The Standard Model: Electroweak Theory & Higgs Physics

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A Decade of Discovery Past . . .

- EW theory → law of nature \([Z, e^+ e^-, \bar{pp}, \nu N, (g - 2)_\mu, \ldots]\)
- Higgs-boson influence in the vacuum [EW experiments]
- \(\nu\) oscillations: \(\nu_\mu \rightarrow \nu_\tau, \nu_e \rightarrow \nu_\mu/\nu_\tau\) [\(\nu_\odot, \nu_{atm}, \text{reactors}\)]
- Understanding QCD [heavy flavor, \(Z^0, \bar{pp}, \nu N, ep, \text{ions, lattice}\)]
- Discovery of top quark [\(\bar{pp}\)]
- Direct \(\mathcal{CP}\) violation in \(K \rightarrow \pi\pi\) [fixed-target]
- \(B\)-meson decays violate \(\mathcal{CP}\) [\(e^+ e^- \rightarrow B\bar{B}\)]
- Flat universe: dark matter, energy [SN Ia, CMB, LSS]
- Detection of \(\nu_\tau\) interactions [fixed-target]
- Quarks, leptons structureless at 1 TeV scale [mostly colliders]
Tevatron Collider is breaking new ground in sensitivity
Tevatron Collider in a Nutshell

980-GeV protons, antiprotons (2π km)

*frequency of revolution* \( \approx 45000 \text{ s}^{-1} \)

392 ns between crossings

(36 \( \times \) 36 bunches)

collision rate = \( \mathcal{L} \cdot \sigma_{\text{inelastic}} \approx 10^7 \text{ s}^{-1} \)

\( c \approx 10^9 \text{ km/h}; \quad v_p \approx c - 495 \text{ km/h} \)

Record \( \mathcal{L}_{\text{init}} = 2.85 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1} \)

[CERN ISR: \( pp, 1.4 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1} \)]

Goal: \( \approx 8 \text{ fb}^{-1} \text{ by } 10.2009 \)
The World’s Most Powerful Microscopes

**nanonanophysics**

CDF dijet event ($\sqrt{s} = 1.96$ TeV): $E_T = 1.364$ TeV

$q\bar{q} \rightarrow \text{jet} + \text{jet}$
LHC will operate soon, breaking new ground in $E$ & $\mathcal{L}$
LHC in a nutshell

7-TeV protons on protons (27 km); \( v_p \approx c - 10 \text{ km/h} \)

Heavy ions, e.g. Pb-Pb at \( \sqrt{s} \approx 1 \text{ PeV} \)

Novel two-in-one dipoles (\( \approx 9 \text{ teslas} \))

Beam commissioning May 2008

\( \sim \) First collisions at \( E_{cm} = 14 \text{ TeV} \)

Pilot run: Goal of \( \approx 100 \text{ pb}^{-1} \) by end 2008

First physics run! Goal of a few \( \text{fb}^{-1} \) in 2009

Eventual: \( \mathcal{L} \gtrsim 10^{34} \text{ cm}^{-2} \text{ s}^{-1} \): 100 \( \text{fb}^{-1} \)/year

(Much more from Nural Akchurin)
Why the LHC is so exciting (I)

- Even low luminosity opens vast new realm: 
  \(10 \text{ pb}^{-1}\) (few days at initial \(\mathcal{L}\)) yields 
  8000 top quarks, \(10^5\) \(W\)-bosons, 
  100 QCD dijets beyond Tevatron kinematic limit 
  Supersymmetry hints in a few weeks?

- Essential first step: rediscover the standard model 

- The antithesis of a one-experiment machine; 
  enormous scope and versatility beyond high-\(p_{	ext{\perp}}\)

- \(\mathcal{L}\) upgrade extends \(\gtrsim10\)-year program . . .
Sample event rates in $p^+ p$ collisions

\begin{align*}
\sigma_{\text{jet}}(E_T^{\text{jet}} > \sqrt{s}/20) & \\
\sigma_{\text{Higgs}}(M_H = 150 \text{ GeV}) & \\
\sigma_{\text{Higgs}}(M_H = 500 \text{ GeV}) & \\
\sigma_{\text{cbbar}} & \\
\sigma_{\text{W}} & \\
\sigma_{\text{Z}} & \\
\sigma_{\text{ttbar}} & \\
\sigma_{\text{tot}} & \\
\end{align*}

For $\sqrt{s} = 10^{33}$ cm$^{-2}$s$^{-1}$ and $L = 10^{33}$ cm$^{-2}$s$^{-1}$

\[\text{events/sec} = \frac{1}{2} \times 10^{33} \times \frac{1}{10^{20}} \times 10^{3} \times 10^{9} = 10^{9} \text{ events/sec}
\]
LHC experiments will need a trigger . . .

\[ \frac{d\sigma}{dM_{jj}}, \ |\eta| \leq 1 \]

\[ \int dE_T \frac{d\sigma}{dE_T}, |y_{1,2}| \leq 2.5 \]

Dijet integral cross section, $|\eta| \leq 2.5$ . . .

10 pb$^{-1}$ @ LHC $\sim \gtrsim 10^4$ events with $E_T \gtrsim 1.364$ TeV
The importance of the 1-TeV scale

EW theory does not predict Higgs-boson mass

- Conditional upper bound from Unitarity

Compute amplitudes $\mathcal{M}$ for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s, t) = 16\pi \sum_J (2J + 1) a_J(s) P_J(\cos \theta)$$

Most channels decouple – pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies) – $\forall M_H$.

Four interesting channels:

$$W^+_L W^-_L \quad Z^0_L Z^0_L / \sqrt{2} \quad HH / \sqrt{2} \quad HZ^0_L$$

$L$: longitudinal, $1/\sqrt{2}$ for identical particles
In HE limit, \( s \)-wave amplitudes \( \propto G_F M_H^2 \)

\[
\lim_{s \gg M_H^2} (a_0) \rightarrow \frac{-G_F M_H^2}{4\pi \sqrt{2}} \cdot \begin{pmatrix}
1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\
1/\sqrt{8} & 3/4 & 1/4 & 0 \\
1/\sqrt{8} & 1/4 & 3/4 & 0 \\
0 & 0 & 0 & 1/2
\end{pmatrix}
\]

Require that largest eigenvalue respect pw unitarity condition \( |a_0| \leq 1 \)

\[
\implies M_H \leq \left( \frac{8\pi \sqrt{2}}{3G_F} \right)^{1/2} = 1 \text{ TeV}/c^2
\]

condition for perturbative unitarity

---

\(^1\)Convenient to calculate using Goldstone-boson equivalence theorem, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by \( \mathcal{L}_{\text{int}} = -\lambda v h (2w^+ w^- + z^2 + h^2) - (\lambda/4)(2w^+ w^- + z^2 + h^2)^2 \), with \( 1/v^2 = G_F \sqrt{2} \) and \( \lambda = G_F M_H^2/\sqrt{2} \).
If the bound is respected

- weak interactions remain weak at all energies
- perturbation theory is everywhere reliable

If the bound is violated

- perturbation theory breaks down
- weak interactions among $W^\pm$, $Z$, $H$ become strong on 1-TeV scale

⇒ features of strong interactions at GeV energies will characterize electroweak gauge boson interactions at TeV energies

*New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV*

Threshold behavior of the pw amplitudes $a_{IJ}$ follows from chiral symmetry

\[
\begin{align*}
  a_{00} &\approx G_F s / 8\pi \sqrt{2} \quad \text{attractive} \\
  a_{11} &\approx G_F s / 48\pi \sqrt{2} \quad \text{attractive} \\
  a_{20} &\approx -G_F s / 16\pi \sqrt{2} \quad \text{repulsive}
\end{align*}
\]

What the LHC is not really for . . .

- Find the Higgs boson, the Holy Grail of particle physics, the source of all mass in the Universe.
- Celebrate.
- Then particle physics will be over.

_We are not ticking off items on a shopping list . . ._

We are exploring a vast new terrain . . . and reaching the Fermi scale
The Origins of Mass

(masses of nuclei “understood”)

\( p, [\pi], \rho \) understood: QCD

*confinement energy* is the source


We understand the visible mass of the Universe

\[ \text{... without the Higgs mechanism} \]

\( W, Z \) electroweak symmetry breaking

\[ M_W^2 = \frac{1}{2} g^2 v^2 = \pi \alpha / G_F \sqrt{2} \sin^2 \theta_W \]

\[ M_Z^2 = M_W^2 / \cos^2 \theta_W \]

\( q, \ell^\mp \) EWSB + Yukawa couplings

\( \nu_\ell \) EWSB + Yukawa couplings; new physics?

All fermion masses \( \leftrightarrow \) physics beyond standard model

\( H \) ?? fifth force ??
Challenge: Understanding the Everyday

- Why are there atoms?
- Why chemistry?
- Why stable structures?
- What makes life possible?

