

*The Standard Model  
(Electroweak Theory)  
and Higgs Physics*

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# A Decade of Discovery Past . . .

- ▷ Electroweak theory → law of nature
- ▷ Higgs-boson influence observed in the vacuum
- ▷ Neutrino flavor oscillations:  $\nu_\mu \rightarrow \nu_\tau$ ,  
 $\nu_e \rightarrow \nu_\mu / \nu_\tau$
- ▷ Understanding QCD
- ▷ Discovery of top quark
- ▷ Direct  $\mathcal{CP}$  violation in  $K \rightarrow \pi\pi$
- ▷  $B$ -meson decays violate  $\mathcal{CP}$
- ▷ Flat universe dominated by dark matter, energy
- ▷ Detection of  $\nu_\tau$  interactions
- ▷ Quarks & leptons structureless at TeV scale

# A Decade of Discovery Past . . .

- ▷ Electroweak theory → law of nature  
[ $Z$ ,  $e^+e^-$ ,  $\bar{p}p$ ,  $\nu N$ ,  $(g - 2)_\mu$ , . . .]
- ▷ Higgs-boson influence observed in the vacuum  
[EW experiments]
- ▷ Neutrino flavor oscillations:  $\nu_\mu \rightarrow \nu_\tau$ ,  
 $\nu_e \rightarrow \nu_\mu/\nu_\tau$  [ $\nu_\odot$ ,  $\nu_{\text{atm}}$ , reactors]
- ▷ Understanding QCD  
[heavy flavor,  $Z^0$ ,  $\bar{p}p$ ,  $\nu N$ ,  $ep$ , ions, lattice]
- ▷ Discovery of top quark [ $\bar{p}p$ ]
- ▷ Direct  $\mathcal{CP}$  violation in  $K \rightarrow \pi\pi$  [fixed-target]
- ▷  $B$ -meson decays violate  $\mathcal{CP}$  [ $e^+e^- \rightarrow B\bar{B}$ ]
- ▷ Flat universe dominated by dark matter, energy  
[SN Ia, CMB, LSS]
- ▷ Detection of  $\nu_\tau$  interactions [fixed-target]
- ▷ Quarks & leptons structureless at TeV scale  
[mainly colliders]

## Goal: Understanding the Everyday

- ▷ Why are there atoms?
- ▷ Why chemistry?
- ▷ Why stable structures?
- ▷ What makes life possible?

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- ▷ Why are there atoms?
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*What would the world be like,  
without a (Higgs) mechanism to hide  
electroweak symmetry and give  
masses to the quarks and leptons?*

Searching for the mechanism of electroweak symmetry breaking, we seek to understand

*why the world is the way it is.*

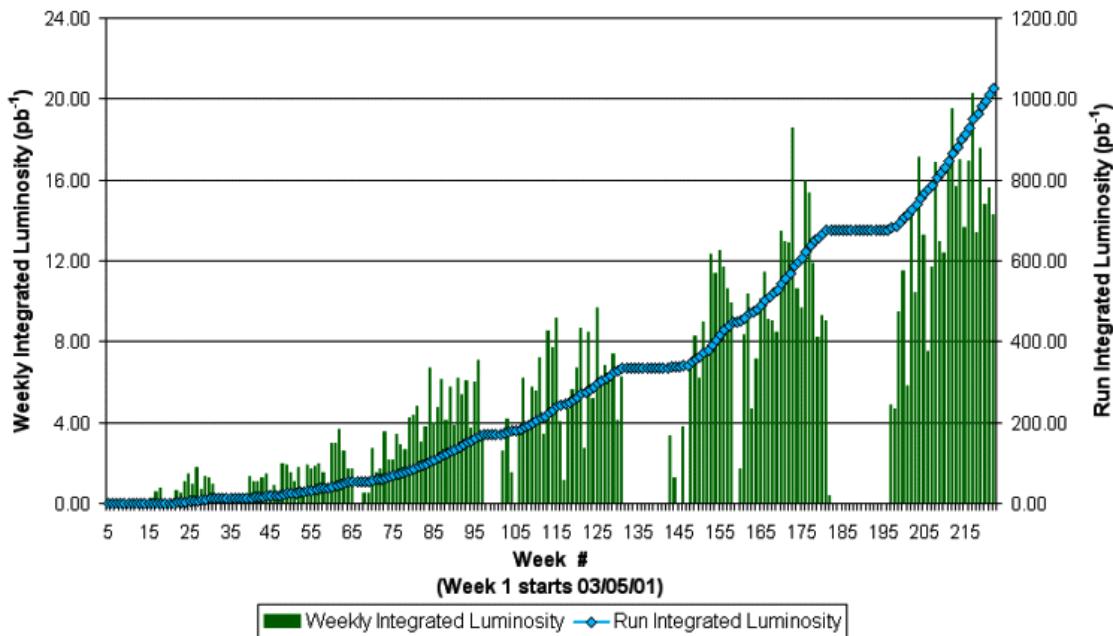
This is one of the deepest questions humans have ever pursued, and

*it is coming within the reach of particle physics.*

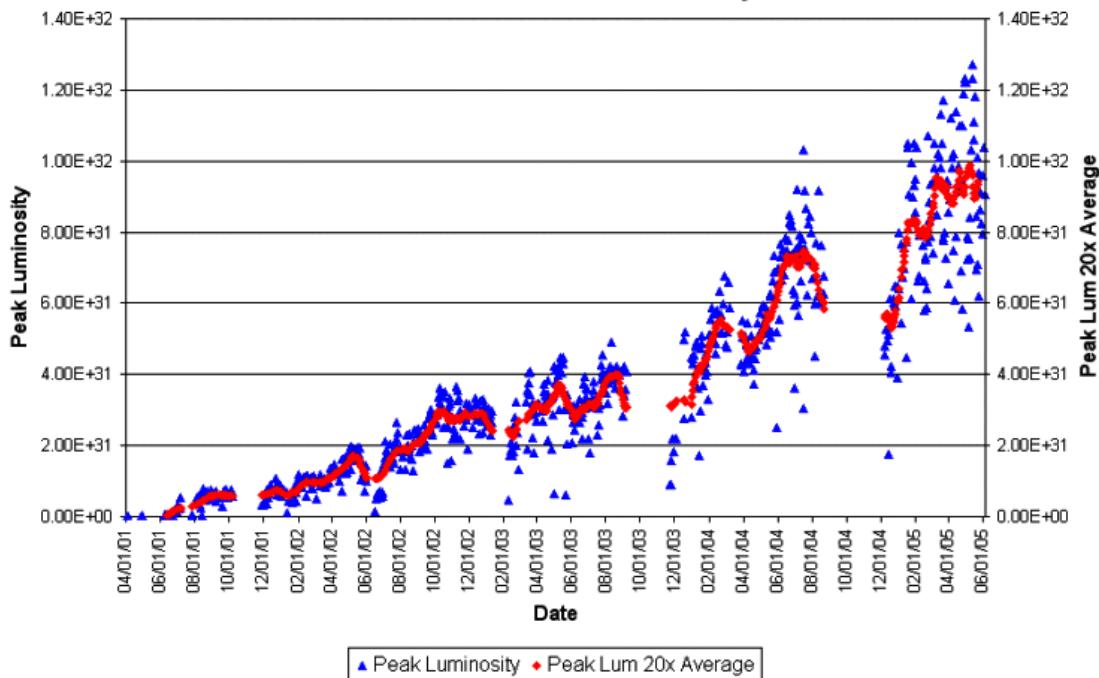
Tevatron Collider is running *now*,  
breaking new ground in sensitivity



### Collider Run II Integrated Luminosity



### Collider Run II Peak Luminosity



## Tevatron Collider in a Nutshell

980-GeV protons, antiprotons  
 $(2\pi \text{ km})$

*frequency of revolution*  $\approx 45\,000 \text{ s}^{-1}$

392 ns between crossings  
 $(36 \otimes 36 \text{ bunches})$

collision rate  $= \mathcal{L} \cdot \sigma_{\text{inelastic}} \approx 10^7 \text{ s}^{-1}$

$c \approx 10^9 \text{ km/h}; \quad v_p \approx c - 495 \text{ km/h}$

Record  $\mathcal{L}_{\text{init}} = 1.27 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$

[ISR:  $pp, 1.4$ ]

Maximum  $\bar{p}$  at Low  $\beta$ :  $1.661 \times 10^{12}$

The Large Hadron Collider will  
operate soon, breaking new ground  
in energy and sensitivity



## LHC in a nutshell

7-TeV protons on protons (27 km);

$$v_p \approx c - 10 \text{ km/h}$$

Novel two-in-one dipoles ( $\approx 9$  teslas)

Startup:  $43 \otimes 43$  bunches,

$$\mathcal{L} \approx 6 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$$

Early: 936 bunches,

$$\mathcal{L} \gtrsim 5 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1} [75 \text{ ns}]$$

First year? 2808 bunches,

$$\mathcal{L} \rightarrow 2 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$$

25 ns bunch spacing

Eventual  $\mathcal{L} \gtrsim 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ :

$$100 \text{ fb}^{-1}/\text{year}$$

# Tentative Outline . . .

## ▷ Preliminaries

Current state of particle physics

A few words about QCD

Sources of mass

## ▷ Antecedents of the electroweak theory

What led to EW theory

What EW theory needs to explain

## ▷ Some consequences of the Fermi theory

$\mu$  decay

$\nu e$  scattering

# ... Outline ...

▷  $SU(2)_L \otimes U(1)_Y$  theory

Gauge theories

Spontaneous symmetry breaking

Consequences:  $W^\pm$ ,  $Z^0/\text{NC}$ ,  $H$ ,  $m_f$ ?

Measuring  $\sin^2 \theta_W$  in  $\nu e$  scattering

GIM / CKM

▷ Phenomena at tree level and beyond

$Z^0$  pole

$W$  mass and width

Atomic parity violation

Looking for trouble

$m_t$ ,  $M_W$ ,  $M_Z$  correlation

Vacuum energy problem

# ... Outline

- ▷ The Higgs boson and the 1-TeV scale
  - Why the Higgs boson must exist
  - Higgs properties, constraints
  - How well can we anticipate  $M_H$ ?
  - Higgs searches
- ▷ The problems of mass
- ▷ The EW scale and beyond
  - Hierarchy problem
  - Why is the EW scale so small?
  - Why is the Planck scale so large?
- ▷ Outlook

# General References

- ▷ C. Quigg, “Nature’s Greatest Puzzles,”  
hep-ph/0502070
- ▷ C. Quigg, “The Electroweak Theory,”  
hep-ph/0204104 (TASI 2000 Lectures)
- ▷ C. Quigg, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions*
- ▷ R. N. Cahn & G. Goldhaber, *Experimental Foundations of Particle Physics*
- ▷ G. Altarelli & M. Grünewald, “Precision Electroweak Tests of the Standard Model,”  
hep-ph/0404165
- ▷ F. Teubert, “Electroweak Physics,” ICHEP04
- ▷ S. Eidelman et al., “Review of Particle Physics,”  
*Phys. Lett.* **B592**, 1 (2004)

Problem sets: <http://lutece.fnal.gov/TASI/default.html>

# Our picture of matter

Pointlike constituents ( $r < 10^{-18}$  m)

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

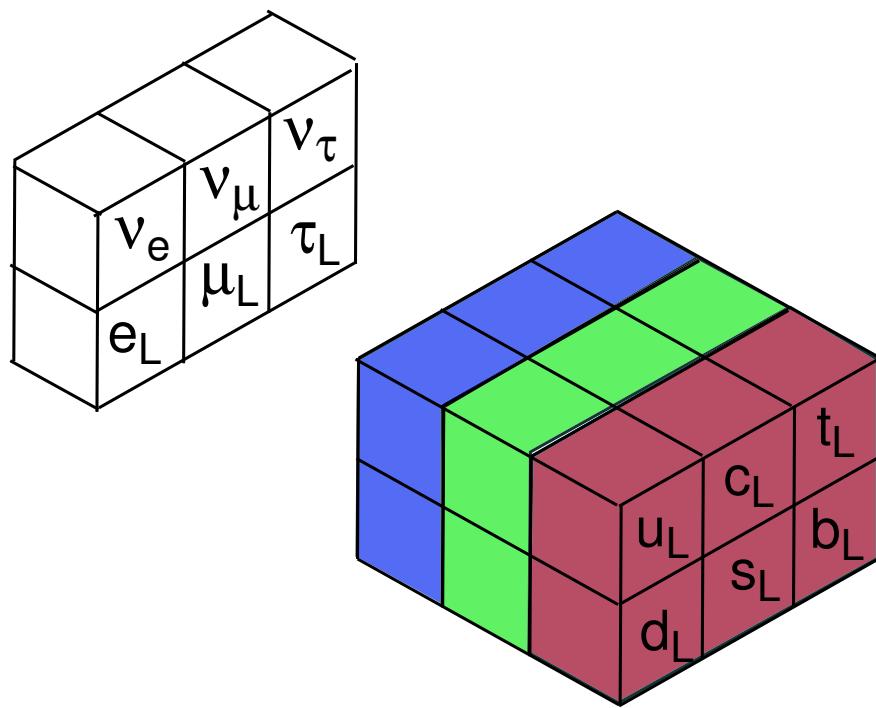
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

