The Electroweak Theory: Issues for the LHC Era

Chris Quigg
Fermilab
quigg@fnal.gov

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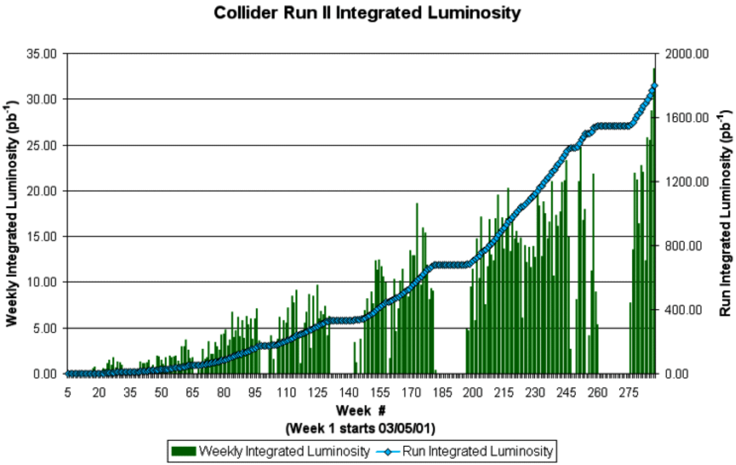
A Decade of Discovery Past . . .

- EW theory $\rightarrow$ law of nature [Z, e^+e^-, $\bar{p}p$, νN, $(g-2)_\mu, \ldots$]
- Higgs-boson influence in the vacuum [EW experiments]
- ν oscillations: $\nu_\mu \rightarrow \nu_\tau$, $\nu_e \rightarrow \nu_\mu/\nu_\tau$ [$\nu_\odot$, ν_{atm}, reactors]
- Understanding QCD [heavy flavor, Z^0, $\bar{p}p$, νN, ep, ions, lattice]
- Discovery of top quark [$\bar{p}p$]
- Direct $\mathcal{CP}$ violation in $K \rightarrow \pi\pi$ [fixed-target]
- B-meson decays violate $\mathcal{CP}$ [$e^+e^- \rightarrow B\bar{B}$]
- Flat universe: dark matter, energy [SN Ia, CMB, LSS]
- Detection of ν_τ interactions [fixed-target]
- Quarks, leptons structureless at 1 TeV scale [mostly colliders]

Tevatron Collider is breaking new ground in sensitivity


Collider Run II Integrated Luminosity
Weekly Integrated Luminosity (pb-1)
Run Integrated Luminosity (pb-1)
35.00
30.00
25.00
20.00
15.00
10.00
5.00
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2000.00
1600.00
1200.00
800.00
400.00
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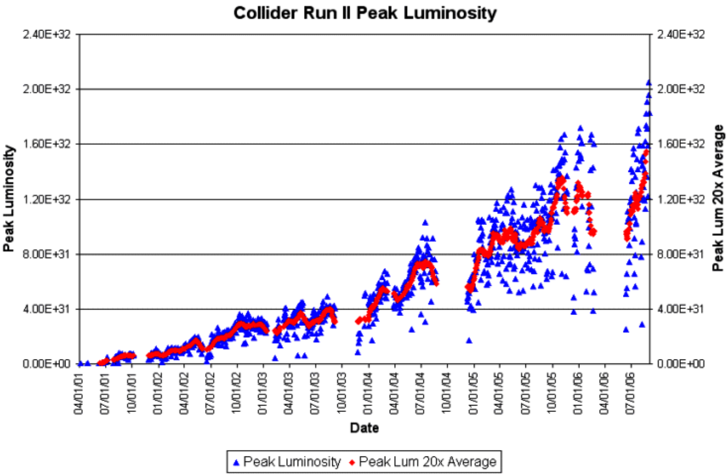
5 20 35 50 65 80 95 110 125 140 155 170 185 200 215 230 245 260 275
Week #
(Week 1 starts 03/05/01)
Weekly Integrated Luminosity
Run Integrated Luminosity


Collider Run II Peak Luminosity
Peak Luminosity
Peak Lum 20x Average
Date
2.40E+32
2.00E+32
1.60E+32
1.20E+32
8.00E+31
4.00E+31
0.00E+00
04/01/01
07/01/01
10/01/01
01/01/02
04/01/02
07/01/02
10/01/02
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▲ Peak Luminosity ◆ Peak Lum 20x Average

Tevatron Collider in a Nutshell

980-GeV protons, antiprotons (2π km)

frequency of revolution $\approx 45\,000\ \mathrm{s}^{-1}$

392 ns between crossings

($36 \otimes 36$ bunches)

collision rate $= \mathcal{L} \cdot \sigma_{\text{inelastic}} \approx 10^7\ \mathrm{s}^{-1}$

$c \approx 10^9$ km/h; $\quad v_p \approx c - 495$ km/h

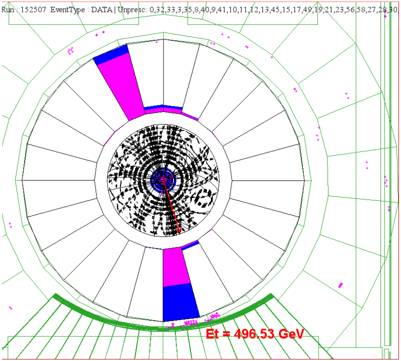
Record $\mathcal{L}_{\text{init}} = 2.28 \times 10^{32}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$

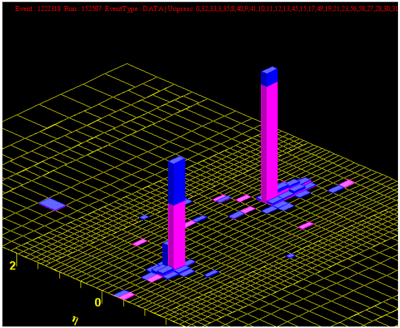
[CERN ISR: pp, $1.4 \times 10^{32}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$]

Goal: $\approx 8\ \mathrm{fb}^{-1}$ by 10.2009

The World's Most Powerful Microscopes

nanonanophysics

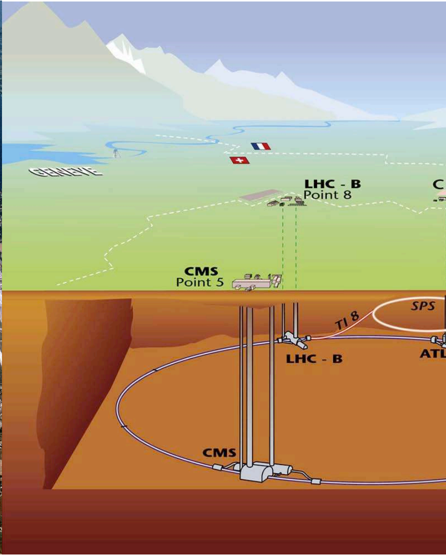




CDF dijet event ($\sqrt{s} = 1.96$ TeV): $E_T = 1.364$ TeV

$$\boxed{q\bar{q} \longrightarrow \text{jet} + \text{jet}}$$

LHC will operate *soon*, breaking new ground in E & $\mathcal{L}$



LHC in a nutshell

7-TeV protons on protons (27 km); $v_p \approx c - 10$ km/h

Novel two-in-one dipoles (≈ 9 teslas)

Collisions! ($E_{cm} = 900$ GeV, $\mathcal{L} \approx 10^{29}$ cm^{-2} s^{-1}): 11.07

Commissioning run until end 2007, then shutdown

First collisions at $E_{cm} = 14$ TeV: Spring 2008

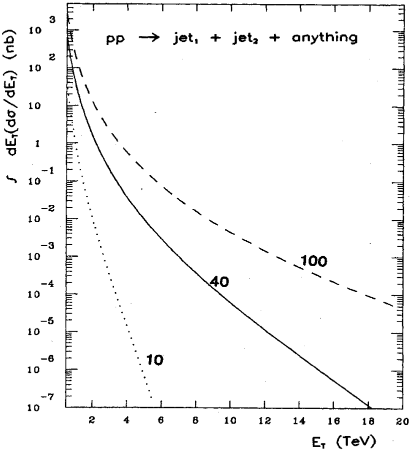
First physics run! Goal of few fb^{-1} by end 2008

Eventual: $\mathcal{L} \gtrsim 10^{34}$ cm^{-2} s^{-1}: 100 fb^{-1}/year

Why the LHC is so exciting (I)

- Even low luminosity opens vast new realm:
  10 pb^{-1} *(few days at initial $\mathcal{L}$)* yields
  8000 top quarks, 10^5 W-bosons,
  100 QCD dijets beyond Tevatron kinematic limit
  Supersymmetry hints in a few weeks ?
- Essential first step: rediscover the standard model
- The antithesis of a one-experiment machine;
  enormous scope and versatility beyond high-$p_\perp$
- $\mathcal{L}$ upgrade extends $\gtrsim$10-year program . . .

You will need a trigger ...



Dijet integral cross section, $|\eta| \leq 2.5$ ...

$10\ \text{pb}^{-1}$ @ LHC $\rightsquigarrow \gtrsim 10^4$ events with $E_T \gtrsim 1.364$ TeV

The importance of the 1-TeV scale

EW theory does not predict Higgs-boson mass

▷ Conditional *upper bound* from Unitarity

Compute amplitudes $\mathcal{M}$ for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta)$$

Most channels decouple – pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies) – $\forall M_H$.

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0/\sqrt{2} \quad HH/\sqrt{2} \quad HZ_L^0$$

L: longitudinal, $1/\sqrt{2}$ for identical particles

In HE limit,[1] s-wave amplitudes $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2}(a_0) \rightarrow \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect pw unitarity condition $|a_0| \leq 1$

$$\Longrightarrow \quad M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F}\right)^{1/2} = 1\ \mathrm{TeV}/c^2$$

condition for perturbative unitarity

[1] Convenient to calculate using *Goldstone-boson equivalence theorem*, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by $\mathcal{L}_{\mathrm{int}} = -\lambda v h(2w^+w^- + z^2 + h^2) - (\lambda/4)(2w^+w^- + z^2 + h^2)^2$, with $1/v^2 = G_F\sqrt{2}$ and $\lambda = G_F M_H^2/\sqrt{2}$.

- If the bound is respected
  - weak interactions remain weak at all energies
  - perturbation theory is everywhere reliable

- If the bound is violated
  - perturbation theory breaks down
  - weak interactions among $W^{\pm}$, Z, H become strong on 1-TeV scale

$\Rightarrow$ features of *strong* interactions at GeV energies will characterize *electroweak* gauge boson interactions at TeV energies

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

Threshold behavior of the pw amplitudes a_{IJ} follows from chiral symmetry

$$\begin{array}{lcl} a_{00} \approx & G_F s/8\pi\sqrt{2} & \text{attractive} \\ a_{11} \approx & G_F s/48\pi\sqrt{2} & \text{attractive} \\ a_{20} \approx & -G_F s/16\pi\sqrt{2} & \text{repulsive} \end{array}$$

Lee, Quigg, Thacker, *Phys. Rev. D* **16**, 1519 (1977)

What the LHC *is not really* for . . .

- Find the Higgs boson,
  the Holy Grail of particle physics,
  the source of all mass in the Universe.
- Celebrate.
- Then particle physics will be over.

We are not ticking off items on a shopping list . . .

We are exploring a vast new terrain
. . . and reaching the Fermi scale

The Origins of Mass

(masses of nuclei "understood")

$p, [\pi], \rho$ understood: QCD

confinement energy is the source

"Mass without mass" Wilczek, *Phys. Today* (November 1999)

We understand the visible mass of the Universe
. . . without the Higgs mechanism

W, Z electroweak symmetry breaking

$$M_W^2 = \tfrac{1}{2}g^2v^2 = \pi\alpha/G_F\sqrt{2}\sin^2\theta_W$$

$$M_Z^2 = M_W^2/\cos^2\theta_W$$

$q, \ell^{\mp}$ EWSB + Yukawa couplings

ν_ℓ EWSB + Yukawa couplings; new physics?

All fermion masses $\Leftrightarrow$ physics beyond standard model

H ?? fifth force ??

Challenge: Understanding the Everyday

- Why are there atoms?
- Why chemistry?
- Why stable structures?
- What makes life possible?

What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?

Searching for the mechanism of electroweak symmetry breaking, we seek to understand

why the world is the way it is.

This is one of the deepest questions humans have ever pursued, and

it is coming within the reach of particle physics.