*What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?*
Searching for the mechanism of electroweak symmetry breaking, we seek to understand

*why the world is the way it is.*

This is one of the deepest questions humans have ever pursued, and

*it is coming within the reach of particle physics.*
Our picture of matter

Pointlike constituents \((r < 10^{-18} \text{ m})\)

\[
\begin{pmatrix}
  u \\
  d
\end{pmatrix}_L, \quad
\begin{pmatrix}
  c \\
  s
\end{pmatrix}_L, \quad
\begin{pmatrix}
  t \\
  b
\end{pmatrix}_L
\]

\[
\begin{pmatrix}
  \nu_e \\
  e^-
\end{pmatrix}_L, \quad
\begin{pmatrix}
  \nu_\mu \\
  \mu^-
\end{pmatrix}_L, \quad
\begin{pmatrix}
  \nu_\tau \\
  \tau^-
\end{pmatrix}_L
\]

Few fundamental forces, derived from gauge symmetries

\[\text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y\]

Electroweak symmetry breaking: Higgs mechanism?
Formulate electroweak theory

Three crucial clues from experiment:

- Left-handed weak-isospin doublets,

\[
\begin{pmatrix}
\nu_e \\
e \\
\nu_\mu \\
\mu \\
\nu_\tau \\
\tau \\
u_{\nu} \\
u_{d'} \\
u_{s'} \\
u_{b'}
\end{pmatrix}_L
\]

- Universal strength of the (charged-current) weak interactions;

- Idealization that neutrinos are massless.

First two clues suggest \( SU(2)_L \) gauge symmetry
A theory of leptons

\[ L = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \]

\[ R \equiv e_R \]

weak hypercharges \( Y_L = -1, \ Y_R = -2 \)

Gell-Mann–Nishijima connection, \( Q = I_3 + \frac{1}{2} Y \)

SU(2)_L \otimes U(1)_Y \text{ gauge group} \implies \text{gauge fields:}

- weak isovector \( \vec{b}_\mu \), coupling \( g \)

- weak isoscalar \( A_\mu \), coupling \( g'/2 \)

Field-strength tensors

\[ F_{\mu\nu}^\ell = \partial_\nu b_\mu^\ell - \partial_\mu b_\nu^\ell + g \varepsilon_{j\ell} b_j^i b_k^k, \ SU(2)_L \]

\[ f_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu, \ U(1)_Y \]
Interaction Lagrangian

\[ \mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}} \]

\[ \mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu\nu}^\ell F^{\ell\mu\nu} - \frac{1}{4} f_{\mu\nu} f^{\mu\nu}, \]

\[ \mathcal{L}_{\text{leptons}} = \overline{R} i \gamma^\mu \left( \partial_\mu + i \frac{g'}{2} A_\mu Y \right) R \]

\[ + \overline{L} i \gamma^\mu \left( \partial_\mu + i \frac{g'}{2} A_\mu Y + i \frac{g}{2} \vec{\tau} \cdot \vec{b}_\mu \right) L. \]

Mass term \( \mathcal{L}_e = -m_e (\overline{e}_R e_L + \overline{e}_L e_R) = -m_e \bar{e}e \) violates local gauge inv.

Theory: 4 massless gauge bosons \( (A_\mu, b^1_\mu, b^2_\mu, b^3_\mu) \); Nature: 1 (\( \gamma \))
Massive Photon?    *Hiding Symmetry*

Recall miracles of superconductivity:
- No resistance ... ... Meissner effect (exclusion of $B$)

Ginzburg–Landau Phenomenology (not a theory from first principles)

normal, resistive charge carriers ... ... + superconducting charge carriers

\[ B = 0: \quad G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha |\psi|^2 + \beta |\psi|^4 \]

\[
\begin{align*}
T > T_c : \quad & \alpha > 0 \quad \langle |\psi|^2 \rangle_0 = 0 \\
T < T_c : \quad & \alpha < 0 \quad \langle |\psi|^2 \rangle_0 \neq 0
\end{align*}
\]
In a nonzero magnetic field . . .

\[ G_{\text{super}}(B) = G_{\text{super}}(0) + \frac{B^2}{8\pi} + \frac{1}{2m^*} \left| -i\hbar \nabla \psi - \frac{e^*}{c} A \psi \right|^2 \]

\[ e^* = -2 \quad m^* \]

of superconducting carriers

Weak, slowly varying field: \( \psi \approx \psi_0 \neq 0, \nabla \psi \approx 0 \)

Variational analysis \( \leadsto \)

\[ \nabla^2 A - \frac{4\pi e^*}{m^* c^2} |\psi_0|^2 A = 0 \]

wave equation of a massive photon

Photon – gauge boson – acquires mass within superconductor

origin of Meissner effect
Magnet floats (on field lines) above superconductor
Meissner effect levitates Leon Lederman (Snowmass 2001)
Hiding EW Symmetry

Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition

• Introduce a complex doublet of scalar fields

\[ \phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1 \]

• Add to \( \mathcal{L} \) (gauge-invariant) terms for interaction and propagation of the scalars,

\[ \mathcal{L}_{\text{scalar}} = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi^\dagger \phi), \]

where \( D_\mu = \partial_\mu + ig'2 \, A_\mu \, Y + ig \, \vec{\tau} \cdot \vec{b}_\mu \) and

\[ V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2 \]

• Add a Yukawa interaction \( \mathcal{L}_{\text{Yukawa}} = -\zeta_e \left[ \bar{R}(\phi^\dagger L) + (\bar{L} \phi)R \right] \)
• Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^2 < 0$

Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks) $SU(2)_L$ and $U(1)_Y$

but preserves $U(1)_{em}$ invariance

Invariance under $\mathcal{G}$ means $e^{i\alpha \mathcal{G}} \langle \phi \rangle_0 = \langle \phi \rangle_0$, so $\mathcal{G} \langle \phi \rangle_0 = 0$

$$\tau_1 \langle \phi \rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \text{ broken!}$$

$$\tau_2 \langle \phi \rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \text{ broken!}$$

$$\tau_3 \langle \phi \rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \text{ broken!}$$

$$\Upsilon \langle \phi \rangle_0 = \Upsilon \langle \phi \rangle_0 = +1 \langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \text{ broken!}$$
Symmetry of laws $\not\Rightarrow$ symmetry of outcomes
Examine electric charge operator $Q$ on the (neutral) vacuum

\[
Q\langle \phi \rangle_0 = \frac{1}{2}(\tau_3 + Y)\langle \phi \rangle_0
\]

\[
= \frac{1}{2} \begin{pmatrix} Y_{\phi} + 1 & 0 \\ 0 & Y_{\phi} - 1 \end{pmatrix} \langle \phi \rangle_0
\]

\[
= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}
\]

\[
= \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\text{ unbroken!}
\]

Four original generators are broken, \textit{electric charge is not}

- $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}$ (will verify)
- Expect massless photon
- Expect gauge bosons corresponding to

\[
\tau_1, \tau_2, \frac{1}{2}(\tau_3 - Y) \equiv K \text{ to acquire masses}
\]
Expand about the vacuum state

Let $\phi = \begin{pmatrix} 0 \\ (v + \eta)/\sqrt{2} \end{pmatrix}$; in unitary gauge

$$L_{\text{scalar}} = \frac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2 \eta^2$$

$$+ \frac{v^2}{8} [g^2 |b^1_\mu - ib^2_\mu|^2 + (g' A_\mu - gb^3_\mu)^2]$$

$$+ \text{interaction terms}$$

“Higgs boson” $\eta$ has acquired (mass)$^2$ $M_H^2 = -2\mu^2 > 0$

Define $W^{\pm}_\mu = \frac{b^1_\mu \mp ib^2_\mu}{\sqrt{2}}$

$$g^2 \frac{v^2}{8} (|W^{+}_\mu|^2 + |W^{-}_\mu|^2) \iff M_{W^{\pm}} = \frac{gv}{2}$$
\[ (\nu^2/8)(g' A_\mu - g b^{3}_\mu)^2 \ldots \]

Now define orthogonal combinations

\[
Z_\mu = \frac{-g' A_\mu + g b^{3}_\mu}{\sqrt{g^2 + g'^2}} \quad A_\mu = \frac{g A_\mu + g' b^{3}_\mu}{\sqrt{g^2 + g'^2}}
\]

\[
M_{Z^0} = \sqrt{g^2 + g'^2} \nu/2 = M_W \sqrt{1 + g'^2/g^2}
\]

\[ A_\mu \text{ remains massless} \]
\[ \mathcal{L}_{\text{Yukawa}} = -\zeta_e \frac{(v + \eta)}{\sqrt{2}}(\bar{e}_Re_L + \bar{e}_Le_R) \]

\[ = -\frac{\zeta_e v}{\sqrt{2}}\bar{e}e - \frac{\zeta_e \eta}{\sqrt{2}}\bar{e}e \]

Electron acquires \( m_e = \frac{\zeta_e v}{\sqrt{2}} \)

Higgs-boson coupling to electrons: \( m_e/v \propto \text{mass} \)

Desired particle content . . . plus a Higgs scalar

Values of couplings, electroweak scale \( v \)?