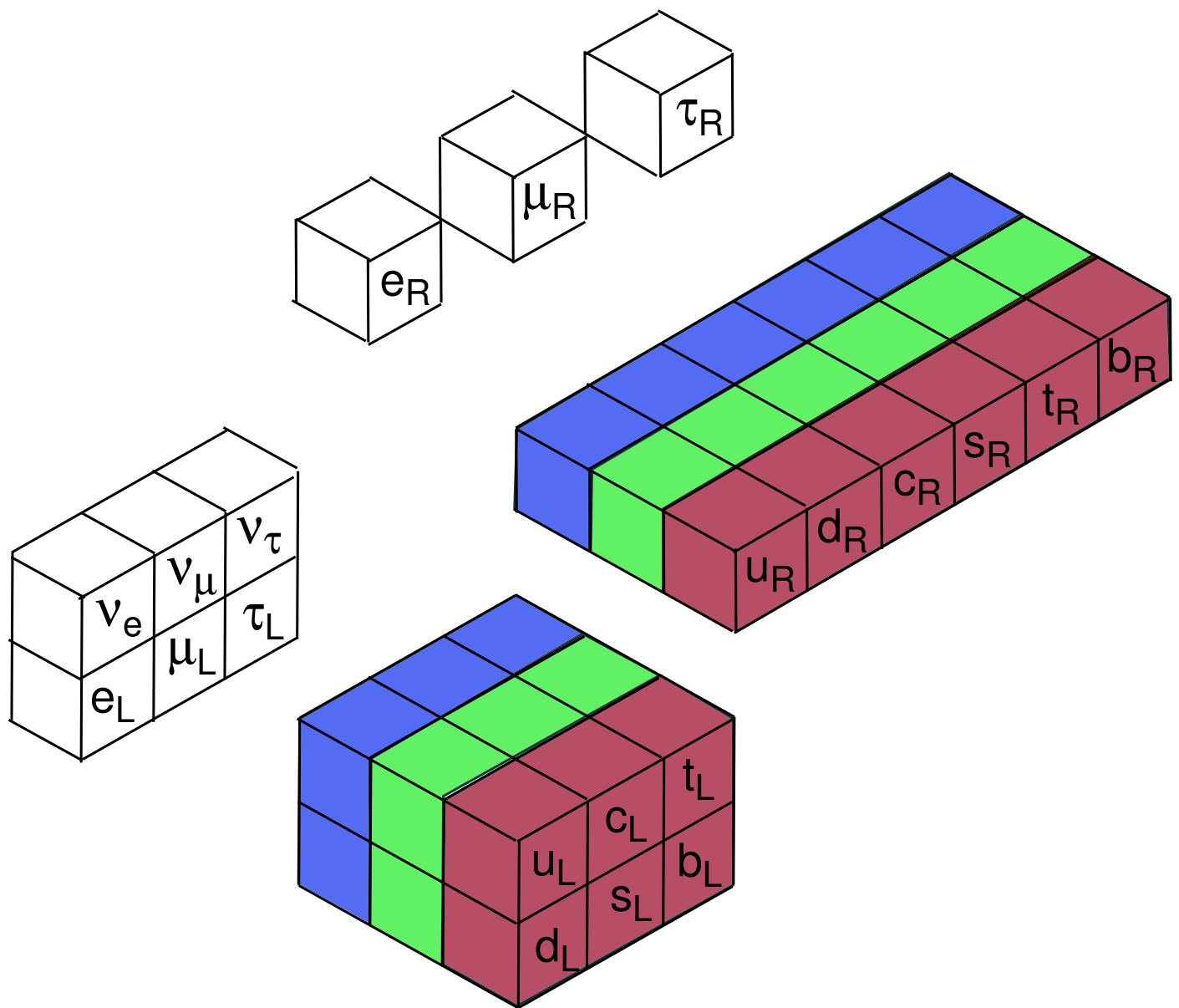
Few fundamental forces, derived from gauge symmetries

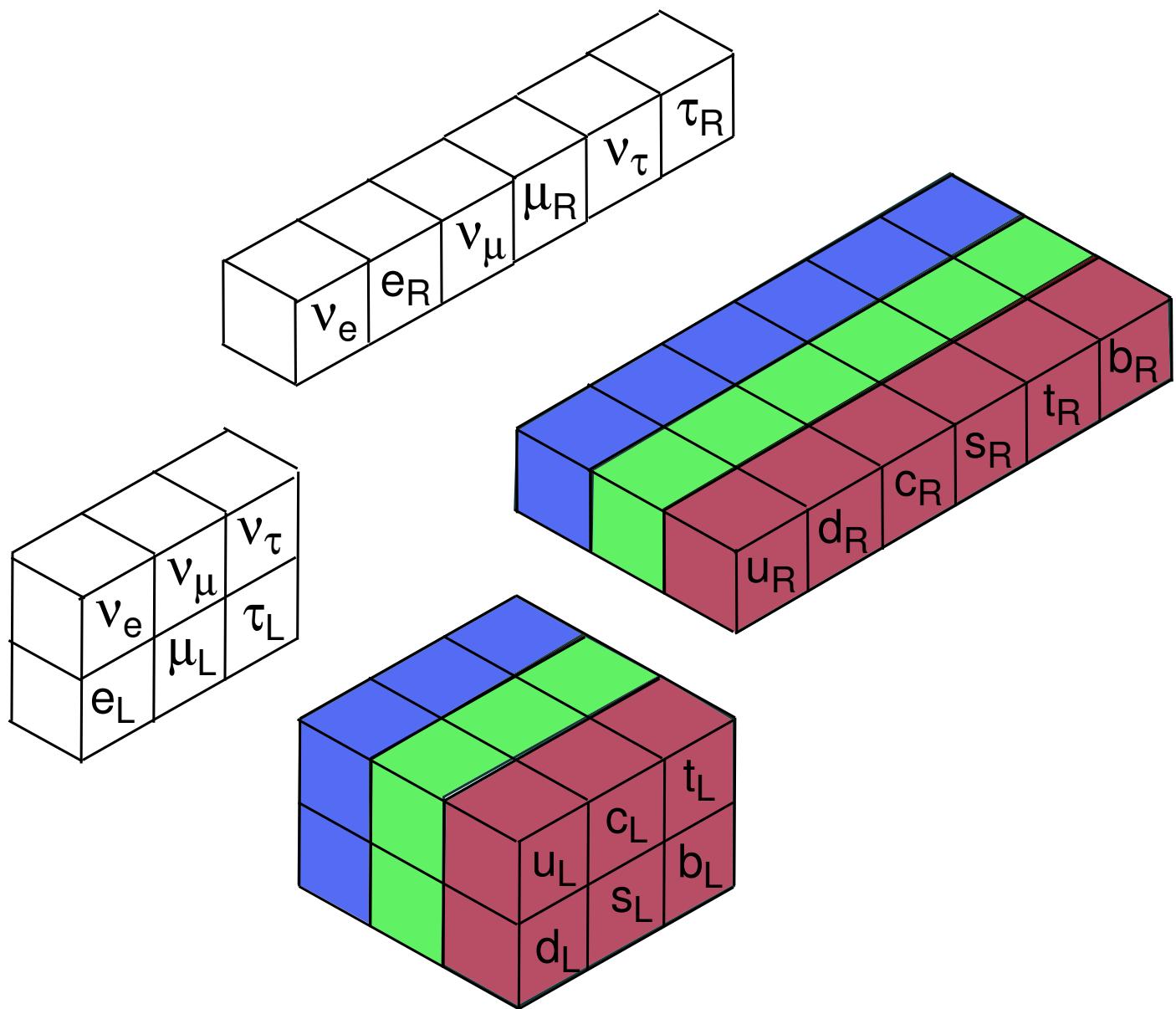
$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

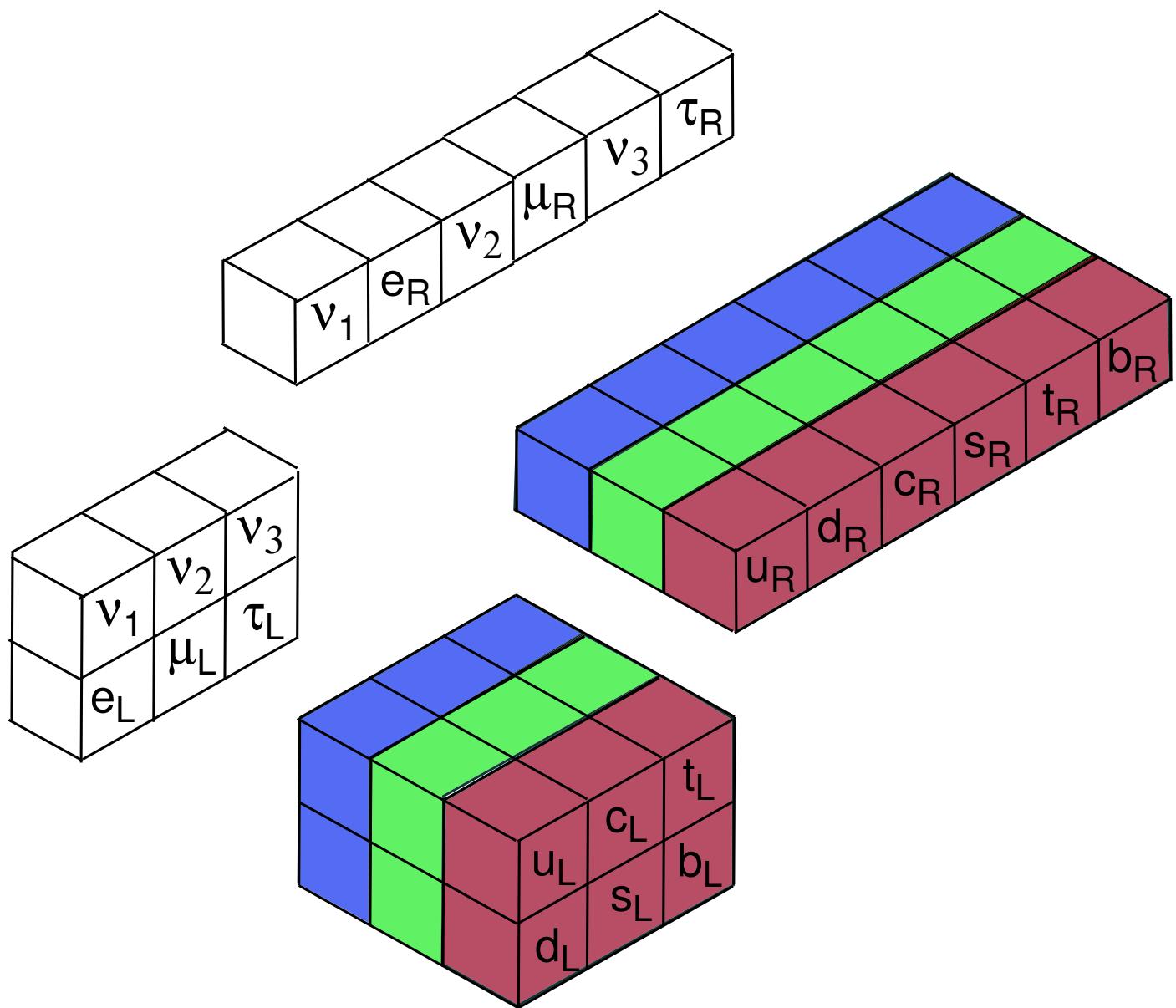
Electroweak symmetry breaking

Higgs mechanism?









## Elementarity

- ▷ Are quarks and leptons structureless?