Our picture of matter

Pointlike constituents ($r < 10^{-18}$ m)

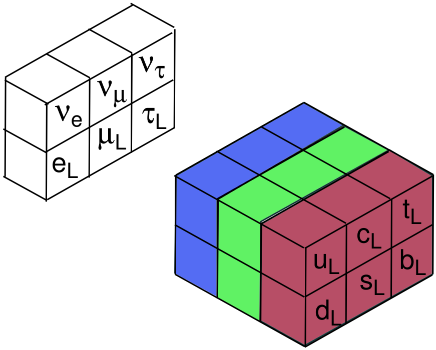
$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

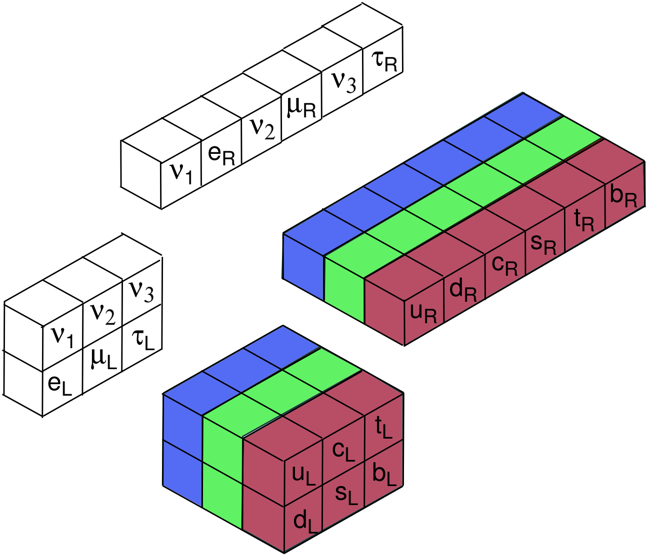
Few fundamental forces, derived from gauge symmetries

$$\mathrm{SU(3)_c} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$$

Electroweak symmetry breaking: Higgs mechanism?



ν_e
ν_μ
ν_τ
e_L
μ_L
τ_L
t_L
c_L
u_L
b_L
s_L
d_L



ν1
eR
ν2
μR
ν3
τR
ν1
ν2
ν3
eL
μL
τL
uR
dR
cR
sR
tR
bR
uL
dL
cL
sL
tL
bL

Formulate electroweak theory

Three crucial clues from experiment:

- Left-handed weak-isospin doublets,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad \begin{pmatrix} t \\ b' \end{pmatrix}_L ;$$

- Universal strength of the (charged-current) weak interactions;
- Idealization that neutrinos are massless.

$\boxed{\text{First two clues suggest } SU(2)_L \text{ gauge symmetry}}$

A theory of leptons

$$\mathsf{L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \mathsf{R} \equiv e_R$$

weak hypercharges $Y_L = -1,\ Y_R = -2$

Gell-Mann–Nishijima connection, $Q = I_3 + \frac{1}{2}Y$

$\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$ gauge group $\Rightarrow$ gauge fields:

- weak isovector $\vec{b}_\mu$, coupling g $\quad \boxed{b^\ell_\mu = b^\ell_\mu - \varepsilon_{jk\ell}\alpha^j b^k_\mu - (1/g)\partial_\mu\alpha^\ell}$
- weak isoscalar $\mathcal{A}_\mu$, coupling $g'/2$ $\quad \boxed{\mathcal{A}_\mu \to \mathcal{A}_\mu - \partial_\mu\alpha}$

Field-strength tensors

$$F^\ell_{\mu\nu} = \partial_\nu b^\ell_\mu - \partial_\mu b^\ell_\nu + g\varepsilon_{jk\ell} b^j_\mu b^k_\nu \ , SU(2)_L$$

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu \ , U(1)_Y$$

Interaction Lagrangian

$$\boxed{\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}}}$$

$$\mathcal{L}_{\text{gauge}} = -\tfrac{1}{4}F^{\ell}_{\mu\nu}F^{\ell\mu\nu} - \tfrac{1}{4}f_{\mu\nu}f^{\mu\nu},$$

$$\begin{aligned}\mathcal{L}_{\text{leptons}} &= \overline{\mathsf{R}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y\right)\mathsf{R} \\ &+ \overline{\mathsf{L}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_{\mu}\right)\mathsf{L}.\end{aligned}$$

Mass term $\mathcal{L}_e = -m_e(\bar{e}_{\mathsf{R}}e_{\mathsf{L}} + \bar{e}_{\mathsf{L}}e_{\mathsf{R}}) = -m_e\bar{e}e$ violates local gauge inv.

Theory: 4 massless gauge bosons ($\mathcal{A}_{\mu}$ $\quad b^1_{\mu}$ $\quad b^2_{\mu}$ $\quad b^3_{\mu}$); Nature: 1 (γ)

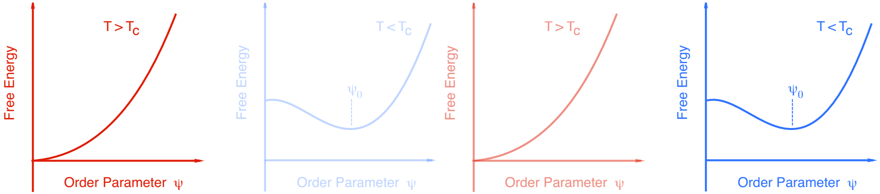
Massive Photon? *Hiding Symmetry*

Recall $\boxed{2}$ miracles of superconductivity:

- No resistance . . . . . . Meissner effect (exclusion of **B**)

Ginzburg–Landau Phenomenology (not a theory from first principles)

normal, resistive charge carriers . . . . . . + superconducting charge carriers



$$\mathbf{B} = 0: \qquad G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha\,|\psi|^2 + \beta\,|\psi|^4$$

$$T > T_c: \quad \alpha > 0 \quad \langle|\psi|^2\rangle_0 = 0$$

$$T < T_c: \quad \alpha < 0 \quad \langle|\psi|^2\rangle_0 \neq 0$$

In a nonzero magnetic field . . .

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{8\pi} + \frac{1}{2m^*}\left|-i\hbar\nabla\psi - \frac{e^*}{c}\mathbf{A}\psi\right|^2$$

$$\left.\begin{array}{c} e^* = -2 \\ m^* \end{array}\right\} \text{of superconducting carriers}$$

Weak, slowly varying field: $\psi \approx \psi_0 \neq 0,\ \nabla\psi \approx 0$

Variational analysis $\rightsquigarrow$

$$\boxed{\nabla^2\mathbf{A} - \frac{4\pi e^{*2}}{m^* c^2}\left|\psi_0\right|^2\mathbf{A} = 0}$$

wave equation of a *massive photon*

Photon – *gauge boson* – acquires mass
within superconductor

origin of Meissner effect

Meissner effect levitates Leon Lederman (Snowmass 2001)

Hiding EW Symmetry

Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition

- Introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1$$

- Add to $\mathcal{L}$ (gauge-invariant) terms for interaction and propagation of the scalars,

$$\mathcal{L}_{\mathrm{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi),$$

  where $\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau} \cdot \vec{b}_\mu$ and

$$\boxed{V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| \, (\phi^\dagger \phi)^2}$$

- Add a Yukawa interaction $\mathcal{L}_{\mathrm{Yukawa}} = -\zeta_e \left[\overline{\mathsf{R}}(\phi^\dagger \mathsf{L}) + (\overline{\mathsf{L}}\phi)\mathsf{R} \right]$

- Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^2 < 0$
  Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks) $SU(2)_L$ and $U(1)_Y$

but preserves $U(1)_{\mathrm{em}}$ invariance

Invariance under $\mathcal{G}$ means $e^{i\alpha\mathcal{G}}\langle\phi\rangle_0 = \langle\phi\rangle_0$, so $\mathcal{G}\langle\phi\rangle_0 = 0$

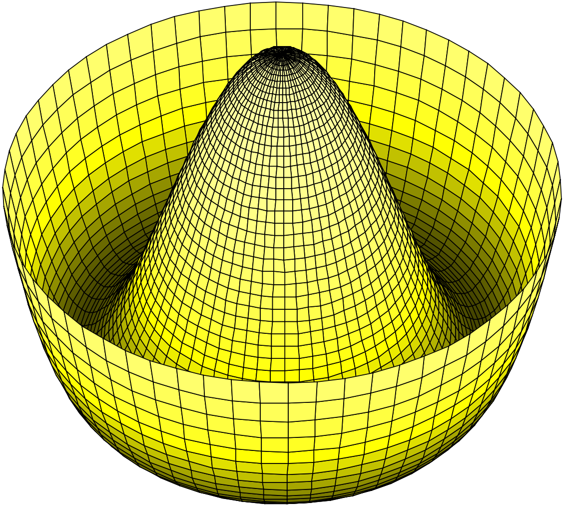
$$\tau_1\langle\phi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

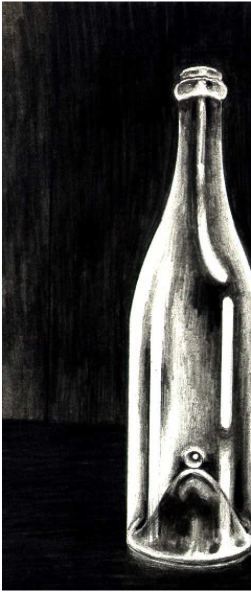
$$\tau_2\langle\phi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_3\langle\phi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$Y\langle\phi\rangle_0 = Y_\phi\langle\phi\rangle_0 = +1\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

Symmetry of laws $\not\Rightarrow$ symmetry of outcomes





Examine electric charge operator Q on the (neutral) vacuum

$$\begin{aligned} Q\langle\phi\rangle_0 &= \tfrac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 \\ &= \tfrac{1}{2}\begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix}\langle\phi\rangle_0 \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \textit{unbroken!} \end{aligned}$$

Four original generators are broken, *electric charge is not*

- $\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y \to U(1)_{\mathrm{em}}$ (will verify)
- Expect massless photon
- Expect gauge bosons corresponding to

$$\tau_1,\ \tau_2,\ \tfrac{1}{2}(\tau_3 - Y) \equiv K \quad \text{to acquire masses}$$

Expand about the vacuum state

Let $\phi = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}$; in *unitary gauge*

$$\begin{aligned} \mathcal{L}_{\text{scalar}} &= \tfrac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2 \eta^2 \\ &+ \frac{v^2}{8}[g^2 \left| b_\mu^1 - i b_\mu^2 \right|^2 + (g' \mathcal{A}_\mu - g b_\mu^3)^2] \\ &+ \text{interaction terms} \end{aligned}$$

"Higgs boson" η has acquired $(\text{mass})^2$ $M_H^2 = -2\mu^2 > 0$

$$\text{Define } W_\mu^\pm = \frac{b_\mu^1 \mp i b_\mu^2}{\sqrt{2}}$$

$$\frac{g^2 v^2}{8}(\left|W_\mu^+\right|^2 + \left|W_\mu^-\right|^2) \Longleftrightarrow M_{W^\pm} = gv/2$$

$(v^2/8)(g'\mathcal{A}_\mu - g b_\mu^3)^2 \ldots$

Now define orthogonal combinations

$$Z_\mu = \frac{-g'\mathcal{A}_\mu + g b_\mu^3}{\sqrt{g^2 + g'^2}} \qquad A_\mu = \frac{g\mathcal{A}_\mu + g' b_\mu^3}{\sqrt{g^2 + g'^2}}$$

$$M_{Z^0} = \sqrt{g^2 + g'^2}\, v/2 = M_W \sqrt{1 + g'^2/g^2}$$

$\boxed{A_\mu \text{ remains massless}}$

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= -\zeta_e \frac{(v+\eta)}{\sqrt{2}}(\bar{e}_{\mathsf{R}} e_{\mathsf{L}} + \bar{e}_{\mathsf{L}} e_{\mathsf{R}}) \\ &= -\frac{\zeta_e v}{\sqrt{2}}\bar{e}e - \frac{\zeta_e \eta}{\sqrt{2}}\bar{e}e\end{aligned}$$

electron acquires $m_e = \zeta_e v/\sqrt{2}$

Higgs-boson coupling to electrons: m_e/v ($\propto$ mass)

Desired particle content . . . plus a Higgs scalar

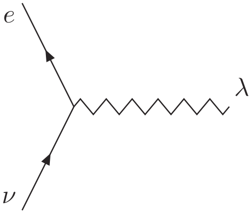
Values of couplings, electroweak scale v?

What about interactions?

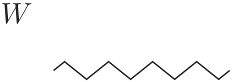
Interactions . . .

$$\mathcal{L}_{W\text{-}\ell} = -\frac{g}{2\sqrt{2}}[\bar{\nu}\gamma^{\mu}(1-\gamma_5)eW_{\mu}^{+} + \bar{e}\gamma^{\mu}(1-\gamma_5)\nu W_{\mu}^{-}]$$

$+$ similar terms for μ and τ



$$\frac{-ig}{2\sqrt{2}}\gamma_\lambda(1-\gamma_5)$$



$$= \frac{-i(g_{\mu\nu} - k_\mu k_\nu / M_W^2)}{k^2 - M_W^2} \, .$$

Compute $\nu_\mu e \to \mu\nu_e$

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu / M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\frac{g^4}{16 M_W^4} = 2G_F^2 \Rightarrow \frac{g}{2\sqrt{2}} = \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}}$$

Using $M_W = gv/2$, determine the electroweak scale

$$\boxed{v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV}}$$

$$\Rightarrow \langle\phi^0\rangle_0 = (G_F\sqrt{8})^{-\frac{1}{2}} \approx 174 \text{ GeV}$$

W-propagator modifies HE behavior

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

$$\lim_{E_\nu \to \infty} \sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\sqrt{2}}$$

independent of energy!

