutrinos made in the Sun pass through your body.
- Each second, about $10^{3}$ neutrinos made in Earth's atmosphere by cosmic rays pass through your body.
- Other neutrinos reach us from natural (radioactive decays of elements inside the Earth) and artificial (nuclear reactors, particle accelerators) sources.
- Inside your body are more than $10^{7}$ neutrino relics from the early universe.

$\nu_i,\ \bar{\nu}_i$ number density now: 56 $\mathrm{cm}^{-3}$, $\propto (1+z)^3$

# Neutrinos in the electroweak theory



# Neutrinos in the electroweak theory



# Leptons are observed as free particles

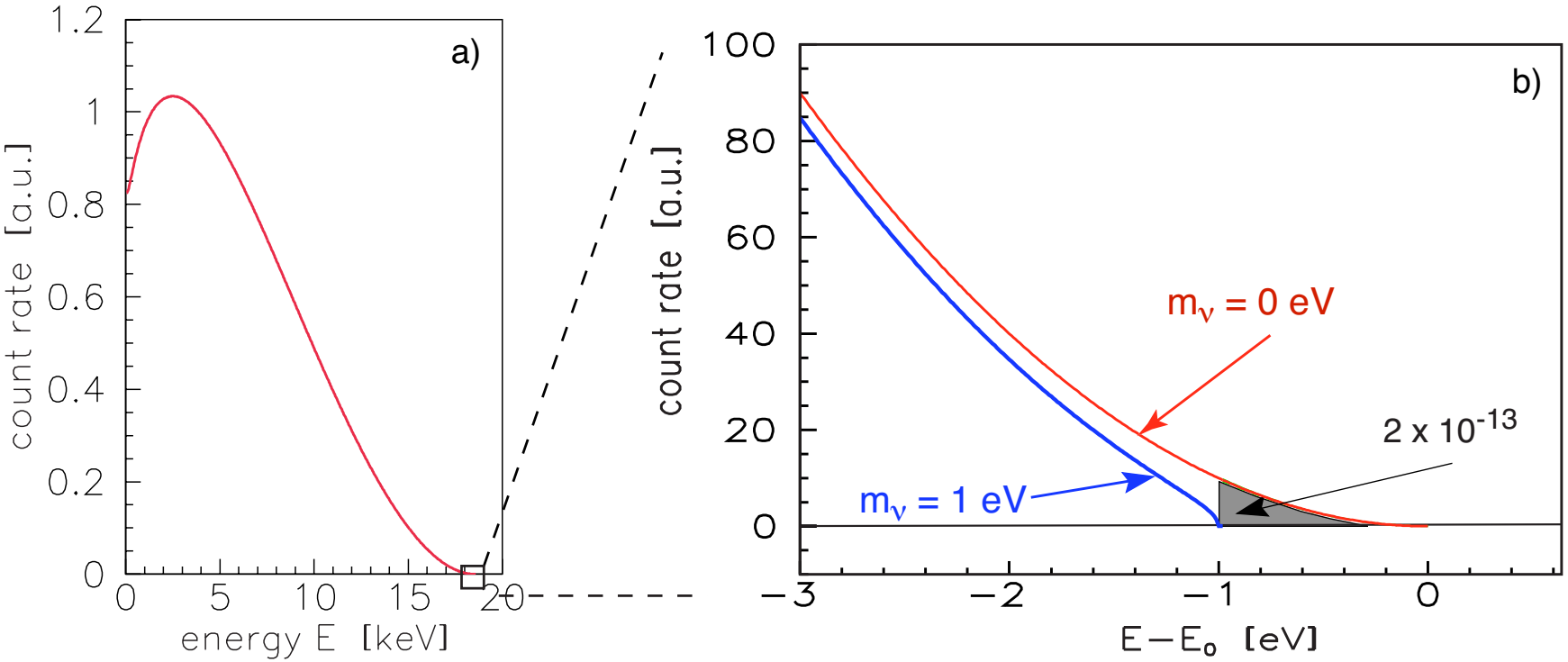
| Lepton | Mass | Lifetime |
|---|---|---|
| $\nu_e$ | $< 2$ eV | |
| $e^-$ | 0.510 998 918(44) MeV | $> 4.6 \times 10^{26}$ y (90% CL) |
| $\nu_\mu$ | $< 0.19$ MeV (90% CL) | |
| $\mu^-$ | 105.658 369 2(94) MeV | $2.197\,03(4) \times 10^{-6}$ s |
| $\nu_\tau$ | $< 18.2$ MeV (95% CL) | |
| $\tau^-$ | $1776.90 \pm 0.20$ MeV | $290.6 \pm 1.0 \times 10^{-15}$ s |

All spin-$\frac{1}{2}$, pointlike ($\lesssim$ few $\times 10^{-17}$ cm)

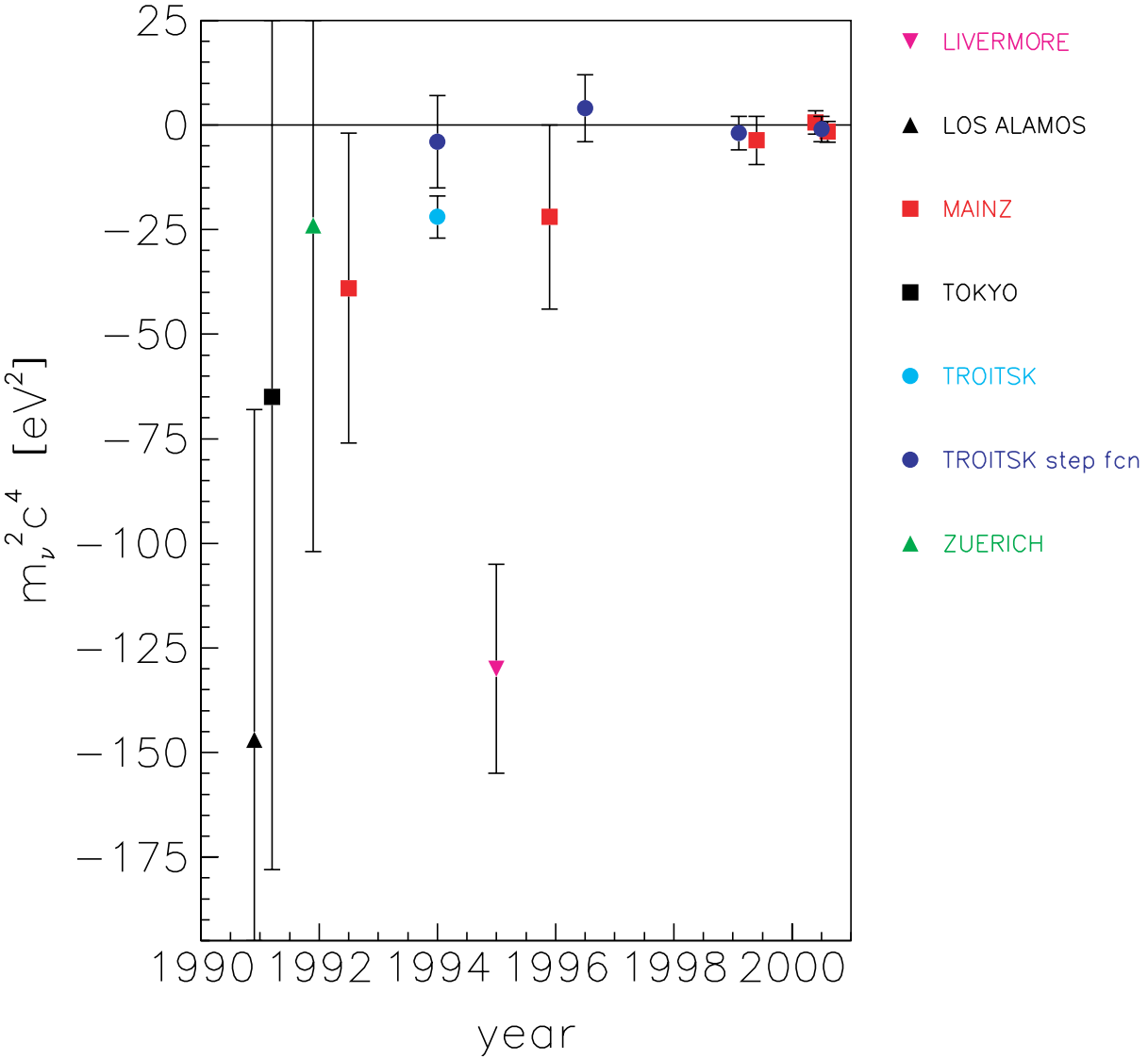
*kinematically determined* $\nu$ masses consistent with 0
($\nu$ oscillations $\Rightarrow$ nonzero, unequal masses)

# Tritium $\beta$-decay spectrum

$$^3\mathrm{H} \rightarrow {}^3\mathrm{He}\ e^-\ \bar{\nu}_e\text{:}\ Q \approx 18.57\ \mathrm{keV}$$

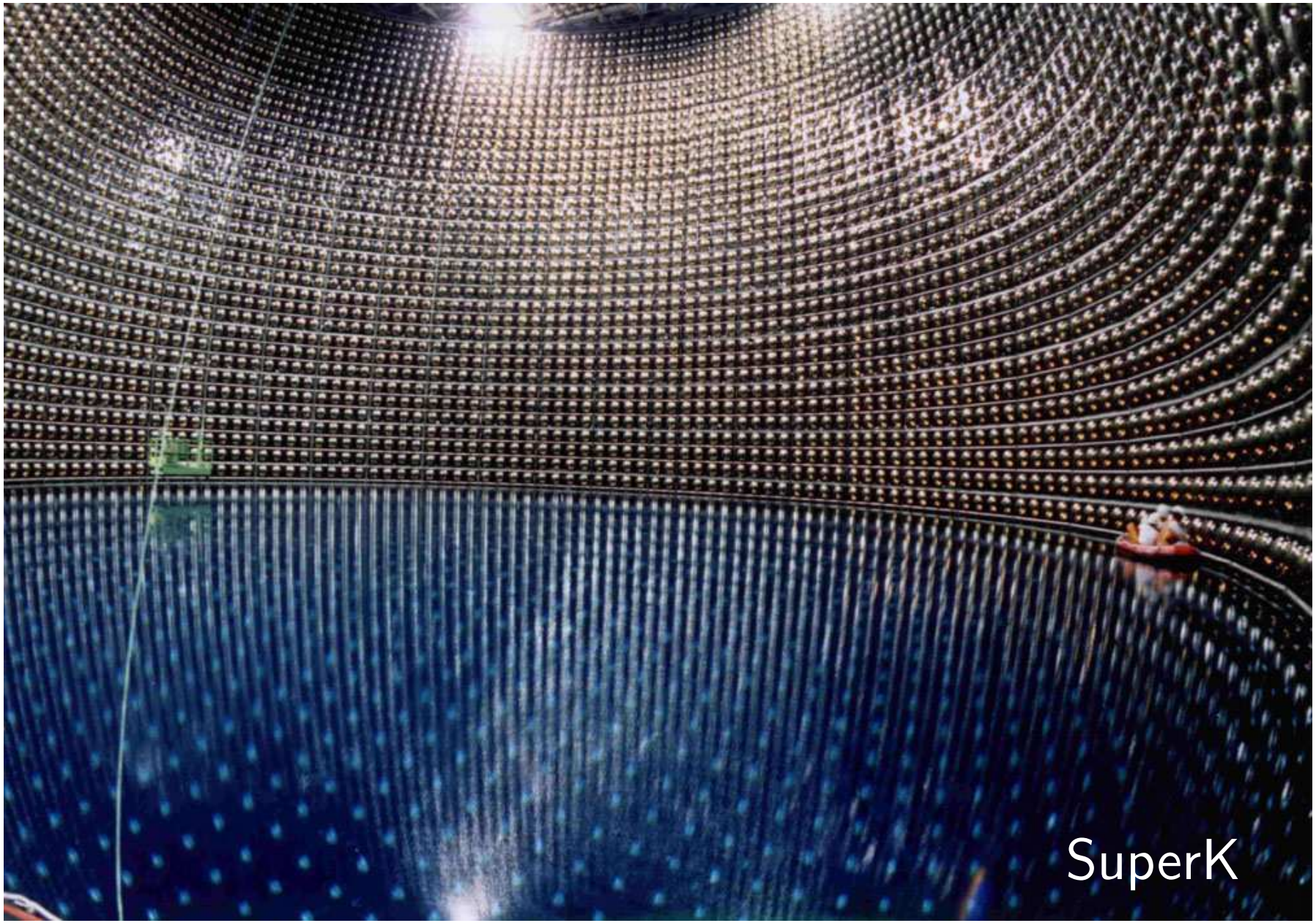


# $\nu$ mass limits ($e$-associated)



# KATRIN aims for $m_\nu \lesssim 0.2$ eV

# Neutrino flavor change $\rightsquigarrow$ neutrinos have mass



# The essence of neutrino oscillations

If neutrinos of definite flavor $(\nu_e, \nu_\mu, \nu_\tau)$ are superpositions of different mass eigenstates $(\nu_1, \nu_2, \nu_3)$
the mass eigenstates evolve in time with different frequencies . . . and so the superposition changes in time

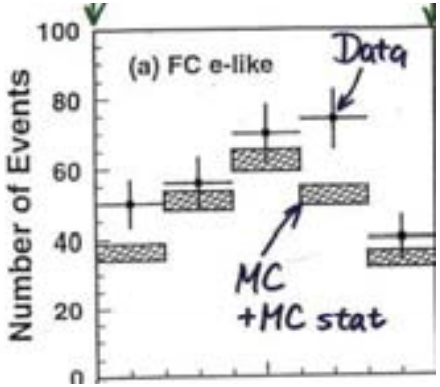
$\boxed{\text{Beam created as flavor } \nu_\alpha \text{ evolves into a flavor mixture}}$

*Suppose* $\nu_\alpha = \nu_1 \cos\theta + \nu_2 \sin\theta$; $\nu_\beta = -\nu_1 \sin\theta + \nu_2 \cos\theta$:

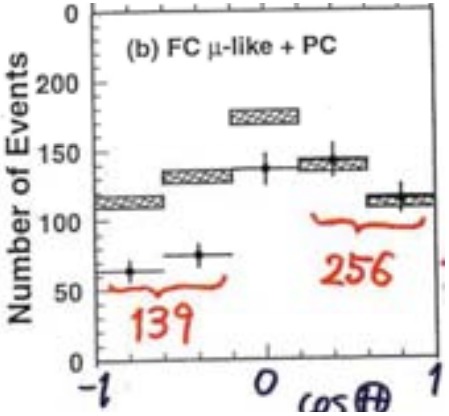
After distance $L$, beam created as $\nu_\alpha$ with energy $E$ has

$$P_{\alpha\to\beta} = \sin^2 2\theta \sin^2\left(\Delta m^2 L/4E\right)$$

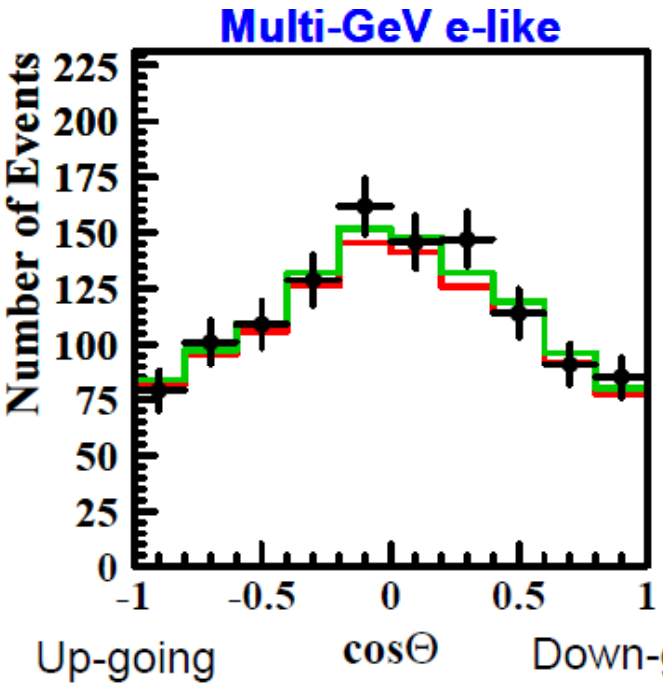
# SuperK Atmospheric Neutrinos

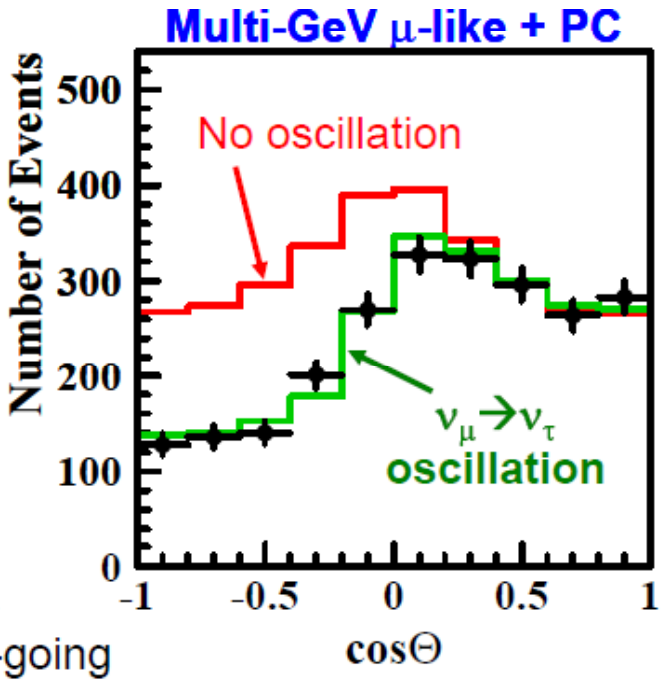



@Neutrino98
(535 day)