What about interactions?
Interactions . . .

\[ \mathcal{L}_{W-\ell} = -\frac{g}{2\sqrt{2}}[\bar{\nu}\gamma^\mu(1 - \gamma_5)eW^+_{\mu} + \bar{e}\gamma^\mu(1 - \gamma_5)\nu W^-_{\mu}] \]

+ similar terms for \( \mu \) and \( \tau \)

\[ = \frac{-i g}{2\sqrt{2}} \gamma_\lambda (1 - \gamma_5) \]

\[ = \frac{-i (g_{\mu\nu} - k_\mu k_\nu / M_W^2)}{k^2 - M_W^2} . \]
Compute $\nu_\mu e \to \mu \nu_e$

$$\sigma(\nu_\mu e \to \mu \nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\frac{g^4}{16M_W^4} = 2G_F^2 \Rightarrow \frac{g}{2\sqrt{2}} = \left( \frac{G_F M_W^2}{\sqrt{2}} \right)^{1/2}$$

Using $M_W = g v/2$, determine the electroweak scale

$$v = \left( G_F \sqrt{2} \right)^{-\frac{1}{2}} \approx 246 \text{ GeV}$$

$$\Rightarrow \langle \phi^0 \rangle_0 = \left( G_F \sqrt{8} \right)^{-\frac{1}{2}} \approx 174 \text{ GeV}$$
$W$-propagator modifies HE behavior

$$\sigma(\nu_\mu e \rightarrow \mu \nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \left[1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu}\right]^2 \frac{1}{(1 + 2m_e E_\nu/M_W^2)}$$

$$\lim_{E_\nu \rightarrow \infty} \sigma(\nu_\mu e \rightarrow \mu \nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\sqrt{2}}$$

independent of energy!

Partial-wave unitarity respected for

$$s < M_W^2 \left[\exp \left(\pi \sqrt{2}/G_F M_W^2\right) - 1\right]$$
**W-boson properties**

No prediction yet for $M_W$ (haven't determined $g$)

Leptonic decay $W^- \rightarrow e^- \nu_e$

\[ e(p) \quad p \approx \left( \frac{M_W}{2}; \frac{M_W \sin \theta}{2}, 0, \frac{M_W \cos \theta}{2} \right) \]

\[ \bar{\nu}_e(q) \quad q \approx \left( \frac{M_W}{2}; -\frac{M_W \sin \theta}{2}, 0, -\frac{M_W \cos \theta}{2} \right) \]

\[
\mathcal{M} = -i \left( \frac{G_F M_W^2}{\sqrt{2}} \right)^{\frac{1}{2}} \bar{u}(e, p) \gamma_\mu (1 - \gamma_5) \nu(\nu, q) \varepsilon^\mu
\]

$\varepsilon^\mu = (0; \hat{\varepsilon})$: $W$ polarization vector in its rest frame

\[
|\mathcal{M}|^2 = \frac{G_F M_W^2}{\sqrt{2}} \text{tr} [\not\varepsilon (1 - \gamma_5) \not\varepsilon (1 + \gamma_5) \not\varepsilon^* \not p] ;
\]

\[
\text{tr}[\cdots] = [\varepsilon \cdot q \varepsilon^* \cdot p - \varepsilon \cdot \varepsilon^* q \cdot p + \varepsilon \cdot p \varepsilon^* \cdot q + i \epsilon_{\mu\nu\rho\sigma} \varepsilon^\mu q^\nu \varepsilon^* \rho p^\sigma] \]
\[ \text{tr}[\cdots] = [\varepsilon \cdot q \varepsilon^* \cdot p - \varepsilon \cdot \varepsilon^* \cdot q \cdot p + \varepsilon \cdot p \varepsilon^* \cdot q + i\varepsilon_{\mu\nu\rho\sigma} \varepsilon^\mu q^\nu \varepsilon^* p^\sigma] \]

decay rate is independent of \( W \) polarization; look first at longitudinal pol.
\( \varepsilon^\mu = (0; 0, 0, 1) = \varepsilon^{*\mu} \), eliminate \( \varepsilon_{\mu\nu\rho\sigma} \)

\[
|M|^2 = \frac{4G_F M^4_W}{\sqrt{2}} \sin^2 \theta
\]

\[
\frac{d\Gamma_0}{d\Omega} = \frac{|M|^2}{64\pi^2} \frac{S_{12}}{M^3_W}
\]

\[
S_{12} = \sqrt{[M^2_W - (m_e + m_\nu)^2][M^2_W - (m_e - m_\nu)^2]} = M^2_W
\]

\[
\Gamma(W \to e\nu) = \frac{G_F M^3_W}{6\pi\sqrt{2}}
\]
Other helicities: \( \varepsilon^\mu_{\pm 1} = (0; -1, \mp i, 0) / \sqrt{2} \)

\[
\frac{d\Gamma_{\pm 1}}{d\Omega} = \frac{G_F M_W^3}{32\pi^2 \sqrt{2}} (1 \mp \cos \theta)^2
\]

Extinctions at \( \cos \theta = \pm 1 \) are consequences of angular momentum conservation:

\( W^- \quad \uparrow \downarrow \quad (\theta = 0) \) forbidden
\( (\theta = \pi) \) allowed

\((\text{situation reversed for } W^+ \rightarrow e^+ \nu_e)\)

\( e^+ \) follows polarization direction of \( W^+ \)
\( e^- \) avoids polarization direction of \( W^- \)

*important* for discovery of \( W^\pm \) in \( \bar{p}p \ (\bar{q}q) \) \( C \) violation
Fig. 2. The W decay angular distribution of the emission angle $\theta^*$ of the electron (positron) with respect to the proton (anti-proton) direction in the rest frame of the W. Only those events for which the lepton charge and the decay kinematics are well determined have been used. The curve shows the $(V^- A)$ expectation of $(1 + \cos \theta^*)^2$. 

UA1
75 EVENTS
Background subtracted and acceptance corrected

\[ \frac{1}{N} \frac{dN}{d\cos \theta^*} \]

\[ (1+\cos \theta^*)^2 \]
Interactions . . .

\[ \mathcal{L}_{A-\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e} \gamma^\mu e A^\mu \]

. . . vector interaction; \( \Rightarrow A^\mu \) as \( \gamma \), provided we identify

\[ \frac{gg'}{\sqrt{g^2 + g'^2}} \equiv e \]

Define \( g' = g \tan \theta_W \)

\( \theta_W \): weak mixing angle

\[ g = e / \sin \theta_W \geq e \]

\[ g' = e / \cos \theta_W \geq e \]

\[ Z^\mu = b^3_\mu \cos \theta_W - A^\mu \sin \theta_W \]

\[ A^\mu = A^\mu_\cos \theta_W + b^3_\mu \sin \theta_W \]

\[ \mathcal{L}_{Z-\nu} = \frac{-g}{4 \cos \theta_W} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu \ Z^\mu \]

Purely left-handed!
Interactions . . .

\[ \mathcal{L}_{Z-e} = \frac{-g}{4 \cos \theta_W} \bar{e} [L_e \gamma^\mu (1 - \gamma_5) + R_e \gamma^\mu (1 + \gamma_5)] e Z_\mu \]

\[
\begin{align*}
L_e & = 2 \sin^2 \theta_W - 1 = 2x_W + \tau_3 \\
R_e & = 2 \sin^2 \theta_W = 2x_W
\end{align*}
\]

Z-decay calculation analogous to \( W^\pm \)

\[
\Gamma(Z \rightarrow \nu \bar{\nu}) = \frac{G_F M_Z^3}{12\pi \sqrt{2}} \\
\Gamma(Z \rightarrow e^+ e^-) = \Gamma(Z \rightarrow \nu \bar{\nu}) [L_e^2 + R_e^2]
\]

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Neutral-current interactions

New $\nu e$ reaction, not present in $V - A$

\[
\sigma(\nu_\mu e \rightarrow \nu_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[ L_e^2 + R_e^2 / 3 \right]
\]
\[
\sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[ L_e^2 / 3 + R_e^2 \right]
\]
\[
\sigma(\nu_e e \rightarrow \nu_e e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[ (L_e + 2)^2 + R_e^2 / 3 \right]
\]
\[
\sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[ (L_e + 2)^2 / 3 + R_e^2 \right]
\]
Gargamelle $\nu_\mu e$ event (1973)
Electromagnetism is mediated by a massless photon, coupled to the electric charge;

Mediator of charged-current weak interaction acquires a mass \( M_W^2 = \frac{\pi \alpha}{G_F \sqrt{2}} \sin^2 \theta_W \),

Mediator of (new!) neutral-current weak interaction acquires mass \( M_Z^2 = M_W^2 / \cos^2 \theta_W \);

Massive neutral scalar particle, the Higgs boson, appears, but its mass is not predicted;

Fermions can acquire mass—values not predicted.