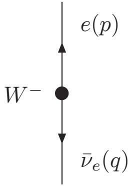
Partial-wave unitarity respected for

$$s < M_W^2[\exp(\pi\sqrt{2}/G_F M_W^2) - 1]$$

W-boson properties

No prediction yet for M_W (haven't determined g)

Leptonic decay $W^- \to e^- \nu_e$



$$p \approx \left(\frac{M_W}{2}; \frac{M_W \sin\theta}{2}, 0, \frac{M_W \cos\theta}{2} \right)$$

$$q \approx \left(\frac{M_W}{2}; -\frac{M_W \sin\theta}{2}, 0, -\frac{M_W \cos\theta}{2} \right)$$

$$\mathcal{M} = -i \left(\frac{G_F M_W^2}{\sqrt{2}} \right)^{\frac{1}{2}} \bar{u}(e,p)\gamma_\mu(1-\gamma_5)v(\nu,q)\,\varepsilon^\mu$$

$\varepsilon^\mu = (0;\hat{\varepsilon})$: W polarization vector in its rest frame

$$\begin{aligned} |\mathcal{M}|^2 &= \frac{G_F M_W^2}{\sqrt{2}} \mathrm{tr}\left[\not{\varepsilon}(1-\gamma_5)\not{q}(1+\gamma_5)\not{\varepsilon}^*\not{p}\right] ; \\ \mathrm{tr}[\cdots] &= \left[\varepsilon \cdot q\, \varepsilon^* \cdot p - \varepsilon \cdot \varepsilon^*\, q \cdot p + \varepsilon \cdot p\, \varepsilon^* \cdot q + i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma\right] \end{aligned}$$

$$\mathrm{tr}[\cdots] \;=\; [\varepsilon\cdot q\,\varepsilon^*\cdot p - \varepsilon\cdot\varepsilon^*\,q\cdot p + \varepsilon\cdot p\,\varepsilon^*\cdot q + i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma]$$

decay rate is independent of W polarization; look first at longitudinal pol.
$\varepsilon^\mu = (0;0,0,1) = \varepsilon^{*\mu}$, eliminate $\epsilon_{\mu\nu\rho\sigma}$

$$|\mathcal{M}|^2 = \frac{4G_F M_W^4}{\sqrt{2}}\sin^2\theta$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2}\,\frac{\mathcal{S}_{12}}{M_W^3}$$

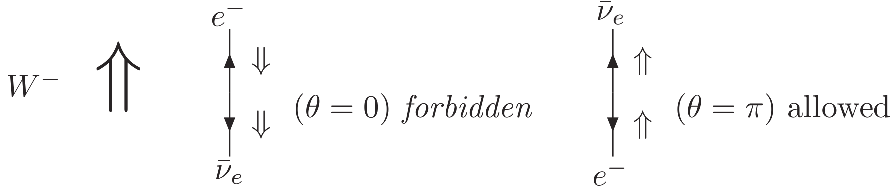
$$\mathcal{S}_{12} = \sqrt{[M_W^2 - (m_e + m_\nu)^2][M_W^2 - (m_e - m_\nu)^2]} = M_W^2$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2\sqrt{2}}\sin^2\theta \qquad \boxed{\Gamma(W\to e\nu) = \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

Other helicities: $\varepsilon^{\mu}_{\pm 1} = (0; -1, \mp i, 0)/\sqrt{2}$

$$\frac{d\Gamma_{\pm 1}}{d\Omega} = \frac{G_F M_W^3}{32\pi^2\sqrt{2}}(1 \mp \cos\theta)^2$$

Extinctions at $\cos\theta = \pm 1$ are consequences of angular momentum conservation:



(situation reversed for $W^+ \to e^+\nu_e$)

e^+ follows polarization direction of W^+

e^- avoids polarization direction of W^-

important for discovery of $W^\pm$ in $\bar{p}p$ $(\bar{q}q)$ C violation

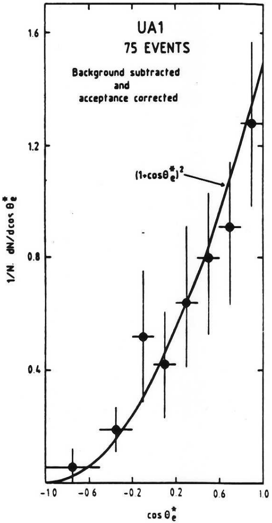


Fig. 2. The W decay angular distribution of the emission angle θ^* of the electron (positron) with respect to the proton (antiproton) direction in the rest frame of the W. Only those events for which the lepton charge and the decay kinematics are well determined have been used. The curve shows the $(V - A)$ expectation of $(1 + \cos\theta^*)^2$.

Interactions . . .

$$\mathcal{L}_{A\text{-}\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e}\gamma^\mu e A_\mu$$

. . . vector interaction; $\Rightarrow A_\mu$ as γ, provided we identify

$$\boxed{gg'/\sqrt{g^2 + g'^2} \equiv e}$$

Define $g' = g \tan\theta_W$ θ_W: weak mixing angle

$$\begin{aligned} g &= e/\sin\theta_W \geq e \\ g' &= e/\cos\theta_W \geq e \end{aligned}$$

$$Z_\mu = b_\mu^3 \cos\theta_W - \mathcal{A}_\mu \sin\theta_W \quad A_\mu = \mathcal{A}_\mu \cos\theta_W + b_\mu^3 \sin\theta_W$$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4\cos\theta_W} \bar{\nu}\gamma^\mu(1-\gamma_5)\nu \, Z_\mu$$

Purely left-handed!

Interactions . . .

$$\mathcal{L}_{Z-e} = \frac{-g}{4\cos\theta_W}\bar{e}\left[L_e\gamma^\mu(1-\gamma_5) + R_e\gamma^\mu(1+\gamma_5)\right]e\,Z_\mu$$

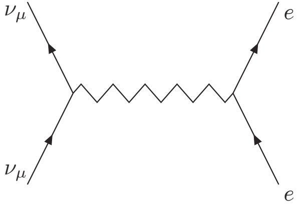
$$\begin{aligned} L_e &= 2\sin^2\theta_W - 1 = 2x_W + \tau_3 \\ R_e &= 2\sin^2\theta_W = 2x_W \end{aligned}$$

Z-decay calculation analogous to $W^\pm$

$$\begin{aligned} \Gamma(Z \to \nu\bar{\nu}) &= \frac{G_F M_Z^3}{12\pi\sqrt{2}} \\ \Gamma(Z \to e^+e^-) &= \Gamma(Z \to \nu\bar{\nu})\left[L_e^2 + R_e^2\right] \end{aligned}$$

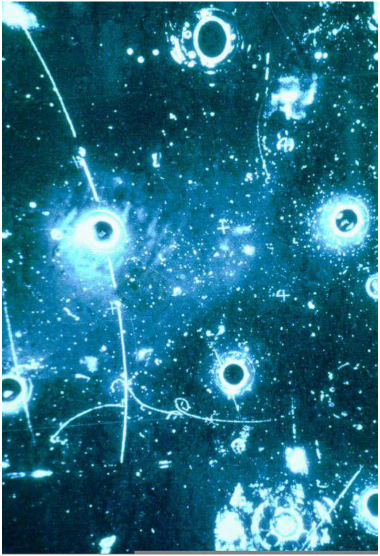
Neutral-current interactions

New νe reaction, not present in $V - A$



$$
\begin{aligned}
\sigma(\nu_\mu e \to \nu_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2 + R_e^2/3\right] \\
\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2/3 + R_e^2\right] \\
\sigma(\nu_e e \to \nu_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2 + R_e^2/3\right] \\
\sigma(\bar{\nu}_e e \to \bar{\nu}_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2/3 + R_e^2\right]
\end{aligned}
$$

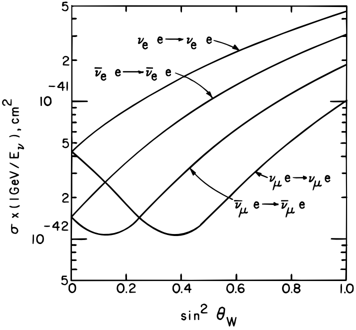
Gargamelle $\bar{\nu}_\mu e$ event (1973)



- Electromagnetism is mediated by a massless photon, coupled to the electric charge;
- Mediator of charged-current weak interaction acquires a mass $M_W^2 = \pi\alpha / G_F\sqrt{2}\sin^2\theta_W$,
- Mediator of (new!) neutral-current weak interaction acquires mass $M_Z^2 = M_W^2/\cos^2\theta_W$;
- Massive neutral scalar particle, the Higgs boson, appears, but its mass is not predicted;
- Fermions can acquire mass—values not predicted.

Determine $\sin^2\theta_W$ to predict M_W, M_Z

"Model-independent" analysis



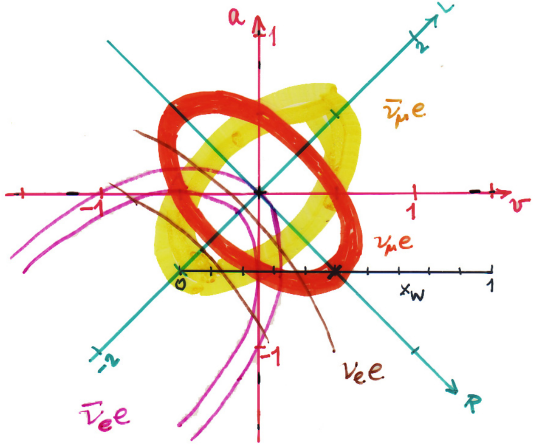
Measure all cross sections to determine chiral couplings L_e and R_e or traditional vector and axial couplings v and a

$$a = \tfrac{1}{2}(L_e - R_e) \qquad v = \tfrac{1}{2}(L_e - R_e)$$

$$L_e = v + a \qquad R_e = v - a$$

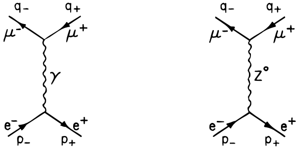
model-independent in V, A framework

Neutrino–electron scattering



Twofold ambiguity remains even after measuring all four cross sections: same cross sections result if we interchange $R_e \leftrightarrow -R_e$ ($v \leftrightarrow a$)

Consider $e^+e^- \to \ \mu^+\mu^-$



$$\begin{aligned}
\mathcal{M} &= -ie^2\bar{u}(\mu, q_-)\gamma_\lambda Q_\mu v(\mu, q_+)\frac{g^{\lambda\nu}}{s}\bar{v}(e, p_+)\gamma_\nu u(e.p_-) \\
&+\frac{i}{2}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)\bar{u}(\mu, q_-)\gamma_\lambda[R_\mu(1+\gamma_5) + L_\mu(1-\gamma_5)]v(\mu, q_+) \\
&\times\frac{g^{\lambda\nu}}{s - M_Z^2}\bar{v}(e, p_+)\gamma_\nu[R_e(1+\gamma_5) + L_e(1-\gamma_5)]u(e, p_-)
\end{aligned}$$

muon charge $Q_\mu = -1$

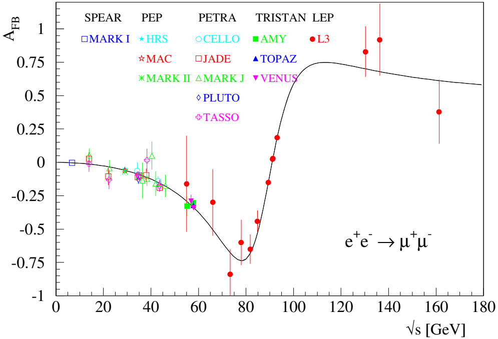
$e^+e^- \longrightarrow \mu^+\mu^- \ldots$

$$\begin{aligned}
\frac{d\sigma}{dz} \;=\; & \frac{\pi\alpha^2 Q_\mu^2}{2s}(1+z^2) \\
& -\frac{\alpha Q_\mu G_F M_Z^2(s-M_Z^2)}{8\sqrt{2}[(s-M_Z^2)^2+M_Z^2\Gamma^2]} \\
& \times[(R_e+L_e)(R_\mu+L_\mu)(1+z^2)+2(R_e-L_e)(R_\mu-L_\mu)z] \\
& +\frac{G_F^2 M_Z^4 s}{64\pi[(s-M_Z^2)^2+M_Z^2\Gamma^2]} \\
& \times[(R_e^2+L_e^2)(R_\mu^2+L_\mu^2)(1+z^2)+2(R_e^2-L_e^2)(R_\mu^2-L_\mu^2)z]
\end{aligned}$$