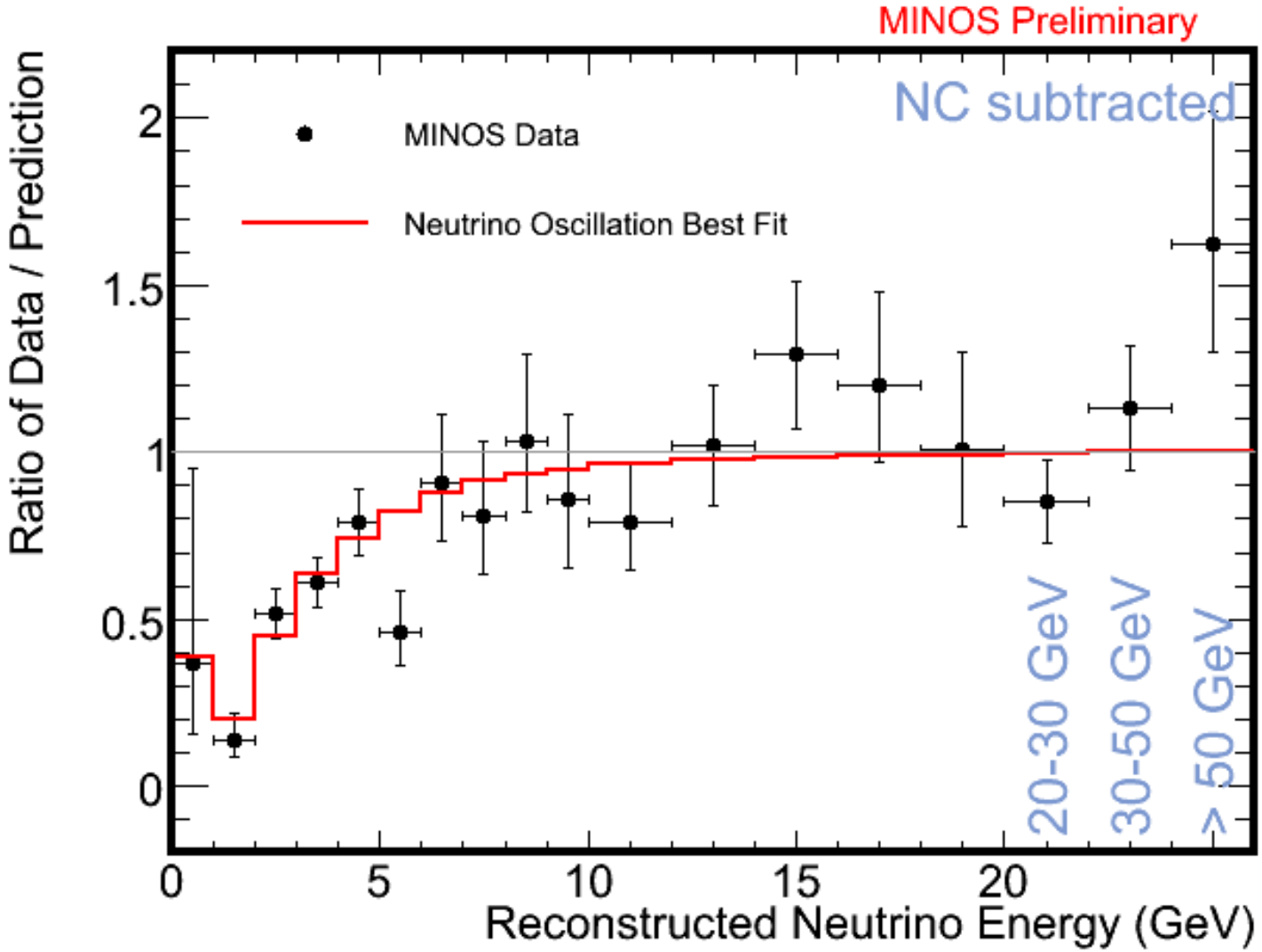

Now
(2293 day)



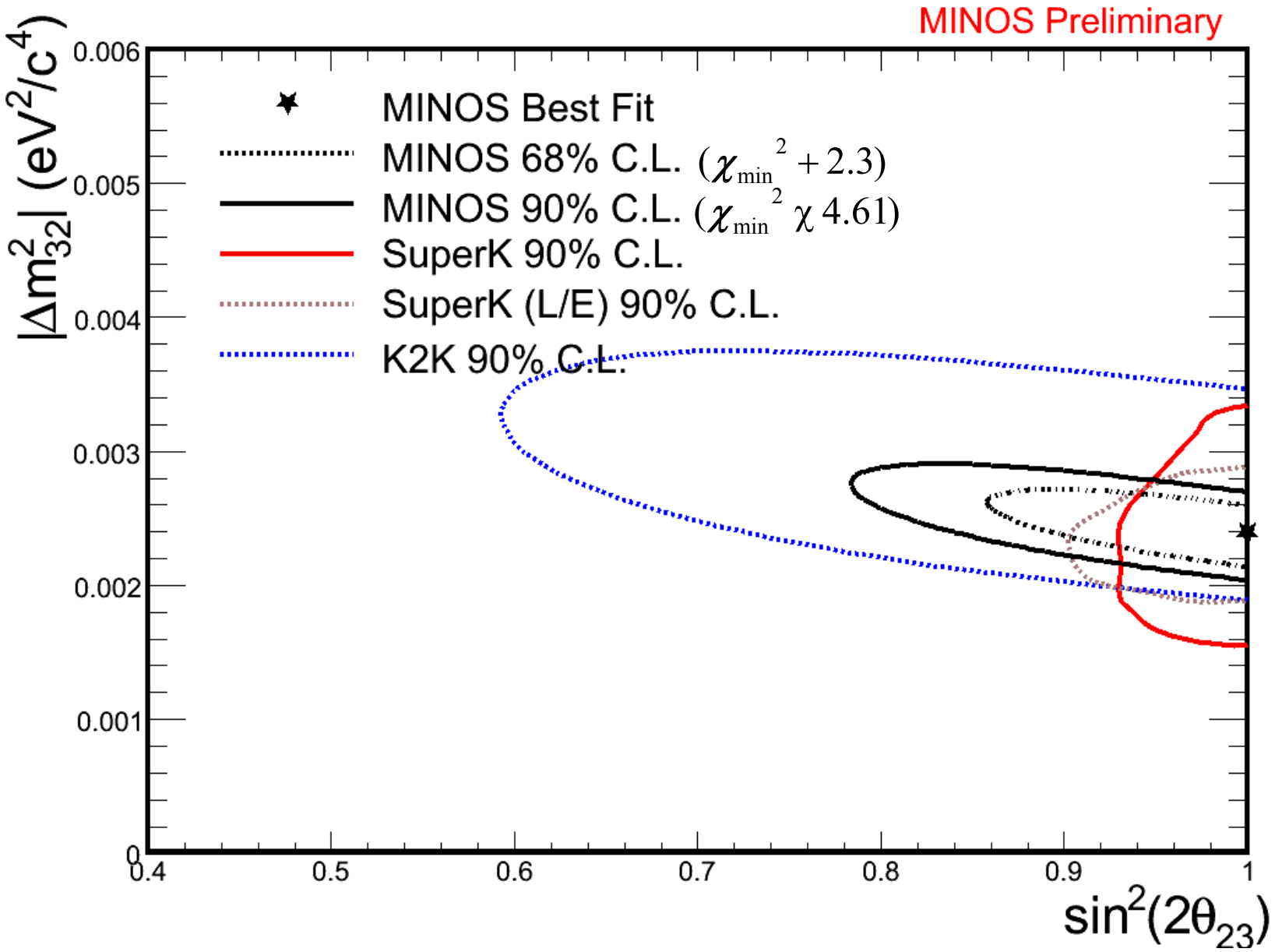




Downward $\nu$ travel 15 km, upward $\nu$ up to 13 000 km

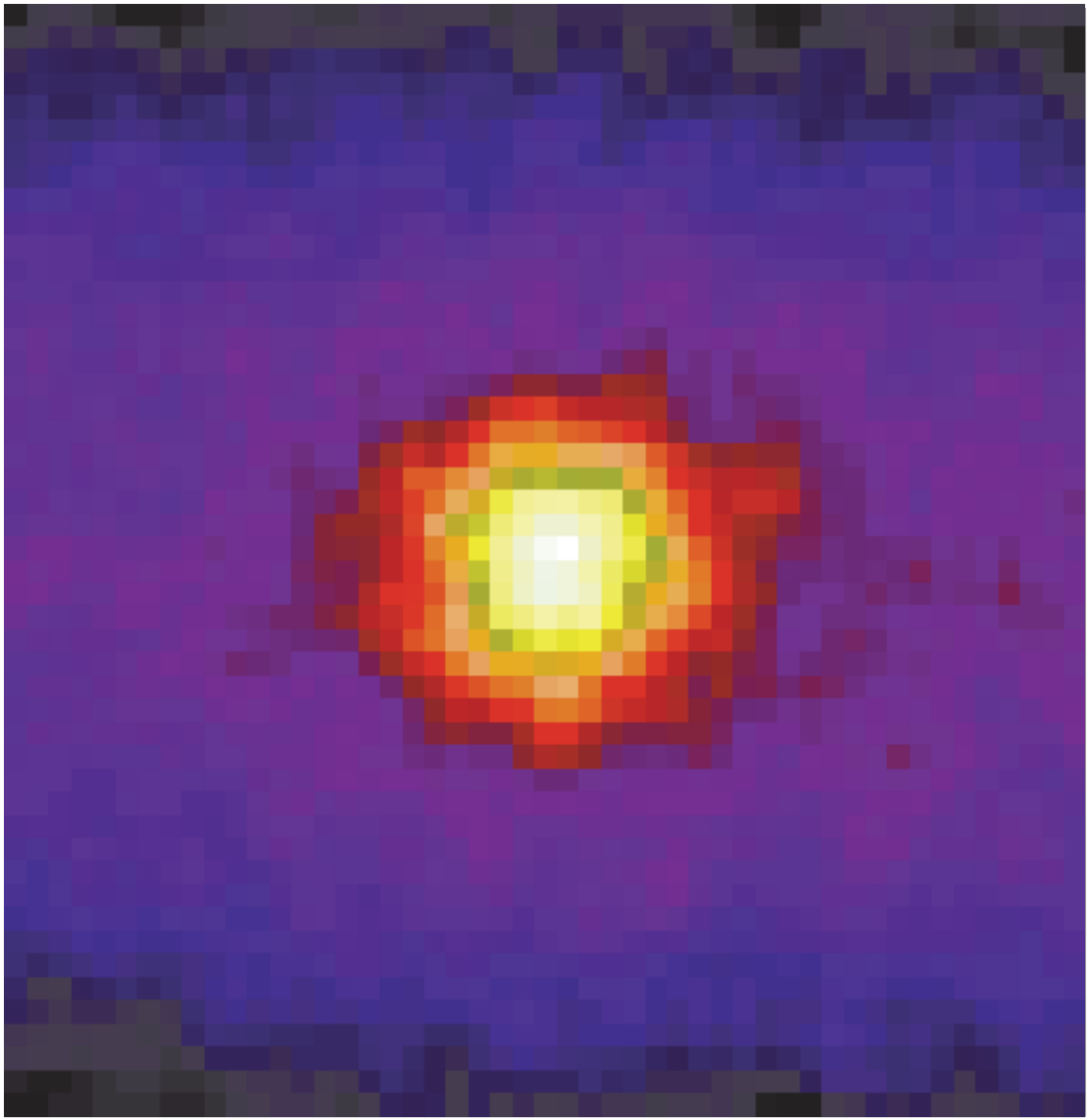
# MINOS $\nu_\mu$ Deficit



# Atmospheric Neutrino Summary



# SuperK All-Sky Map in Neutrinos ($\nu e \rightarrow \nu e$)



$$4p \rightarrow {}^4\mathrm{He} + 2e^+ + 2\nu_e + 25\ \mathrm{MeV}$$

# Solar neutrinos: measure three reaction rates

[CC] Charged-current dissociation: sensitive to $\nu_e$ flux

$$\nu_e \mathrm{d} \to e^- \; p \; p \quad W\text{-exchange}$$

[NC] Neutral-current dissociation: sensitive to total $\nu$ flux

$$\nu_\ell \mathrm{d} \to \nu_\ell \; p \; n \quad Z\text{-exchange}$$

[ES] Elastic scattering: sensitive to $\approx \nu_e + \frac{1}{7}(\nu_\mu + \nu_\tau)$ flux

$$\sigma(\nu_{\mu,\tau} e \to \nu_{\mu,\tau} e) = \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2 + R_e^2/3\right] \quad Z\text{-exchange}$$

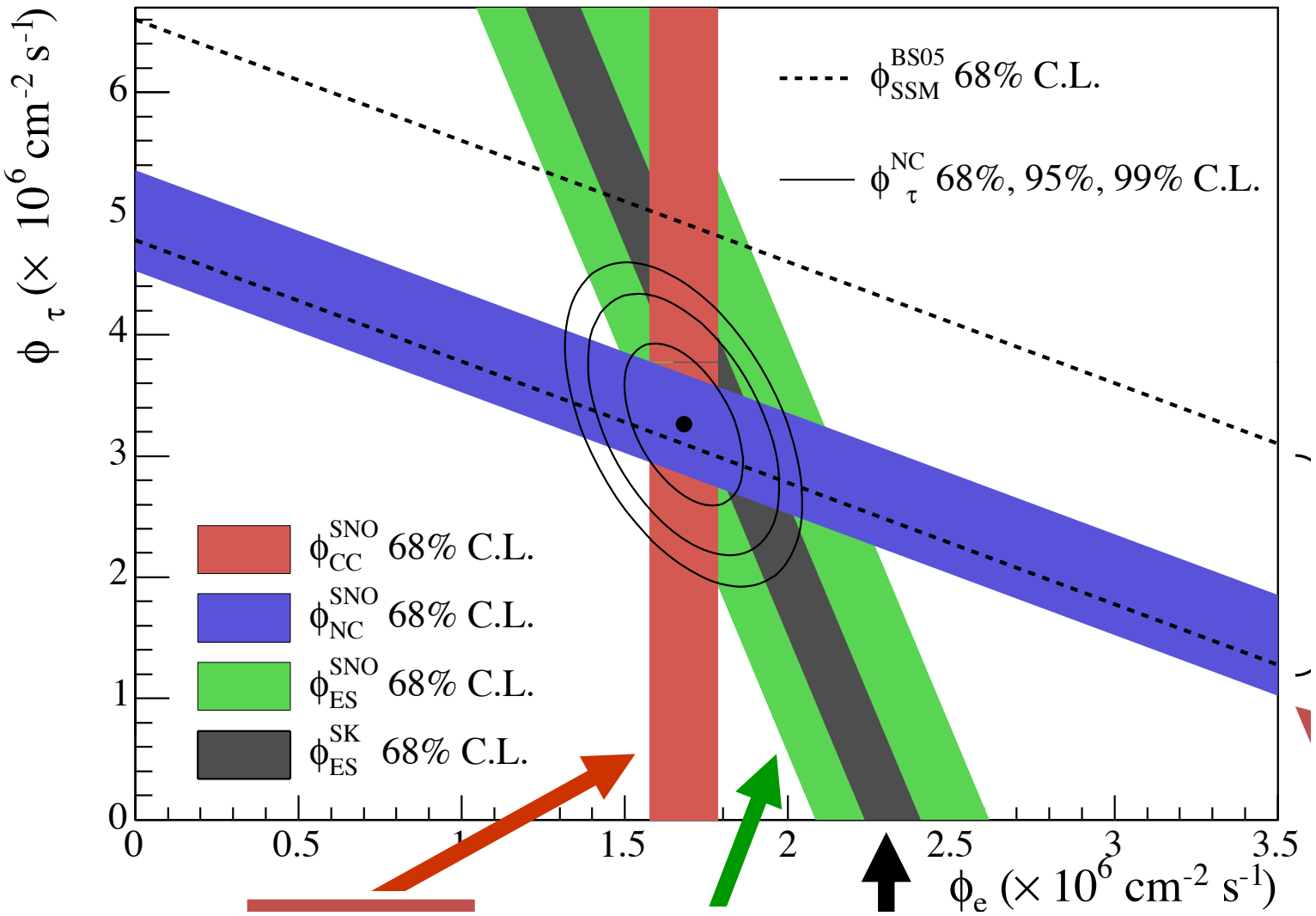
$$\sigma(\nu_e e \to \nu_e e) = \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2 + R_e^2/3\right] \quad W + Z\text{-exchange}$$

$$L_\ell = 2\sin^2\theta_W - 1 \approx -\tfrac{1}{2};\; R_\ell = 2\sin^2\theta_W \approx \tfrac{1}{2}$$

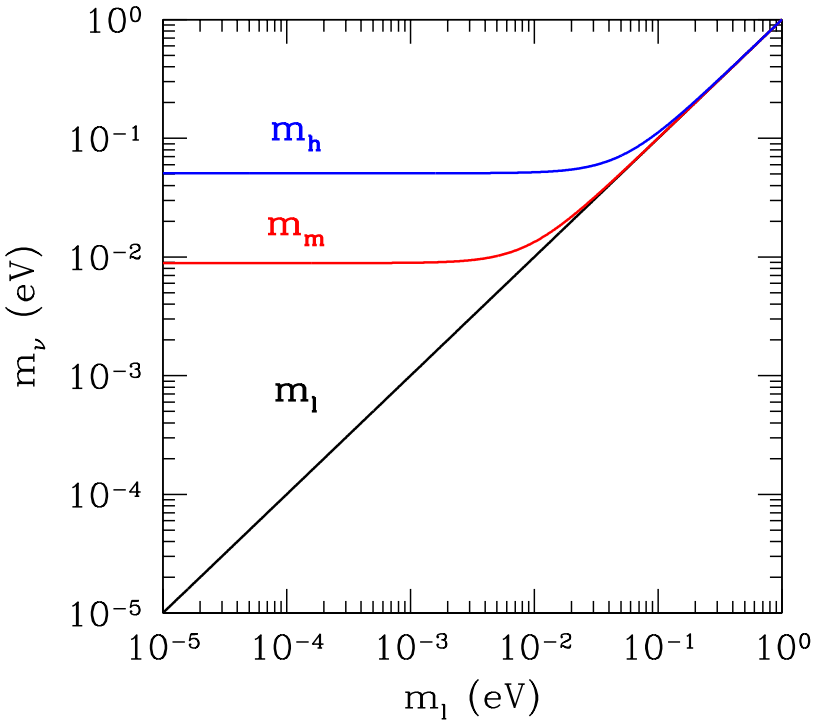
# Solar neutrino observations: SuperK & SNO