## Symmetry

- ▷ Electroweak symmetry breaking and the 1-TeV scale
- ▷ Origin of gauge symmetries
- ▷ Meaning of discrete symmetries

## Unity

- ▷ Coupling constant unification
- ▷ Unification of quarks and leptons
  - (neutrality of atoms  $\Rightarrow$  new forces!);  
of constituents and force particles
- ▷ Incorporation of gravity

## Identity

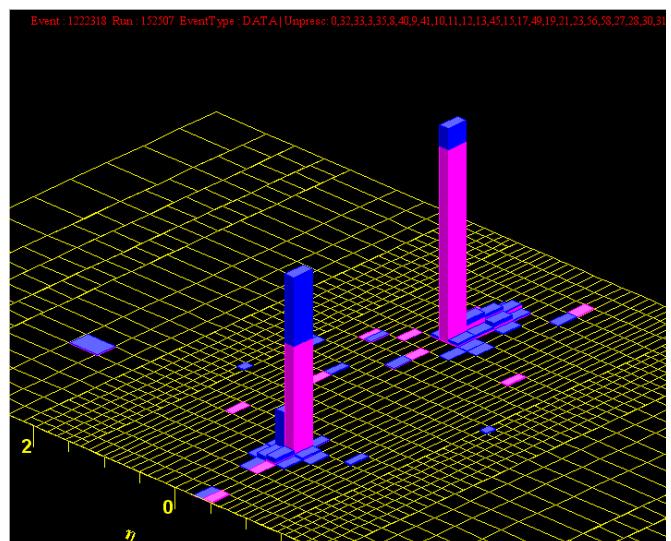
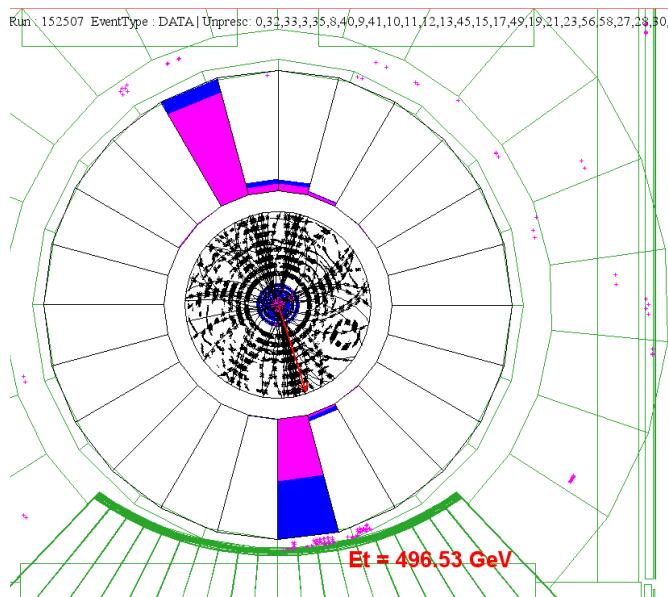
- ▷ Fermion masses and mixings; CP violation;  $\nu$  oscillations
- ▷ What makes an electron an  $e$  and a top quark a  $t$ ?

## Topography

- ▷ What is the fabric of space and time?  
... the origin of space and time?

# Elementarity

## The World's Most Powerful Microscopes nanonanophysics



CDF dijet event ( $\sqrt{s} = 1.96 \text{ TeV}$ ):

$$E_T = 1.364 \text{ TeV}$$



# Elementarity

If the Lagrangian has the form  $\pm \frac{g^2}{2\Lambda^2} \bar{\psi}_L \gamma_\mu \psi_L \bar{\psi}_L \gamma^\mu \psi_L$  (with  $g^2/4\pi$  set equal to 1), then we define  $\Lambda \equiv \Lambda_{LL}^\pm$ . For the full definitions and for other forms, see the Note in the Listings on Searches for Quark and Lepton Compositeness in the full *Review* and the original literature.

$\Lambda_{LL}^+(eeee)$	> 8.3 TeV, CL = 95%
$\Lambda_{LL}^-(eeee)$	> 10.3 TeV, CL = 95%
$\Lambda_{LL}^+(eee\mu\mu)$	> 8.5 TeV, CL = 95%
$\Lambda_{LL}^-(eee\mu\mu)$	> 6.3 TeV, CL = 95%
$\Lambda_{LL}^+(eee\tau\tau)$	> 5.4 TeV, CL = 95%
$\Lambda_{LL}^-(eee\tau\tau)$	> 6.5 TeV, CL = 95%
$\Lambda_{LL}^+(\ell\ell\ell\ell)$	> 9.0 TeV, CL = 95%
$\Lambda_{LL}^-(\ell\ell\ell\ell)$	> 7.8 TeV, CL = 95%
$\Lambda_{LL}^+(eeeuu)$	> 23.3 TeV, CL = 95%
$\Lambda_{LL}^-(eeeuu)$	> 12.5 TeV, CL = 95%
$\Lambda_{LL}^+(eeedd)$	> 11.1 TeV, CL = 95%
$\Lambda_{LL}^-(eeedd)$	> 26.4 TeV, CL = 95%
$\Lambda_{LL}^+(eeecc)$	> 1.0 TeV, CL = 95%
$\Lambda_{LL}^-(eeecc)$	> 2.1 TeV, CL = 95%
$\Lambda_{LL}^+(eee bb)$	> 5.6 TeV, CL = 95%
$\Lambda_{LL}^-(eee bb)$	> 4.9 TeV, CL = 95%
$\Lambda_{LL}^+(\mu\mu qq)$	> 2.9 TeV, CL = 95%
$\Lambda_{LL}^-(\mu\mu qq)$	> 4.2 TeV, CL = 95%
$\Lambda(\ell\nu\ell\nu)$	> 3.10 TeV, CL = 90%
$\Lambda(e\nu qq)$	> 2.81 TeV, CL = 95%
$\Lambda_{LL}^+(qqqq)$	> 2.7 TeV, CL = 95%
$\Lambda_{LL}^-(qqqq)$	> 2.4 TeV, CL = 95%
$\Lambda_{LL}^+(\nu\nu qq)$	> 5.0 TeV, CL = 95%
$\Lambda_{LL}^-(\nu\nu qq)$	> 5.4 TeV, CL = 95%

# Two views of Symmetry

## 1. *Indistinguishability*

One object transformed into another

Familiar (and useful!) from

Global Symmetries: isospin,  $SU(3)_f$ , ...

Spacetime Symmetries

Gauge Symmetries

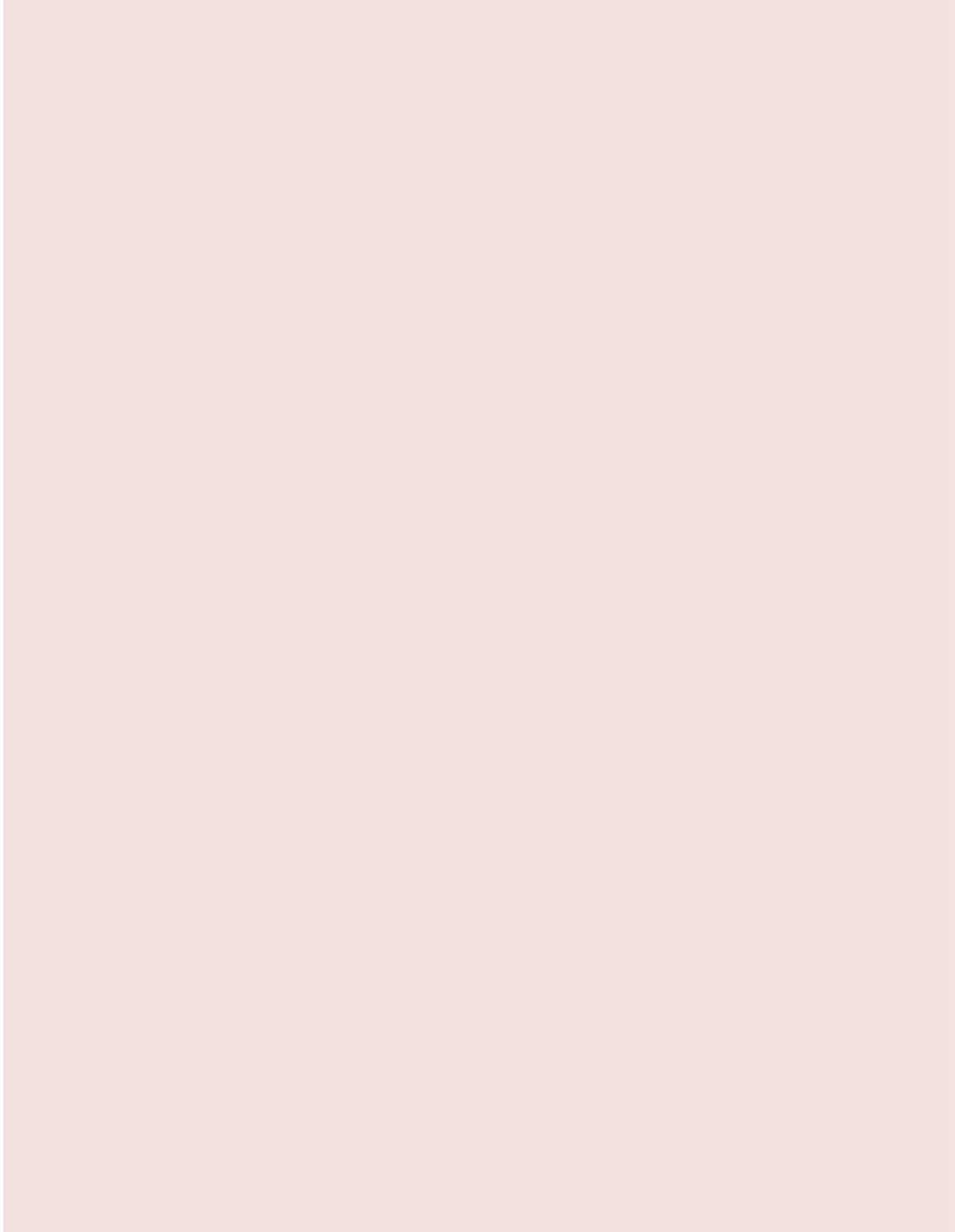
“EQUIVALENCE”

Idealize more perfect worlds, the better  
to understand our diverse, changing world

Unbroken unified theory: perfect world of  
equivalent forces, interchangeable massless  
particles ... *Perfectly boring?*

*Symmetry  $\Leftrightarrow$  Disorder*

# A Perfect World



# Two views of Symmetry

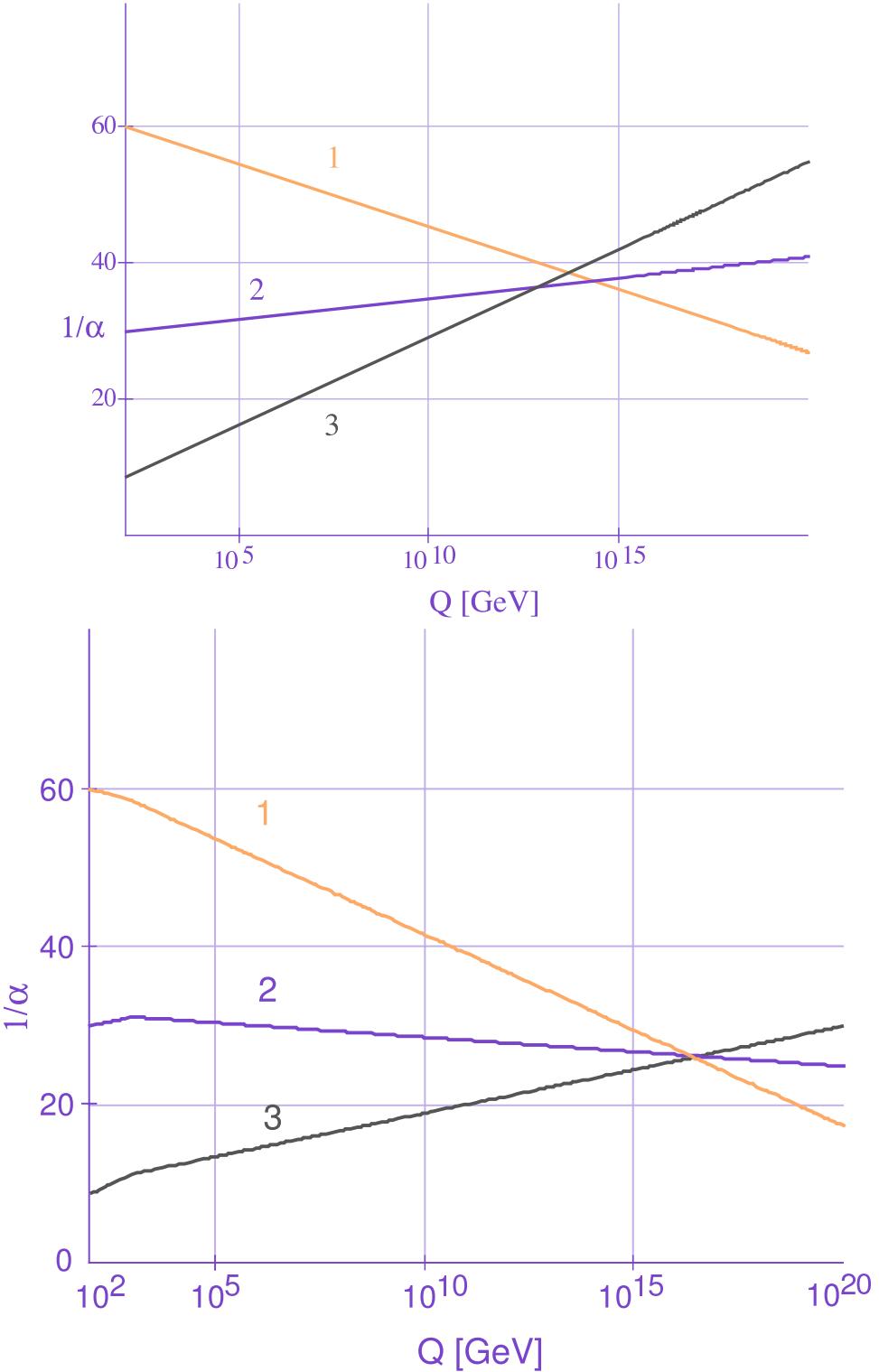
## 2. *Unobservable*

Goodness of a symmetry means something cannot be measured

e.g., vanishing asymmetry

Un observable	Transformation	Conserved
Absolute position	$\vec{r} \rightarrow \vec{r} + \vec{\Delta}$	$\vec{p}$
Absolute time	$t \rightarrow t + \delta$	$E$
Absolute orientation	$\hat{r} \rightarrow \hat{r}'$	$\vec{L}$
Absolute velocity	$\vec{v} \rightarrow \vec{v} + \vec{w}$	
Absolute right	$\vec{r} \rightarrow -\vec{r}$	P
Absolute future	$t \rightarrow -t$	T
Absolute charge	$Q \rightarrow -Q$	C
Absolute phase		
	:	