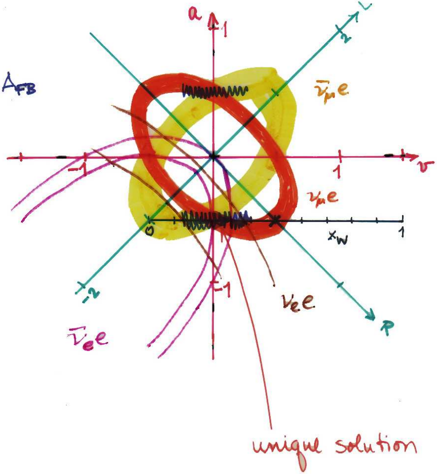
Measuring A resolves ambiguity

$$\text{Forward-backward asymmetry } A \equiv \frac{\int_0^1 dz\, d\sigma/dz - \int_{-1}^0 dz\, d\sigma/dz}{\int_{-1}^1 dz\, d\sigma/dz}$$

$$\begin{aligned}
\lim_{s/M_Z^2 \ll 1} A &= \frac{3G_F s}{16\pi\alpha Q_\mu \sqrt{2}}(R_e - L_e)(R_\mu - L_\mu) \\
&\approx -6.7 \times 10^{-5}\left(\frac{s}{1\ \text{GeV}^2}\right)(R_e - L_e)(R_\mu - L_\mu) = -3G_F s a^2/4\pi\alpha\sqrt{2}
\end{aligned}$$



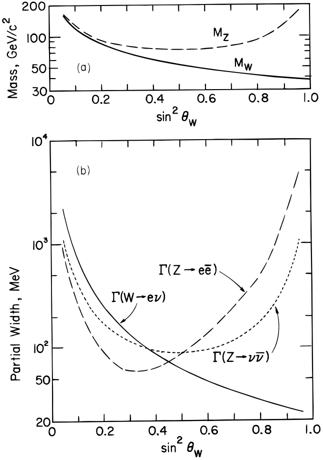
Neutrino–electron scattering $e^+e^- \rightarrow \mu^+\mu^-$



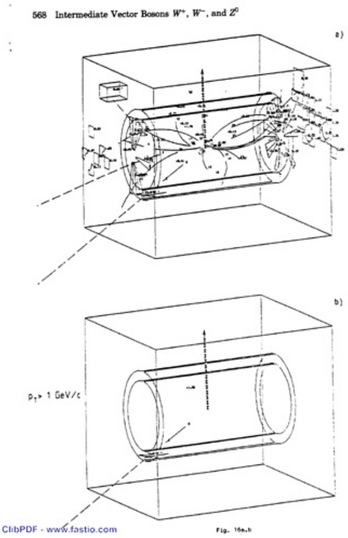
$\boxed{\text{Validate EW theory, measure } \sin^2\theta_W}$

With a measurement of $\sin^2\theta_W$, predict

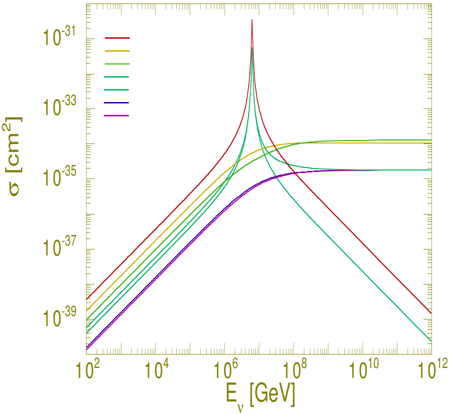
$M_W^2 = \pi\alpha/G_F\sqrt{2}\sin^2\theta_W \approx (37.28\ \mathrm{GeV}/c^2)^2/\sin^2\theta_W \quad M_Z^2 = M_W^2/\cos^2\theta_W$



First Z from UA1



νe cross sections . . .



At low energies: $\sigma(\bar{\nu}_e e \rightarrow \text{ hadrons}) > \sigma(\nu_\mu e \rightarrow \mu\nu_e) > \sigma(\nu_e e \rightarrow \nu_e e) > \sigma(\bar{(}\nu_e e \rightarrow \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) > \sigma(\nu_\mu e \rightarrow \nu_\mu e) > \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e)$

Electroweak interactions of quarks

- Left-handed doublet

$$\begin{array}{cccc} & I_3 & Q & Y = 2(Q - I_3) \\ \mathsf{L}_q = \begin{pmatrix} u \\ d \end{pmatrix}_L & \begin{array}{c} \frac{1}{2} \\ -\frac{1}{2} \end{array} & \begin{array}{c} +\frac{2}{3} \\ -\frac{1}{3} \end{array} & \frac{1}{3} \end{array}$$

- two right-handed singlets

$$\begin{array}{cccc} & I_3 & Q & Y = 2(Q - I_3) \\ \mathsf{R}_u = u_R & 0 & +\frac{2}{3} & +\frac{4}{3} \\ \mathsf{R}_d = d_R & 0 & -\frac{1}{3} & -\frac{2}{3} \end{array}$$

Electroweak interactions of quarks

- CC interaction

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{u}_e \gamma^\mu (1 - \gamma_5) d \, W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u \, W_\mu^- \right]$$

identical in form to $\mathcal{L}_{W\text{-}\ell}$: universality $\Leftrightarrow$ weak isospin

- NC interaction

$$\mathcal{L}_{Z\text{-}q} = \frac{-g}{4 \cos \theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu \left[L_i (1 - \gamma_5) + R_i (1 + \gamma_5) \right] q_i \, Z_\mu$$

$$L_i = \tau_3 - 2Q_i \sin^2 \theta_W \quad R_i = -2Q_i \sin^2 \theta_W$$

equivalent in form (not numbers) to $\mathcal{L}_{Z\text{-}\ell}$

Trouble in Paradise

Universal $u \leftrightarrow d$, $\nu_e \leftrightarrow e$ *not quite right*

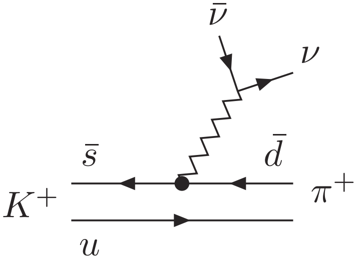
$$\text{Good:} \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow \text{Better:} \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

$$d_\theta \equiv d \, \cos\theta_C + s \, \sin\theta_C \qquad \cos\theta_C = 0.9736 \pm 0.0010$$

"Cabibbo-rotated" doublet perfects CC interaction (up to small third-generation effects) but $\Rightarrow$ serious trouble for NC

$$\begin{aligned} \mathcal{L}_{Z\text{-}q} &= \frac{-g}{4\cos\theta_W} Z_\mu \{ \bar{u}\gamma^\mu [L_u(1-\gamma_5) + R_u(1+\gamma_5)] u \\ &\quad + \bar{d}\gamma^\mu [L_d(1-\gamma_5) + R_d(1+\gamma_5)] d \, \cos^2\theta_C \\ &\quad + \bar{s}\gamma^\mu [L_d(1-\gamma_5) + R_d(1+\gamma_5)] s \, \sin^2\theta_C \\ &\quad + \bar{d}\gamma^\mu [L_d(1-\gamma_5) + R_d(1+\gamma_5)] s \, \sin\theta_C \cos\theta_C \\ &\quad + \bar{s}\gamma^\mu [L_d(1-\gamma_5) + R_d(1+\gamma_5)] d \, \sin\theta_C \cos\theta_C \} \end{aligned}$$

Strangeness-changing NC interactions highly suppressed!



BNL E-787/E-949 has three $K^+ \to \pi^+ \nu\bar{\nu}$ candidates, with $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) = 1.47^{+1.30}_{-0.89} \times 10^{-10}$
Phys. Rev. Lett. **93**, 031801 (2004)

(SM: 0.78 ± 0.11: U. Haisch, hep-ph/0605170)

Glashow–Iliopoulos–Maiani

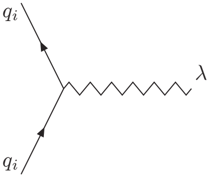
two LH doublets: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$

$$(s_\theta = s\cos\theta_C - d\sin\theta_C)$$

$+$ right-handed singlets, e_R, μ_R, u_R, d_R, c_R, s_R

$\boxed{\text{Required new charmed quark, } c}$

Cross terms vanish in $\mathcal{L}_{Z\text{-}q}$,



$$\frac{-ig}{4\cos\theta_W}\gamma_\lambda[(1-\gamma_5)L_i + (1+\gamma_5)R_i] \quad ,$$

$$L_i = \tau_3 - 2Q_i\sin^2\theta_W \quad R_i = -2Q_i\sin^2\theta_W$$

flavor-diagonal interaction!

Straightforward generalization to n quark doublets

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}}\left[\bar{\Psi}\gamma^{\mu}(1-\gamma_5)\mathcal{O}\Psi\, W_{\mu}^{+} + \text{h.c.}\right]$$

composite $\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ \\ d \\ s \\ \vdots \end{pmatrix}$

flavor structure $\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$

U: unitary quark mixing matrix

Weak-isospin part: $\mathcal{L}^{\text{iso}}_{Z\text{-}q} = \dfrac{-g}{4\cos\theta_W}\bar{\Psi}\gamma^{\mu}(1-\gamma_5)\left[\mathcal{O},\mathcal{O}^{\dagger}\right]\Psi$

Since $\left[\mathcal{O},\mathcal{O}^{\dagger}\right] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

$\boxed{\Rightarrow \text{NC interaction is flavor-diagonal}}$

General $n \times n$ mixing matrix U: $n(n-1)/2$ real $\angle$, $(n-1)(n-2)/2$ complex phases

3×3 (Cabibbo–Kobayashi-Maskawa): $3\ \angle + 1$ phase $\Rightarrow$ CP violation

Fermion mass is accommodated, not explained

- All fermion masses $\sim$ physics beyond the standard model!
- $\zeta_t \approx 1 \qquad \zeta_e \approx 3 \times 10^{-6} \qquad \zeta_\nu \approx 10^{-10}$??

  What accounts for the range and values of the Yukawa couplings?
- There may be *other sources* of neutrino mass

Successful predictions of $SU(2)_L \otimes U(1)_Y$ theory:

- neutral-current interactions
- necessity of charm
- existence and properties of $W^{\pm}$ and Z^0

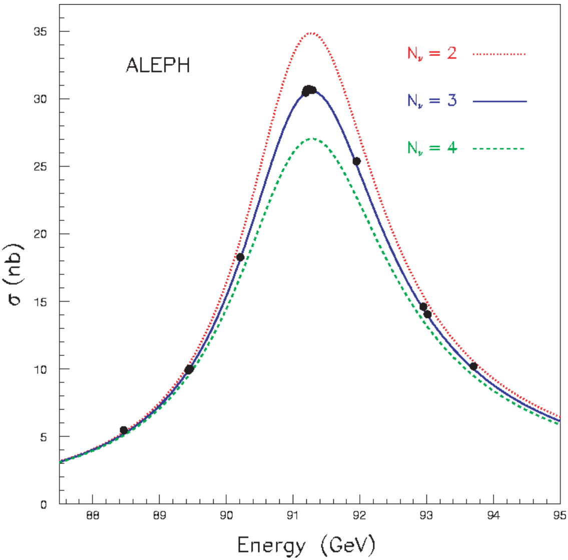
$+$ a decade of precision EW tests (one-per-mille)

M_Z	$91\,187.6 \pm 2.1$ MeV/c^2
Γ_Z	2495.2 ± 2.3 MeV
$\sigma^0_{\text{hadronic}}$	41.541 ± 0.037 nb
Γ_{hadronic}	1744.4 ± 2.0 MeV
Γ_{leptonic}	83.984 ± 0.086 MeV
$\Gamma_{\text{invisible}}$	499.0 ± 1.5 MeV

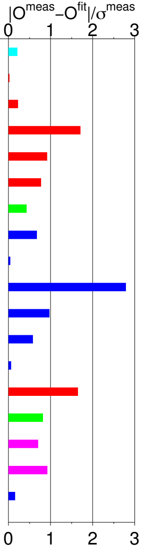
$$\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$$

light ν : $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \to \nu_i\bar{\nu}_i) = 2.994 \pm 0.012 \quad (\nu_e, \nu_\mu, \nu_\tau)$

Three light neutrinos



	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02766
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ_{had}^{0} [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01640
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1479
R_b	0.21629 ± 0.00066	0.21585
R_c	0.1721 ± 0.0030	0.1722
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0741
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1479
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.392 ± 0.029	80.371
Γ_W [GeV]	2.147 ± 0.060	2.091
m_t [GeV]	171.4 ± 2.1	171.7