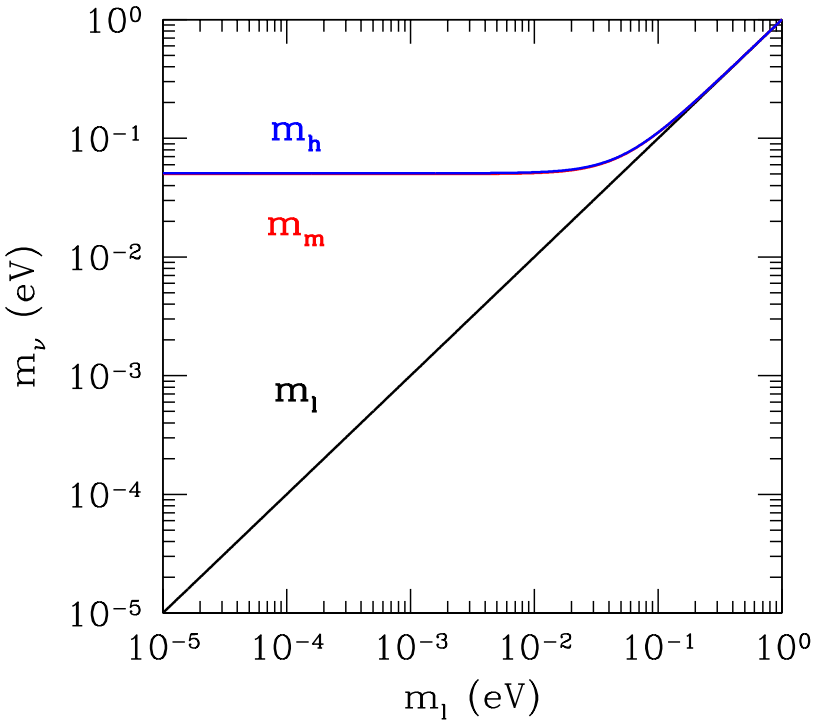
*Total* flux agrees with solar model, but only 30% arrive as $\nu_e$

Solar $\nu$ emerge as $\nu_2$



# Absolute scale of neutrino masses is not yet known




Normal spectrum

Inverted spectrum

$$m_2^2 - m_1^2 = \Delta m_\odot^2 = 7.9 \times 10^{-5}\ \text{eV}^2 \qquad |m_3^2 - m_1^2| = \Delta m_{\text{atm}}^2 = 2.5 \times 10^{-3}\ \text{eV}^2$$

$$\rho_c \equiv 3H_0^2/8\pi G_N = 1.05h^2 \times 10^4\ \text{eV cm}^{-3} = 5.6 \times 10^3\ \text{eV cm}^{-3}$$

$$\boxed{(56\nu_i + 56\bar{\nu}_i)\ \text{cm}^{-3} \rightsquigarrow \sum_i m_{\nu_i} \lesssim 50\ \text{eV}}$$

# Expectations for relic neutrinos

Relate to Cosmic Microwave Background:

$$\frac{dn_\gamma(T)}{d^3p} = \frac{1}{(2\pi)^3}\frac{1}{\exp(p/T)-1},$$

$p$: relic momentum, $T$: photon temperature

$$n_\gamma(T) = \frac{1}{(2\pi)^3}\int d^3p\,\frac{1}{\exp(p/T)-1} = \frac{2\zeta(3)}{\pi^2}\,T^3,$$

$T_0 = (2.725 \pm 0.002)$ K: $n_{\gamma 0} \equiv n_\gamma(T_0) \approx 410\ \mathrm{cm}^{-3}$

# Expectations for relic neutrinos

$\nu$ decoupled around 1 MeV (0.1 s), were not reheated by $e^+e^-$ annihilation: $T_\nu/T = \left(\frac{4}{11}\right)^{1/3}$ (below $m_e$).

$$T_{\nu 0} = \left(\tfrac{4}{11}\right)^{1/3} T_0 = 1.945 \text{ K} \rightsquigarrow 1.697 \times 10^{-4} \text{ eV} .$$

$$\frac{dn_{\nu_i}(T_\nu)}{d^3p} = \frac{1}{(2\pi)^3} \frac{1}{\exp\left(p/T_\nu\right) + 1} .$$

$$\begin{aligned} n_{\nu_i}(T_\nu) &= \frac{1}{(2\pi)^3} \int d^3p \, \frac{1}{\exp\left(p/T_\nu\right) + 1} \\ &= \frac{3\zeta(3)}{4\pi^2} T_\nu^3 = \tfrac{3}{22} n_\gamma(T) . \end{aligned}$$

# Expectations for relic neutrinos

In the present Universe,

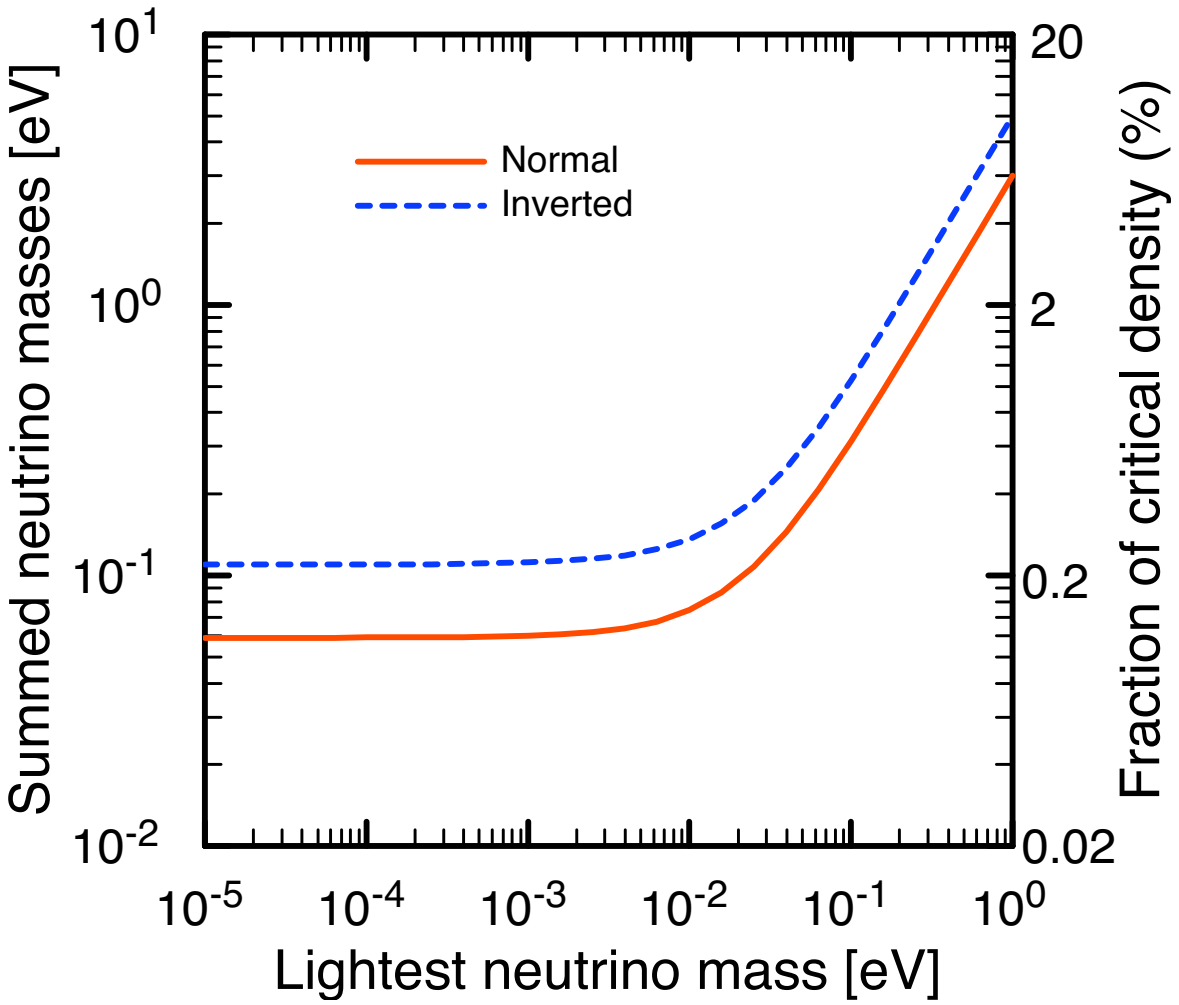
$$n_{\nu_i 0} = n_{\nu_i^c 0} \equiv n_{\nu_i}(T_{\nu 0}) \approx 56 \text{ cm}^{-3}$$

rms neutrino momentum:

$$\langle p_{\nu 0}^2 \rangle^{\frac{1}{2}} \approx 3.597\, T_{\nu 0} \approx 6.044 \times 10^{-4} \text{ eV}$$

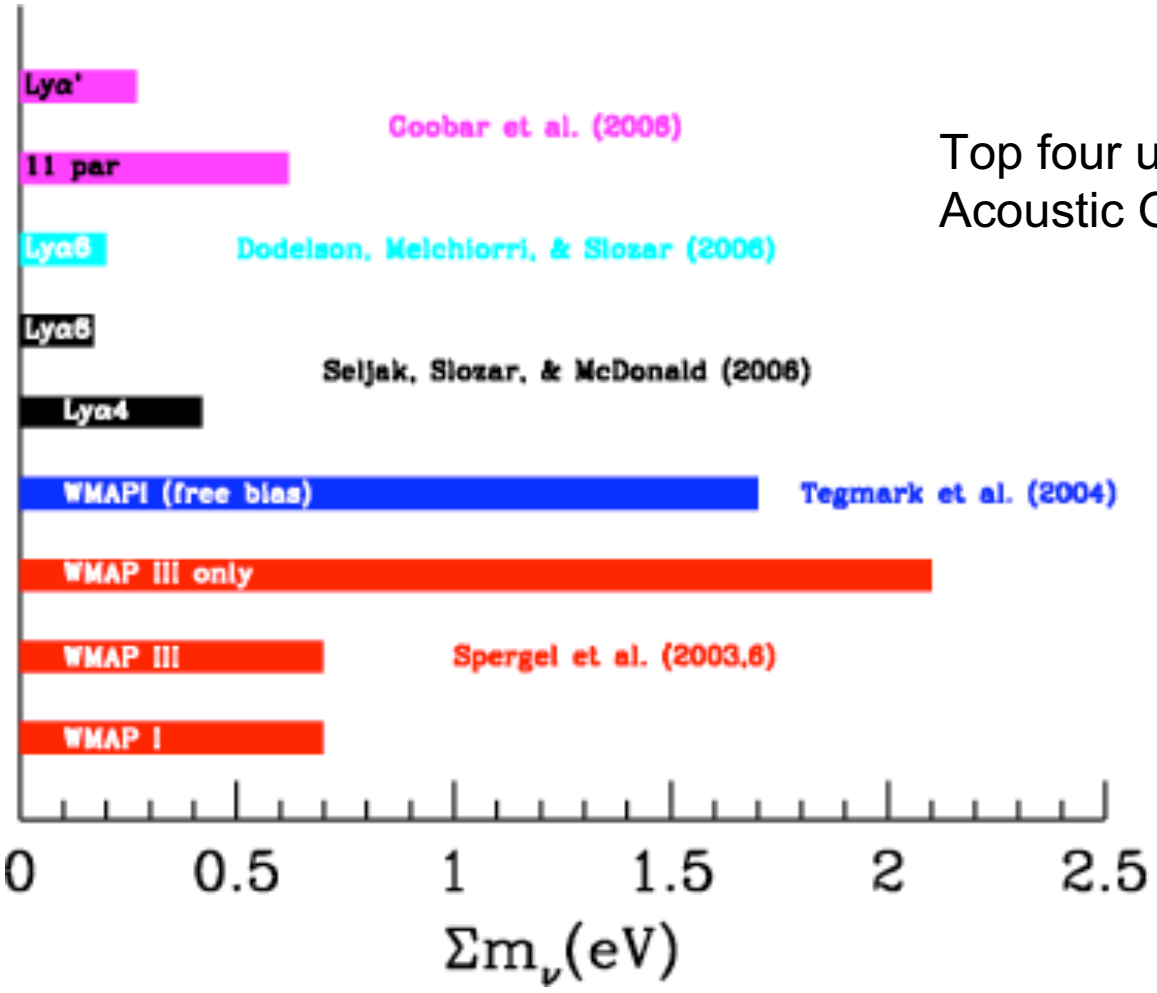
not necessarily small on scale of $m_\nu$

## $\sum_i m_{\nu_i}$




$\Omega_\nu \gtrsim (1.2, 2.2) \times 10^{-3}$ for (normal, inverted) spectrum

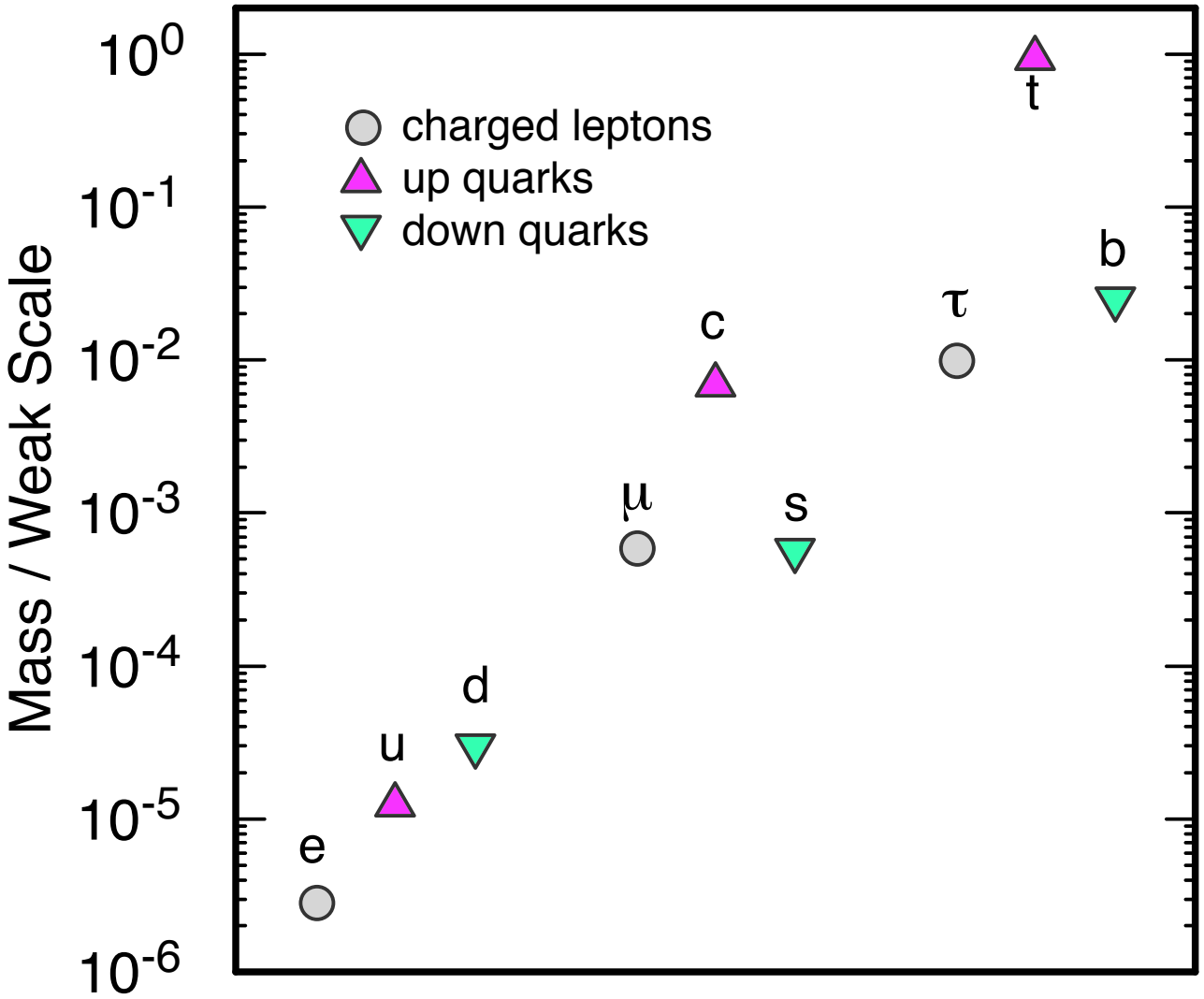
# $\sum_i m_{\nu_i}$: cosmological constraints




Top four use Baryon Acoustic Oscillations

S. Dodelson, Fermilab–KEK $\nu$ School: http://nuss.fnal.gov

# Quark & charged-lepton masses




If neutrinos have Dirac masses, $\nu$ Yukawa couplings $\lesssim 10^{-11}$

# Summary of $\nu$ mixing parameters

$$\text{Define } \boldsymbol{\nu} = (\nu_1, \nu_2, \nu_3) \qquad \boldsymbol{\ell}_{\mathsf{L}} = (e_{\mathsf{L}}, \mu_{\mathsf{L}}, \tau_{\mathsf{L}})$$

$$\mathcal{L}_{\mathsf{CC}}^{(q)} = -\frac{g}{\sqrt{2}}\, \bar{\boldsymbol{\nu}}\, \gamma^{\mu} \mathcal{V}^{\dagger} \boldsymbol{\ell}_{\mathsf{L}} W_{\mu}^{+} + \text{ h.c. },$$

$\mathcal{V}$: $\nu$ mixing matrix

$$\mathcal{V} = \begin{pmatrix} \mathcal{V}_{e1} & \mathcal{V}_{e2} & \mathcal{V}_{e3} \\ \mathcal{V}_{\mu 1} & \mathcal{V}_{\mu 2} & \mathcal{V}_{\mu 3} \\ \mathcal{V}_{\tau 1} & \mathcal{V}_{\tau 2} & \mathcal{V}_{\tau 3} \end{pmatrix}$$

*Convention:* $\nu_1, \nu_2$: solar pair, $m_1 < m_2$
$\nu_3$ separated by $\Delta m^2_{\mathsf{atm}}$; above or below?