Determine \( \sin^2 \theta_W \) to predict \( M_W, M_Z \)
“Model-independent” analysis

Measure all cross sections to determine chiral couplings $L_e$ and $R_e$ or traditional vector and axial couplings $v$ and $a$

$$a = \frac{1}{2}(L_e - R_e) \quad v = \frac{1}{2}(L_e - R_e)$$

$$L_e = v + a \quad R_e = v - a$$

model-independent in $V, A$ framework
Neutrino–electron scattering
Twofold ambiguity remains even after measuring all four cross sections: same cross sections result if we interchange $R_e \leftrightarrow -R_e \ (\nu \leftrightarrow a)$

Consider $e^+ e^- \rightarrow \mu^+ \mu^-$

\[
\mathcal{M} = -ie^2 \bar{u}(\mu, q_-)\gamma^\lambda Q_\mu \nu(\mu, q_+)\frac{g^{\lambda\nu}}{s}\bar{v}(e, p_+)\gamma^\nu u(e, p_-) \\
+ \frac{i}{2} \left( \frac{G_F M_Z^2}{\sqrt{2}} \right) \bar{u}(\mu, q_-)\gamma^\lambda [R_\mu (1 + \gamma_5) + L_\mu (1 - \gamma_5)] \nu(\mu, q_+) \\
\times \frac{g^{\lambda\nu}}{s - M_Z^2} \bar{v}(e, p_+)\gamma^\nu [R_e (1 + \gamma_5) + L_e (1 - \gamma_5)] u(e, p_-)
\]

muon charge $Q_\mu = -1$
\[ e^+ e^- \rightarrow \mu^+ \mu^- \ldots \]

\[
\frac{d\sigma}{dz} = \frac{\pi \alpha^2 Q^2_\mu}{2s} (1 + z^2) \\
- \frac{\alpha Q_\mu G_F M^2_Z (s - M^2_Z)}{8\sqrt{2} [(s - M^2_Z)^2 + M^2_Z \Gamma^2]} \\
\times [(R_e + L_e)(R_\mu + L_\mu)(1 + z^2) + 2(R_e - L_e)(R_\mu - L_\mu)z] \\
+ \frac{G^2_F M^4_Z s}{64\pi [(s - M^2_Z)^2 + M^2_Z \Gamma^2]} \\
\times [(R^2_e + L^2_e)(R^2_\mu + L^2_\mu)(1 + z^2) + 2(R^2_e - L^2_e)(R^2_\mu - L^2_\mu)z]
\]
Measuring $A$ resolves ambiguity

Forward-backward asymmetry $A \equiv \frac{\int_{0}^{1} dz \frac{d\sigma}{dz} - \int_{-1}^{0} dz \frac{d\sigma}{dz}}{\int_{-1}^{1} dz \frac{d\sigma}{dz}}$

$$\lim_{s/M_Z^2 \ll 1} A = \frac{3G_F s}{16\pi\alpha Q_\mu \sqrt{2}} (R_e - L_e)(R_\mu - L_\mu)$$

$$\approx -6.7 \times 10^{-5} \left( \frac{s}{1 \text{ GeV}^2} \right) (R_e - L_e)(R_\mu - L_\mu) = -3G_F s a^2 / 4\pi\alpha \sqrt{2}$$
Neutrino–electron scattering $e^+ e^- \rightarrow \mu^+ \mu^-$

Validate EW theory, measure $\sin^2 \theta_W$
With a measurement of $\sin^2 \theta_W$, predict

$$M_W^2 = \frac{\pi \alpha}{G_F \sqrt{2}} \sin^2 \theta_W \approx \left(37.28 \text{ GeV/c}^2\right)^2 / \sin^2 \theta_W$$

$$M_Z^2 = M_W^2 / \cos^2 \theta_W$$
First $Z$ from UA1
At low energies: \( \sigma(\bar{\nu}_e e \rightarrow \text{hadrons}) > \sigma(\nu_\mu e \rightarrow \mu \nu_e) > \sigma(\nu_e e \rightarrow \nu_e e) > \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) > \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) > \sigma(\nu_\mu e \rightarrow \nu_\mu e) > \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e) \)
Electroweak interactions of quarks

- Left-handed doublet

\[
L_q = \begin{pmatrix} u \\ d \end{pmatrix}_L
\]

\[
\begin{array}{ccc}
l_3 & Q & Y = 2(Q - l_3) \\
\frac{1}{2} & +\frac{2}{3} & \frac{1}{3} \\
-\frac{1}{2} & -\frac{1}{3} & \\
\end{array}
\]

- two right-handed singlets

\[
R_u = u_R \quad 0 \quad +\frac{2}{3} \quad +\frac{4}{3}
\]

\[
R_d = d_R \quad 0 \quad -\frac{1}{3} \quad -\frac{2}{3}
\]
Electroweak interactions of quarks

- CC interaction

\[ \mathcal{L}_{W-q} = \frac{-g}{2\sqrt{2}} \left[ \bar{u}_e \gamma^\mu (1 - \gamma_5) d \, W^+_\mu + \bar{d} \gamma^\mu (1 - \gamma_5) u \, W^-_\mu \right] \]

identical in form to \( \mathcal{L}_{W-\ell} \): universality \( \Leftrightarrow \) weak isospin

- NC interaction

\[ \mathcal{L}_{Z-q} = \frac{-g}{4 \cos \theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu \left[ L_i (1 - \gamma_5) + R_i (1 + \gamma_5) \right] q_i \, Z_\mu \]

\[ L_i = \tau_3 - 2Q_i \sin^2 \theta_W \quad R_i = -2Q_i \sin^2 \theta_W \]

equivalent in form (not numbers) to \( \mathcal{L}_{Z-\ell} \)
Trouble in Paradise

Universal $u \leftrightarrow d, \nu_e \leftrightarrow e$ not quite right

Good: $\begin{pmatrix} u \\ d \end{pmatrix}_L$ → Better: $\begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$

$d_\theta \equiv d \cos \theta_C + s \sin \theta_C \quad \cos \theta_C = 0.9736 \pm 0.0010$

“Cabibbo-rotated” doublet perfects CC interaction (up to small third-generation effects) but $\Rightarrow$ serious trouble for NC

$$L_{Z-q} = \frac{-g}{4 \cos \theta_W} Z_\mu \left\{ \bar{u} \gamma^\mu \left[ L_u (1 - \gamma_5) + R_u (1 + \gamma_5) \right] u \\
+ \bar{d} \gamma^\mu \left[ L_d (1 - \gamma_5) + R_d (1 + \gamma_5) \right] d \cos^2 \theta_C \\
+ \bar{s} \gamma^\mu \left[ L_d (1 - \gamma_5) + R_d (1 + \gamma_5) \right] s \sin^2 \theta_C \\
+ \bar{d} \gamma^\mu \left[ L_d (1 - \gamma_5) + R_d (1 + \gamma_5) \right] s \sin \theta_C \cos \theta_C \\
+ \bar{s} \gamma^\mu \left[ L_d (1 - \gamma_5) + R_d (1 + \gamma_5) \right] d \sin \theta_C \cos \theta_C \right\}$$
Strangeness-changing NC interactions highly suppressed!

BNL E-787/E-949 has three $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ candidates, with $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1.47^{+1.30}_{-0.89} \times 10^{-10}$


(SM: 0.78 ± 0.11: U. Haisch, hep-ph/0605170)
Glashow–Iliopoulos–Maiani

two LH doublets: \( \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L \)

\( s_\theta = s \cos \theta_C - d \sin \theta_C \)

+ right-handed singlets, \( e_R, \mu_R, u_R, d_R, c_R, s_R \)

**Required new charmed quark, \( c \)**

Cross terms vanish in \( \mathcal{L}_{Z-q} \),

\[
\lambda \frac{-ig}{4 \cos \theta_W} \gamma_\lambda [(1 - \gamma_5)L_i + (1 + \gamma_5)R_i] ,
\]

\[
L_i = \tau_3 - 2Q_i \sin^2 \theta_W \quad R_i = -2Q_i \sin^2 \theta_W
\]

flavor-diagonal interaction!
Straightforward generalization to $n$ quark doublets

$$\mathcal{L}_{W-q} = \frac{-g}{2\sqrt{2}} [\bar{\Psi} \gamma^\mu (1 - \gamma_5) \mathcal{O} \Psi \ W_\mu^+ + \text{h.c.}]$$

composite $\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix}$

flavor structure $\mathcal{O} = \begin{pmatrix} 0 & \mathcal{U} \\ 0 & 0 \end{pmatrix}$