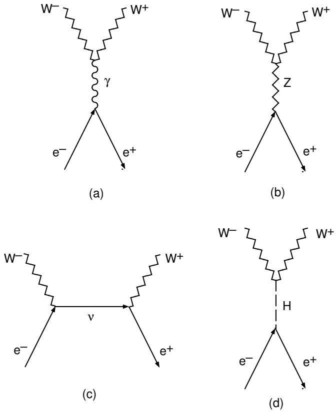


LEP Electroweak Working Group, Summer 2006

Why a Higgs boson must exist

▷ Role in canceling high-energy divergences

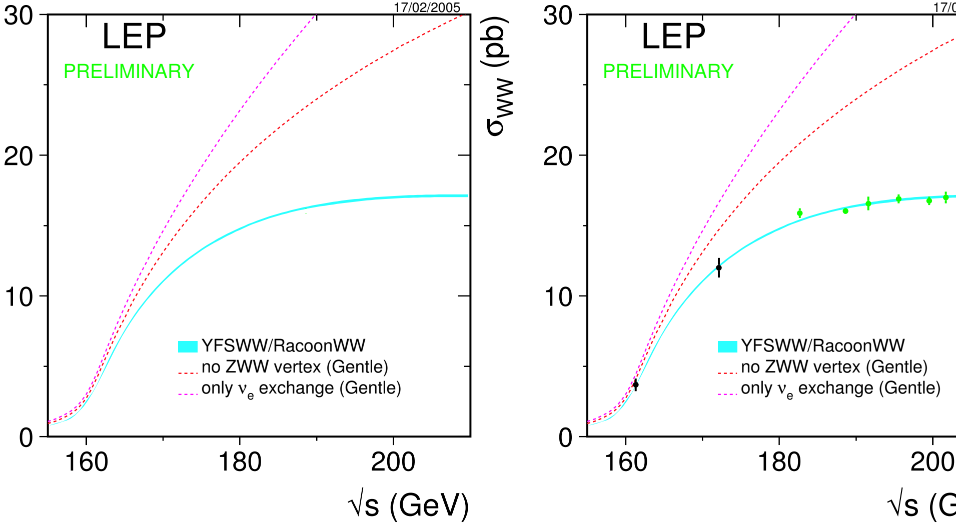
S-matrix analysis of $e^+e^- \to W^+W^-$



Individual $J = 1$ partial-wave amplitudes $\mathcal{M}_{\gamma}^{(1)}$, $\mathcal{M}_{Z}^{(1)}$, $\mathcal{M}_{\nu}^{(1)}$ have unacceptable high-energy behavior ($\propto s$)

. . . But sum is well-behaved

"Gauge cancellation" observed at LEP2 (Tevatron)



$J = 0$ amplitude exists because electrons have mass, and can be found in "wrong" helicity state

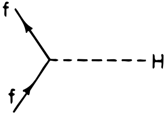
$$\mathcal{M}_{\nu}^{(0)} \propto s^{\frac{1}{2}} \quad \text{: unacceptable HE behavior}$$

(no contributions from γ and Z)

This divergence is canceled by the Higgs-boson contribution

$$\boxed{\Rightarrow He\bar{e} \text{ coupling must be} \propto m_e,}$$

because "wrong-helicity" amplitudes $\propto m_e$



$$\frac{-im_f}{v} = -im_f(G_F\sqrt{2})^{1/2}$$

If the Higgs boson did not exist, something else would have to cure divergent behavior

If gauge symmetry were unbroken . . .

- no Higgs boson
- no longitudinal gauge bosons
- no extreme divergences
- no wrong-helicity amplitudes

. . . and no viable low-energy phenomenology

In spontaneously broken theory . . .

- gauge structure of couplings eliminates the most severe divergences
- lesser—but potentially fatal—divergence arises because the electron has mass . . . due to the Higgs mechanism
- SSB provides its own cure—the Higgs boson

Similar interplay & compensation *must exist* in any acceptable theory

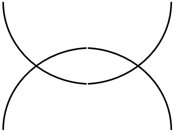
Bounding M_H from above ...

Triviality of scalar field theory

- Only *noninteracting* scalar field theories make sense on all energy scales
- Quantum field theory vacuum is a dielectric medium that screens charge
- $\Rightarrow$ *effective charge* is a function of the distance or, equivalently, of the energy scale

$$\boxed{\text{running coupling constant}}$$

In $\lambda\phi^4$ theory, calculate variation of coupling constant λ in perturbation theory by summing bubble graphs



$\lambda(\mu)$ is related to a higher scale Λ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log{(\Lambda/\mu)}$$

(Perturbation theory reliable only when λ is small, lattice field theory treats strong-coupling regime)

For stable Higgs potential (*i.e.*, for vacuum energy not to race off to $-\infty$), *require* $\lambda(\Lambda) \geq 0$

Rewrite RGE as an inequality

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log\left(\Lambda/\mu\right)$$

. . . implies an *upper bound*

$$\boxed{\lambda(\mu) \leq 2\pi^2/3 \log\left(\Lambda/\mu\right)}$$

If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit $\Lambda \to \infty$ while holding μ fixed at some reasonable physical scale. In this limit, the $\boxed{\text{bound}}$ forces $\lambda(\mu)$ to zero.

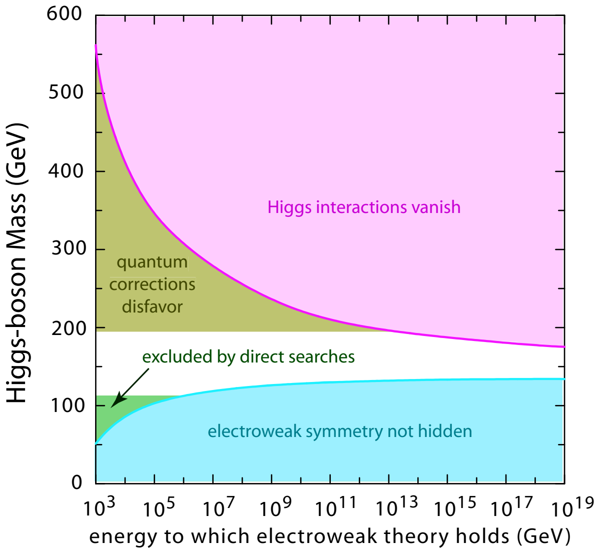
$\longrightarrow$ free field theory "trivial"

Rewrite as bound on M_H:

$$\Lambda \le \mu \exp\left(\frac{2\pi^2}{3\lambda(\mu)}\right)$$

Choose $\mu = M_H$, and recall $M_H^2 = 2\lambda(M_H)v^2$

$$\boxed{\Lambda \le M_H \exp\left(4\pi^2 v^2 / 3M_H^2\right)}$$


600
500
400
300
200
100
0
Higgs-boson Mass (GeV)
Higgs interactions vanish
quantum
corrections
disfavor
excluded by direct searches
electroweak symmetry not hidden
10^3
10^5
10^7
10^9
10^{11}
10^{13}
10^{15}
10^{17}
10^{19}
energy to which electroweak theory holds (GeV)

Moral: For any M_H, there is a *maximum energy scale* $\Lambda^\star$ at which the theory ceases to make sense.

The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies

Perturbative analysis breaks down when $M_H \to 1\ \mathrm{TeV}/c^2$ and interactions become strong

Lattice analyses $\Longrightarrow M_H \lesssim 710 \pm 60\ \mathrm{GeV}/c^2$ if theory describes physics to a few percent up to a few TeV

$\boxed{\text{If } M_H \to 1 \text{ TeV EW theory lives on brink of instability}}$

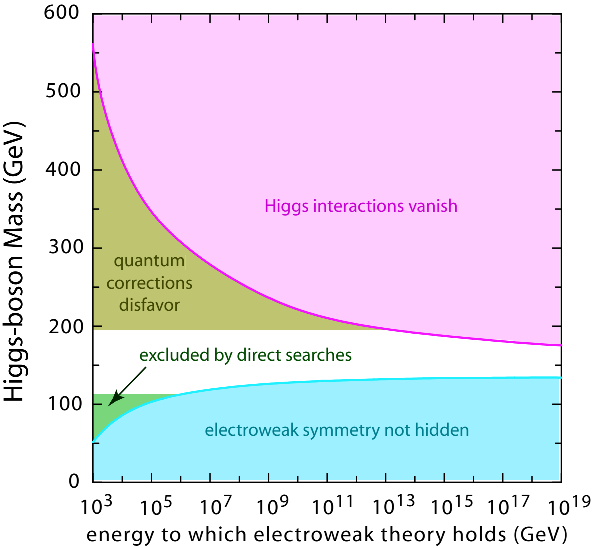
Lower bound by requiring EWSB vacuum $V(v) < V(0)$

Requiring that $\langle\phi\rangle_0 \neq 0$ be an absolute minimum of the one-loop potential up to a scale Λ yields the vacuum-stability condition . . . (for $m_t \lesssim M_W$)

$$M_H^2 > \frac{3G_F\sqrt{2}}{8\pi^2}(2M_W^4 + M_Z^4 - 4m_t^4)\log(\Lambda^2/v^2)$$

(No illuminating analytic form for heavy m_t)

If the Higgs boson is relatively light (which would require explanation) then the theory can be self-consistent up to very high energies



If EW theory is to make sense all the way up to a unification scale $\Lambda^\star = 10^{16}$ GeV, then $134\ \mathrm{GeV}/c^2 \lesssim M_H \lesssim 177$ GeV

Higgs-Boson Properties

$$\Gamma(H \to f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

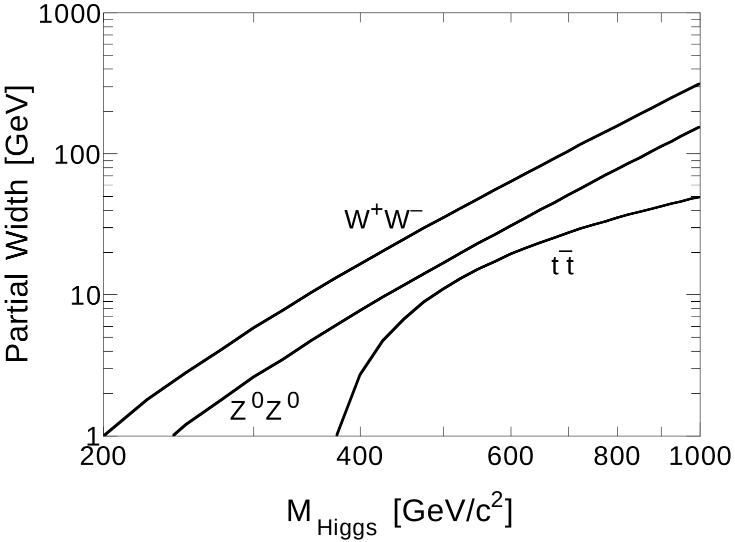
$\propto M_H$ in the limit of large Higgs mass; $\propto \beta^3$ for scalar

$$\Gamma(H \to W^+W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}}(1-x)^{1/2}(4-4x+3x^2) \quad x \equiv 4M_W^2/M_H^2$$

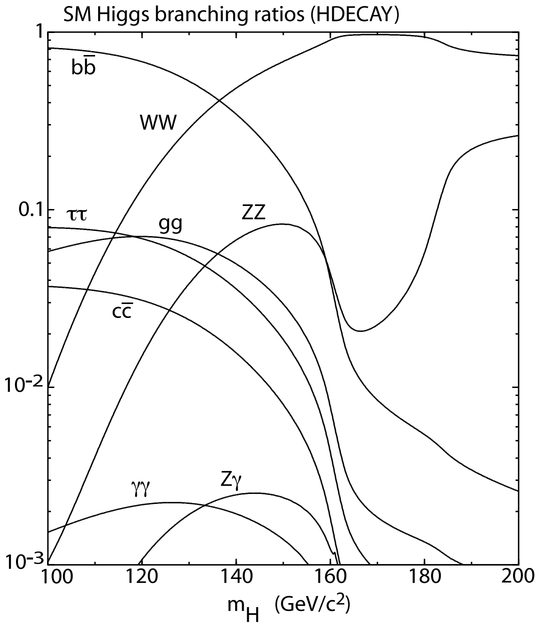
$$\Gamma(H \to Z^0Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}}(1-x')^{1/2}(4-4x'+3x'^2) \quad x' \equiv 4M_Z^2/M_H^2$$

asymptotically $\propto M_H^3$ and $\frac{1}{2}M_H^3$, respectively ($\frac{1}{2}$ from weak isospin)