# Unity



# QCD is part of the standard model

*... a remarkably simple, successful, and rich theory*

Wilczek, hep-ph/9907340

## ▷ Perturbative QCD

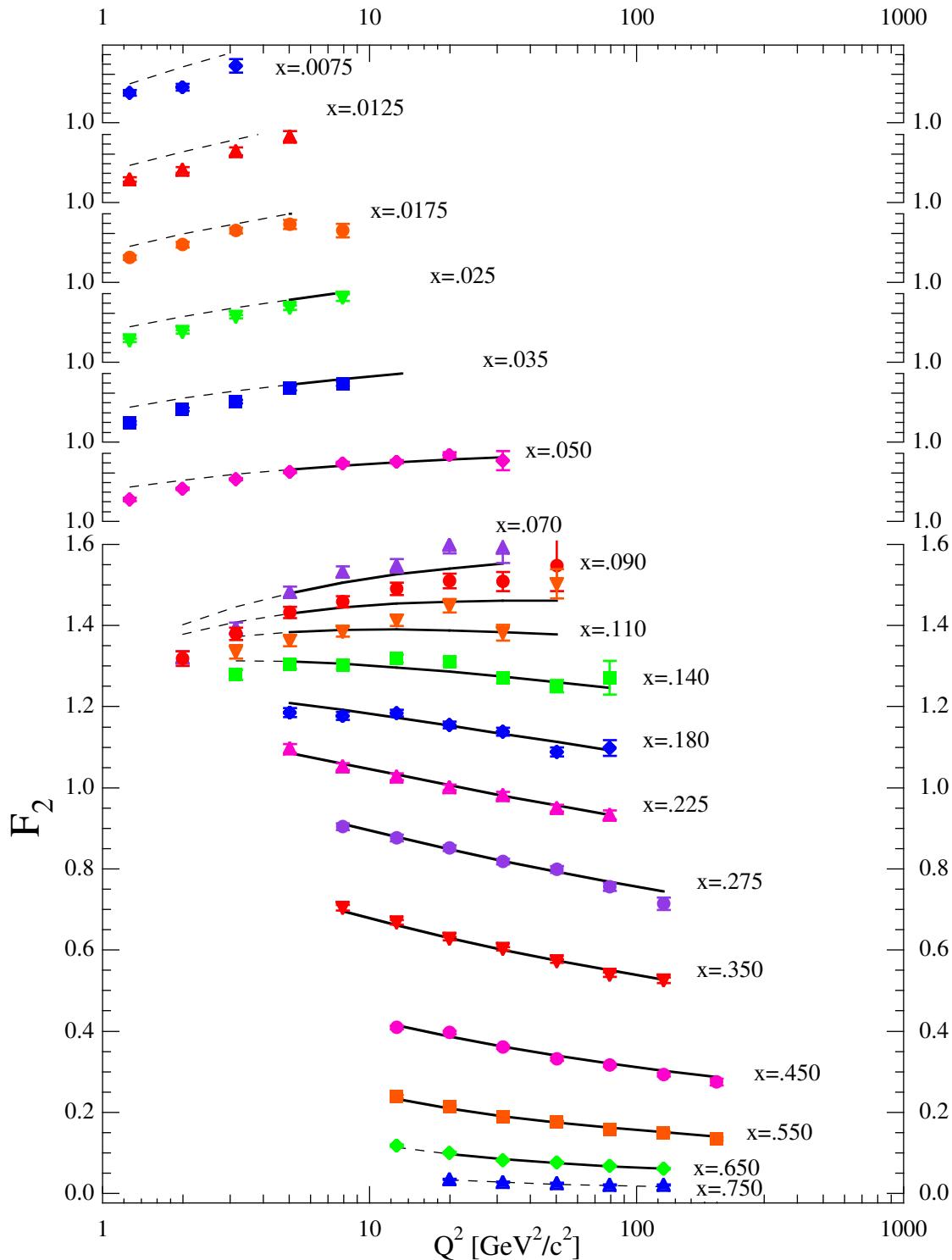
- Exists, thanks to asymptotic freedom
- Describes many phenomena in quantitative detail:
  - ▷  $Q^2$ -evolution of structure functions
  - ▷ Jet production in  $\bar{p}p$  collisions
  - ▷ Many decays, event shapes, ...
- We can measure the running of  $\alpha_s$   
*(engineering value for unification)*

## ▷ Nonperturbative (lattice) QCD

- Predicts the hadron spectrum
- Gives our best information on quark masses, etc.

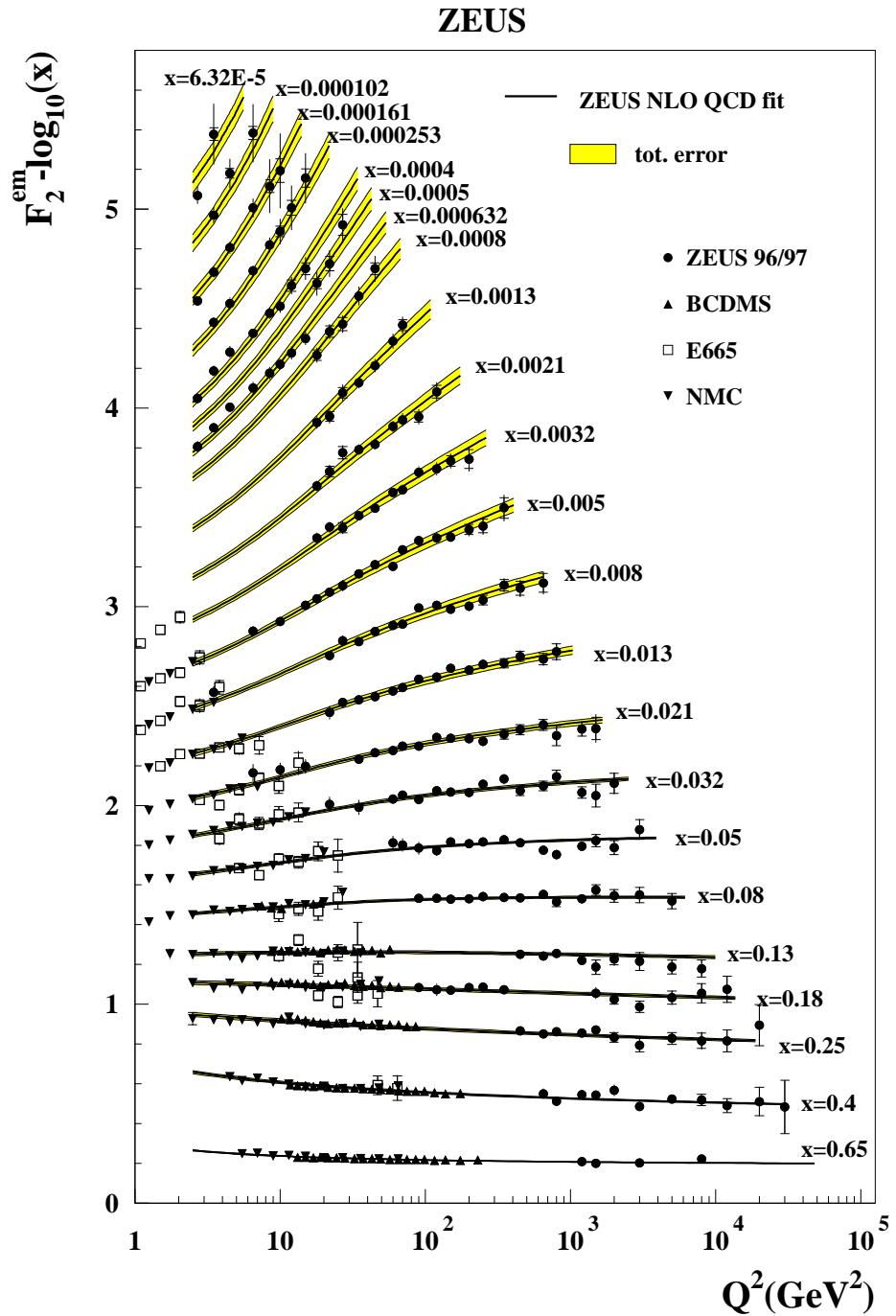
El-Khadra & Luke, hep-ph/0208114

# $F_2(x, Q^2)$ in $\nu N$ interactions (CCFR)



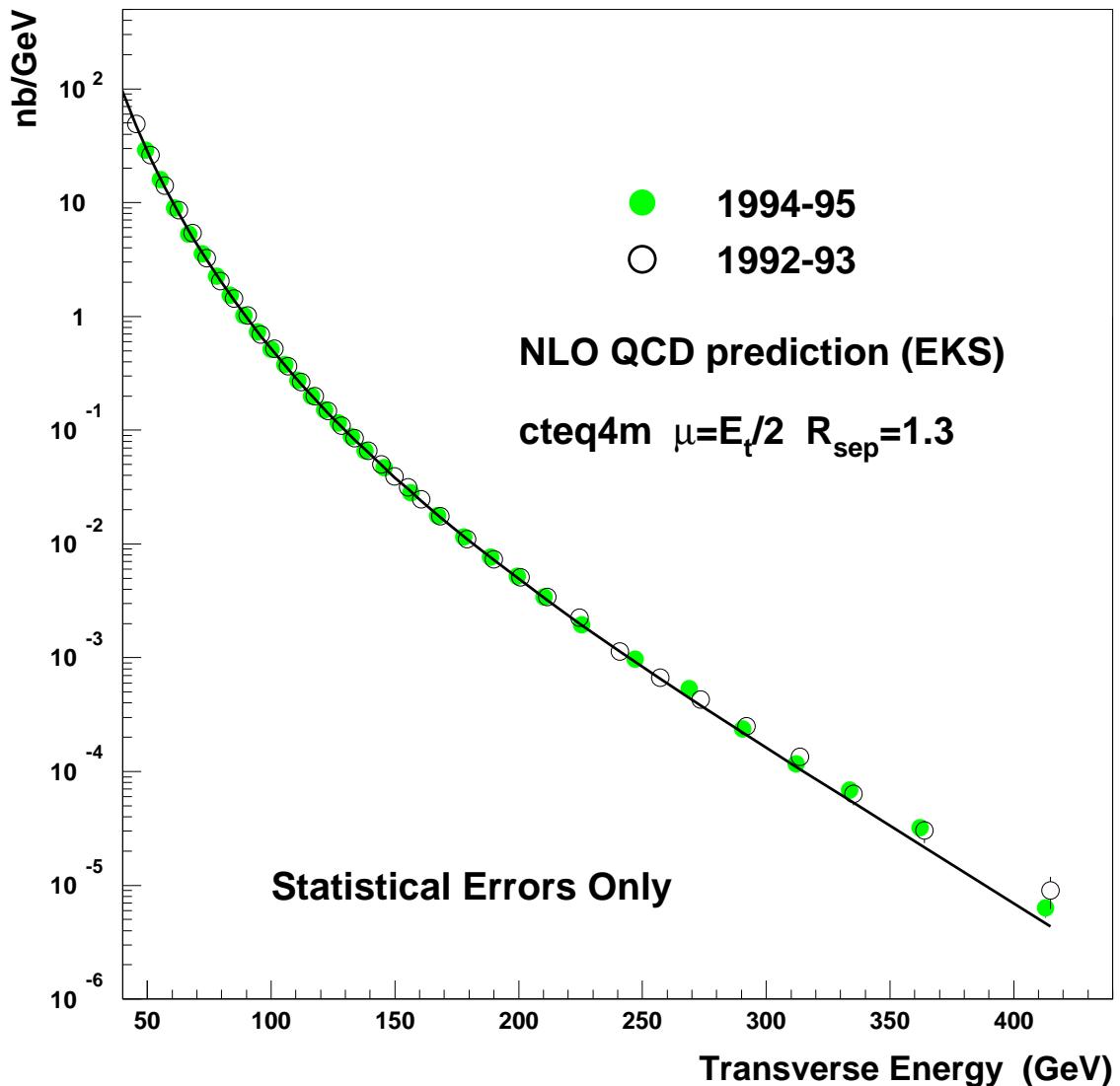
Conrad, Shaevitz, Bolton, *Rev. Mod. Phys.* **70**, 1341 (1998).