# Experiment tells us . . .

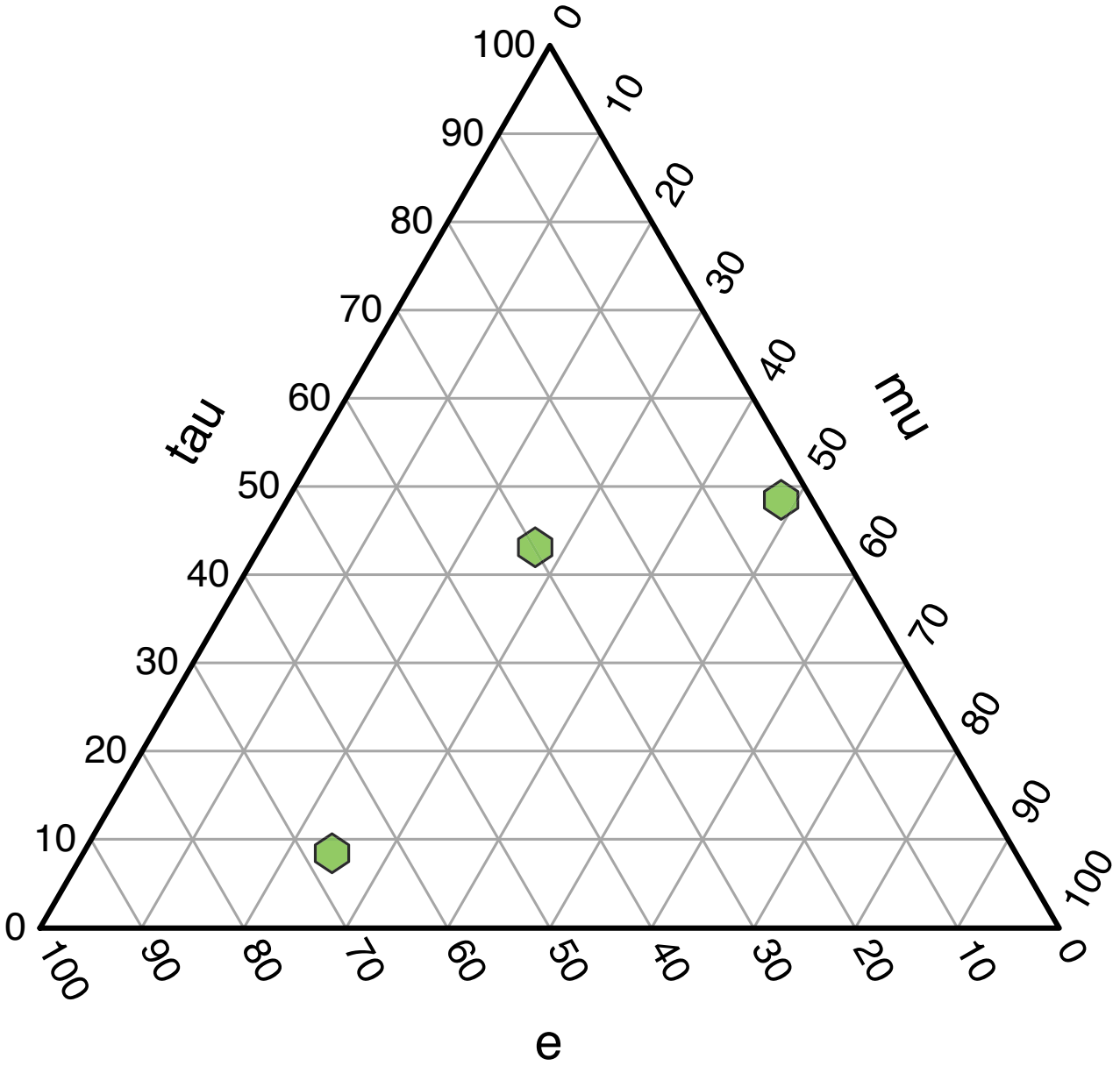
$$|\mathcal{V}| = \begin{pmatrix} 0.79 - 0.88 & 0.47 - 0.61 & < 0.20 \\ 0.19 - 0.52 & 0.42 - 0.73 & 0.58 - 0.82 \\ 0.20 - 0.53 & 0.44 - 0.74 & 0.56 - 0.81 \end{pmatrix}$$

$$\mathcal{V} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

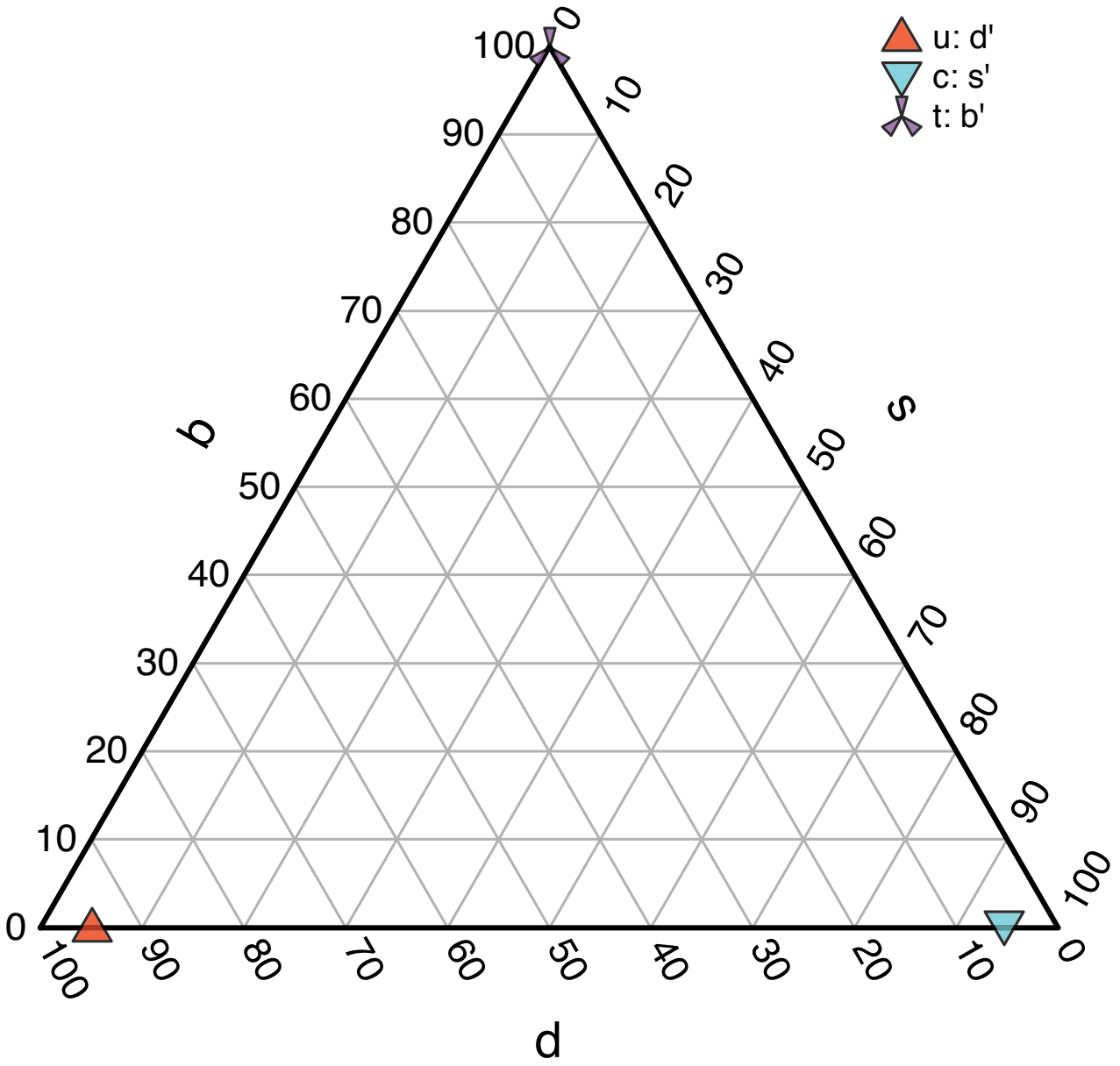
$s_{ij} = \sin\theta_{ij},\ c_{ij} = \cos\theta_{ij}$

$30^\circ \lesssim \theta_{12} \lesssim 38^\circ$: solar
$35^\circ \lesssim \theta_{23} \lesssim 55^\circ$: atm
$\theta_{13} \lesssim 10^\circ$
CP phase $\delta$ unconstrained

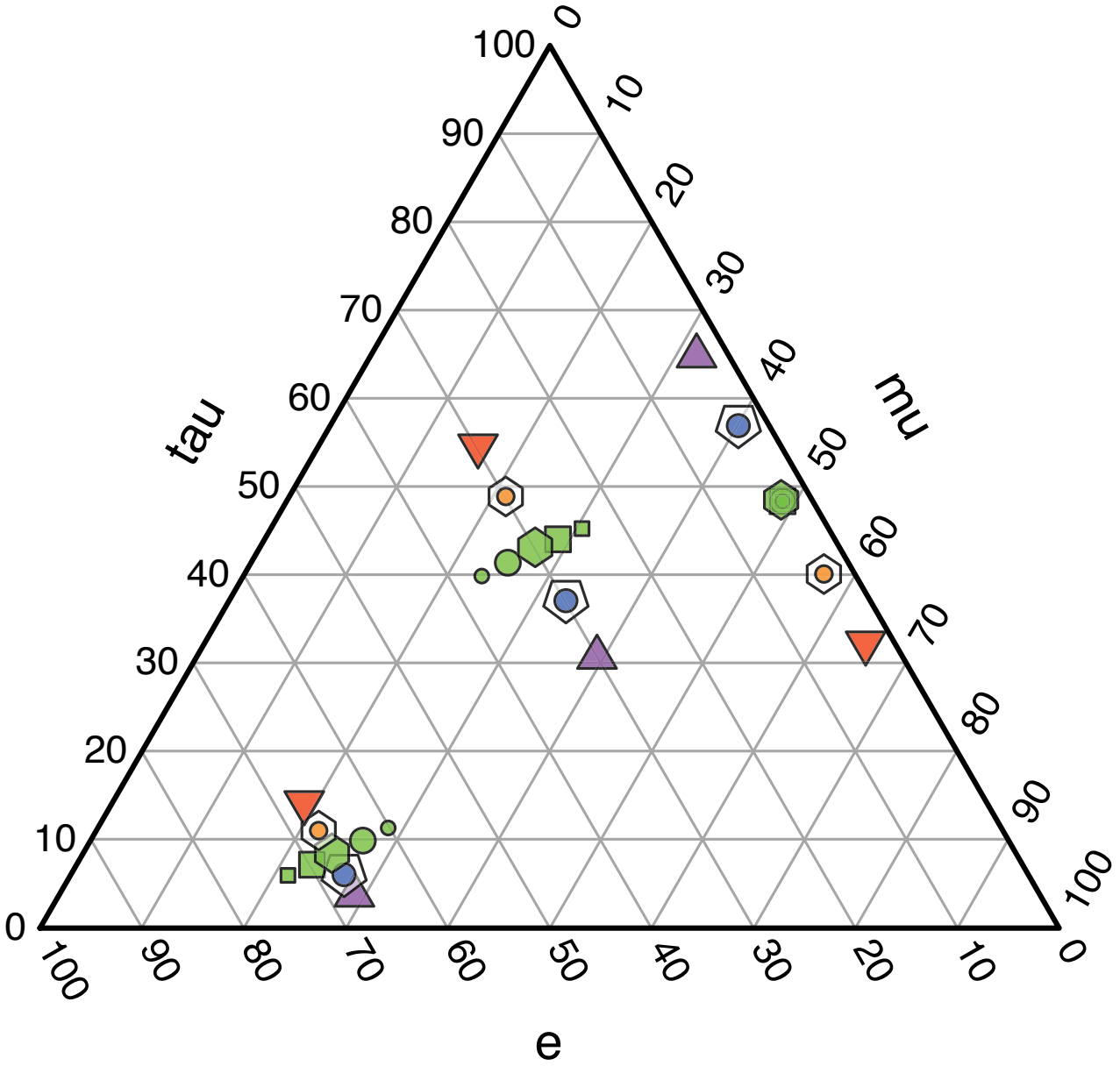
# Neutrino Mixing (representative $\theta_{12}, \theta_{23}$ values, $\theta_{13} = 10^\circ$)



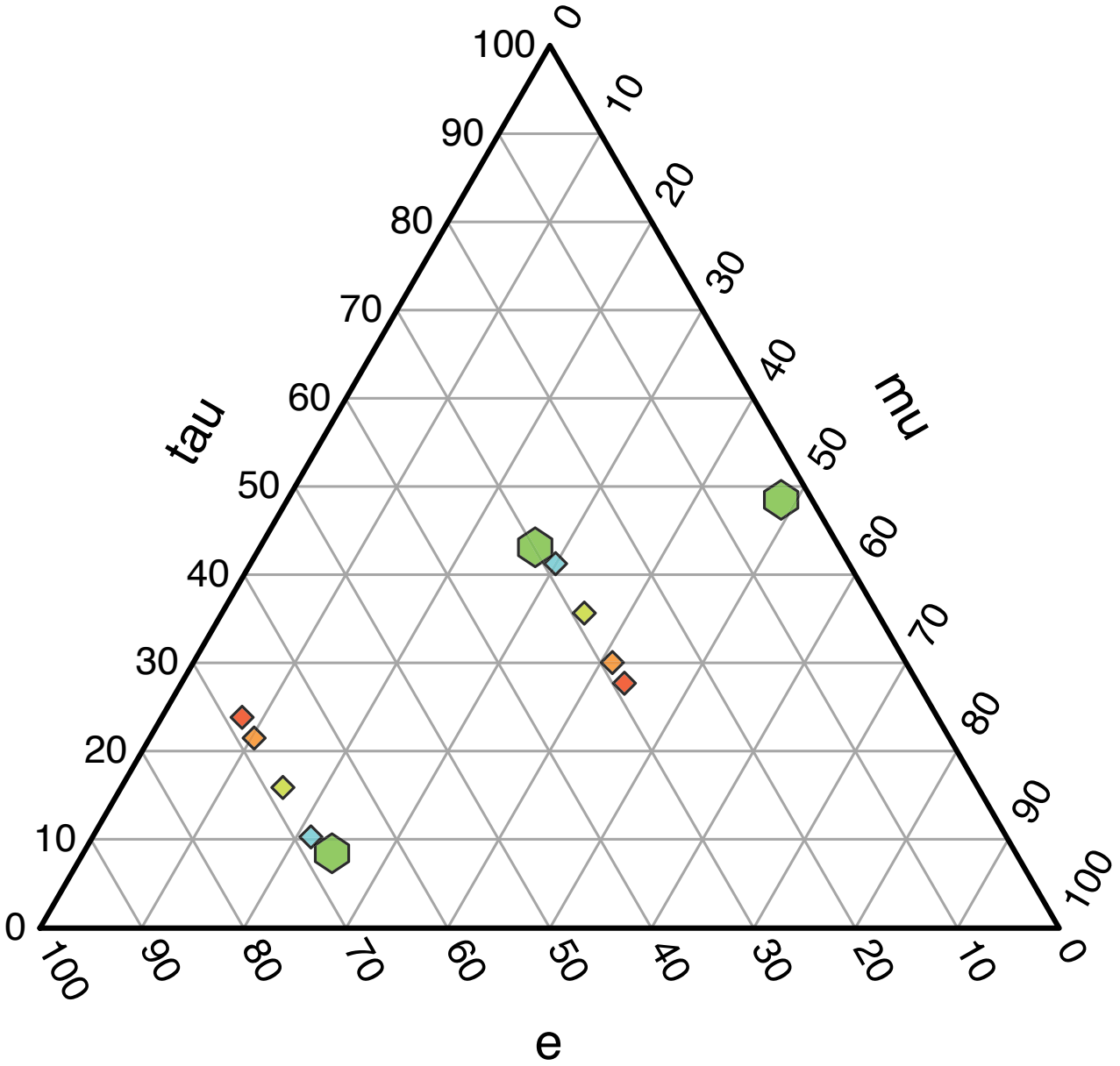
# Quark Mixing (Components: $|V_{u\alpha}|^2$, etc.)



# Neutrino Mixing ($\theta_{12}, \theta_{23}$ variations, $\theta_{13} = 10^\circ$)



# Neutrino Mixing ($\theta_{12}, \theta_{23}$ fixed, $\delta$ variations, $\theta_{13} = 10^\circ$)



# Varieties of neutrino mass: Dirac mass

Chiral decomposition of Dirac spinor:

$$\psi = \tfrac{1}{2}(1-\gamma_5)\psi + \tfrac{1}{2}(1+\gamma_5)\psi \equiv \psi_{\mathsf{L}} + \psi_{\mathsf{R}}$$

Dirac mass connects LH, RH components of *same field*

$$\mathcal{L}_D = -D(\bar{\psi}_{\mathsf{L}}\psi_{\mathsf{R}} + \bar{\psi}_{\mathsf{R}}\psi_{\mathsf{L}}) = -D\bar{\psi}\psi$$

$$\Longrightarrow \text{mass eigenstate } \psi = \psi_{\mathsf{L}} + \psi_{\mathsf{R}}$$

*(invariant under global phase rotation $\nu \to e^{i\theta}\nu$, $\ell \to e^{i\theta}\ell$, so that lepton number is conserved)*

# Varieties of neutrino mass: Dirac mass

Add RH neutrino $N_{\rm R}$ to the standard-model spectrum

$N_{\rm R}$: $SU(2)_{\rm L}$ singlet with $Y = 0$, so sterile

$$\mathcal{L}_{\rm D}^{(\nu)} = -\zeta_\nu \left[ (\bar{\rm L}_\ell \bar{\phi}) N_{\rm R} + \bar{N}_{\rm R} (\bar{\phi}^\dagger {\rm L}_\ell) \right] \to -m_{\rm D} \left( \bar{\nu}_{\rm L} N_{\rm R} + \bar{N}_{\rm R} \nu_{\rm L} \right)$$

$$m_{\rm D} = \zeta_\nu v / \sqrt{2}$$

*Some argue that $\zeta_\nu \lesssim 10^{-11}$ is unnatural, while the range $\zeta_t \approx 1$ to $\zeta_e \approx$ few $\times 10^{-6}$ is only puzzling. But all Dirac masses involve physics beyond the standard model.*

# Varieties of neutrino mass: Majorana mass

Neutrinos: no color, no $Q$: their own antiparticles?