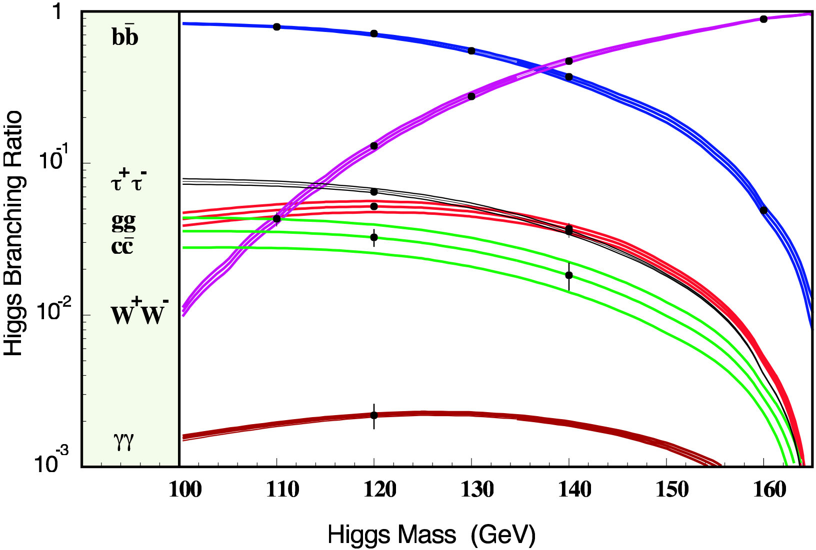
$2x^2$ and $2x'^2$ terms $\Leftrightarrow$ decays into transverse gauge bosons
Dominant decays for large M_H: pairs of longitudinal weak bosons



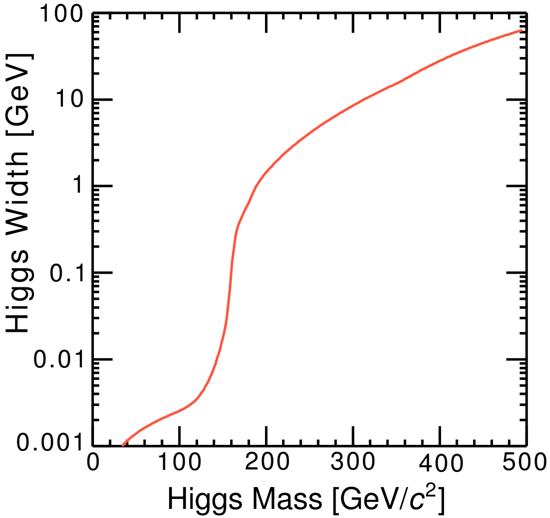
For $M_H \to 1$ TeV, Higgs boson is *ephemeral:* $\Gamma_H \to M_H$.


SM Higgs branching ratios (HDECAY)
1
0.1
10-2
10-3
100
120
140
160
180
200
mH (GeV/c2)
bb
WW
ττ
gg
ZZ
cc
γγ
Zγ

ILC would measure light Higgs-boson couplings precisely



Points: 500 fb^{-1} @ 350 GeV   Bands: theory uncertainty (m_b)



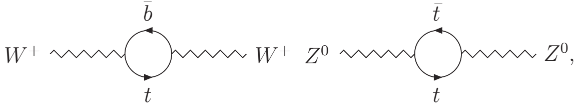
Below W^+W^- threshold, $\Gamma_H \lesssim 1$ GeV

Far above W^+W^- threshold, $\Gamma_H \propto M_H^3$

Experimental clues to the Higgs-boson mass

Sensitivity of EW observables to m_t gave early indications for massive top

Quantum corrections to SM predictions for M_W and M_Z arise from different quark loops



$$\ldots\text{alter the link } \underbrace{M_W^2}_{(80.404 \pm 0.030\ \text{GeV})^2} = \underbrace{M_Z^2\left(1 - \sin^2\theta_W\right)}_{(80.939\ \text{GeV})^2}(1-\Delta\rho)$$

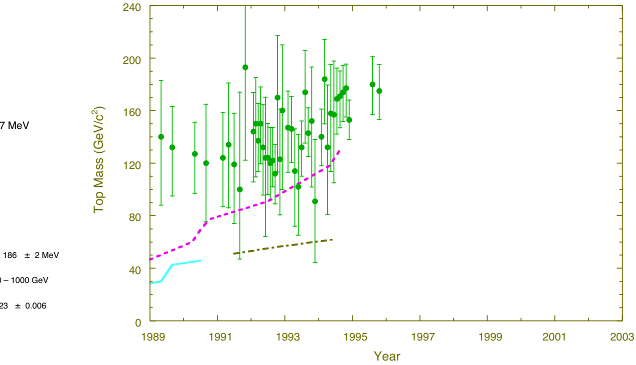
where $\Delta\rho \approx \Delta\rho^{(\text{quarks})} = 3G_F m_t^2/8\pi^2\sqrt{2}$

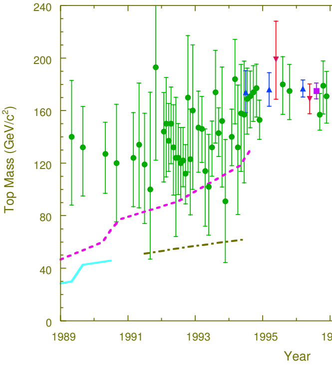
Strong dependence on m_t^2 accounts for precision of m_t estimates derived from EW observables

Tevatron: $\delta m_t/m_t \approx 1.28\%$*. . . Look beyond quark loops to next most important quantum corrections:* $\boxed{\textit{Higgs-boson effects}}$

Global fits to precision EW measurements

- precision improves with time / calculations improve with time





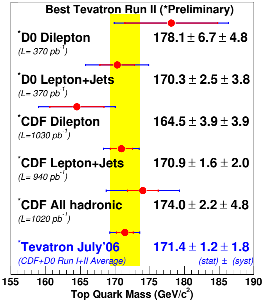
11.94, LEPEWWG: $m_t = 178 \pm 11^{+18}_{-19}$ GeV/c^2

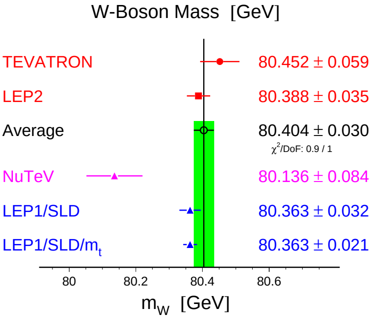
Direct measurements: $m_t = 171.4 \pm 2.2$ GeV/c^2

H quantum corrections smaller than t corrections, exhibit more subtle dependence on M_H than the m_t^2 dependence of the top-quark corrections

$$\Delta\rho^{(\mathrm{Higgs})} = \mathcal{C} \cdot \ln\left(\frac{M_H}{v}\right)$$

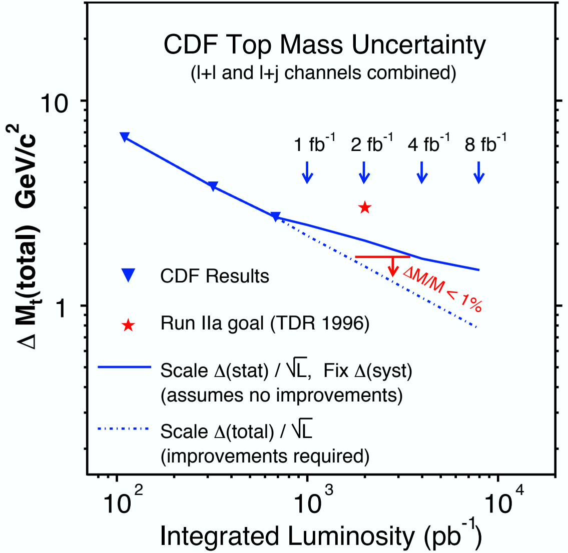
M_Z known to 23 ppm, m_t and M_W well measured

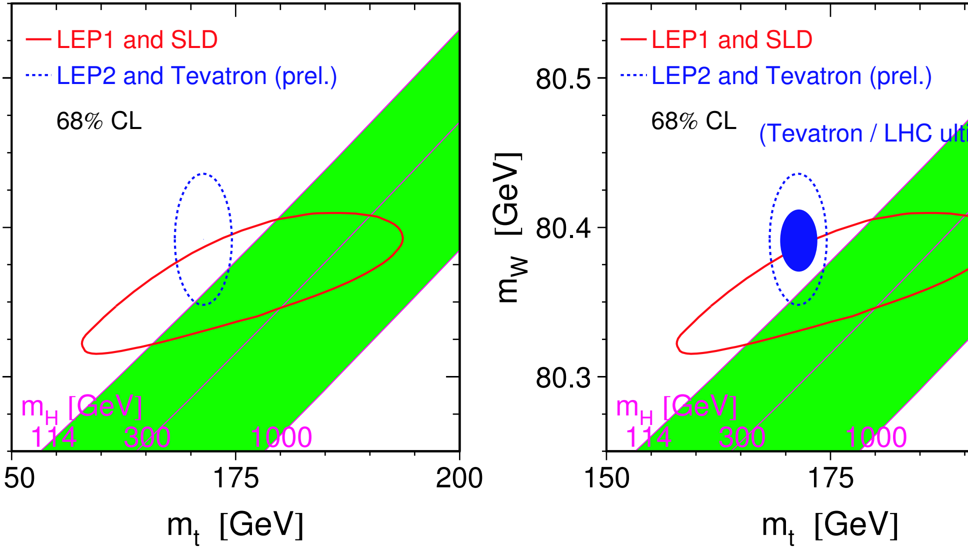




. . . so examine dependence of M_W upon m_t and M_H

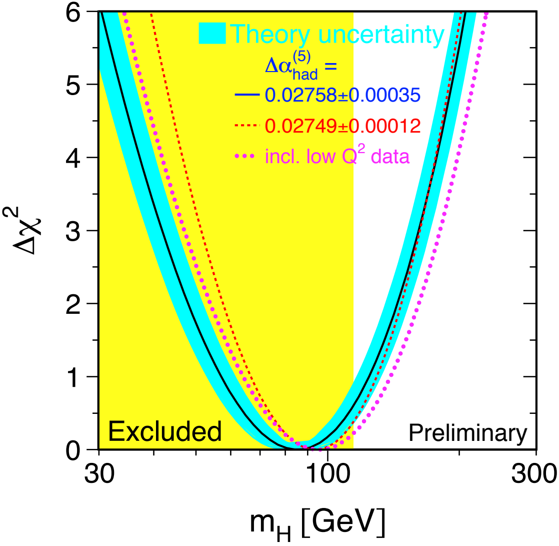
CDF's top-mass projections . . .





Direct, indirect determinations agree reasonably
Both favor a light Higgs boson, . . . *within framework of SM analysis.*

Fit to a universe of data



Standard-Model $M_H \lesssim 200$ GeV at 95% CL

- *Within SM,* LEP EWWG deduce a 95% CL upper limit, $M_H \lesssim 200$ GeV/c^2.
- Direct searches at LEP $\Rightarrow M_H > 114.4$ GeV/c^2, excluding much of the favored region
- Either the Higgs boson is just around the corner, or SM analysis is misleading

Things will soon be popping!

- Tevatron, LHC measurements will determine m_t within 1 or 2 GeV
  ...and improve δM_W to about 15 MeV
- As the Tevatron's integrated luminosity approaches 10 fb^{-1}, CDF and DØ will explore the region of M_H not excluded by LEP
- ATLAS and CMS will carry on the exploration of the Higgs sector at the LHC;
  could require a few years, at low mass;
  full range accessible, $\gamma\gamma, \ell\ell\nu\nu, b\bar{b}, \ell^+\ell^-\ell^+\ell^-, \ell\nu jj, \tau\tau$ channels.

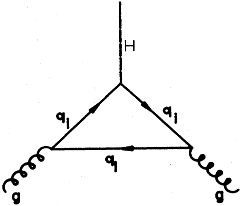
A few words on Higgs production ...