# $F_2(x, Q^2)$ in $\ell N$ interactions (ZEUS)



ZEUS, hep-ex/0208023.

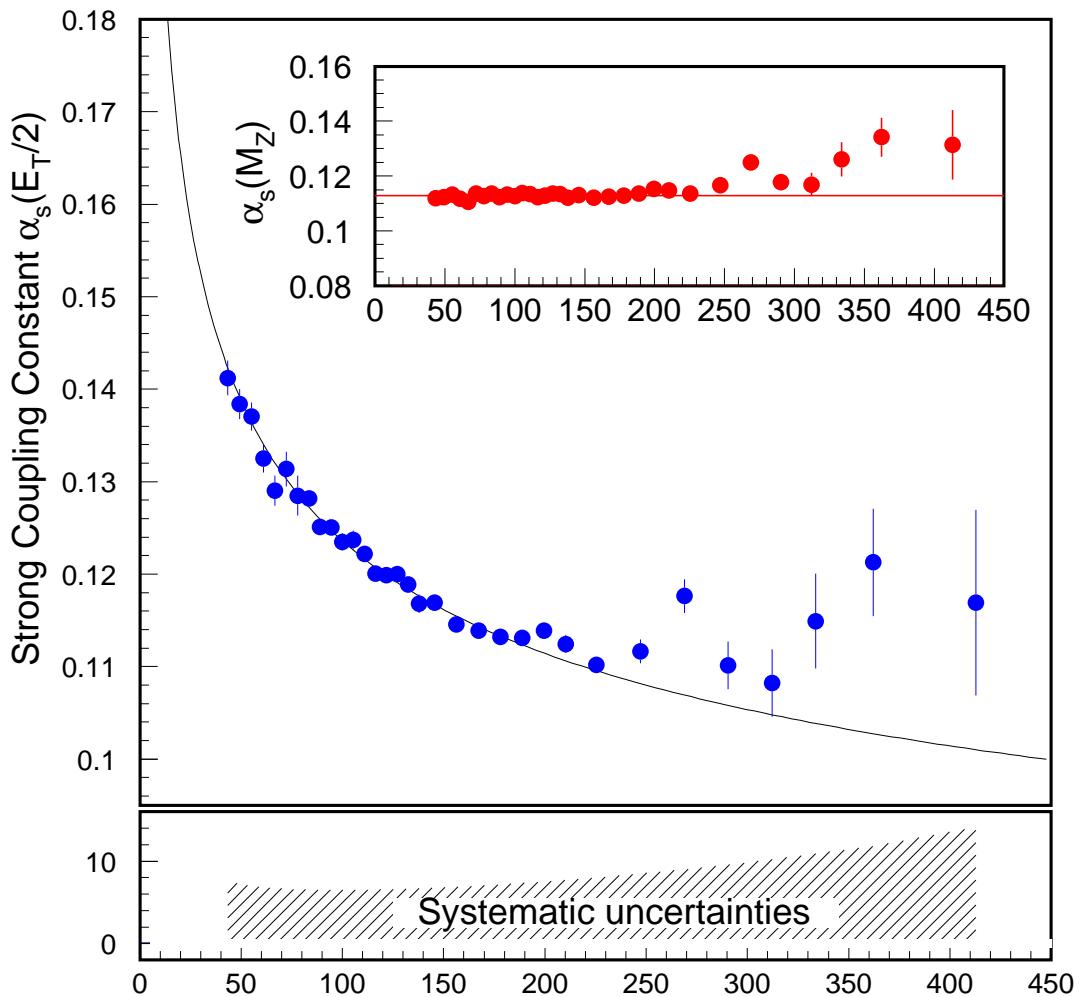
# Inclusive jet cross section at $\sqrt{s} = 1.8 \text{ TeV}$ (CDF)



T. Affolder et al. (CDF), *Phys. Rev. D64*, 032001 (2001)

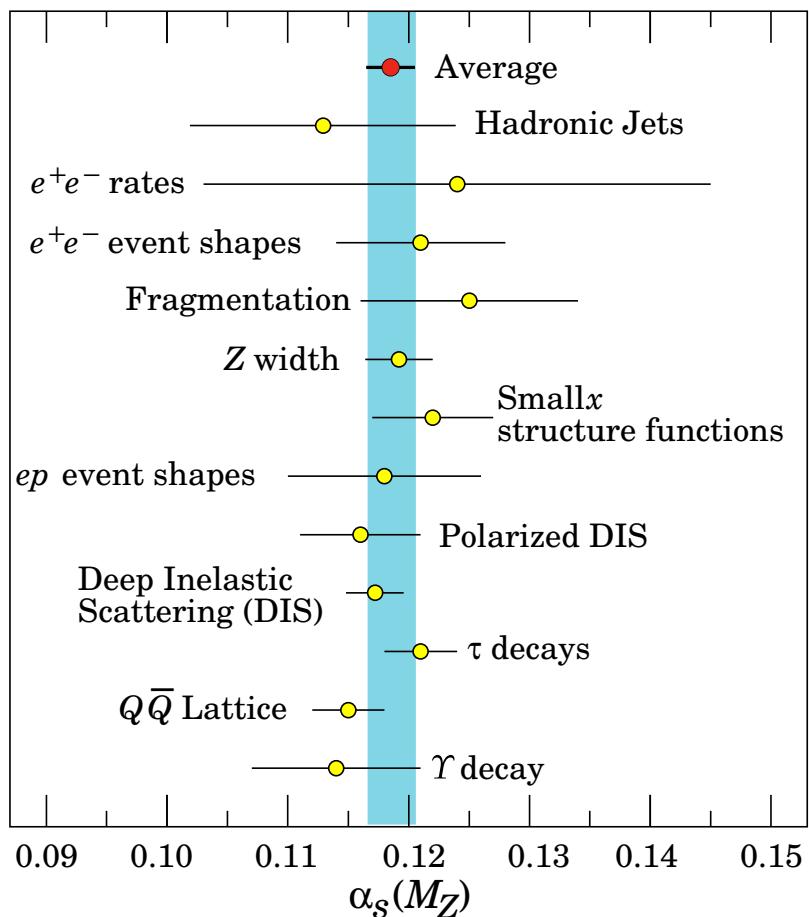
## $\alpha_s(E_T/2)$ from $\bar{p}p \rightarrow \text{jets}$

CDF Preliminary

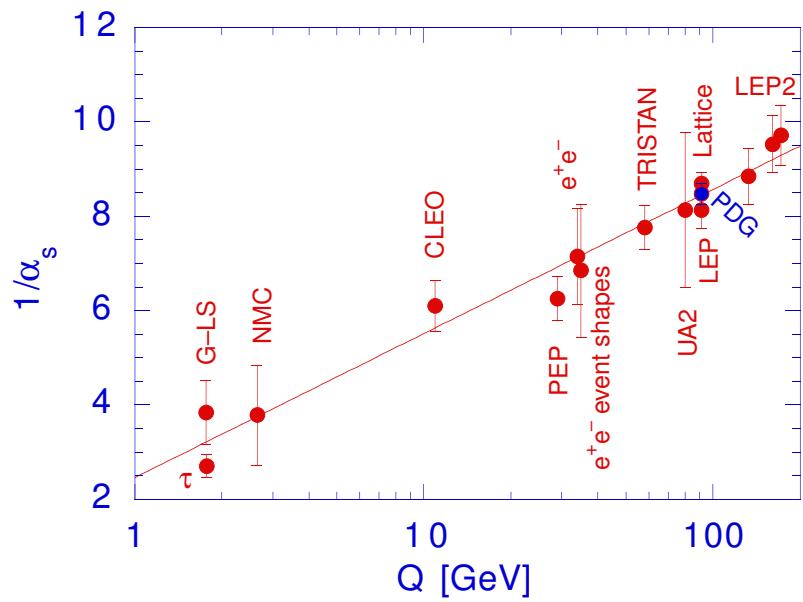


T. Affolder et al. (CDF), *Phys. Rev. Lett.* **88**, 042001 (2002)

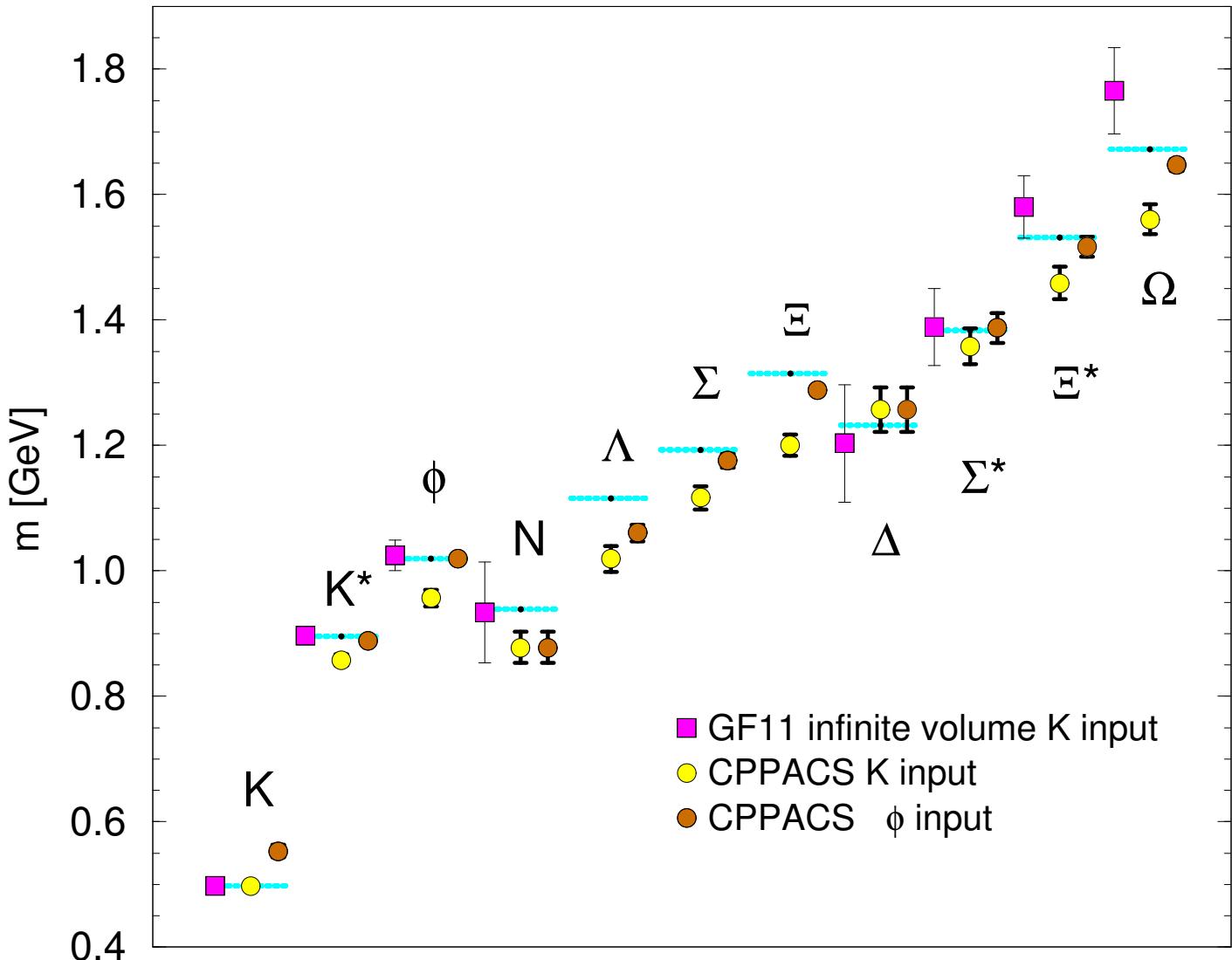
Comprehensive survey: W. de Boer, hep-ph/0407021



PDG



# Quenched hadron spectrum



S. Aoki, et al. (CP-PACS), *Phys. Rev. Lett.* **84**, 238 (2000)

(No dynamical fermions)

# The Origins of Mass

(masses of nuclei “understood” )

$p, [\pi], \rho$  understood: QCD

*confinement energy* is the source  
“Mass without mass”

We understand the visible mass of the Universe  
. . . without the Higgs mechanism

$W, Z$  electroweak symmetry breaking

$$M_W^2 = \frac{1}{2}g^2 v^2 = \pi\alpha/G_F \sqrt{2} \sin^2 \theta_W$$

$$M_Z^2 = M_W^2 / \cos^2 \theta_W$$

$q, \ell^\mp$  EWSB + Yukawa couplings

$\nu_\ell$  EWSB + Yukawa couplings; new physics?