Majorana fermions

Charge conjugate of RH field is LH: $\psi_{\mathsf{L}}^c \equiv (\psi^c)_{\mathsf{L}} = (\psi_{\mathsf{R}})^c$
Majorana joins LH, RH components of *conjugate fields*

$$\begin{aligned} -\mathcal{L}_{\mathsf{MA}} &= A(\bar{\nu}_{\mathsf{R}}^c \nu_{\mathsf{L}} + \bar{\nu}_{\mathsf{L}} \nu_{\mathsf{R}}^c) = A\bar{\chi}\chi \\ -\mathcal{L}_{\mathsf{MB}} &= B(\bar{\nu}_{\mathsf{L}}^c \nu_{\mathsf{R}} + \bar{\nu}_{\mathsf{R}} \nu_{\mathsf{L}}^c) = B\bar{\omega}\omega \end{aligned}$$

for which the mass eigenstates are

$$\begin{aligned} \chi &\equiv \nu_{\mathsf{L}} + \nu_{\mathsf{R}}^c = \chi^c = \nu_{\mathsf{L}} + (\nu_{\mathsf{L}})^c \\ \omega &\equiv \nu_{\mathsf{R}} + \nu_{\mathsf{L}}^c = \omega^c = \nu_{\mathsf{R}} + (\nu_{\mathsf{R}})^c \end{aligned}$$

# Lepton number violation

Majorana $\nu$: no conserved additive quantum number

$\mathcal{L}_{\mathrm{M}}$ violates lepton number by two units

$\Rightarrow$ Majorana $\nu$ can mediate $\beta\beta_{0\nu}$ decays

$$(Z, A) \rightarrow (Z + 2, A) + e^- + e^-$$

Detecting $\beta\beta_{0\nu}$ would offer decisive evidence for Majorana nature of $\nu$

Active $\nu_{\mathrm{L}}$ mass generated by $I = 1$ Higgs with vev or effective operator containing two $I = \frac{1}{2}$ Higgs combined to transform as $I = 1$.

# Neutrino observatories: expectations

Cosmic $\nu$ flux may exceed atmospheric background at $E_\nu \approx$ few TeV

*prospect for sources · characterize sources · study $\nu$ properties*

*Sources* include AGN (at $\sim 10^2$ Mpc) 1 Mpc $\approx 3.1 \times 10^{22}$ m

*pp* or $p\gamma \Rightarrow$ $\approx$ numbers of $\pi^+$ $\pi^0$ $\pi^-$

$$\pi^+ + \pi^0 + \pi^- \Rightarrow 2\gamma + 2\nu_\mu + 2\bar{\nu}_\mu + 1\nu_e + 1\bar{\nu}_e$$

$$\boxed{\Phi^0_{\rm std} = \{\varphi^0_e = \tfrac{1}{3}, \varphi^0_\mu = \tfrac{2}{3}, \varphi^0_\tau = 0\} \quad (\nu = \bar{\nu})}$$

*Detection* (in volumes $\rightarrow$ 1 km$^3$)

$$\boxed{(\nu_\mu, \bar{\nu}_\mu) N \rightarrow (\mu^-, \mu^+) + \text{anything}}$$

Can we achieve efficient, calibrated $(\nu_e, \bar{\nu}_e)$ detection?

Good $(\nu_\tau, \bar{\nu}_\tau)$ detection, NC capability desirable

# $\nu_\mu N \to \mu^- +$ anything

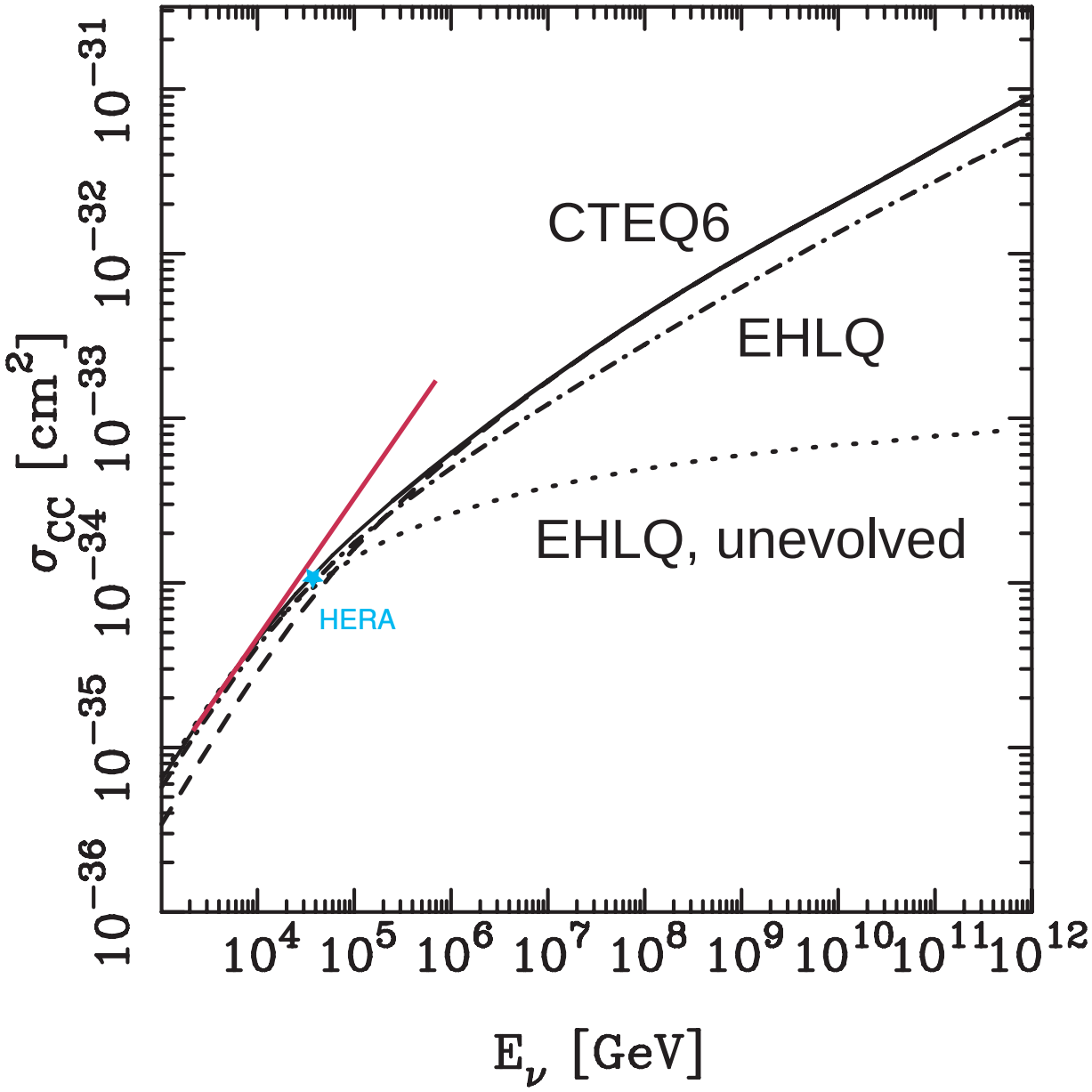
$$\frac{d^2\sigma}{dxdy} = \frac{2G_F^2 M E_\nu}{\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 \left[xq(x,Q^2) + x\bar{q}(x,Q^2)(1-y)^2\right]$$

$$\begin{aligned} q(x,Q^2) &= \frac{u_v(x,Q^2)+d_v(x,Q^2)}{2} + \frac{u_s(x,Q^2)+d_s(x,Q^2)}{2} \\ &\quad + s_s(x,Q^2) + b_s(x,Q^2) \\ \bar{q}(x,Q^2) &= \frac{u_s(x,Q^2)+d_s(x,Q^2)}{2} + c_s(x,Q^2) + t_s(x,Q^2), \end{aligned}$$

. . . isoscalar nucleon

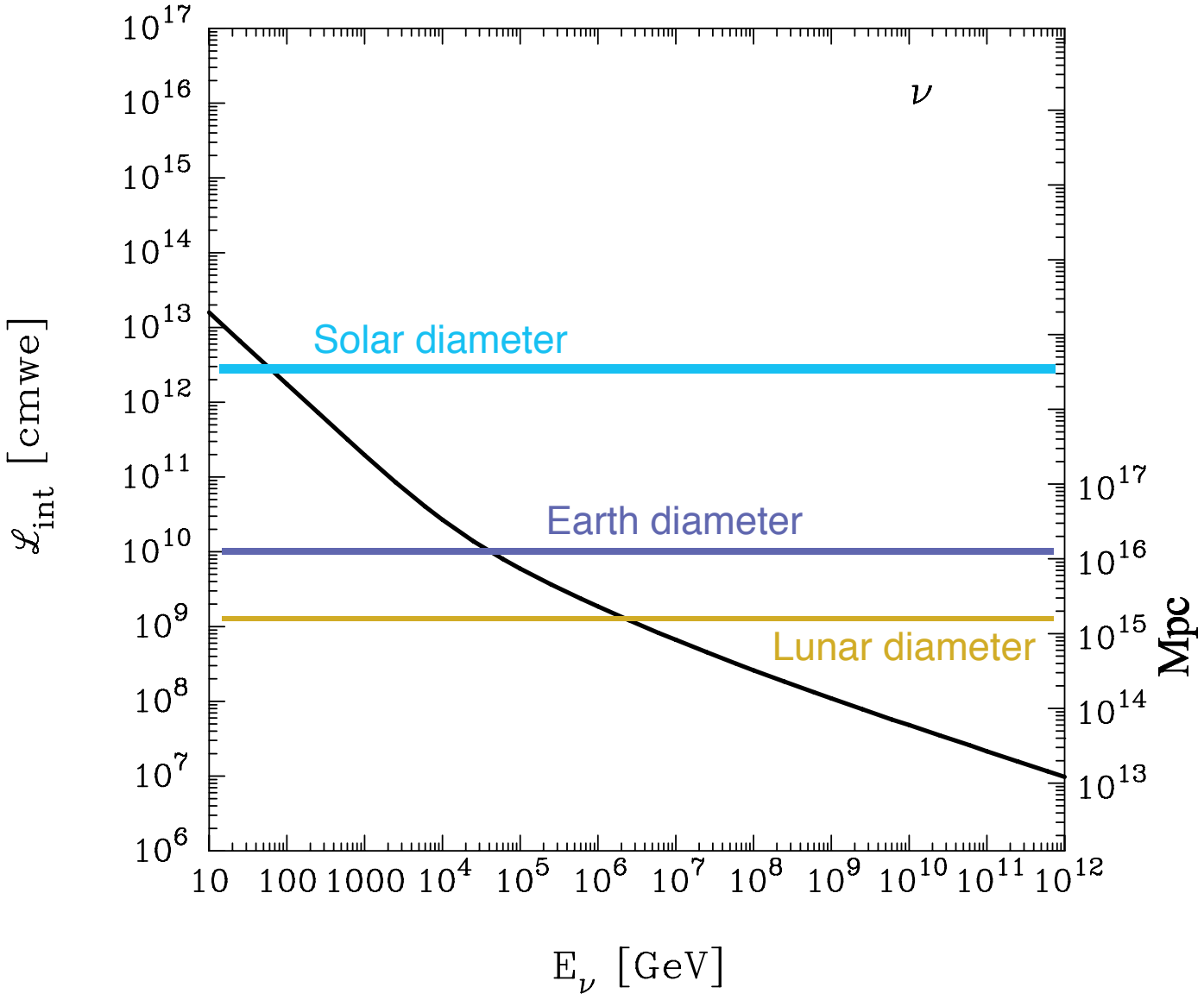
$$x = Q^2/2M\nu \qquad y \equiv \nu/E_\nu \qquad \nu \equiv E_\nu - E_\mu$$

# $\nu N \to \mu + \ldots$ Cross Sections



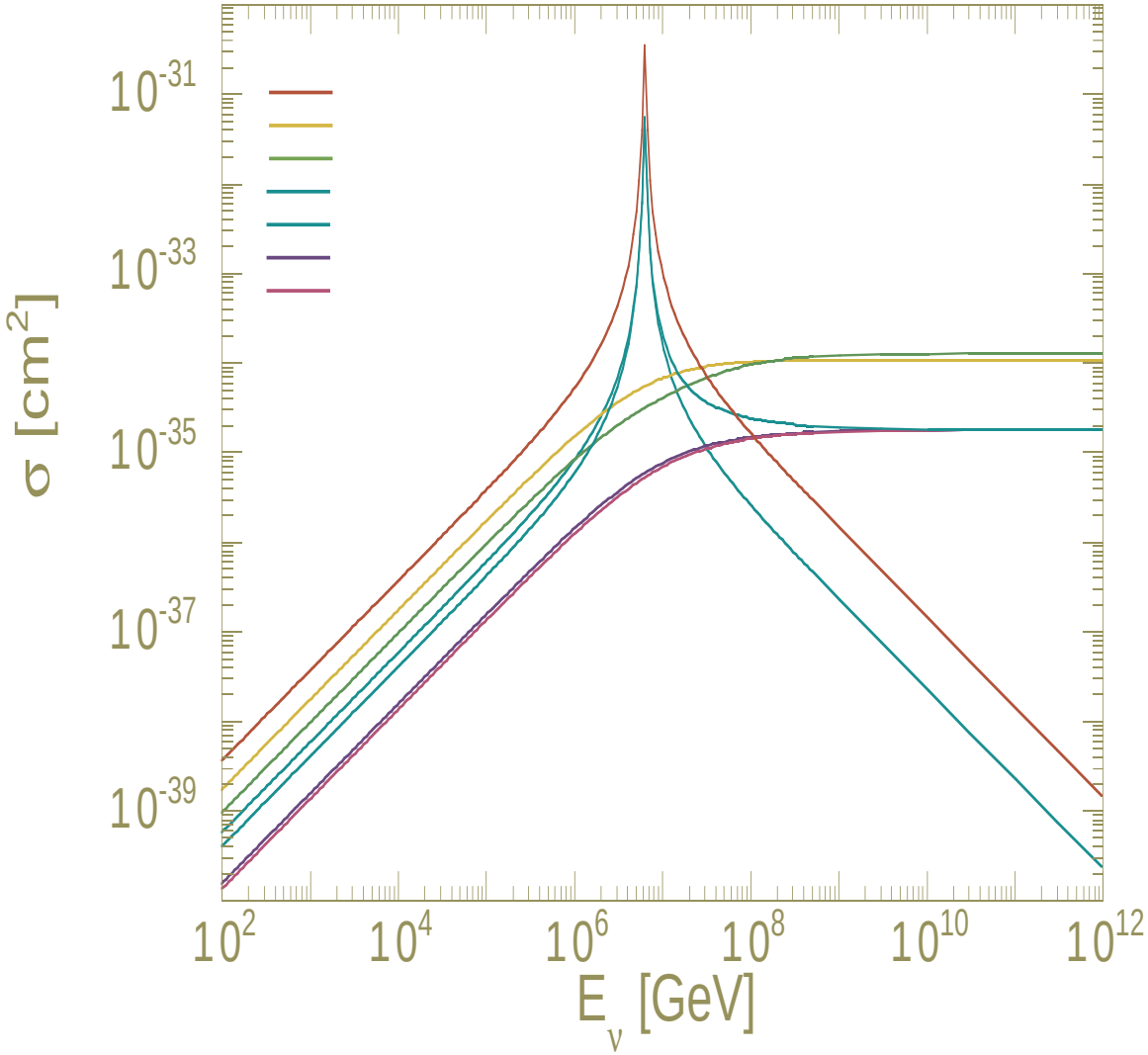

M. H. Reno

# $\nu N \to \mu + \ldots$ Interaction Lengths




$n_{B0} \approx 2.5 \times 10^{-7}$ cm$^{-3}$

# $\nu e$ cross sections . . .




At low energies: $\sigma(\bar{\nu}_e e \rightarrow$ hadrons$) > \sigma(\nu_\mu e \rightarrow \mu\nu_e) > \sigma(\nu_e e \rightarrow \nu_e e) > \sigma(\bar{(}\nu_e e \rightarrow \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) > \sigma(\nu_\mu e \rightarrow \nu_\mu e) > \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e)$

# Influence of neutrino oscillations

Flux at Earth $\Phi = \{\varphi_e, \varphi_\mu, \varphi_\tau\} \neq \Phi^0 = \{\varphi_e^0, \varphi_\mu^0, \varphi_\tau^0\}$ source fluxes

Vacuum oscillation length is short; for $|\Delta m^2| = 10^{-5}$ eV$^2$,

$$L_{\rm osc} \;=\; 4\pi E_\nu/|\Delta m^2| \approx 2.5 \times 10^{-24}\ \text{Mpc} \cdot (E_\nu/1\ \text{eV})$$

...a fraction of Mpc even for $E_\nu = 10^{20}$ eV

*$\nu$ oscillate many times between cosmic source and terrestrial detector*

*Also*, over long paths, cosmic neutrinos are vulnerable to decay processes that would not affect terrestrial or solar experiments.

# . . . Neutrino Oscillations

$$\text{(flavor) } \nu_\alpha = \sum_i \mathcal{V}_{\alpha i}\nu_i \text{ (mass)}$$

Idealize $\sin\theta_{13} = 0$, $\sin 2\theta_{23} = 1$, write $x = \sin^2\theta_{12}\cos^2\theta_{12}$.