$\mathcal{U}$: unitary quark mixing matrix

Weak-isospin part: $\mathcal{L}_{Z-q}^{\text{iso}} = \frac{-g}{4 \cos \theta_W} \bar{\Psi} \gamma^\mu (1 - \gamma_5) \begin{pmatrix} \mathcal{O} & \mathcal{O}^\dagger \end{pmatrix} \Psi$

Since $[\mathcal{O}, \mathcal{O}^\dagger] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

$\implies$ NC interaction is flavor-diagonal

General $n \times n$ mixing matrix $\mathcal{U}$: $n(n-1)/2$ real $\angle$, $(n-1)(n-2)/2$ complex phases

$3 \times 3$ (Cabibbo–Kobayashi–Maskawa): $3 \angle + 1$ phase $\implies$ CP violation
Quark Mixing (Components: $|U_{u\alpha}|^2$, etc.)
Quark & charged-lepton masses

If neutrinos have Dirac masses, $\nu$ Yukawa couplings $\lesssim 10^{-11}$
Fermion mass is accommodated, not explained

- All fermion masses $\sim$ physics beyond the standard model!
- $\zeta_t \approx 1 \quad \zeta_e \approx 3 \times 10^{-6} \quad \zeta_\nu \approx 10^{-11}$

What accounts for the range and values of the Yukawa couplings?

- There may be *other sources* of neutrino mass
Neutrino Mixing (representative $\theta_{12}, \theta_{23}$ values, $\theta_{13} = 10^\circ$)
Absolute scale of neutrino masses is not yet known

\[ m_2 - m_1 = \Delta m^2_\odot = 7.9 \times 10^{-5} \text{ eV}^2 \]
\[ |m_3 - m_1| = \Delta m^2_{\text{atm}} = 2.5 \times 10^{-3} \text{ eV}^2 \]

\[ \rho_c \equiv \frac{3H_0^2}{8\pi G_N} = 1.05h^2 \times 10^4 \text{ eV cm}^{-3} = 5.6 \times 10^3 \text{ eV cm}^{-3} \]

\[(56\nu_i + 56\bar{\nu}_i) \text{ cm}^{-3} \sim \sum_i m_{\nu_i} \lesssim 50 \text{ eV}\]
Successful predictions of $SU(2)_L \otimes U(1)_Y$ theory:

- neutral-current interactions
- necessity of charm
- existence and properties of $W^\pm$ and $Z^0$

+ a decade of precision EW tests (one-per-mille)

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_W$</td>
<td>$80,398 \pm 25$ MeV</td>
</tr>
<tr>
<td>$\Gamma_W$</td>
<td>$2,140 \pm 60$ MeV</td>
</tr>
<tr>
<td>$M_Z$</td>
<td>$91,187.6 \pm 2.1$ MeV</td>
</tr>
<tr>
<td>$\Gamma_Z$</td>
<td>$2,495.2 \pm 2.3$ MeV</td>
</tr>
<tr>
<td>$\sigma^0_{\text{hadronic}}$</td>
<td>$41.541 \pm 0.037$ nb</td>
</tr>
<tr>
<td>$\Gamma_{\text{hadronic}}$</td>
<td>$1,744.4 \pm 2.0$ MeV</td>
</tr>
<tr>
<td>$\Gamma_{\text{leptonic}}$</td>
<td>$83.984 \pm 0.086$ MeV</td>
</tr>
<tr>
<td>$\Gamma_{\text{invisible}}$</td>
<td>$499.0 \pm 1.5$ MeV</td>
</tr>
</tbody>
</table>

\[ \Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}} \]

light $\nu$: $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{SM}(Z \rightarrow \nu_i \bar{\nu}_i) = 2.994 \pm 0.012 \quad (\nu_e, \nu_\mu, \nu_\tau)$
Three light neutrinos

![Graph showing energy vs cross-section for different numbers of neutrinos: N_\nu = 2, N_\nu = 3, N_\nu = 4.](image)

- $N_\nu = 2$ (dashed red line)
- $N_\nu = 3$ (solid blue line)
- $N_\nu = 4$ (dashed green line)

**ALEPH**

\[\sigma \text{ (nb)} \]

**Energy (GeV)**

- 88
- 89
- 90
- 91
- 92
- 93
- 94
- 95

Chris Quigg (Fermilab)  
Electroweak & Higgs  
ISSCSMB · Muğla 2007
### Measurement–best fit differences

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \alpha_{\text{had}}^{(S)}(m_Z)$</td>
<td>$0.02758 \pm 0.00035$</td>
</tr>
<tr>
<td>$m_Z$ [GeV]</td>
<td>$91.1875 \pm 0.0021$</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>$2.4952 \pm 0.0023$</td>
</tr>
<tr>
<td>$\sigma_{\text{had}}^0$ [nb]</td>
<td>$41.540 \pm 0.037$</td>
</tr>
<tr>
<td>$R_l$</td>
<td>$20.767 \pm 0.025$</td>
</tr>
<tr>
<td>$A_{\text{fb}}^{0,l}$</td>
<td>$0.01714 \pm 0.00095$</td>
</tr>
<tr>
<td>$A_l(P_\tau)$</td>
<td>$0.1465 \pm 0.0032$</td>
</tr>
<tr>
<td>$R_b$</td>
<td>$0.21629 \pm 0.00066$</td>
</tr>
<tr>
<td>$R_c$</td>
<td>$0.1721 \pm 0.0030$</td>
</tr>
<tr>
<td>$A_{\text{fb}}^{0,b}$</td>
<td>$0.0992 \pm 0.0016$</td>
</tr>
<tr>
<td>$A_{\text{fb}}^{0,c}$</td>
<td>$0.0707 \pm 0.0035$</td>
</tr>
<tr>
<td>$A_b$</td>
<td>$0.923 \pm 0.020$</td>
</tr>
<tr>
<td>$A_c$</td>
<td>$0.670 \pm 0.027$</td>
</tr>
<tr>
<td>$A_l$(SLD)</td>
<td>$0.1513 \pm 0.0021$</td>
</tr>
<tr>
<td>$\sin^2\theta_{\text{eff}}^{\text{lep}}(Q_{\text{fb}})$</td>
<td>$0.2324 \pm 0.0012$</td>
</tr>
<tr>
<td>$m_W$ [GeV]</td>
<td>$80.398 \pm 0.025$</td>
</tr>
<tr>
<td>$\Gamma_W$ [GeV]</td>
<td>$2.140 \pm 0.060$</td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>$170.9 \pm 1.8$</td>
</tr>
</tbody>
</table>

**LEP Electroweak Working Group, Winter 2007**
Why a Higgs boson must exist

- Role in canceling high-energy divergences

*S*-matrix analysis of $e^+e^- \rightarrow W^+W^-$

Individual $J = 1$ partial-wave amplitudes $\mathcal{M}_{\gamma}^{(1)}$, $\mathcal{M}_{Z}^{(1)}$, $\mathcal{M}_{\nu}^{(1)}$ have unacceptable high-energy behavior ($\propto s$)
... But sum is well-behaved

“Gauge cancellation” observed at LEP2 (Tevatron)
$J = 0$ amplitude exists because electrons have mass, and can be found in “wrong” helicity state

$$\mathcal{M}_\nu^{(0)} \propto s^{\frac{1}{2}} : \text{unacceptable HE behavior}$$

(no contributions from $\gamma$ and $Z$)

This divergence is canceled by the Higgs-boson contribution

⇒ $He\bar{e}$ coupling must be $\propto m_e$,

because “wrong-helicity” amplitudes $\propto m_e$

If the Higgs boson did not exist, something else would have to cure divergent behavior
If gauge symmetry were unbroken . . .

- no Higgs boson
- no longitudinal gauge bosons
- no extreme divergences
- no wrong-helicity amplitudes

. . . and no viable low-energy phenomenology

In spontaneously broken theory . . .

- gauge structure of couplings eliminates the most severe divergences
- lesser—but potentially fatal—divergence arises because the electron has mass . . . due to the Higgs mechanism
- SSB provides its own cure—the Higgs boson

Similar interplay & compensation must exist in any acceptable theory
Bounding $M_H$ from above . . .