All fermion masses  $\Leftrightarrow$  physics beyond standard model

$H$  ?? fifth force ??

# Antecedents of EW Theory

Commins & Bucksbaum, *Weak Int<sup>ns</sup> of Leptons & Quarks*

• 1896 - Sophie Joliot-Curie & Pierre Curie  
Paper mica. Curie de Curie mica  
Effet de la radioactivité sur les minéraux  
Vidage à l'air.



1896: Becquerel radioactivity

Several varieties, including  $\beta^-$  decay



Examples:

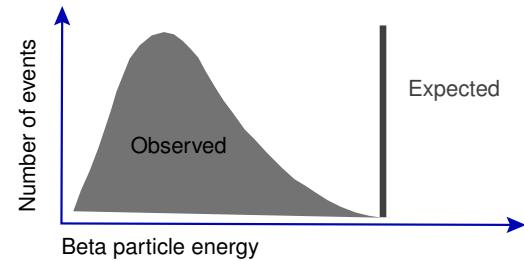


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$\beta^+$ -emitters,  ${}^A Z \rightarrow {}^A(Z-1) + \beta^+$ , are rare among naturally occurring isotopes. Radio-phosphorus produced 1934 by the Joliot-Curie, *after* positron discovery in cosmic rays.

${}^{19} Ne \rightarrow {}^{19} F + \beta^+$  studied for right-handed charged currents and time reversal invariance; *positron-emission tomography*

1914: Chadwick  $\beta$  spectrum

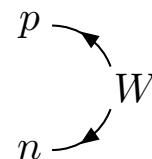


Energy conservation in question

1930: Pauli  $\approx$  massless, neutral, penetrating particle  
nuclear spin & statistics

↳ neutrino  $\nu$

$\beta$  decay first hint for flavor



charged-current, flavor-changing interactions

1932: Chadwick neutron

↳ isospin symmetry

## Neutron & flavor symmetry

$$M(n) = 939.565\,63 \pm 0.000\,28 \text{ MeV}/c^2$$

$$M(p) = 938.272\,31 \pm 0.000\,28 \text{ MeV}/c^2$$

$$\Delta M = 1.293318 \pm 0.000\,009 \text{ MeV}/c^2$$

$$\boxed{\Delta M/M \approx 1.4 \times 10^{-3}}$$

## Charge-independent nuclear forces?

$$^3\text{H}(pnn) = 8.481\,855 \pm 0.000\,013 \text{ MeV}$$

$$^3\text{He}(ppn) = 7.718\,109 \pm 0.000\,013 \text{ MeV}$$

$$\Delta(\text{B.E.}) = 0.763\,46 \text{ MeV}$$

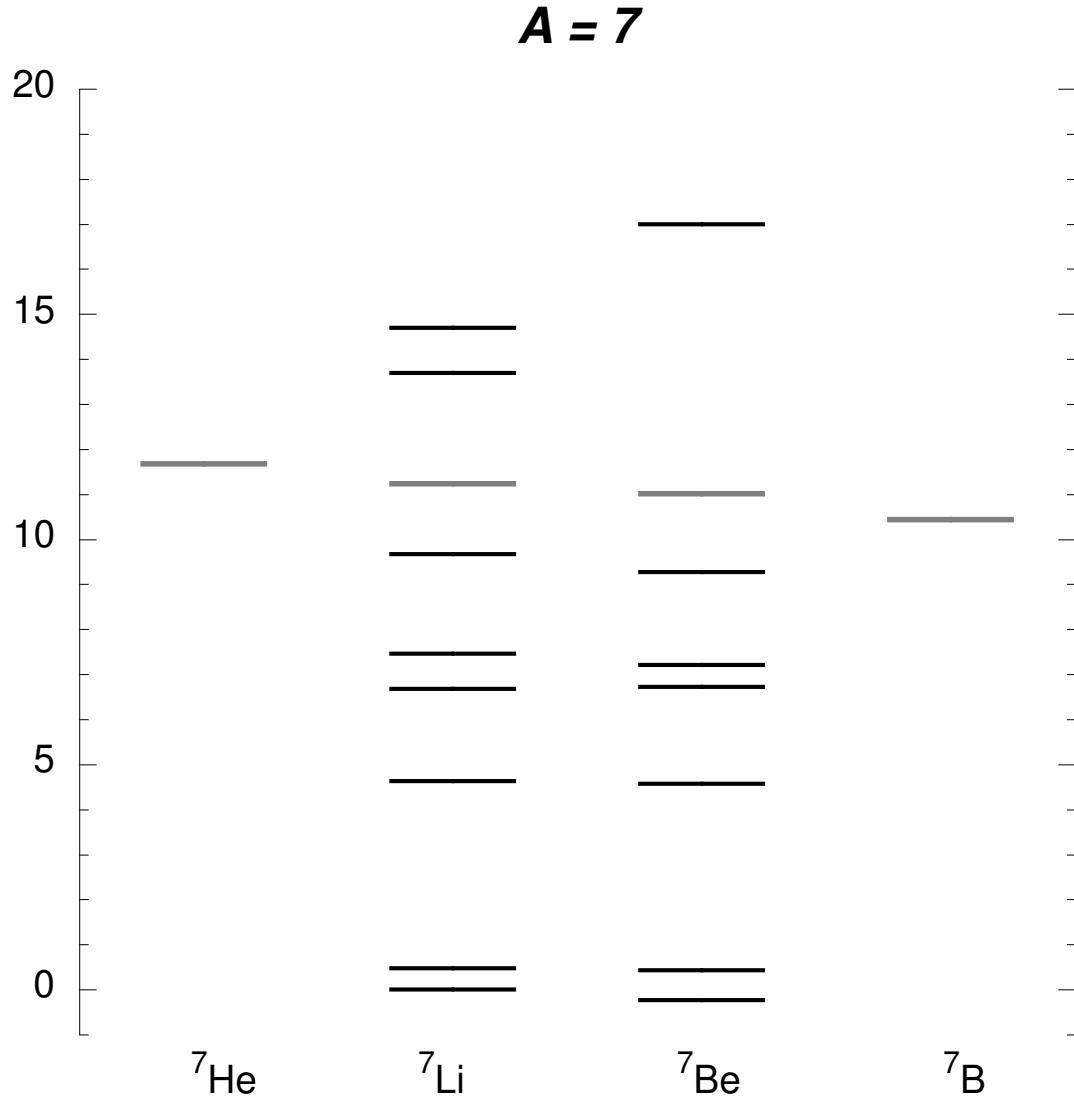
$$^3\text{He} \text{ charge radius } r = 1.97 \pm 0.015 \text{ fm}$$

$$\boxed{\text{Coulomb energy: } \alpha/r \approx 0.731 \text{ MeV}}$$

## Level structures in mirror nuclei. 1

$$I_3 = -\frac{1}{2} : {}^7\text{Li}(3p + 4n) \quad {}^7\text{Be}(4p + 3n) : I_3 = \frac{1}{2}$$

$$I_3 = -\frac{3}{2} : {}^7\text{He}(2p + 3n) \quad {}^7\text{B}(5p + 2n) : I_3 = \frac{3}{2}$$



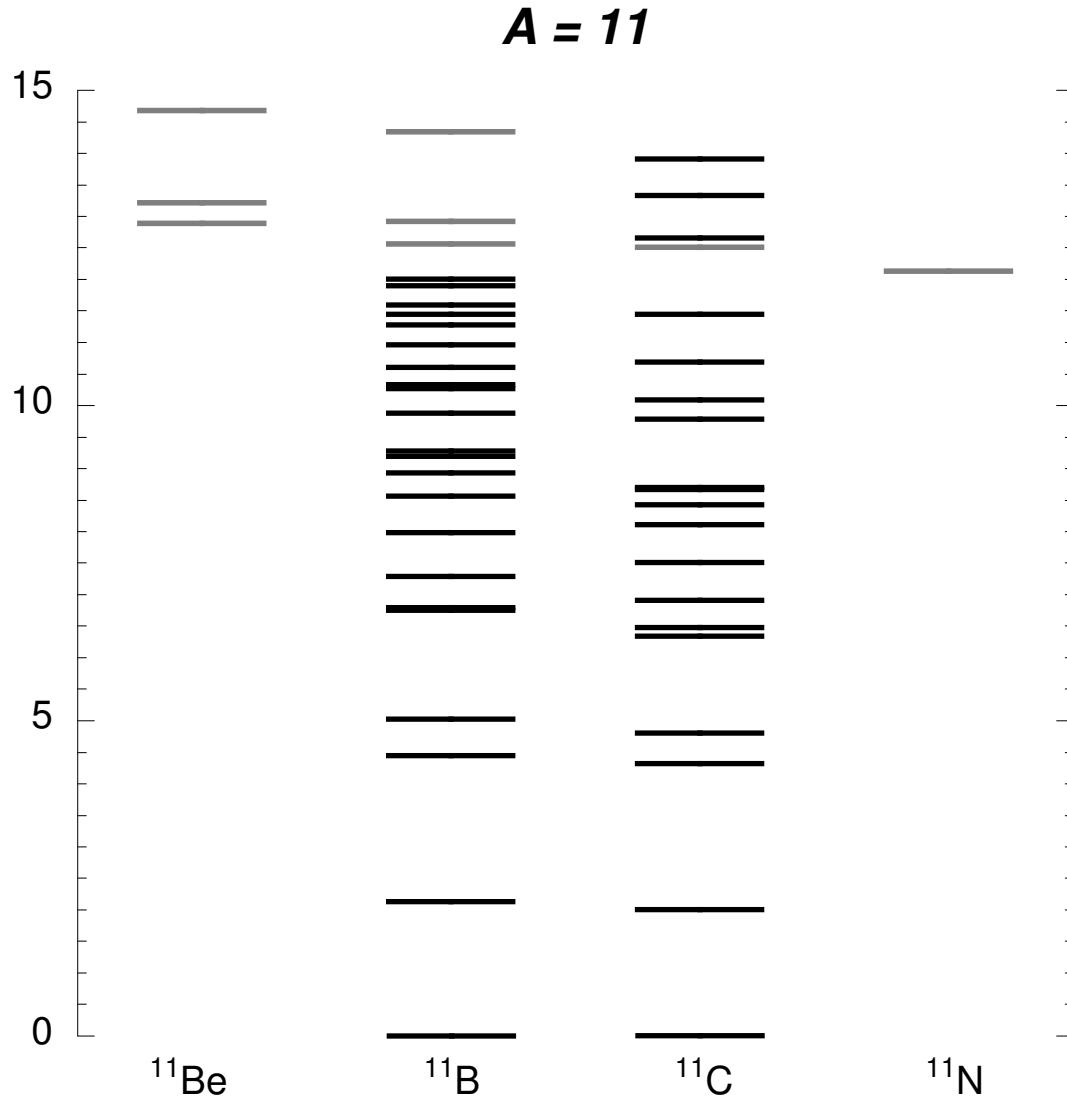
$n - p$  mass difference, Coulomb energy removed

**(isobaric analogue states)**

## Level structures in mirror nuclei. 2

$$I_3 = -\frac{1}{2} : {}^{11}\text{B}(5p + 6n) \quad {}^{11}\text{C}(6p + 5n) : I_3 = \frac{1}{2}$$

$$I_3 = -\frac{3}{2} : {}^{11}\text{Be}(4p + 7n) \quad {}^{11}\text{N}(7p + 4n) : I_3 = \frac{3}{2}$$



${}^{11}\text{Li}(3p + 8n)$  ground state (34.4 MeV)  $I = \frac{5}{2}$  isobaric analogue