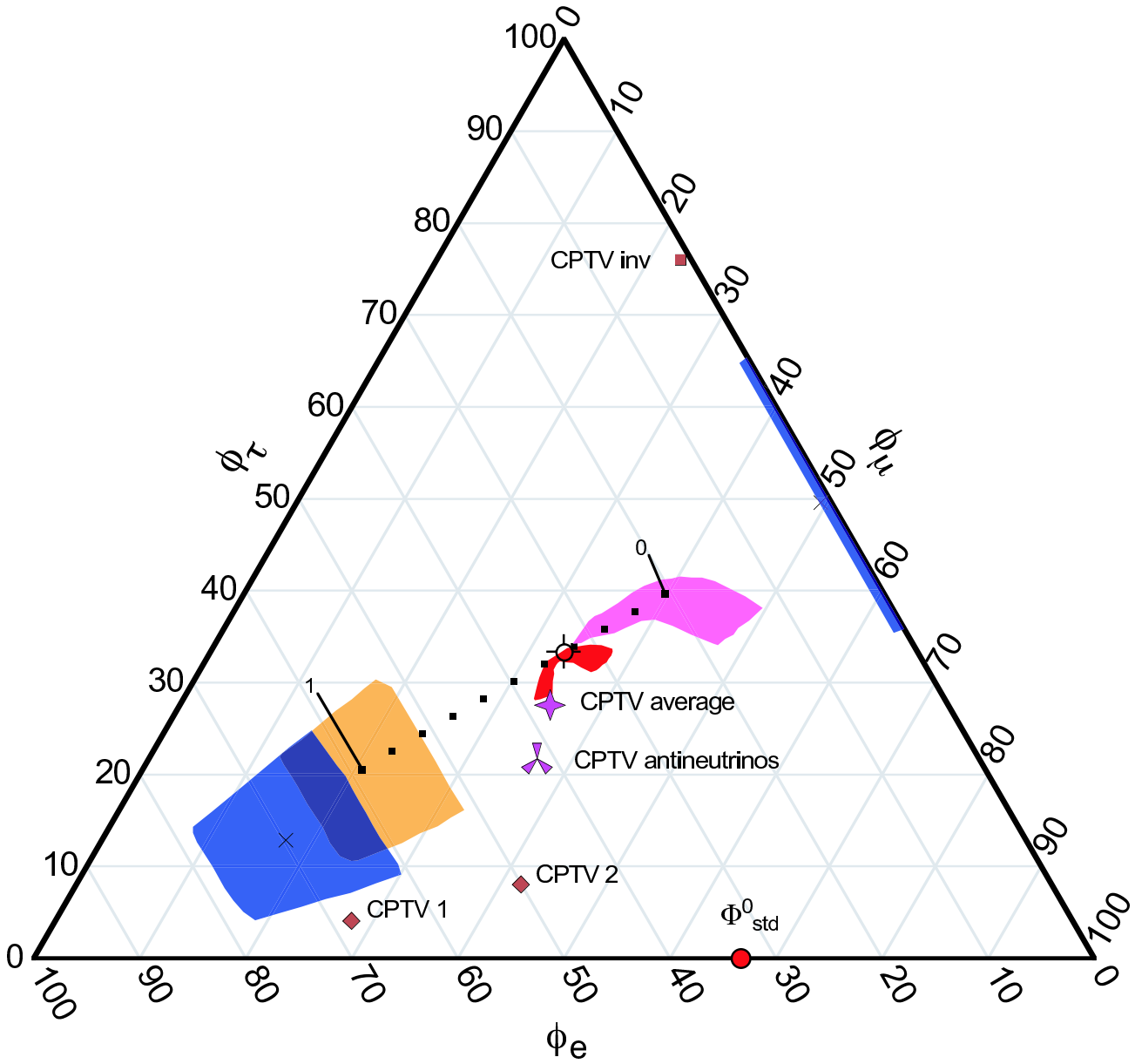
$$\mathcal{V}_{\text{ideal}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ s_{12}/\sqrt{2} & -c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Transfer matrix $\mathcal{X}$: $\Phi^0$ (source) $\rightarrow \Phi$ (detector); Over many oscillations,

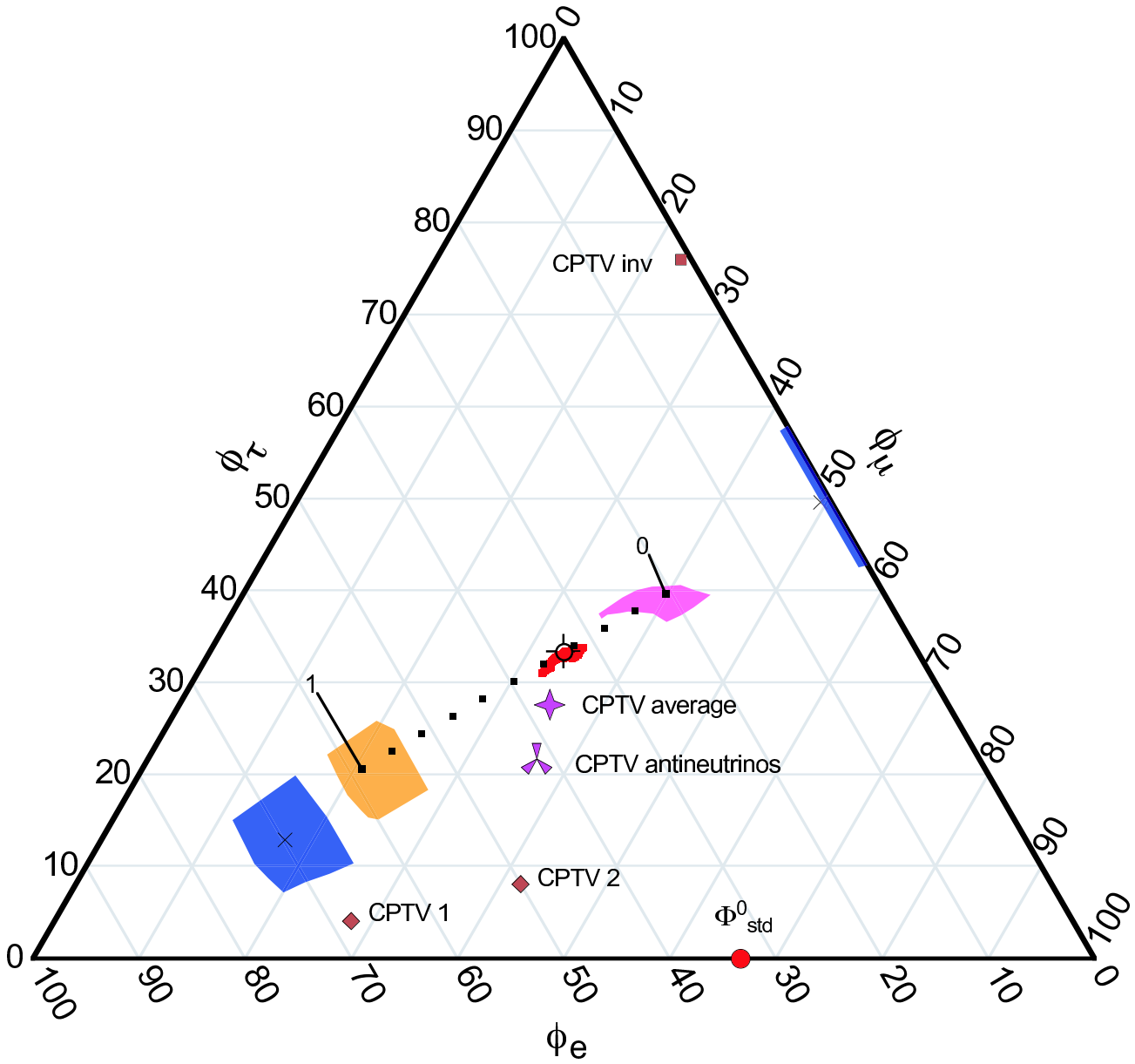
$$\mathcal{X}_{\text{ideal}} = \begin{pmatrix} 1-2x & x & x \\ x & \frac{1}{2}(1-x) & \frac{1}{2}(1-x) \\ x & \frac{1}{2}(1-x) & \frac{1}{2}(1-x) \end{pmatrix}$$

$$\boxed{\mathcal{X}_{\text{ideal}} : \Phi^0_{\text{std}} \rightarrow \{\varphi_e = \tfrac{1}{3}, \varphi_\mu = \tfrac{1}{3}, \varphi_\tau = \tfrac{1}{3}\}}$$

# With current constraints . . .



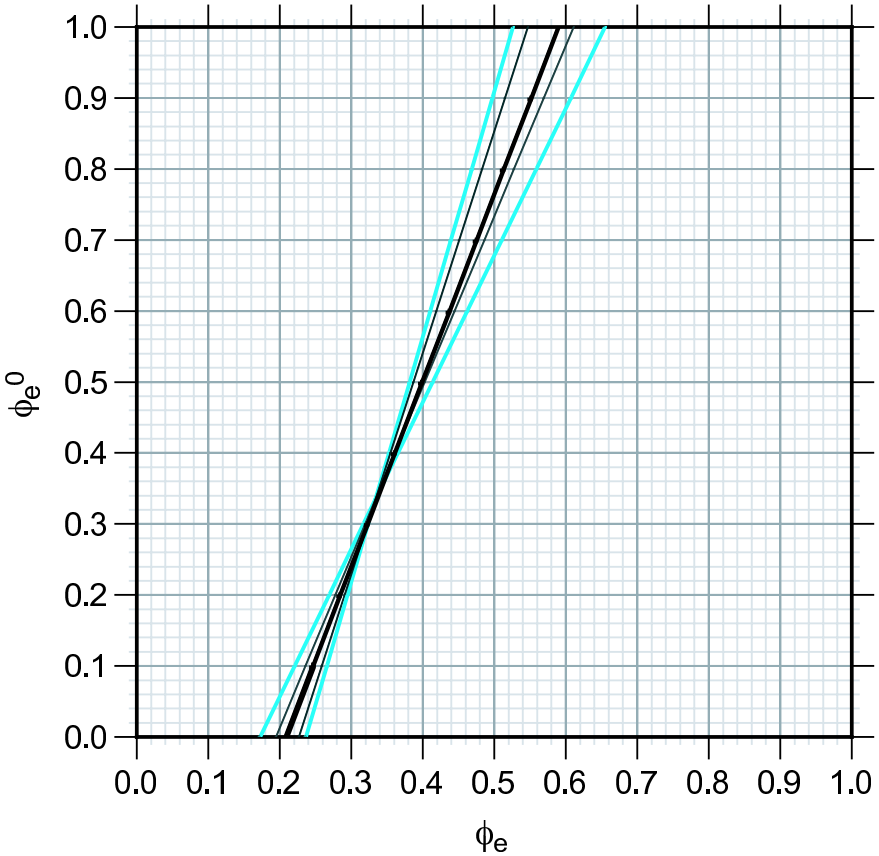
# After current generation . . .



# Reconstructing the $\nu$ Mixture at the Source

$\nu_\mu$, $\nu_\tau$ fully mixed $\Rightarrow$ can't fully characterize $\Phi^0$

Reconstruct $\nu_e$ fraction at source using $\mathcal{X}_{\rm ideal}$: $\varphi_e^0 = (\varphi_e - x)/(1-3x)$




Extreme $\varphi_e$ implicates unconventional physics

# Influence of neutrino decays

Nonradiative decays $\nu_i \to (\nu_j, \bar{\nu}_j) + X$ not very constrained:

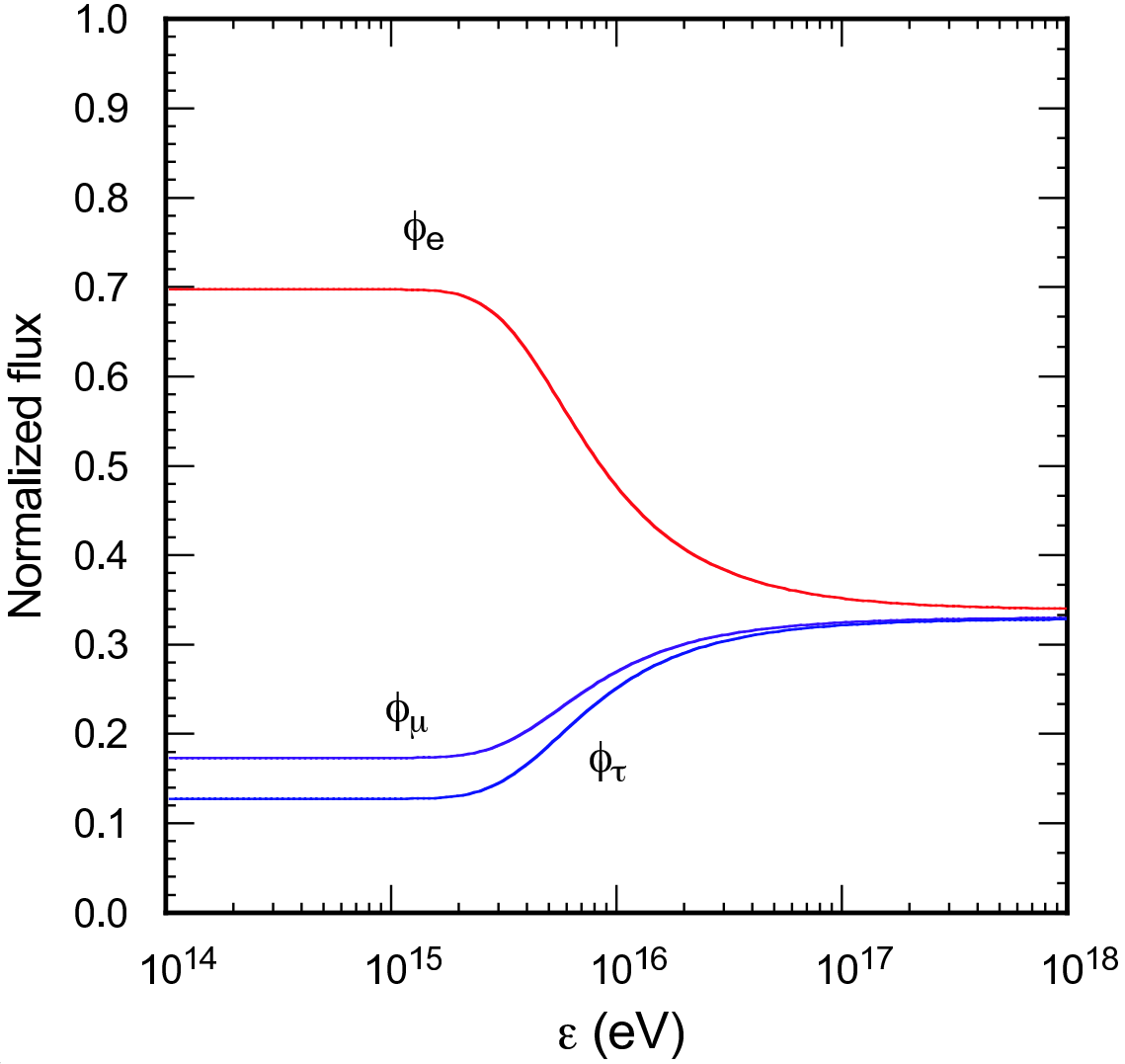
$$\tau/m \gtrsim 10^{-4}\ \mathrm{s/eV}$$

If only lightest neutrino survives, flavor mix at Earth is independent of composition at source

| Normal hierarchy | Inverted hierarchy |
|---|---|
| $m_1 < m_2 < m_3$: | $m_1 > m_2 > m_3$: |
| $\varphi_\alpha = \lvert U_{\alpha 1}\rvert^2$ | $\varphi_\alpha = \lvert U_{\alpha 3}\rvert^2$ |
| $\Phi_{\mathrm{normal}} \approx \{0.70, 0.17, 0.13\}$ | $\Phi_{\mathrm{inverted}} \approx \{0, 0.5, 0.5\}$ |

$$\boxed{\text{far from } \Phi_{\mathrm{std}} = \{\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\}}$$

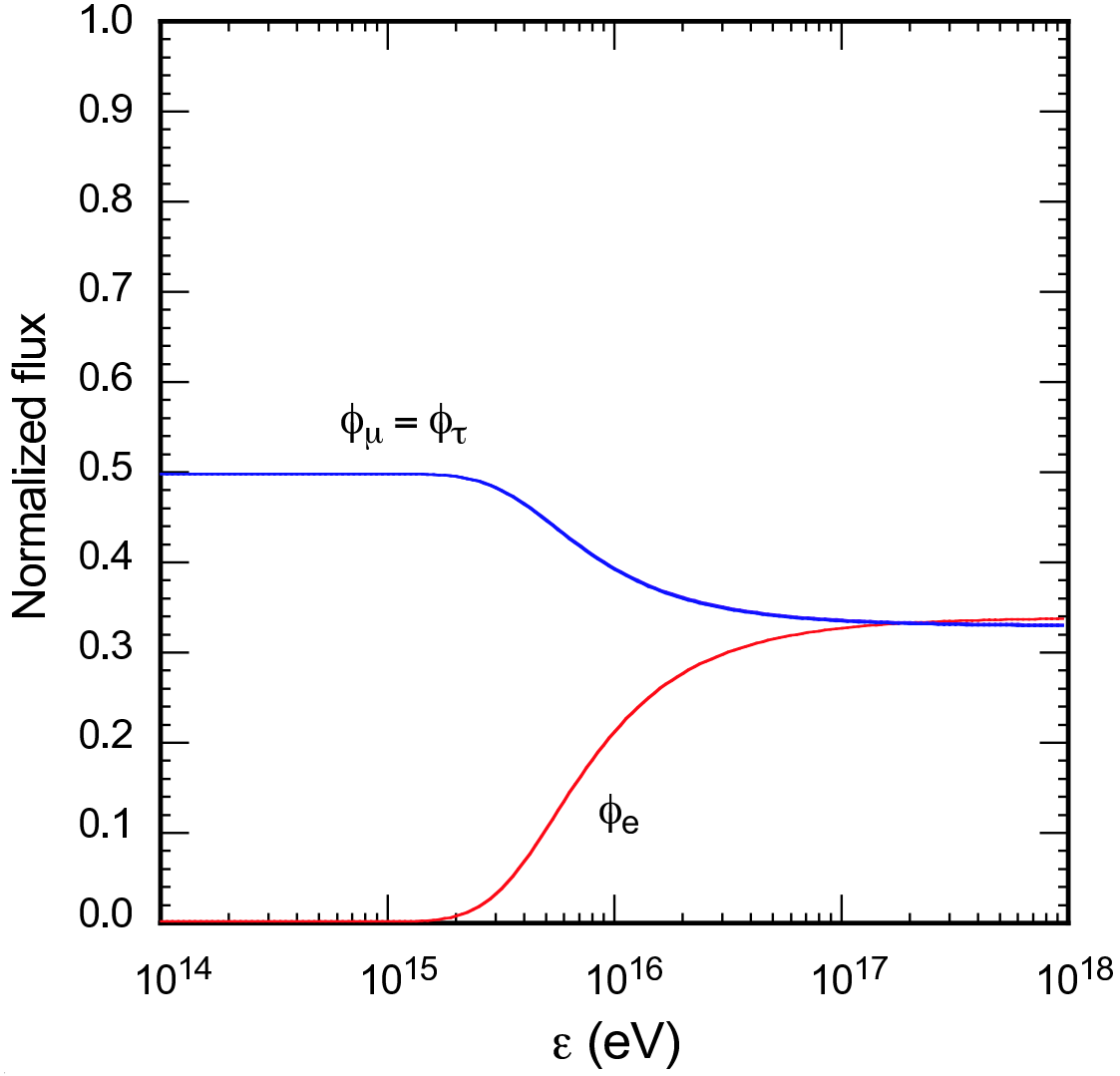
# Energy-dependent composition as marker of $\nu$ decays




$E_\nu = \varepsilon(1\ \mathrm{s/eV})/(\tau_\nu/m_\nu) \cdot L/(100\ \mathrm{Mpc})$