$e^+e^- \to H$: hopelessly small
$\mu^+\mu^- \to H$: scaled by $(m_\mu/m_e)^2 \approx 40\,000$
$e^+e^- \to HZ$: prime channel

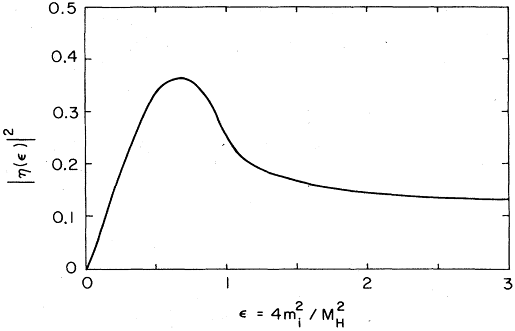
Hadron colliders:
$gg \to H \to b\bar{b}$: background ?!
$gg \to H \to \gamma\gamma$: rate ?!
$\bar{p}p \to H(W, Z)$: prime Tevatron channel

At the LHC:
Many channels become accessible, expect sensitive search up to 1 TeV

H couples to gluons through quark loops



Only heavy quarks matter:

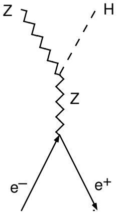


Higgs search in e^+e^- collisions

$\sigma(e^+e^- \to H \to \text{all})$ is *minute*, $\propto m_e^2$

Even narrowness of low-mass H is not enough to make it visible ... Sets aside a traditional strength of e^+e^- machines—*pole physics*

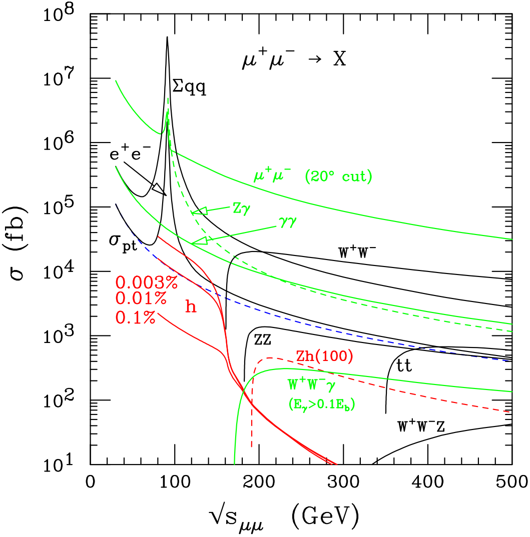
Most promising:
associated production $e^+e^- \to HZ$
(has no small couplings)



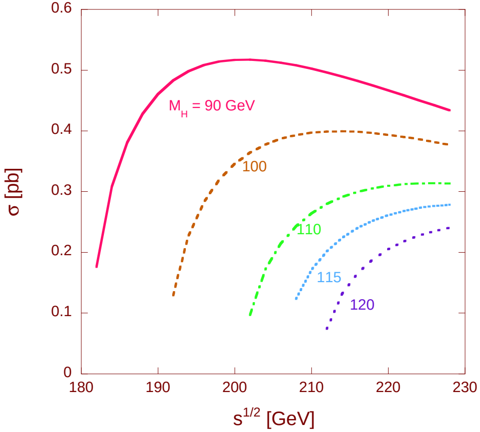
$$\boxed{\sigma = \frac{\pi\alpha^2}{24\sqrt{s}} \frac{K(K^2+3M_Z^2)[1+(1-4x_W)^2]}{(s-M_Z^2)^2 \quad x_W^2(1-x_W)^2}}$$

K: c.m. momentum of H $\qquad$ $x_W \equiv \sin^2\theta_W$

$\ell^+\ell^- \to X \ldots$



$$\sigma(e^+e^- \to H) = (m_e/m_\mu)^2\sigma(\mu^+\mu^- \to H) \approx \sigma(\mu^+\mu^- \to H)/40\,000$$



$+$ important effect of ISR

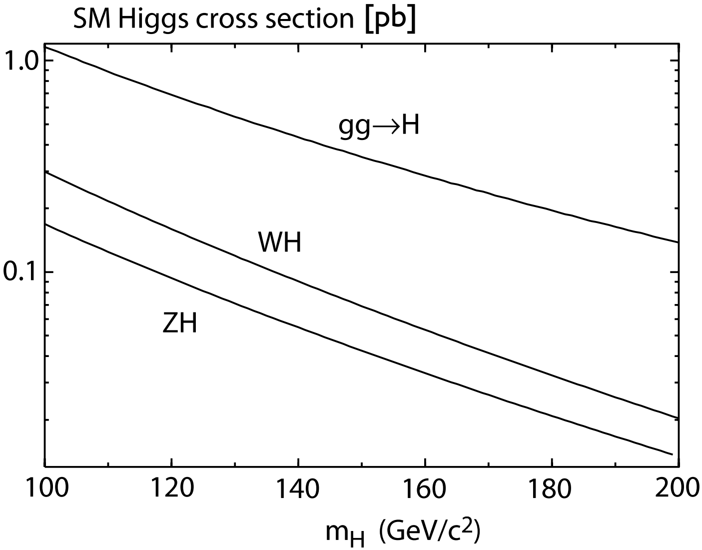
LEP 2: sensitive nearly to kinematical limit $\boxed{M_H^{\max} = \sqrt{s} - M_Z}$

LC: sensitive for $M_H \lesssim 0.7\sqrt{s}$

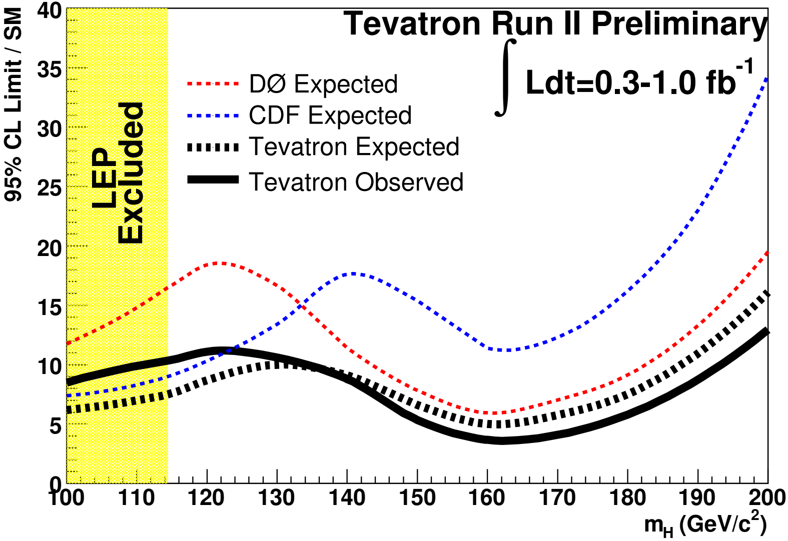
& measure excitation curve to determine

$$\delta M_H \approx 60\ \text{MeV}\ \sqrt{100\ \text{fb}^{-1}/\mathcal{L}}\ \text{for}\ M_H = 100\ \text{GeV}$$

Higgs-boson production at the Tevatron



Current Tevatron Sensitivity



combining experiments, channels

The agent of electroweak symmetry breaking represents

a novel fundamental interaction

at an energy of a few hundred GeV.

We do not know the nature of the new force.

Inspired by the Meissner effect, we describe the EWSB interaction as an analogue of the Ginzburg–Landau picture of superconductivity.

light Higgs boson $\Leftrightarrow$ perturbative dynamics

heavy Higgs boson $\Leftrightarrow$ strong dynamics

What is the nature of the mysterious new force that hides electroweak symmetry?

- A fundamental force of a new character, based on interactions of an elementary scalar
- A new gauge force, perhaps acting on undiscovered constituents
- A residual force that emerges from strong dynamics among the weak gauge bosons
- An echo of extra spacetime dimensions

We have explored examples of all four, theoretically.

$\boxed{\text{Which path has Nature taken?}}$

Essential step toward understanding the new force that shapes our world:

Find the Higgs boson and explore its properties.

- Is it there? How many?
- Verify $J^{PC} = 0^{++}$
- Does H generate mass for gauge bosons, fermions?
- How does H interact with itself?

Finding the Higgs boson starts a new adventure!

10 years precise measurements: no significant deviations

Quantum corrections tested at $\pm 10^{-3}$

No "new physics" ... yet!

Theory tested at distances from 10^{-17} cm to $\sim 10^{22}$ cm

origin Coulomb's law (tabletop experiments)

smaller $\begin{cases} \text{Atomic physics} \rightarrow \text{QED} \\ \text{high-energy expts.} \rightarrow \text{EW theory} \end{cases}$

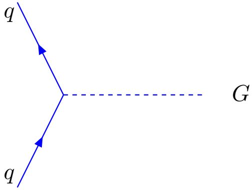
larger $M_\gamma \approx 0$ in planetary ... measurements

Is EW theory true? Is it complete ??

But what about gravity?

Natural to neglect gravity in particle physics ...

$$G_{\text{Newton}} \textit{ small} \iff M_{\text{Planck}} = \left(\frac{\hbar c}{G_{\text{Newton}}}\right)^{\frac{1}{2}} \approx 1.22 \times 10^{19} \text{ GeV } \textit{large}$$



$$\sim \frac{E}{M_{\text{Planck}}}$$

$$\text{Estimate } B(K \to \pi G) \sim \left(\frac{M_K}{M_{\text{Planck}}}\right)^2 \sim 10^{-38}$$

300 years after Newton: Why is gravity weak?

But gravity is not always negligible . . .

The vacuum energy problem

$$\text{Higgs potential } V(\varphi^\dagger\varphi) = \mu^2(\varphi^\dagger\varphi) + |\lambda|\,(\varphi^\dagger\varphi)^2$$

At the minimum,

$$V(\langle\varphi^\dagger\varphi\rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda|\,v^4}{4} < 0.$$

$$\text{Identify } M_H^2 = -2\mu^2$$

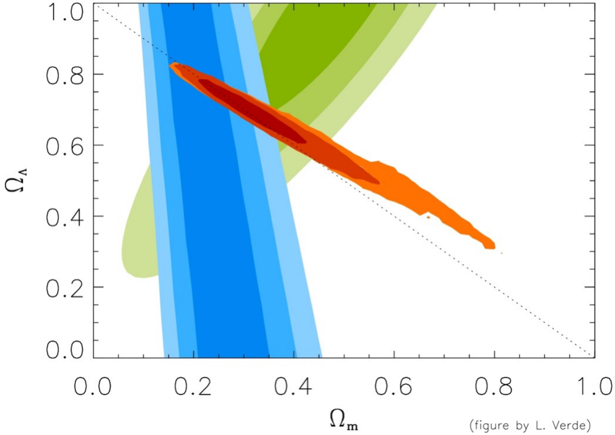
$V \neq 0$ contributes position-independent vacuum energy density

$$\boxed{\varrho_H \equiv \frac{M_H^2 v^2}{8} \geq 10^8\ \text{GeV}^4} \quad \approx 10^{24}\ \text{g cm}^{-3}$$

Adding vacuum energy density $\varrho_{\text{vac}} \Leftrightarrow$ adding cosmological constant Λ to Einstein's equation

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = \frac{8\pi G_N}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu} \qquad \Lambda = \frac{8\pi G_N}{c^4}\varrho_{\text{vac}}$$

Observed vacuum energy density $\varrho_{\text{vac}} \lesssim 10^{-46}\ \text{GeV}^4$



(figure by L. Verde)

$\varrho_H \gtrsim 10^8\ \text{GeV}^4$: mismatch by 10^{54}

A chronic dull headache for thirty years . . .

EWSB: another path?

Modeled EWSB on Ginzburg–Landau description of superconducting phase transition;

. . . had to introduce new, elementary scalars

GL is not the last word on superconductivity:

dynamical Bardeen–Cooper–Schrieffer theory

The elementary fermions – electrons – and gauge interactions – QED – needed to generate the scalar bound states are already present in the case of superconductivity.

Could a scheme of similar economy account for EWSB?

$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ + massless u and d

(treat $SU(2)_L \otimes U(1)_Y$ as perturbation)

$m_u = m_d = 0$:
QCD has exact $SU(2)_L \otimes SU(2)_R$ chiral symmetry.

At an energy scale $\sim \Lambda_{QCD}$, strong interactions become strong, fermion condensates appear, and

$$SU(2)_L \otimes SU(2)_R \to SU(2)_V$$

$\rightsquigarrow$ 3 Goldstone bosons, one for each broken generator:
3 massless pions (Nambu)

Broken generators: 3 axial currents; couplings to π measured by pion decay constant f_π.

Turn on $\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$: EW gauge bosons couple to axial currents, acquire masses of order $\sim gf_\pi$.

$$\mathcal{M}^2 = \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & gg' \\ 0 & 0 & gg' & g'^2 \end{pmatrix} \frac{f_\pi^2}{4} \quad (W^+, W^-, W_3, \mathcal{A})$$

same structure as standard EW theory.

Diagonalize: $M_W^2 = g^2 f_\pi^2/4$, $M_Z^2 = (g^2 + g'^2) f_\pi^2/4$, $M_A^2 = 0$, so

$$\frac{M_Z^2}{M_W^2} = \frac{(g^2 + g'^2)}{g^2} = \frac{1}{\cos^2\theta_W}$$

Massless pions disappear from physical spectrum, to become longitudinal components of weak bosons. $\boxed{M_W \approx 30\ \mathrm{MeV}/c^2}$ No fermion masses ...

Parameters of the Standard Model

3	coupling parameters $\alpha_s, \alpha_{\mathrm{EM}}, \sin^2\theta_W$
2	parameters of the Higgs potential
1	vacuum phase (QCD)
6	quark masses
3	quark mixing angles
1	CP-violating phase
3	charged-lepton masses
3	neutrino masses
3	leptonic mixing angles
1	leptonic CP-violating phase (+ Majorana . . .)
26^+	arbitrary parameters

parameter count not improved by unification

The EW scale and beyond

EWSB scale, $v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246$ GeV, sets

$$M_W^2 = g^2 v^2/2 \qquad M_Z^2 = M_W^2/\cos^2\theta_W$$

But it is not the only scale of physical interest

quasi-certain: $M_{\text{Planck}} = 1.22 \times 10^{19}$ GeV

probable: $\mathrm{SU(3)_c} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$ unification scale $\sim 10^{15-16}$ GeV

somewhere: flavor scale

How to keep the distant scales from mixing in the face of quantum corrections?