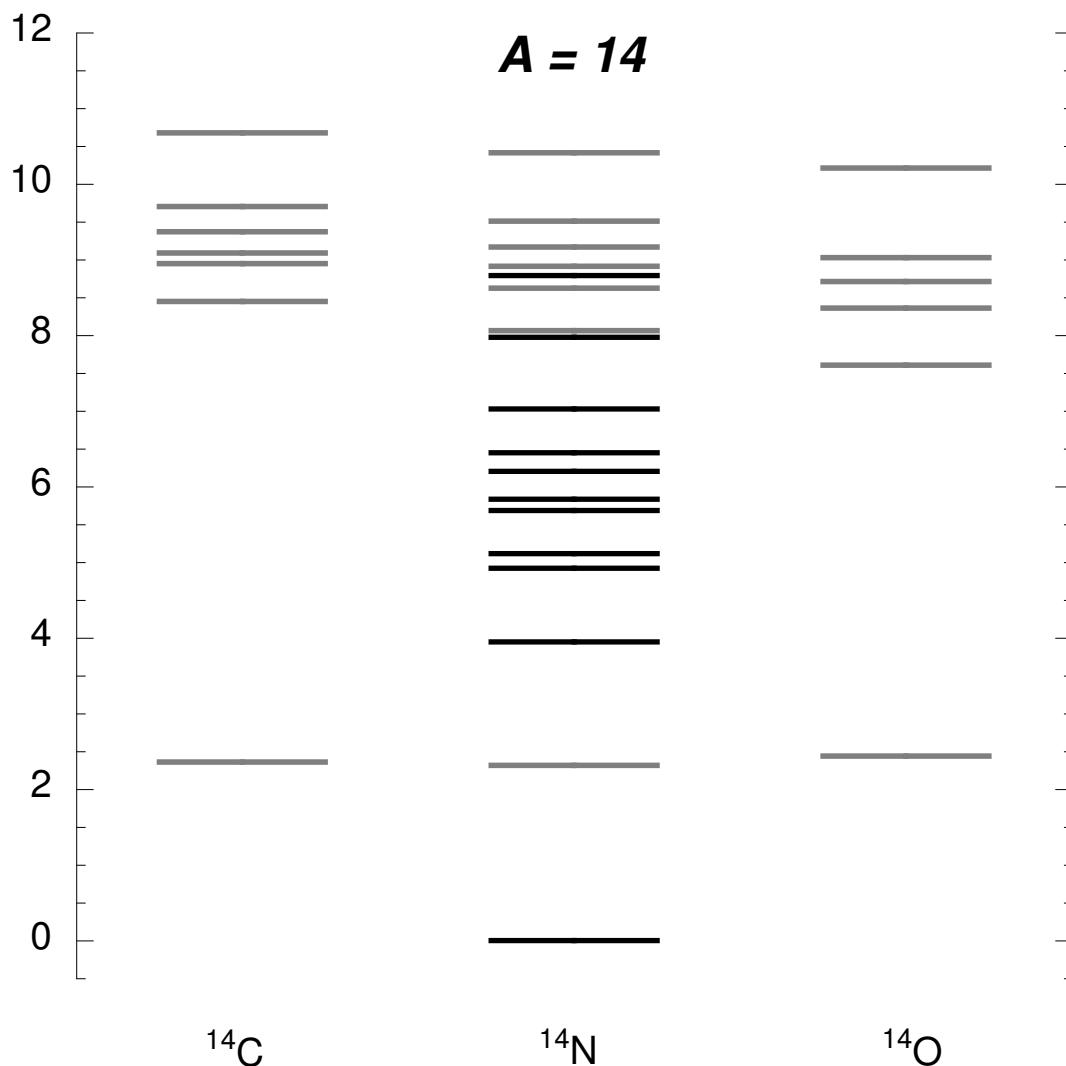
## Level structures in mirror nuclei. 3

$A = 14$ :  $NN$  outside closed core

$^{14}\text{O} : \ ^{12}\text{C} + (\text{pp}) \quad I_3 = +1$

$^{14}\text{N} : \ ^{12}\text{C} + (\text{pn}) \quad I_3 = 0$

$^{14}\text{C} : \ ^{12}\text{C} + (\text{nn}) \quad I_3 = -1$



## The first flavor symmetry

isospin invariance  $\begin{pmatrix} p \\ n \end{pmatrix}$  isospin rotations

In the absence of EM, *convention* determines which (combination) is up

Aside: *Without EM*, how would we know there are two species of nucleons?

# Parity violation in weak decays

1956 Wu *et al.*: correlation between  
spin vector  $\vec{J}$  of polarized  ${}^{60}\text{Co}$  and  
direction  $\hat{p}_e$  of outgoing  $\beta$  particle

Parity leaves spin (axial vector) unchanged  $\mathcal{P} : \vec{J} \rightarrow \vec{J}$

Parity reverses electron direction  $\mathcal{P} : \hat{p}_e \rightarrow -\hat{p}_e$

Correlation  $\vec{J} \cdot \hat{p}_e$  is *parity violating*

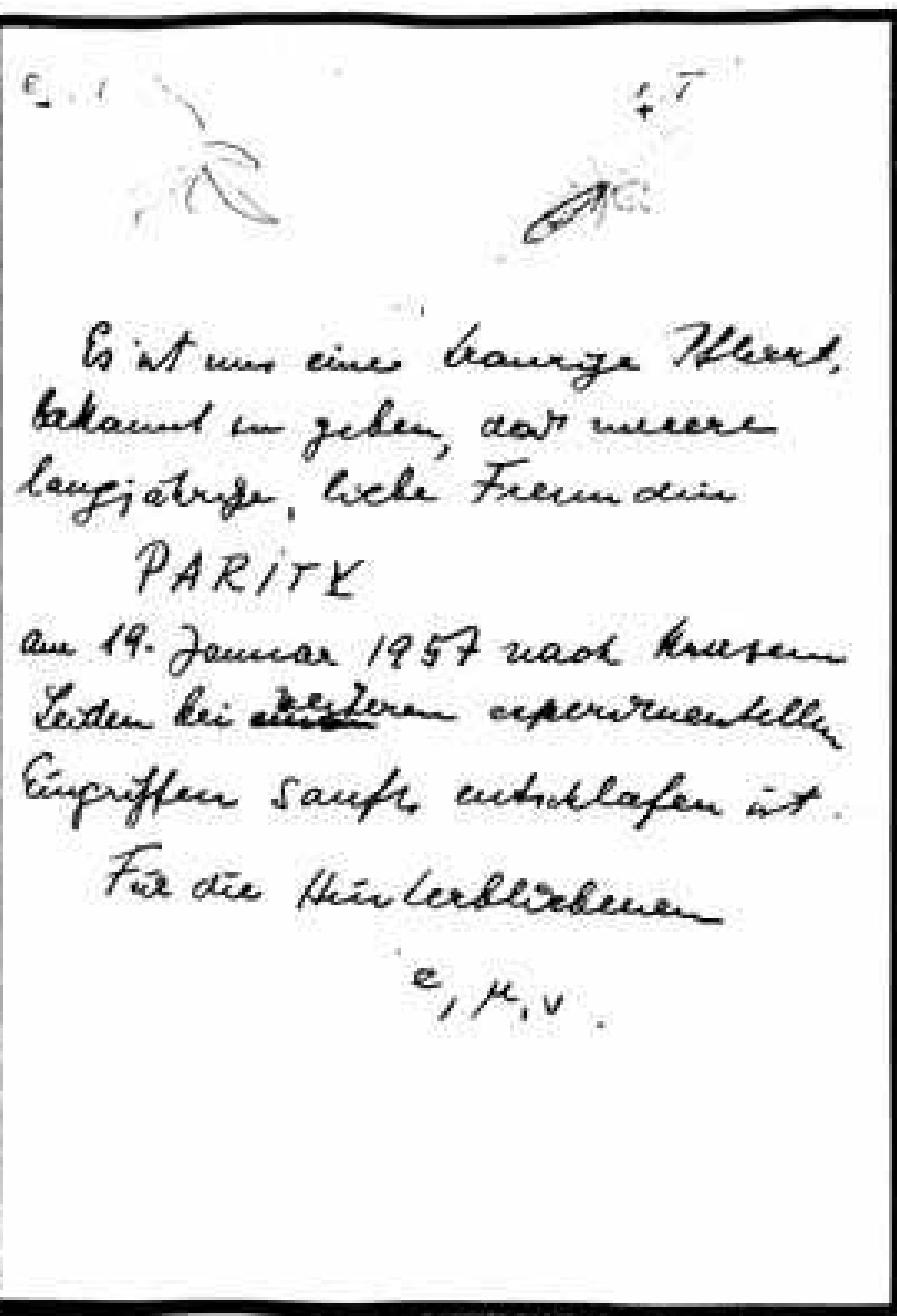
Experiments in late 1950s established that  
(charged-current) weak interactions are left-handed

Parity links left-handed, right-handed neutrinos,

$$\nu_L \xleftarrow{\quad} \mathcal{P} \xleftarrow{\quad} \cancel{\nu_R}$$

$\Rightarrow$  build a manifestly parity-violating theory with only  $\nu_L$ .

# Pauli's Reaction to the Downfall of Parity



# **Pauli's Reaction to the Downfall of Parity**

*Es ist uns eine traurige Pflicht,  
bekannt zu geben, daß unsere  
langjährige ewige Freundin*

**PARITY**

*den 19. Januar 1957 nach  
kurzen Leiden bei weiteren  
experimentellen Eingriffen  
sanfte entschlafen ist.*

*Für die hinterbliebenen*

*e    μ    ν*

*It is our sad duty to announce  
that our loyal friend of many  
years*

**PARITY**

*went peacefully to her eternal  
rest on the nineteenth of  
January 1957, after a short  
period of suffering in the face  
of further experimental  
interventions.*

*For those who survive her,*

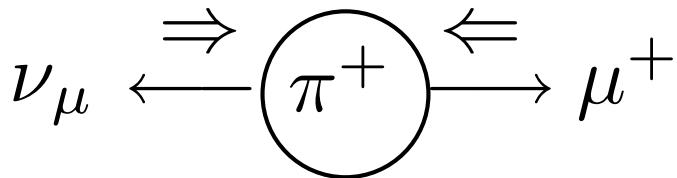
*e    μ    ν*

## Pauli's assertiveness training . . .



## How do we know $\nu$ is LH?

- ▷ Measure  $\mu^+$  helicity in (spin-zero)  $\pi^+ \rightarrow \mu^+ \nu_\mu$



$$h(\nu_\mu) = h(\mu^+)$$

Bardon, *Phys. Rev. Lett.* **7**, 23 (1961)

Possoz, *Phys. Lett.* **70B**, 265 (1977)

$\mu^+$  forced to have “wrong” helicity

... inhibits decay, and inhibits  $\pi^+ \rightarrow e^+ \nu_e$  more

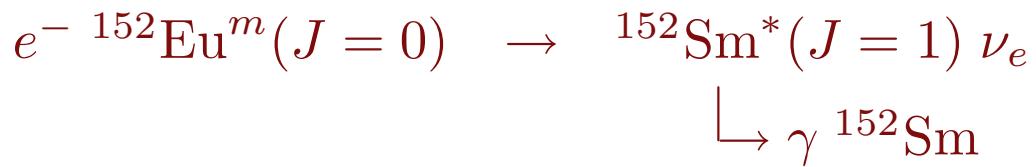
$$\Gamma(\pi^+ \rightarrow e^+ \nu_e) / \Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu) = 1.23 \times 10^{-4}$$

- ▷ Measure longitudinal polarization of recoil nucleus in  $\mu^- {}^{12}\text{C}(J=0) \rightarrow {}^{12}\text{B}(J=1)\nu_\mu$

Infer  $h(\nu_\mu)$  by angular momentum conservation

Roesch, *Am. J. Phys.* **50**, 931 (1981)

- ▷ Measure longitudinal polarization of recoil nucleus in



Infer  $h(\nu_e)$  from  $\gamma$  polarization

Goldhaber, *Phys. Rev.* **109**, 1015 (1958)

## Charge conjugation is also violated . . .

$$\nu_L \xrightleftharpoons[\mathcal{C}]{\leftarrow, \rightarrow} \overline{\nu}_L$$