Triviality of scalar field theory

- Only *noninteracting* scalar field theories make sense on all energy scales
- Quantum field theory vacuum is a dielectric medium that screens charge
- $\Rightarrow$ *effective charge* is a function of the distance or, equivalently, of the energy scale

**running coupling constant**
In $\lambda\phi^4$ theory, calculate variation of coupling constant $\lambda$ in perturbation theory by summing bubble graphs

\[
\lambda(\mu) \text{ is related to a higher scale } \Lambda \text{ by}
\]

\[
\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log \left( \frac{\Lambda}{\mu} \right)
\]

(Perturbation theory reliable only when $\lambda$ is small, lattice field theory treats strong-coupling regime)
For stable Higgs potential (i.e., for vacuum energy not to race off to $-\infty$), require $\lambda(\Lambda) \geq 0$

Rewrite RGE as an inequality

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log (\Lambda/\mu)$$

... implies an upper bound

$$\lambda(\mu) \leq \frac{2\pi^2}{3 \log (\Lambda/\mu)}$$
If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit \( \Lambda \to \infty \) while holding \( \mu \) fixed at some reasonable physical scale. In this limit, the bound forces \( \lambda(\mu) \) to zero.

\[ \longrightarrow \text{free field theory “trivial”} \]

Rewrite as bound on \( M_H \):

\[ \Lambda \leq \mu \exp \left( \frac{2\pi^2}{3\lambda(\mu)} \right) \]

Choose \( \mu = M_H \), and recall \( M_H^2 = 2\lambda(M_H)v^2 \)

\[ \Lambda \leq M_H \exp \left( \frac{4\pi^2 v^2}{3M_H^2} \right) \]
Higgs interactions vanish

quantum corrections disfavor

excluded by direct searches

electroweak symmetry not hidden

Chris Quigg (Fermilab)
Electroweak & Higgs
ISSCSMB · Muğla 2007 80 / 136
Moral: For any $M_H$, there is a maximum energy scale $\Lambda^*$ at which the theory ceases to make sense.

The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies.

Perturbative analysis breaks down when $M_H \rightarrow 1$ TeV/c$^2$ and interactions become strong.

Lattice analyses $\Rightarrow M_H \lesssim 710 \pm 60$ GeV/c$^2$ if theory describes physics to a few percent up to a few TeV.

If $M_H \rightarrow 1$ TeV EW theory lives on brink of instability.
Lower bound by requiring EWSB vacuum $V(v) < V(0)$

Requiring that $\langle \phi \rangle_0 \neq 0$ be an absolute minimum of the one-loop potential up to a scale $\Lambda$ yields the vacuum-stability condition . . . (for $m_t \lesssim M_W$)

$$M_H^2 > \frac{3 G_F \sqrt{2}}{8 \pi^2} (2 M_W^4 + M_Z^4 - 4 m_t^4) \log(\Lambda^2/v^2)$$

(No illuminating analytic form for heavy $m_t$)

If the Higgs boson is relatively light (which would require explanation) then the theory can be self-consistent up to very high energies
If EW theory is to make sense all the way up to a unification scale $\Lambda^\ast = 10^{16}$ GeV, then $134 \text{ GeV}/c^2 \lesssim M_H \lesssim 177 \text{ GeV}$.
Metastability of the standard-model vacuum?

\[ m_t = 170.9 \pm 1.8 \text{ GeV} \]

Isidori, et al., hep-ph/0104016
Higgs-Boson Properties

\[ \Gamma(H \rightarrow f \bar{f}) = \frac{G_F m_f^2 M_H}{4\pi \sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2} \]

\(\propto M_H\) in the limit of large Higgs mass; \(\propto \beta^3\) for scalar

\[ \Gamma(H \rightarrow W^+ W^-) = \frac{G_F M_H^3}{32\pi \sqrt{2}} (1 - x)^{1/2} (4 - 4x + 3x^2) \quad x \equiv 4M_W^2 / M_H^2 \]

\[ \Gamma(H \rightarrow Z^0 Z^0) = \frac{G_F M_H^3}{64\pi \sqrt{2}} (1 - x')^{1/2} (4 - 4x' + 3x'^2) \quad x' \equiv 4M_Z^2 / M_H^2 \]

asymptotically \(\propto M_H^3\) and \(\frac{1}{2} M_H^3\), respectively

\(2x^2\) and \(2x'^2\) terms \(\Leftrightarrow\) decays into transverse gauge bosons

Dominant decays for large \(M_H\): pairs of longitudinal weak bosons
SM Higgs Boson Branching Fractions

Djouadi, hep-ph/0503172
ILC would measure light Higgs-boson couplings precisely

Points: 500 fb$^{-1}$ @ 350 GeV    Bands: theory uncertainty (m$_b$)

Battaglia +
Dominant decays at high mass

For $M_H \to 1 \text{ TeV}$, Higgs boson is ephemeral: $\Gamma_H \to M_H$. 
Total width of the standard-model Higgs boson

\[ \Gamma(H) \sim 1 \text{ GeV} \]

Below \( W^+ W^- \) threshold, \( \Gamma_H \ll 1 \text{ GeV} \)

Far above \( W^+ W^- \) threshold, \( \Gamma_H \propto M_H^3 \)

Djouadi, hep-ph/0503172
Experimental clues to the Higgs-boson mass

Sensitivity of EW observables to $m_t$ gave early indications for massive top
Quantum corrections to SM predictions for $M_W$ and $M_Z$ arise from
different quark loops

\[ \begin{array}{c}
\bar{b} \quad W^+ \\
\quad \quad t \\
\quad W^+ \\
\end{array} \quad \quad \quad \quad \quad \begin{array}{c}
\bar{t} \quad W^+ \\
\quad \quad t \\
\quad Z^0 \\
\end{array} \quad \quad \quad \quad \quad \begin{array}{c}
\bar{t} \quad Z^0 \\
\end{array} \]

\[ \cdots \text{alter the link} \quad M_W^2 = M_Z^2 \left(1 - \sin^2 \theta_W\right) (1 - \Delta \rho) \]
\[ (80.398 \pm 0.025 \text{ GeV})^2 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad
\end{array} \]

where $\Delta \rho \approx \Delta \rho^{(\text{quarks})} \approx 3G_F m_t^2 / 8\pi^2 \sqrt{2}$

Strong dependence on $m_t^2$ accounts for precision of $m_t$ estimates derived
from EW observables

\[ \text{Tevatron: } \delta m_t / m_t \approx 1.28\% \quad \quad \text{Look beyond quark loops to next} \]
\[ \text{most important quantum corrections: } \text{Higgs-boson effects} \]
Global fits to precision EW measurements

- precision improves with time / calculations improve with time

11.94, LEPEWWG: \( m_t = 178 \pm 11^{+18}_{-19} \) GeV/c^2

Direct measurements: \( m_t = 170.9 \pm 1.8 \) GeV/c^2
$H$ quantum corrections smaller than $t$ corrections, exhibit more subtle dependence on $M_H$ than the $m_t^2$ dependence of the top-quark corrections

$$\Delta \rho^{(\text{Higgs})} = C \cdot \ln \left( \frac{M_H}{v} \right)$$

$M_Z$ known to 23 ppm, $m_t$ and $M_W$ well measured

... so examine dependence of $M_W$ upon $m_t$ and $M_H$
CDF’s top-mass projections . . .

CDF Top Mass Uncertainty
(l+l and l+j channels combined)

△M_t (total) GeV/c^2

△M/M < 1%

CDF Results

Run IIa goal (TDR 1996)

Scale △(stat) / √L, Fix △(syst)
(assumes no improvements)

Scale △(total) / √L
(improvements required)

Integrated Luminosity (pb^-1)

1 fb^-1 2 fb^-1 4 fb^-1 8 fb^-1
Direct, indirect determinations agree reasonably
Both favor a light Higgs boson, ... within framework of SM analysis.
Fit to a universe of data

Standard-Model $M_H \lesssim 192$ GeV at 95% CL
Within SM, LEP EWWG deduce a 95% CL upper limit, $M_H \lesssim 200 \text{ GeV/c}^2$.

Direct searches at LEP $\Rightarrow M_H > 114.4 \text{ GeV/c}^2$, excluding much of the favored region

Either the Higgs boson is just around the corner, or SM analysis is misleading

Things will soon be popping!
A Cautionary Note

- $A_{FB}^b$, which exerts the greatest “pull” on the global fit [slide 71], is most responsible for raising $M_H$ above the range excluded by direct searches [slide 95].