Normal hierarchy

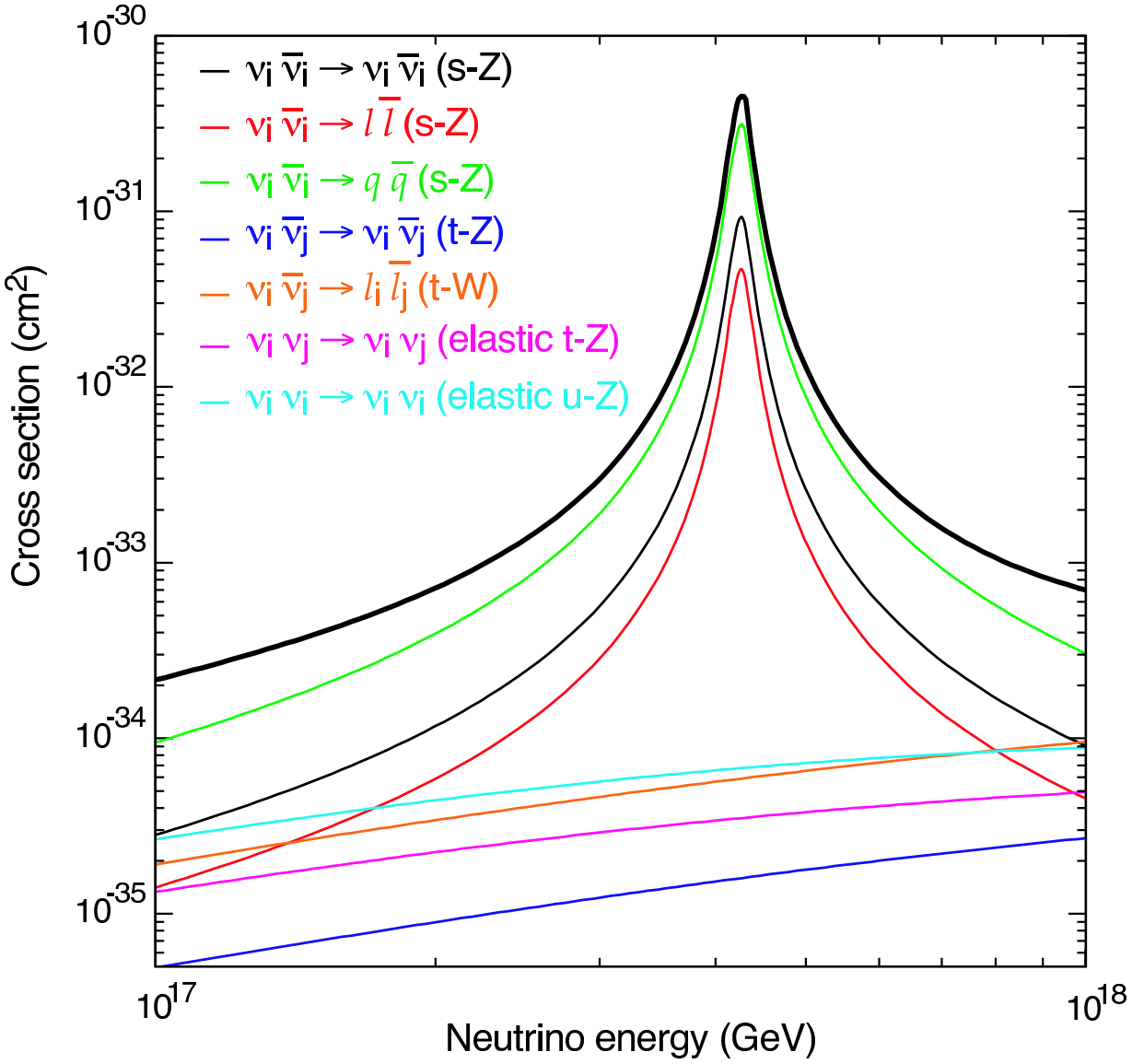
# Energy-dependent composition as marker of $\nu$ decays




$E_\nu = \varepsilon(1\ \mathrm{s/eV})/(\tau_\nu/m_\nu) \cdot L/(100\ \mathrm{Mpc})$

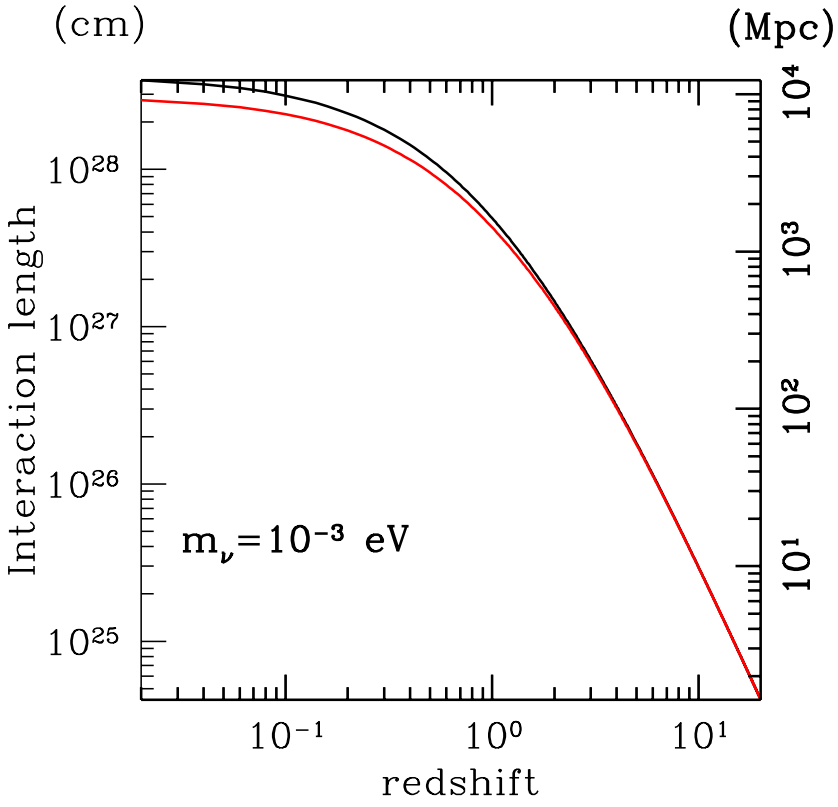
Inverted hierarchy

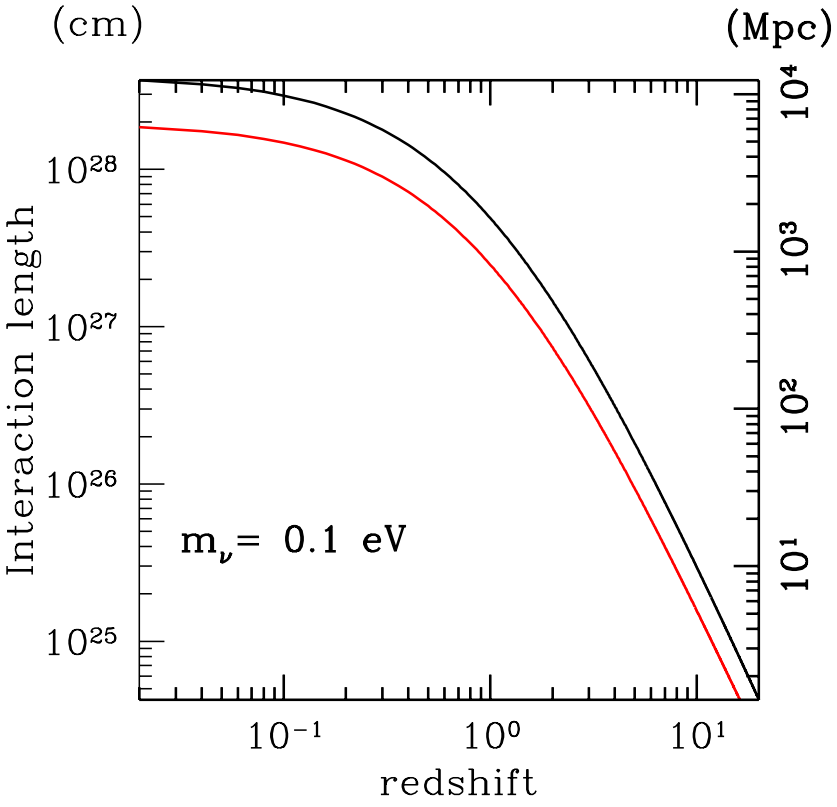
# UHE $\nu$ annihilation on $\nu$ relics




$m_\nu = 10^{-5}$ eV

Interaction lengths on $Z^0$ resonance: Dirac Majorana

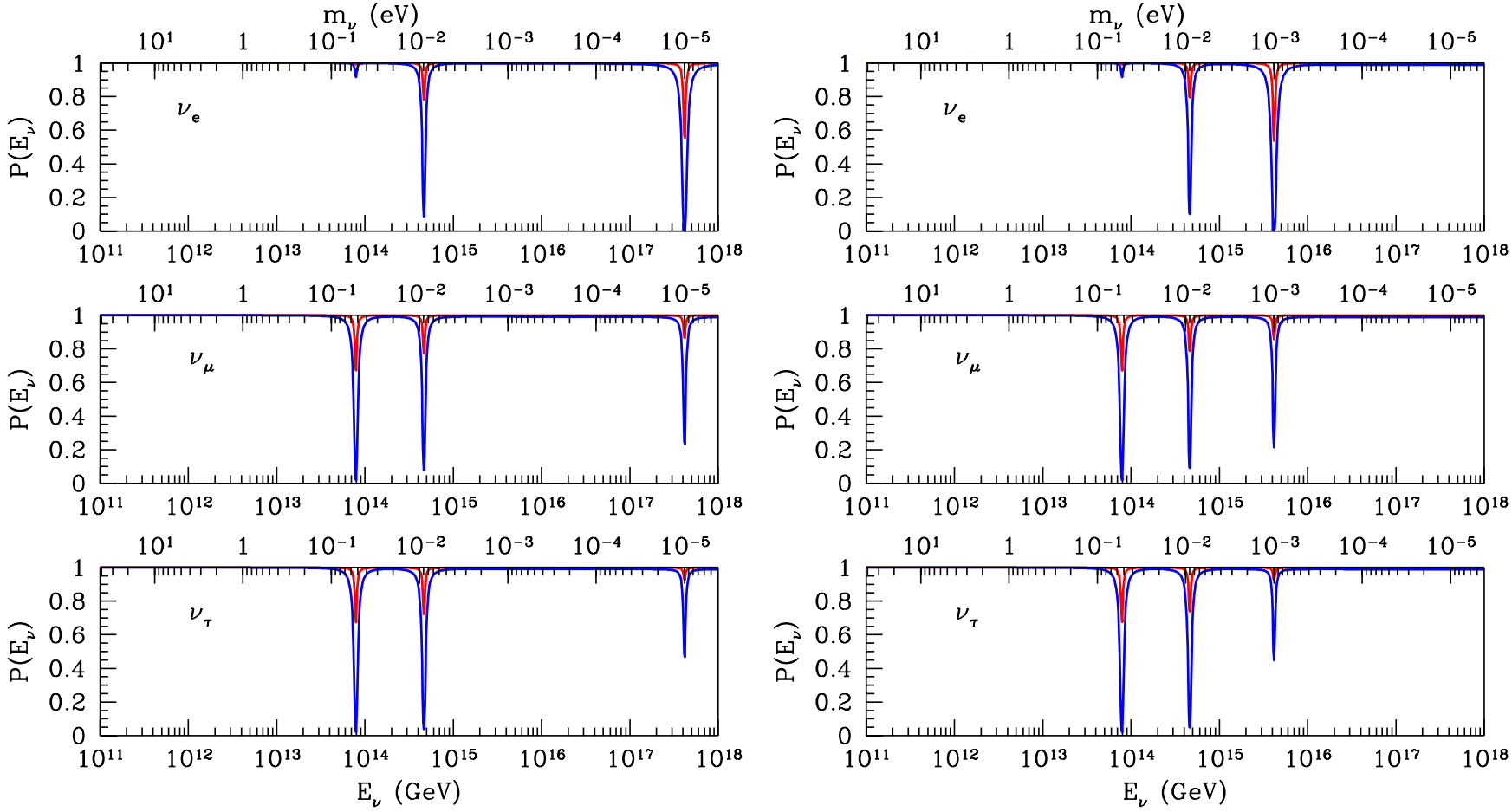



$\nu$ number density now: 56 cm$^{-3}$, $\propto (1+z)^3$

$1.2 \times 10^4$ Mpc $=$ 39 Gly

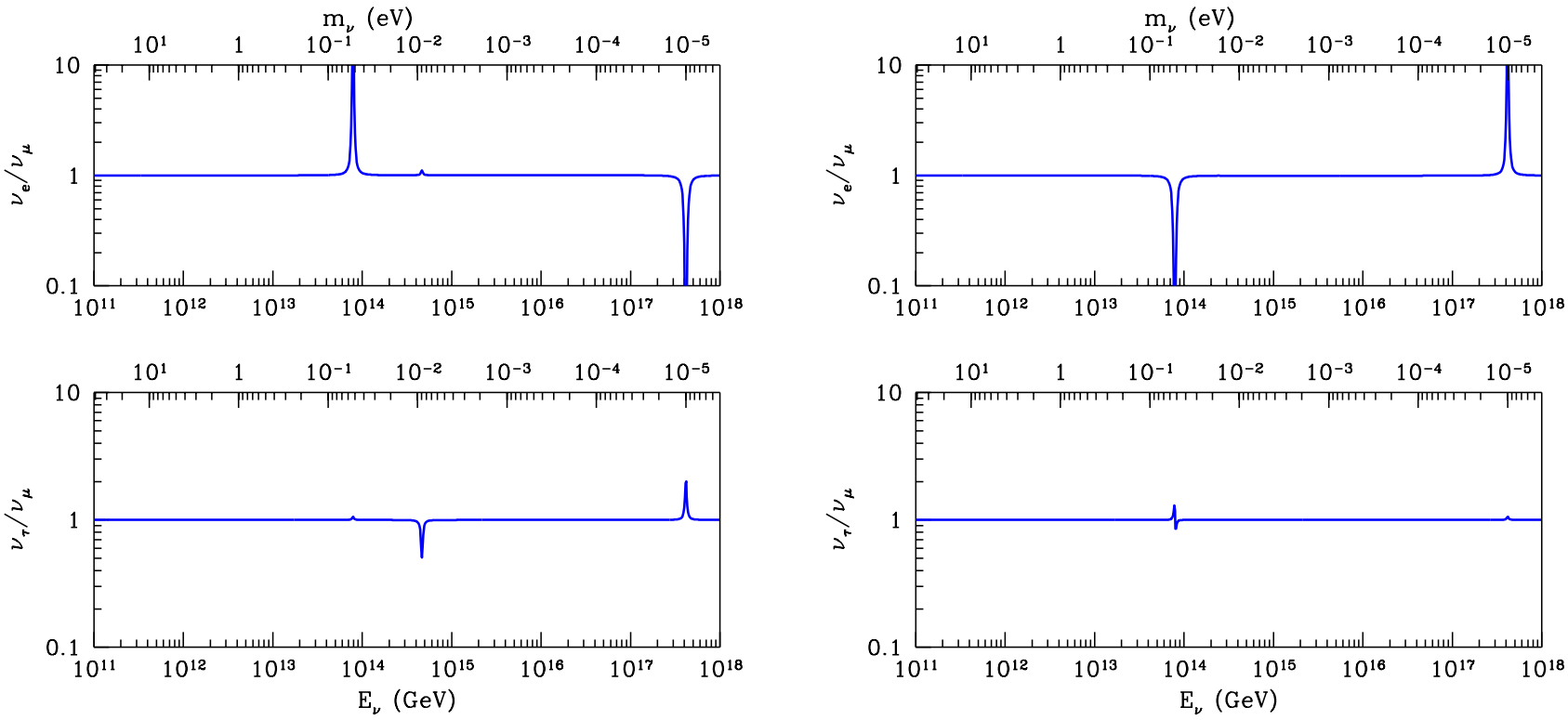
# Fable: l-o-o-o-o-o-n-g path ($10^4$ or $10^5$ Mpc) in current U