OR

How to stabilize the mass of the Higgs boson on the electroweak scale?

OR

Why is the electroweak scale small?

$\boxed{\text{“The hierarchy problem”}}$

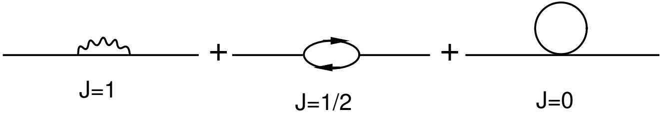
Higgs potential $\quad V(\phi^\dagger\phi) = \mu^2(\phi^\dagger\phi) + |\lambda|\,(\phi^\dagger\phi)^2$

$\mu^2 < 0$: $\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y \to U(1)_{\mathrm{em}}$, as

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ \sqrt{-\mu^2/2|\lambda|} \end{pmatrix} \equiv \begin{pmatrix} 0 \\ \underbrace{(G_F\sqrt{8})^{-1/2}}_{175\ \mathrm{GeV}} \end{pmatrix}$$

Beyond classical approximation, quantum corrections to scalar mass parameters:

$m^2(p^2) = m_0^2 +$



Loop integrals are potentially divergent

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \cdots$$

Λ: reference scale at which m^2 is known

g: coupling constant of the theory

C: coefficient calculable in specific theory

For mass shifts induced by radiative corrections to remain under control (not greatly exceed the value measured on the laboratory scale), *either*

- Λ must be small, *or*
- New Physics must intervene to cut off integral

But natural reference scale for Λ is

$$\Lambda \approx M_{\text{Planck}} = \left(\frac{\hbar c}{G_{\text{Newton}}}\right)^{1/2} \approx 1.22 \times 10^{19} \text{ GeV}$$

for $\text{SU(3)}_{\text{c}} \otimes \text{SU(2)}_{\text{L}} \otimes \text{U(1)}_Y$

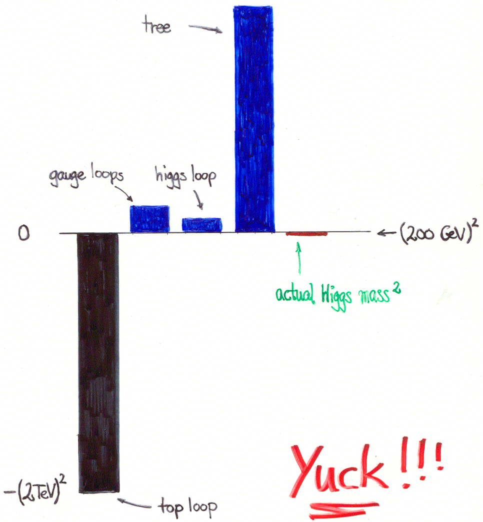
or

$$\Lambda \approx M_U \approx 10^{15}\text{-}10^{16} \text{ GeV} \quad \text{for unified theory}$$

Both $\gg v/\sqrt{2} \approx 175$ GeV $\Longrightarrow$

$\boxed{\text{New Physics at } E \lesssim 1 \text{ TeV}}$

Fine tuning the Higgs
$\Lambda = 10$ TeV



Martin Schmaltz, ICHEP02

Only a few distinct scenarios . . .

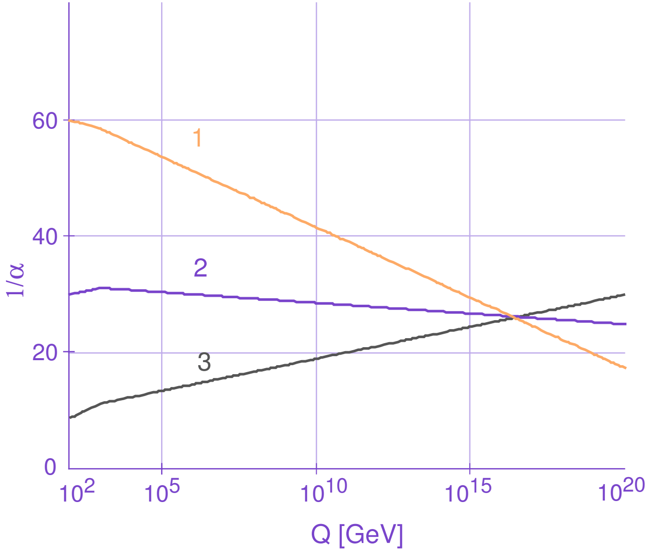
- Supersymmetry: balance contributions of fermion loops (-1) and boson loops $(+1)$
  Exact supersymmetry,

  $$\sum_{i=\substack{\text{fermions}\\+\text{bosons}}} C_i \int dk^2 = 0$$

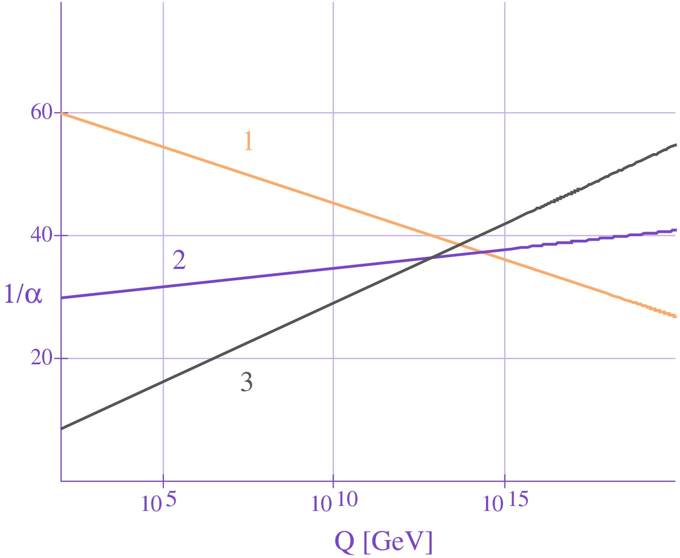
  Broken supersymmetry, shifts acceptably small if superpartner mass splittings are not too large

  $$g^2 \Delta M^2 \text{ “small enough”} \Rightarrow \widetilde{M} \lesssim 1 \text{ TeV}/c^2$$

Coupling constant unification through supersymmetry?



Coupling constant unification by many Higgs doublets?



Only a few distinct scenarios ...

- Composite scalars (technicolor): New physics arises on scale of composite Higgs-boson binding,

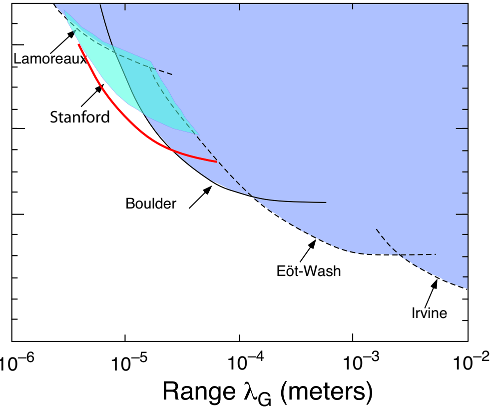
$$\Lambda_{\mathrm{TC}} \simeq O(1\ \mathrm{TeV})$$

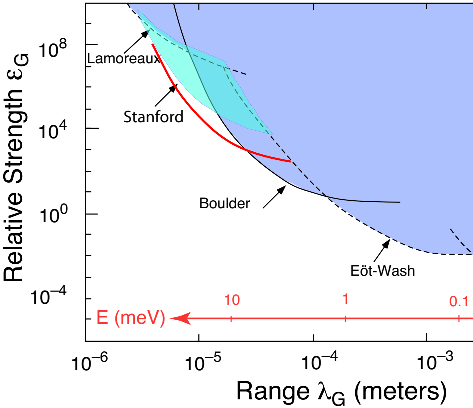
"Form factor" cuts effective range of integration

- Strongly interacting gauge sector: WW resonances, multiple W production, probably scalar bound state "quasiHiggs" with $M < 1$ TeV
- Extra spacetime dimensions: pseudo-Nambu – Goldstone bosons, extra particles cancel integrand ...
- Or maybe the problem is with (our understanding of) *gravity*, not with the electroweak theory?

Gravity follows $1/r^2$ law down to $\lesssim 1$ mm (few meV)

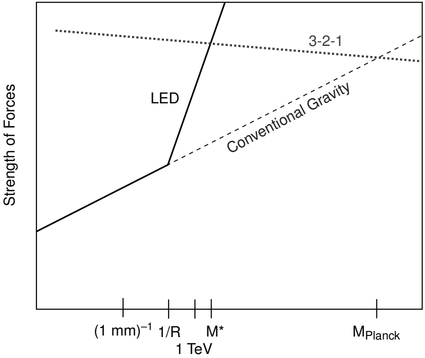
$$V(r) = -\int dr_1 \int dr_2 \frac{G_N \rho(r_1)\rho(r_2)}{r_{12}} \left[1 + \varepsilon_G \exp(-r_{12}/\lambda_G)\right]$$





Experiment leaves us free to consider modifications to Gravity even at (nearly) macroscopic distances

Suppose at scale R Gravity propagates in $3 + n$ spatial dimensions
Force law changes: $F \propto 1/r^{2+n}$



$G_{\mathrm{N}} \sim M_{\mathrm{Pl}}^{-2} \sim M^{\star\,-n-2} R^{-n}$ $\quad$ $M^{\star}$: gravity's true scale

Example: $\quad M^{\star} = 1$ TeV $\quad \Rightarrow R \lesssim 10^{-3}$ m for $n = 2$

$\boxed{M_{\mathrm{P}} \text{ is a mirage (false extrapolation)!}}$

Challenge: Understanding the Everyday (bis)

What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?

Consider the effects of all the $\mathrm{SU(3)_c \otimes SU(2)_L \otimes U(1)}_Y$ *interactions!*

With no Higgs mechanism . . .

- Quarks and leptons would remain massless
- QCD would confine the quarks in color-singlet hadrons
- *N mass little changed,* but p outweighs n
- QCD breaks EW to EM, gives $(1/2500\times$observed$)$ masses to W, Z, so weak-isospin force doesn't confine
- Rapid! β-decay $\Rightarrow$ lightest nucleus is n; no H atom
- Some light elements in BBN (?), but ∞ Bohr radius
- No atoms (as we know them) means no chemistry, no stable composite structures like solids and liquids

. . . the character of the physical world would be profoundly changed

High expectations for the Tevatron

- Biggest changes in the way we think about LHC experiments have come from the Tevatron:

  ▷ the large mass of the top quark and

  ▷ the success of silicon vertex detectors: heavy flavors

- Top quark is a unique window on EWSB and of interest in its own right: single top production
- Entering new terrain for new gauge bosons, strong dynamics, supersymmetry, Higgs, B_s mixing, ...

Why the LHC is so exciting (II)

- Electroweak theory (unitarity) tells us the 1-TeV scale is special: Higgs boson or other new physics (strongly interacting gauge bosons)
- Hierarchy problem $\Rightarrow$ other new physics nearby
- Our ignorance of EWSB obscures our view of other questions (e.g., identity problem). Lifting the veil at 1 TeV will change the face of physics

The cosmic connection

- Observational cosmology is like paleontology: reading the fossil record. Only a few layers are preserved, can we find more?
- Our reading of the fossil record is influenced by our world-view / theoretical framework.
- Cosmology shows us the world we must explain, provides questions and constraints; the answers will come from particle physics.

In a decade or two, we can hope to . . .

Understand electroweak symmetry breaking
Observe the Higgs boson
Measure neutrino masses and mixings
Establish Majorana neutrinos ($\beta\beta_{0\nu}$)
Thoroughly study CP violation in B decay
Exploit rare decays (K, D, . . .)
Observe n EDM, pursue e^- EDM
Use top as a tool
Observe new phases of matter
Understand hadron structure quantitatively
Uncover QCD's full implications
Observe proton decay
Understand the baryon excess
Catalogue matter & energy of universe
Measure dark energy equation of state
Search for new macroscopic forces
Determine GUT symmetry

Detect neutrinos from the universe
Learn how to quantize gravity
Learn why empty space is nearly weightless
Test the inflation hypothesis
Understand discrete symmetry violation
Resolve the hierarchy problem
Discover new gauge forces
Directly detect dark-matter particles
Explore extra spatial dimensions
Understand origin of large-scale structure
Observe gravitational radiation
Solve the strong CP problem
Learn whether supersymmetry is TeV-scale
Seek TeV dynamical symmetry breaking
Search for new strong dynamics
Explain the highest-energy cosmic rays
Formulate problem of identity

. . .

. . . learn the right questions to ask

. . . and rewrite the textbooks!