- Leptonic and hadronic observables point to different best-fit values of $M_H$

- Many subtleties in experimental and theoretical analyses


$\chi^2$ Distributions: Leptonic Asymmetries

$m_H (\text{GeV})$

- $A_L$ Combined
- $A_{FB}^l$
- $A_{LR}$
- $P_\tau$
\( \chi^2 \) Distributions: Hadronic Asymmetries

\[ \chi^2 \]

\[ m_H (\text{GeV}) \]

\[ A_H \text{ combined} \]

\[ A_{FB}^b \]

\[ A_{FB}^c \]

\[ Q_{FB} \]
• Tevatron, LHC measurements will determine $m_t$ within 1 or 2 GeV
  . . . and improve $\delta M_W$ to about 15 MeV
• As the Tevatron’s integrated luminosity approaches $10 \text{ fb}^{-1}$, CDF and DØ will explore the region of $M_H$ not excluded by LEP
• ATLAS and CMS will carry on the exploration of the Higgs sector at the LHC;
  could require a few years, at low mass;
  full range accessible, $\gamma\gamma, \ell\ell\nu\nu, b\bar{b}, \ell^+\ell^-\ell^+\ell^-, \ell\nu jj, \tau\tau$ channels.
A few words on Higgs production . . .

\[ e^+ e^- \rightarrow H: \text{hopelessly small} \]
\[ \mu^+ \mu^- \rightarrow H: \text{scaled by } (m_\mu/m_e)^2 \approx 40\,000 \]
\[ e^+ e^- \rightarrow HZ: \text{prime channel} \]

Hadron colliders:
\[ gg \rightarrow H \rightarrow b\bar{b}: \text{background} \,?! \]
\[ gg \rightarrow H \rightarrow \gamma\gamma: \text{rate} \,?! \]
\[ \bar{p}p \rightarrow H(W,Z): \text{prime Tevatron channel} \]

At the LHC:
Many channels accessible, search sensitive up to 1 TeV
Higgs search in $e^+e^-$ collisions

$\sigma(e^+e^- \to H \to \text{all})$ is minute, $\propto m_e^2$

Even narrowness of low-mass $H$ is not enough to make it visible . . . Sets aside a traditional strength of $e^+e^-$ machines—pole physics

Most promising:
associated production $e^+e^- \to HZ$
(has no small couplings)

\[
\sigma = \frac{\pi \alpha^2}{24\sqrt{s}} \frac{K(K^2 + 3M_Z^2)[1 + (1 - 4x_W)^2]}{(s - M_Z^2)^2 x_W^2(1 - x_W)^2}
\]

$K$: c.m. momentum of $H$ \hspace{1cm} $x_W \equiv \sin^2 \theta_W$
\[ \ell^+ \ell^- \rightarrow X \ldots \]

\[
\sigma(e^+e^- \rightarrow H) = \left( \frac{m_e}{m_\mu} \right)^2 \sigma(\mu^+\mu^- \rightarrow H) \approx \frac{\sigma(\mu^+\mu^- \rightarrow H)}{40\ 000} 
\]

(Much more from Klaus Mönig)

Chris Quigg  (Fermilab)

Electroweak & Higgs

ISSCSMB · Muğla 2007  103 / 136
LEP 2: sensitive nearly to kinematical limit

$M_H^{max} = \sqrt{s} - M_Z$

LC: sensitive for $M_H \ll 0.7\sqrt{s}$

& measure excitation curve to determine

$\delta M_H \approx 60 \text{ MeV} \sqrt{100 \text{ fb}^{-1}}/\mathcal{L}$ for $M_H = 100 \text{ GeV}$

+ important effect of ISR
$H$ couples to gluons through quark loops

Only heavy quarks matter:

\[ \epsilon = \frac{4m_Q^2}{M_H^2} \]
Higgs-boson production at the Tevatron

\[ \sigma(p\bar{p} \rightarrow H + X) \text{ [pb]} \]

\( \sqrt{s} = 1.96 \text{ TeV} \)

MRST/NLO

\( m_t = 178 \text{ GeV} \)

- \( gg \rightarrow H \)
- \( q\bar{q} \rightarrow WH \)
- \( q\bar{q} \rightarrow ZH \)
- \( q\bar{q} \rightarrow q\bar{q}H \)
- \( pp \rightarrow t\bar{t}H \)

\( M_H \text{ [GeV]} \)

Djouadi, hep-ph/0503172
Current Tevatron Sensitivity

Tevatron Run II Preliminary

95% C.L. limit $\sigma$(Higgs) / SM

- D0 Expected
- CDF Expected
- Tevatron Expected
- Tevatron Observed

Excluded by LEP

L = 0.9 - 1.9 fb$^{-1}$

$\sigma$ vs $m_H$(GeV/c$^2$)

combining experiments, channels

Lepton/Photon 2007
Higgs-boson production at the LHC

\[ \sigma(pp \to H + X) \ [\text{pb}] \]
\[ \sqrt{s} = 14 \text{ TeV} \]
MRST/NLO
\[ m_t = 178 \text{ GeV} \]

\[ gg \to H \]
\[ qq \to Hq\bar{q} \]
\[ q\bar{q} \to WH \]
\[ q\bar{q} \to ZH \]
\[ pp \to t\bar{t}H \]

(Much more from Nural Akchurin)

Djouadi, hep-ph/0503172

Chris Quigg (Fermilab)
The agent of electroweak symmetry breaking represents a novel fundamental interaction at an energy of a few hundred GeV.

We do not know the nature of the new force.

Inspired by the Meissner effect, we describe the EWSB interaction as an analogue of the Ginzburg–Landau picture of superconductivity.

light Higgs boson $\Leftrightarrow$ perturbative dynamics
heavy Higgs boson $\Leftrightarrow$ strong dynamics
What is the nature of the mysterious new force that hides electroweak symmetry?

- A fundamental force of a new character, based on interactions of an elementary scalar
- A new gauge force, perhaps acting on undiscovered constituents
- A residual force that emerges from strong dynamics among the weak gauge bosons
- An echo of extra spacetime dimensions

We have explored examples of all four, theoretically.

Which path has Nature taken?
Essential step toward understanding the new force that shapes our world:

Find the Higgs boson and explore its properties.

- Is it there? How many?
- Verify $J^{PC} = 0^{++}$
- Does $H$ generate mass for gauge bosons, fermions?
- How does $H$ interact with itself?

Finding the Higgs boson starts a new adventure!
10 years precise measurements: no significant deviations

Quantum corrections tested at $\pm 10^{-3}$

No “new physics” . . . yet!

Theory tested at distances from $10^{-17}$ cm to $\sim 10^{22}$ cm

origin Coulomb’s law (tabletop experiments)

smaller \[
\begin{cases}
\text{Atomic physics} \rightarrow \text{QED} \\
\text{high-energy expts.} \rightarrow \text{EW theory}
\end{cases}
\]

larger $M_\gamma \approx 0$ in planetary . . . measurements

Is EW theory true? Is it complete ??
But what about gravity?

Natural to neglect gravity in particle physics . . .

\[ G_{\text{Newton}} \text{ small} \iff M_{\text{Planck}} = \left( \frac{\hbar c}{G_{\text{Newton}}} \right)^{\frac{1}{2}} \approx 1.22 \times 10^{19} \text{ GeV} \text{ large} \]

Estimate \( B(K \rightarrow \pi G) \sim \left( \frac{M_K}{M_{\text{Planck}}} \right)^2 \approx 10^{-38} \)

300 years after Newton: Why is gravity weak?
But gravity is not always negligible . . .

The vacuum energy problem

Higgs potential \( V(\varphi^\dagger \varphi) = \mu^2 (\varphi^\dagger \varphi) + |\lambda| (\varphi^\dagger \varphi)^2 \)

At the minimum,

\[
V(\langle \varphi^\dagger \varphi \rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda| v^4}{4} < 0.
\]

Identify \( M_H^2 = -2\mu^2 \)

\( V \neq 0 \) contributes position-independent vacuum energy density

\[
\varrho_H \equiv \frac{M_H^2 v^2}{8} \geq 10^8 \text{ GeV}^4 \approx 10^{24} \text{ g cm}^{-3}
\]

Adding vacuum energy density \( \varrho_{\text{vac}} \) \( \Leftrightarrow \) adding cosmological constant \( \Lambda \) to Einstein’s equation

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \quad \Lambda = \frac{8\pi G_N}{c^4} \varrho_{\text{vac}}
\]
Observed vacuum energy density $\rho_{\text{vac}} \approx 10^{-46}$ GeV$^4$

A chronic dull headache for thirty years . . .

$\rho_H \gtrsim 10^8$ GeV$^4$: mismatch by $10^{54}$
EWSB: another path?

Modeled EWSB on Ginzburg–Landau description of superconducting phase transition; 
... had to introduce new, elementary scalars

GL is not the last word on superconductivity: 
*dynamical* Bardeen–Cooper–Schrieffer theory

The elementary fermions – *electrons* – and gauge interactions – *QED* – needed to generate the scalar bound states are already present in the case of superconductivity.

Could a scheme of similar economy account for EWSB?
SU(3)_c \otimes SU(2)_L \otimes U(1)_Y + \text{massless } u \text{ and } d

(treat SU(2)_L \otimes U(1)_Y \text{ as perturbation})

m_u = m_d = 0:
QCD has exact SU(2)_L \otimes SU(2)_R \text{ chiral symmetry.}

At an energy scale \sim \Lambda_{QCD}, \text{ strong interactions become}
strong, fermion condensates appear, and

\[
SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V
\]

\sim 3 \text{ Goldstone bosons, one for each broken generator:}
3 \text{ massless pions (Nambu)}
Broken generators: 3 axial currents; couplings to $\pi$ measured by pion decay constant $f_\pi$.