$m_\ell = 10^{-5}$ eV

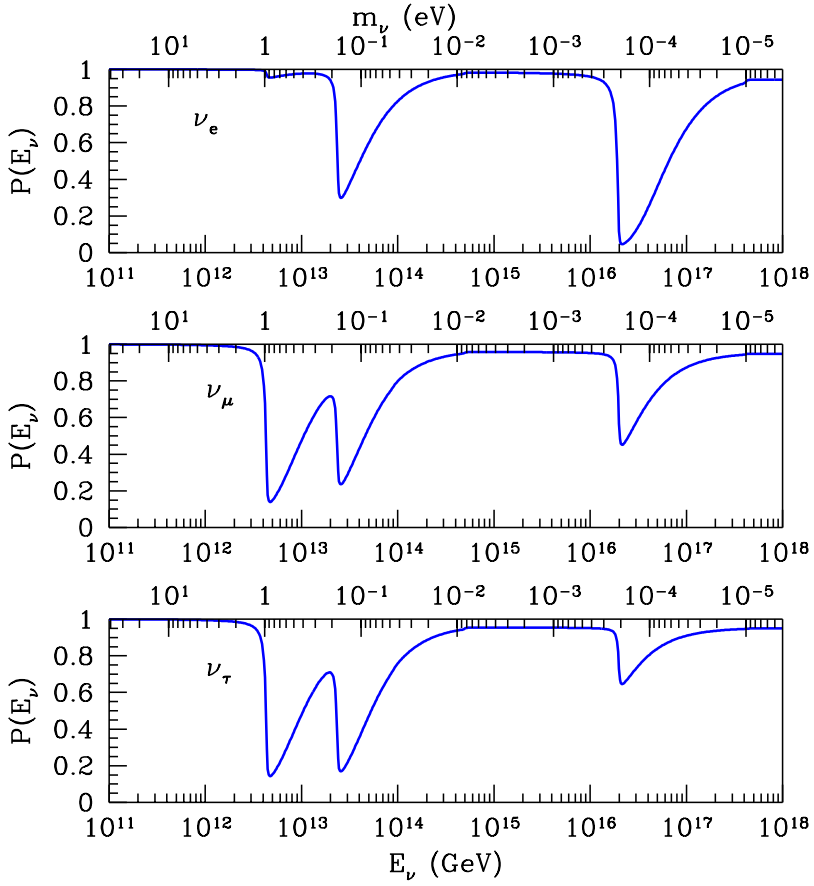
$m_\ell = 10^{-3}$ eV

# Flavor ratios probe the mass hierarchy . . .

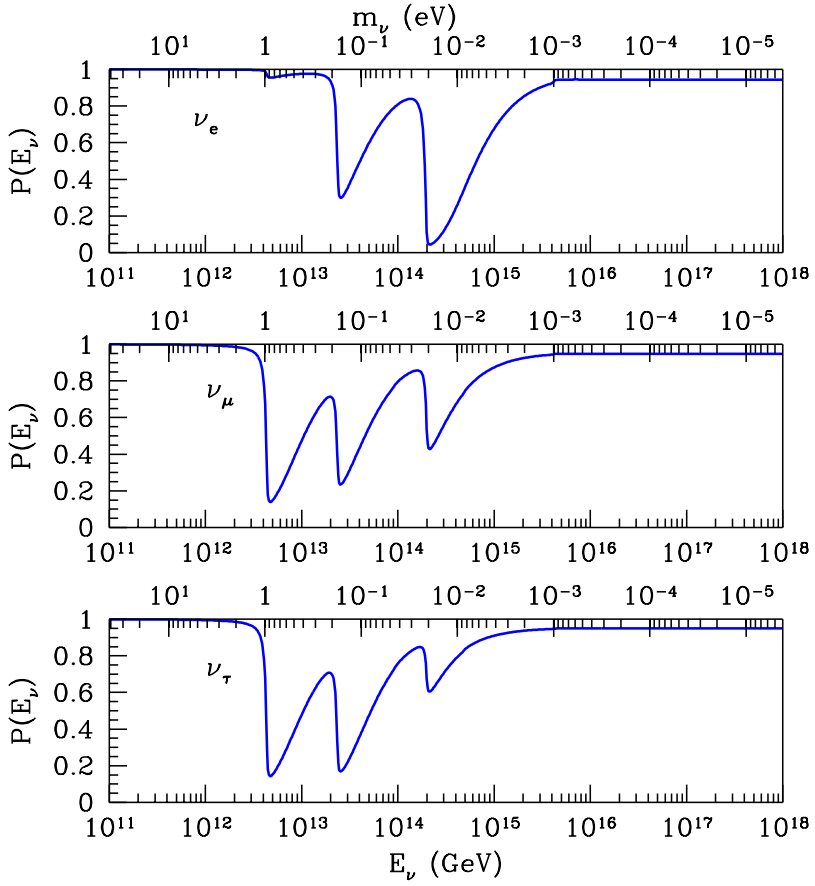



Normal $m_\ell = 10^{-5}$ eV Inverted

# Incorporate evolution of U back to $z = 20$

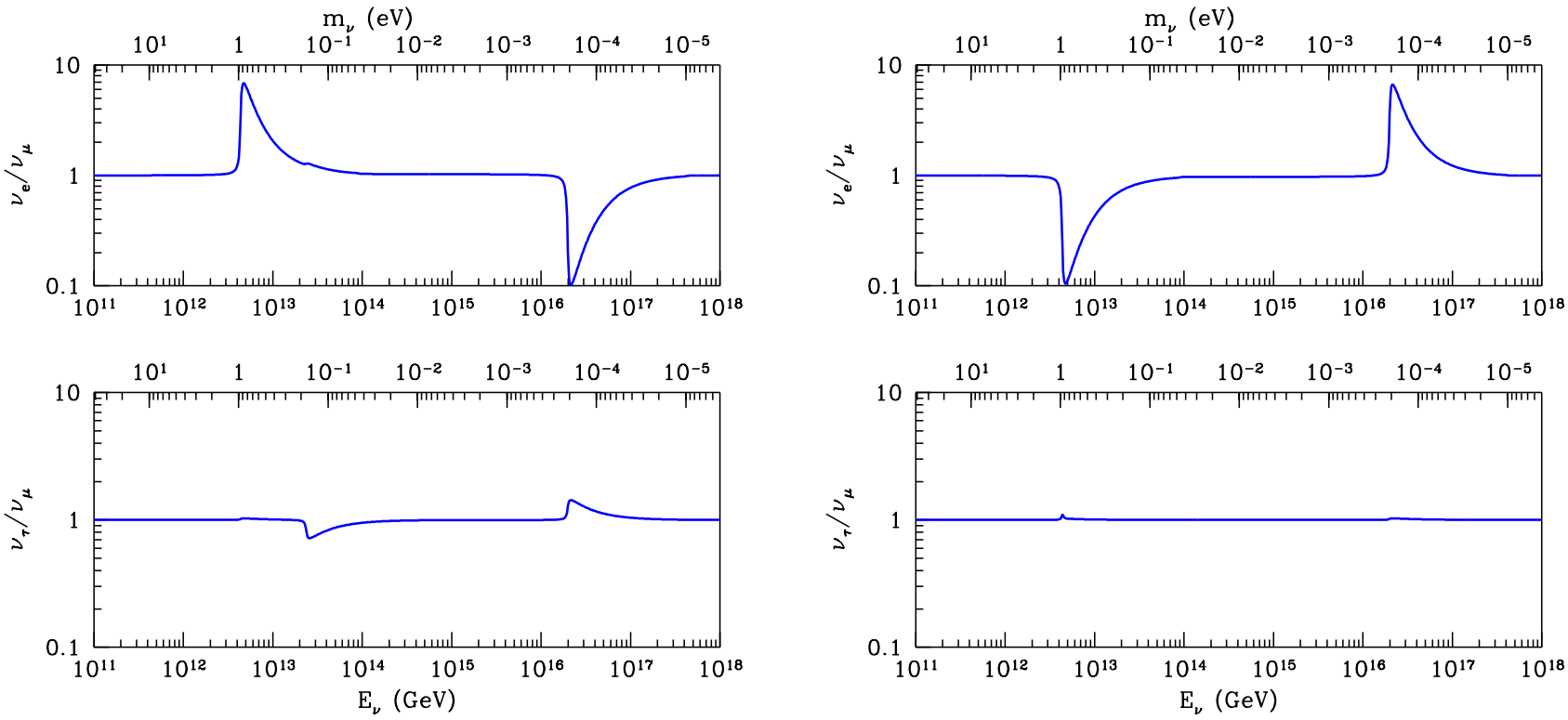



$m_\ell = 10^{-5}$ eV



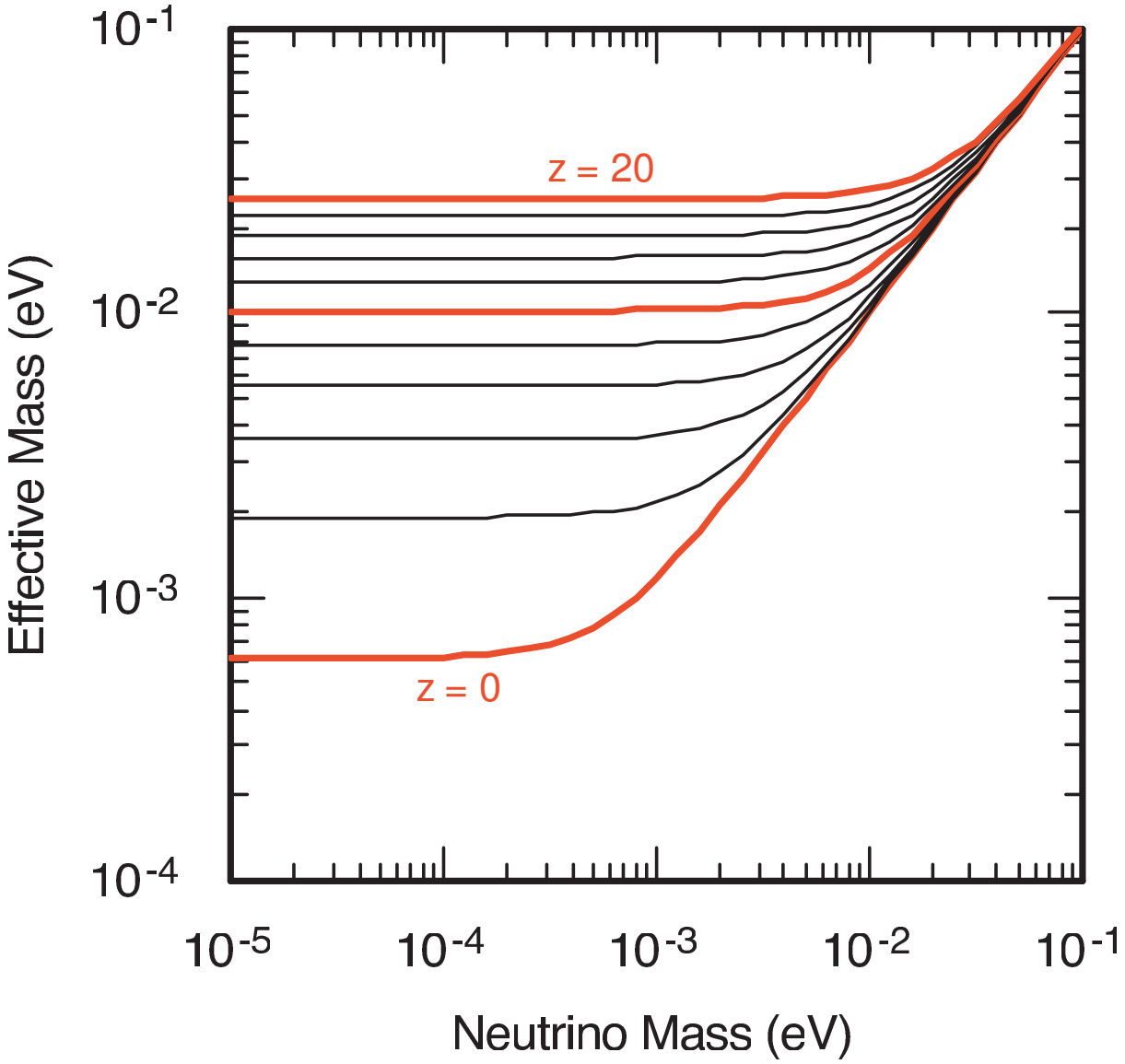

$m_\ell = 10^{-3}$ eV

# Flavor ratios probe the mass hierarchy($z \lesssim 20$)



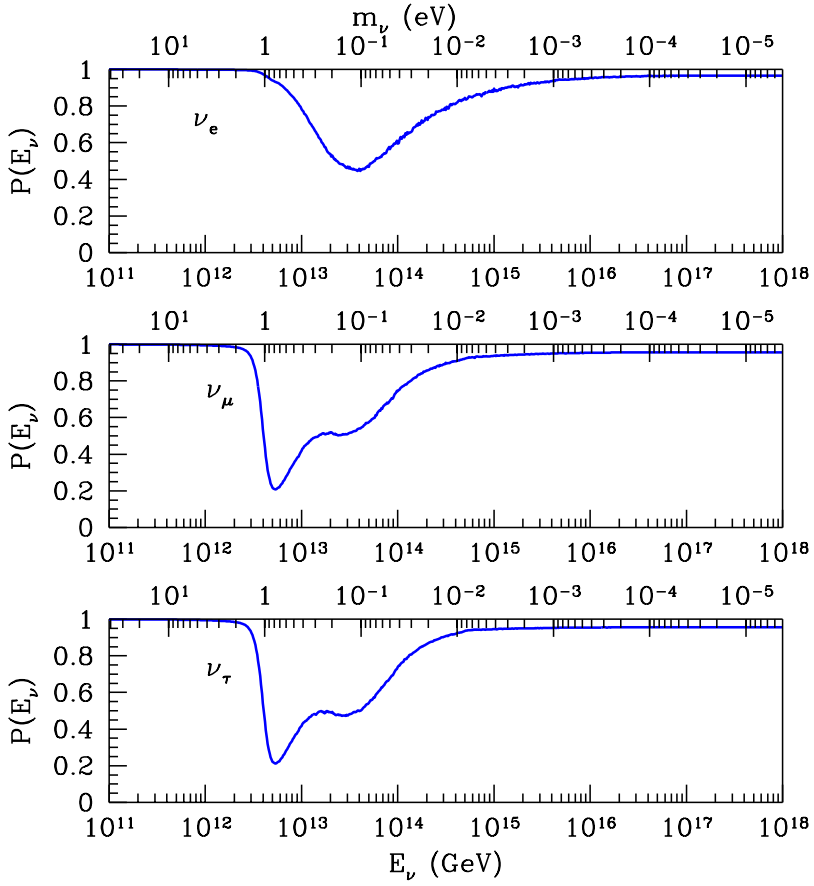

Normal $m_\ell = 10^{-5}$ eV Inverted

# Relic neutrinos are moving targets . . .

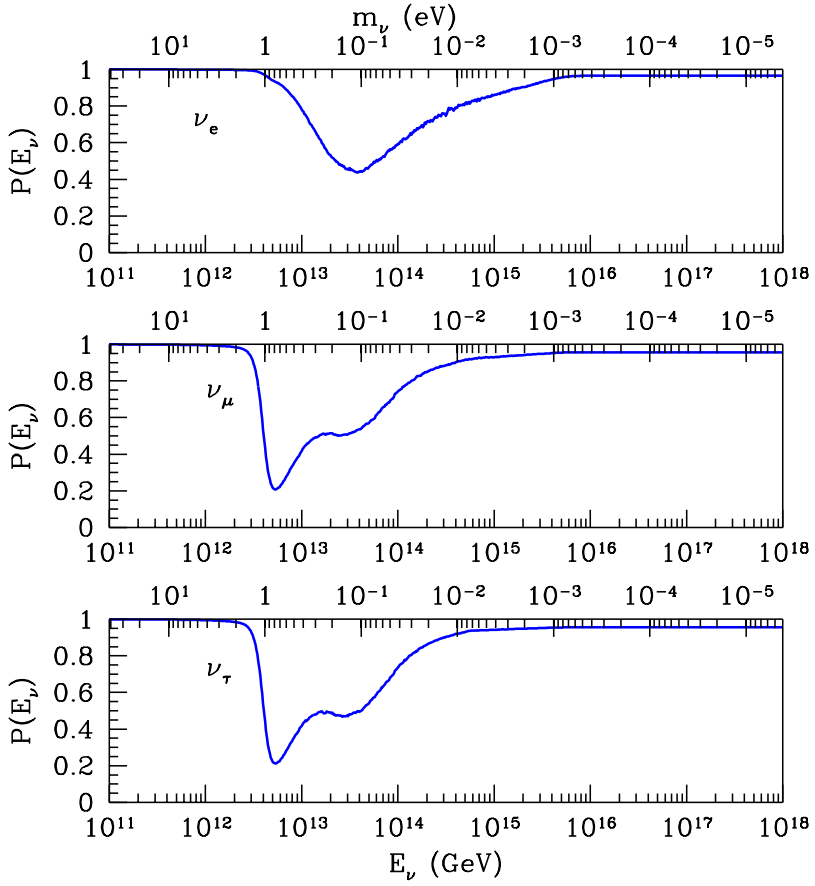



$$T_{\nu 0} = 1.945 \text{ K} = 1.697 \times 10^{-4} \text{ eV} \quad T_\nu \propto (1+z)$$

# Incorporate evolution of U back to $z = 20$, Fermi motion

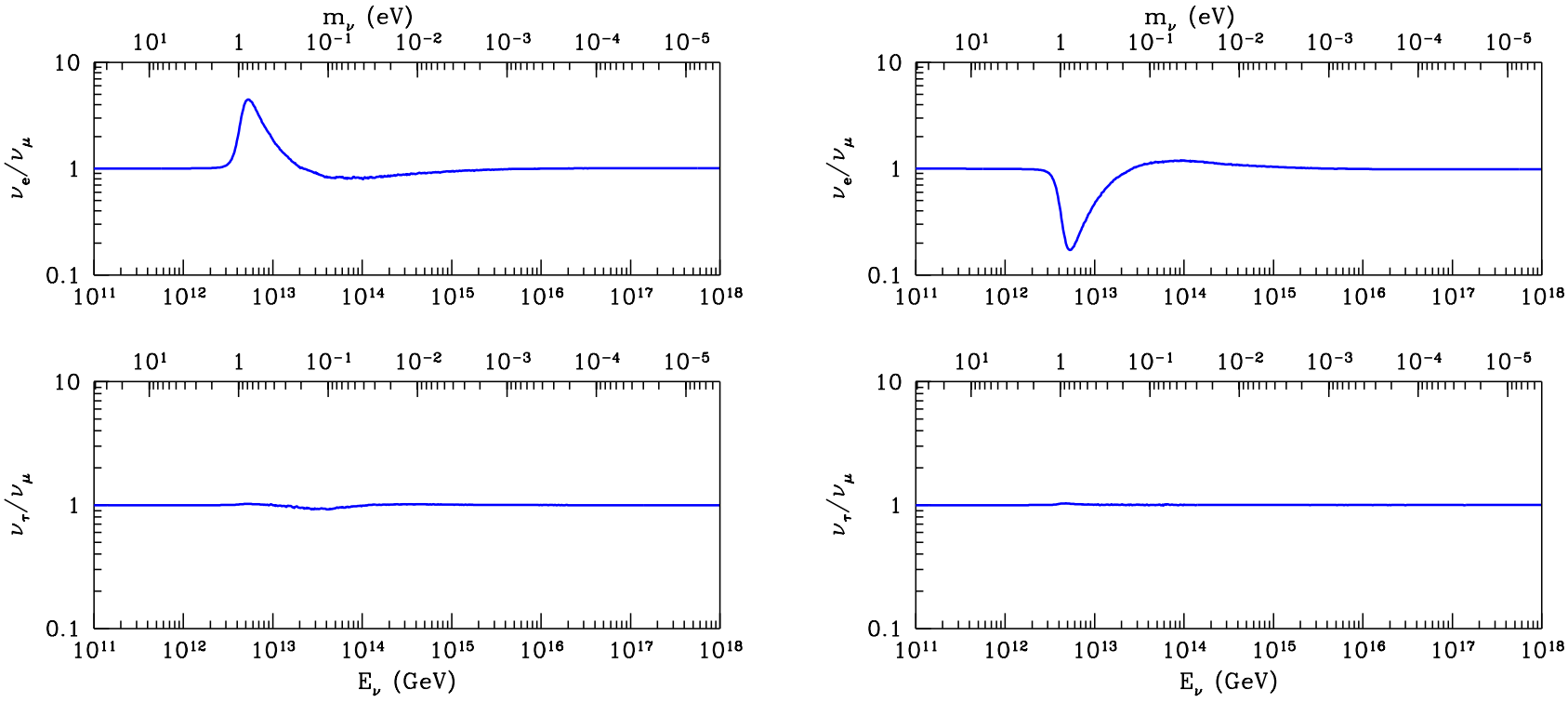



$m_\ell = 10^{-5}$ eV




$m_\ell = 10^{-3}$ eV

# Flavor ratios probe mass hierarchy($z \lesssim 20$, Fermi motion)




Normal $m_\ell = 10^{-5}$ eV Inverted