Turn on $SU(2)_L \otimes U(1)_Y$: EW gauge bosons couple to axial currents, acquire masses of order $\sim gf_\pi$.

$$M^2 = \left(\begin{array}{cccc} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & gg' \\ 0 & 0 & gg' & g'^2 \end{array} \right) \frac{f_\pi^2}{4} \left(W^+, W^-, W_3, A\right)$$

same structure as standard EW theory.

Diagonalize: $M^2_W = g^2 f_\pi^2/4$, $M^2_Z = (g^2 + g'^2) f_\pi^2/4$, $M^2_A = 0$, so

$$\frac{M^2_Z}{M^2_W} = \frac{(g^2 + g'^2)}{g^2} = \frac{1}{\cos^2 \theta_W}$$

Massless pions disappear from physical spectrum, to become longitudinal components of weak bosons. $M_W \approx 30 \text{ MeV}/c^2$  

No fermion masses ...
Parameters of the Standard Model

3 coupling parameters $\alpha_s$, $\alpha_{EM}$, $\sin^2 \theta_W$

2 parameters of the Higgs potential

1 vacuum phase (QCD)

6 quark masses

3 quark mixing angles

1 CP-violating phase

3 charged-lepton masses

3 neutrino masses

3 leptonic mixing angles

1 leptonic CP-violating phase (+ Majorana . . .)

26$^+$ arbitrary parameters

parameter count not improved by unification
The EW scale and beyond

EWSB scale, \( v = (G_F \sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV} \), sets

\[
M_W^2 = \frac{g^2 v^2}{2} \quad M_Z^2 = \frac{M_W^2}{\cos^2 \theta_W}
\]

But it is not the only scale of physical interest

quasi-certain: \( M_{\text{Planck}} = 1.22 \times 10^{19} \text{ GeV} \)

probable: \( SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \) unification scale \( \sim 10^{15-16} \text{ GeV} \)

somewhere: flavor scale

How to keep the distant scales from mixing in the face of quantum corrections?

\textit{OR}

How to stabilize the mass of the Higgs boson on the electroweak scale?

\textit{OR}

Why is the electroweak scale small?

"The hierarchy problem"
Higgs potential

\[ V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2 \]

\[ \mu^2 < 0: \text{SU}(2)_L \otimes \text{U}(1)_{Y} \rightarrow \text{U}(1)_{\text{em}}, \text{ as} \]

\[ \langle \phi \rangle_0 = \left( \begin{array}{c} 0 \\ \sqrt{-\mu^2/2|\lambda|} \end{array} \right) \equiv \left( \begin{array}{c} 0 \\ (G_F \sqrt{8})^{-1/2} \end{array} \right) \]

*Beyond classical approximation,* quantum corrections to scalar mass parameters:

\[ m^2(p^2) = m_0^2 + \left( \begin{array}{c} \text{J=1} \\ \text{J=1/2} \\ \text{J=0} \end{array} \right) \]
Loop integrals are potentially divergent

\[ m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \cdots \]

\(\Lambda\): reference scale at which \(m^2\) is known
\(g\): coupling constant of the theory
\(C\): coefficient calculable in specific theory

For mass shifts induced by radiative corrections to remain under control (not greatly exceed the value measured on the laboratory scale), either

- \(\Lambda\) must be small, or
- New Physics must intervene to cut off integral
But natural reference scale for $\Lambda$ is

$$\Lambda \approx M_{\text{Planck}} = \left( \frac{\hbar c}{G_{\text{Newton}}} \right)^{1/2} \approx 1.22 \times 10^{19} \text{ GeV}$$

for $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

or

$$\Lambda \approx M_U \approx 10^{15}-10^{16} \text{ GeV} \quad \text{for unified theory}$$

Both $\gg v/\sqrt{2} \approx 175 \text{ GeV} \quad \implies \text{New Physics at } E \sim 1 \text{ TeV}$
\[ \delta M_H^2 \text{ for 5-TeV cutoff} \]

\[ \delta M_H^2 = \frac{G_F \Lambda^2}{4\pi^2\sqrt{2}} \left(6M_W^2 + 3M_Z^2 + M_H^2 - 12m_t^2\right) \]
Only a few distinct scenarios . . .

- **Supersymmetry**: balance contributions of fermion loops \((-1)\) and boson loops \((+1)\)

    **Exact supersymmetry**,

    \[
    \sum_{i=\text{fermions}}^\text{bosons} C_i \int dk^2 = 0
    \]

    **Broken supersymmetry**, shifts acceptably small if superpartner mass splittings are not too large

    \[g^2 \Delta M^2 \text{ “small enough” } \Rightarrow \tilde{M} \lesssim 1 \text{ TeV/c}^2\]
Coupling constant unification through supersymmetry?

![Graph showing coupling constant unification through supersymmetry.](image)
Coupling constant unification by many Higgs doublets?
Only a few distinct scenarios . . .

- Composite scalars (technicolor): New physics arises on scale of composite Higgs-boson binding,

\[ \Lambda_{TC} \approx O(1 \text{ TeV}) \]

“Form factor” cuts effective range of integration

- Strongly interacting gauge sector: \( WW \) resonances, multiple \( W \) production, probably scalar bound state “quasiHiggs” with \( M < 1 \text{ TeV} \)

- Extra spacetime dimensions: pseudo-Nambu – Goldstone bosons, extra particles cancel integrand . . .

- Or maybe the problem is with (our understanding of) \textit{gravity}, not with the electroweak theory?
Gravity follows $1/r^2$ law down to $\lesssim 1$ mm (few meV)

$$V(r) = -\int dr_1 \int dr_2 \frac{G_N \rho(r_1) \rho(r_2)}{r_{12}} \left[ 1 + \varepsilon_G \exp\left(-\frac{r_{12}}{\lambda_G}\right) \right]$$

Experiment leaves us free to consider modifications to Gravity even at (nearly) macroscopic distances.
Suppose at scale $R$ Gravity propagates in $3 + n$ spatial dimensions

**Force law changes:** $F \propto 1/r^{2+n}$

\[
G_N \sim M_{Pl}^{-2} \sim M^*^{-n-2} R^{-n} \quad M^*: \text{gravity’s true scale}
\]

Example: $M^* = 1 \text{ TeV} \quad \Rightarrow R \lesssim 10^{-3} \text{ m for } n = 2$

\[
M_P \text{ is a mirage (false extrapolation)!}
\]
Challenge: Understanding the Everyday (bis)

*What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?*

*Consider the effects of all the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ interactions!*
With no Higgs mechanism . . .

- Quarks and leptons would remain massless
- QCD would confine the quarks in color-singlet hadrons
- \( N \) mass little changed, but \( p \) outweighs \( n \)
- QCD breaks EW to EM, gives \((1/2500 \times \text{observed})\) masses to \( W, Z \), so weak-isospin force doesn’t confine
- Rapid! \( \beta \)-decay \(\Rightarrow\) lightest nucleus is \( n \); no H atom
- Some light elements in BBN (?), but \( \infty \) Bohr radius
- No atoms (as we know them) means no chemistry, no stable composite structures like solids and liquids

. . . the character of the physical world would be profoundly changed
Why the LHC is so exciting (II)

- Electroweak theory (unitarity) tells us the 1-TeV scale is special: Higgs boson or other new physics (strongly interacting gauge bosons)
- Hierarchy problem $\Rightarrow$ other new physics nearby
- Our ignorance of EWSB obscures our view of other questions (e.g., identity problem). Lifting the veil at 1 TeV will change the face of physics
In a decade or two, we can hope to . . .

Understand electroweak symmetry breaking
*Observe the Higgs boson*
Measure neutrino masses and mixings
*Establish Majorana neutrinos ($\beta\beta_0\nu$)*
Thoroughly study CP violation in $B$ decay
*Exploit rare decays ($K, D, \ldots$)*
Observe $n$ EDM, pursue $e^-\text{EDM}$
*Use top as a tool*
Observe new phases of matter
*Understand hadron structure quantitatively*
Uncover QCD’s full implications
*Observe proton decay*
Understand the baryon excess
*Catalogue matter & energy of universe*
Measure dark energy equation of state
*Search for new macroscopic forces*
Determine GUT symmetry

Detect neutrinos from the universe
Learn how to quantize gravity
*Learn why empty space is nearly weightless*
Test the inflation hypothesis
*Understand discrete symmetry violation*
Resolve the hierarchy problem
*Discover new gauge forces*
Directly detect dark-matter particles
*Explore extra spatial dimensions*
Understand origin of large-scale structure
*Observe gravitational radiation*
Solve the strong CP problem
*Learn whether supersymmetry is TeV-scale*
Seek TeV dynamical symmetry breaking
*Search for new strong dynamics*
Explain the highest-energy cosmic rays
*Formulate problem of identity*

. . . learn the right questions to ask . . .

. . . and rewrite the textbooks!
General References

My Perspectives on Electroweak Theory . . .