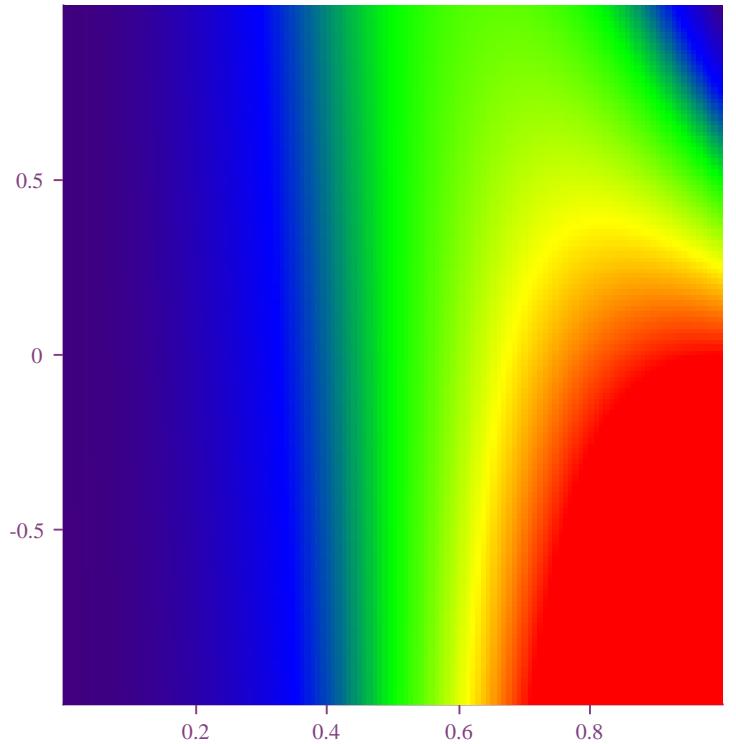
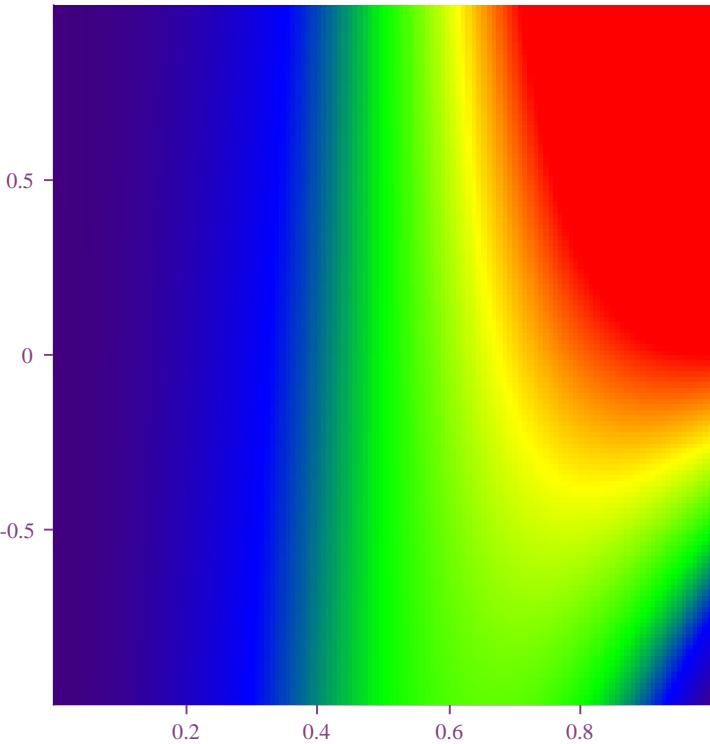
$\mu^\pm$  decay: angular distributions of  $e^\pm$  reversed

$$\frac{dN(\mu^\pm \rightarrow e^\pm + \dots)}{dxdz} = x^2(3 - 2x) \left[ 1 \pm z \frac{(2x - 1)}{(3 - 2x)} \right]$$

$$x \equiv p_e/p_e^{\max}, z \equiv \hat{s}_\mu \cdot \hat{p}_e$$

$e^+$  follows  $\mu^+$  spin

$e^-$  avoids  $\mu^-$  spin



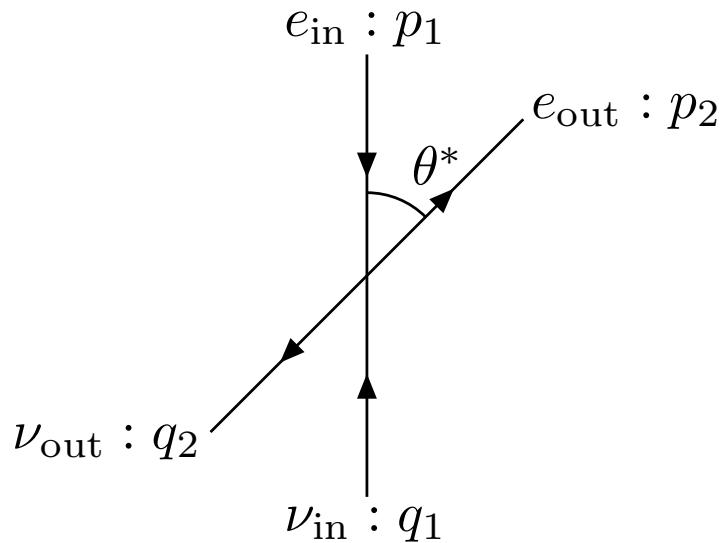
# Effective Lagrangian . . .

Late 1950s: current-current interaction

$$\mathcal{L}_{V-A} = \frac{-G_F}{\sqrt{2}} \bar{\nu} \gamma_\mu (1 - \gamma_5) e \bar{e} \gamma^\mu (1 - \gamma_5) \nu + \text{h.c.}$$

$$G_F = 1.16632 \times 10^{-5} \text{ GeV}^{-2}$$

Compute  $\bar{\nu}e$  scattering amplitude:



$$\begin{aligned} \mathcal{M} = & -\frac{iG_F}{\sqrt{2}} \bar{\nu}(\nu, q_1) \gamma_\mu (1 - \gamma_5) u(e, p_1) \\ & \cdot \bar{u}(e, p_2) \gamma^\mu (1 - \gamma_5) v(\nu, q_2) \end{aligned}$$

$$\bar{\nu}e \rightarrow \bar{\nu}e$$

$$\frac{d\sigma_{V-A}(\bar{\nu}e \rightarrow \bar{\nu}e)}{d\Omega_{\text{cm}}} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s} = \frac{G_F^2 \cdot 2mE_\nu(1-z)^2}{16\pi^2}$$

$$z = \cos \theta^*$$

$$\begin{aligned}\sigma_{V-A}(\bar{\nu}e \rightarrow \bar{\nu}e) &= \frac{G_F^2 \cdot 2mE_\nu}{3\pi} \\ &\approx 0.574 \times 10^{-41} \text{ cm}^2 \left( \frac{E_\nu}{1 \text{ GeV}} \right)\end{aligned}$$

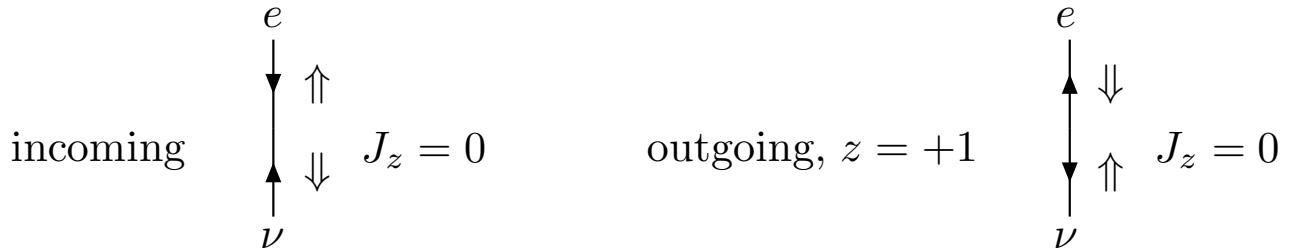
Small!  $\approx 10^{-14} \sigma(pp)$  at 100 GeV

$$\nu e \rightarrow \nu e$$

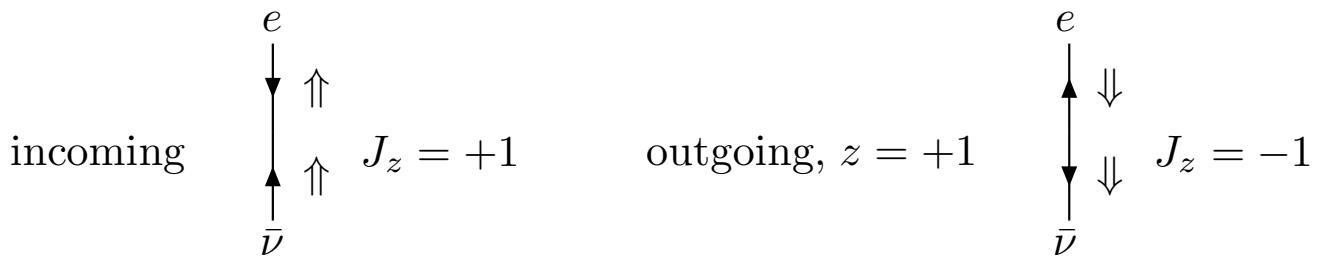
$$\frac{d\sigma_{V-A}(\nu e \rightarrow \nu e)}{d\Omega_{\text{cm}}} = \frac{G_F^2 \cdot 2mE_\nu}{4\pi^2}$$

$$\begin{aligned}\sigma_{V-A}(\nu e \rightarrow \nu e) &= \frac{G_F^2 \cdot 2mE_\nu}{\pi} \\ &\approx 1.72 \times 10^{-41} \text{ cm}^2 \left( \frac{E_\nu}{1 \text{ GeV}} \right)\end{aligned}$$

# Why $3\times$ difference?



allowed at all angles



forbidden (angular momentum) at  $z = +1$

1962: Lederman, Schwartz, Steinberger  $\nu_\mu \neq \nu_e$

- ▷ Make HE  $\pi \rightarrow \mu\nu$  beam
- ▷ Observe  $\nu N \rightarrow \mu + \text{anything}$
- ▷ Don't observe  $\nu N \rightarrow e + \text{anything}$

Danby, et al., *Phys. Rev. Lett.* **9**, 36 (1962)

Suggests family structure

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$$

≈ no interactions known to cross boundaries

Generalize effective (current-current) Lagrangian:

$$\mathcal{L}_{V-A}^{(e\mu)} = \frac{-G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e + \text{h.c.} ,$$

Compute muon decay rate

$$\Gamma(\mu \rightarrow e \bar{\nu}_e \nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

accounts for the 2.2- $\mu\text{s}$  muon lifetime

## TESTS OF NUMBER CONSERVATION LAWS

### LEPTON FAMILY NUMBER

Lepton family number conservation means separate conservation of each of  $L_e$ ,  $L_\mu$ ,  $L_\tau$ .

$\Gamma(Z \rightarrow e^\pm \mu^\mp)/\Gamma_{\text{total}}$	[i] $< 1.7 \times 10^{-6}$ , CL = 95%
$\Gamma(Z \rightarrow e^\pm \tau^\mp)/\Gamma_{\text{total}}$	[i] $< 9.8 \times 10^{-6}$ , CL = 95%
$\Gamma(Z \rightarrow \mu^\pm \tau^\mp)/\Gamma_{\text{total}}$	[i] $< 1.2 \times 10^{-5}$ , CL = 95%
limit on $\mu^- \rightarrow e^-$ conversion	
$\sigma(\mu^- {}^{32}\text{S} \rightarrow e^- {}^{32}\text{S}) /$	$< 7 \times 10^{-11}$ , CL = 90%
$\sigma(\mu^- {}^{32}\text{S} \rightarrow \nu_\mu {}^{32}\text{P}^*)$	
$\sigma(\mu^- \text{Ti} \rightarrow e^- \text{Ti}) /$	$< 4.3 \times 10^{-12}$ , CL = 90%
$\sigma(\mu^- \text{Ti} \rightarrow \text{capture})$	
$\sigma(\mu^- \text{Pb} \rightarrow e^- \text{Pb}) /$	$< 4.6 \times 10^{-11}$ , CL = 90%
$\sigma(\mu^- \text{Pb} \rightarrow \text{capture})$	
limit on muonium $\rightarrow$ antimuonium conversion $R_g = G_C / G_F$	$< 0.0030$ , CL = 90%
$\Gamma(\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu)/\Gamma_{\text{total}}$	[j] $< 1.2 \times 10^{-2}$ , CL = 90%
$\Gamma(\mu^- \rightarrow e^- \gamma)/\Gamma_{\text{total}}$	$< 1.2 \times 10^{-11}$ , CL = 90%
$\Gamma(\mu^- \rightarrow e^- e^+ e^-)/\Gamma_{\text{total}}$	$< 1.0 \times 10^{-12}$ , CL = 90%
$\Gamma(\mu^- \rightarrow e^- 2\gamma)/\Gamma_{\text{total}}$	$< 7.2 \times 10^{-11}$ , CL = 90%
$\Gamma(\tau^- \rightarrow e^- \gamma)/\Gamma_{\text{total}}$	$< 2.7 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \mu^- \gamma)/\Gamma_{\text{total}}$	$< 1.1 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow e^- \pi^0)/\Gamma_{\text{total}}$	$< 3.7 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \mu^- \pi^0)/\Gamma_{\text{total}}$	$< 4.0 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow e^- K_S^0)/\Gamma_{\text{total}}$	$< 9.1 \times 10^{-7}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \mu^- K_S^0)/\Gamma_{\text{total}}$	$< 9.5 \times 10^{-7}$ , CL = 90%
$\Gamma(\tau^- \rightarrow e^- \eta)/\Gamma_{\text{total}}$	$< 8.2 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \mu^- \eta)/\Gamma_{\text{total}}$	$< 9.6 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow e^- \rho^0)/\Gamma_{\text{total}}$	$< 2.0 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \mu^- \rho^0)/\Gamma_{\text{total}}$	$< 6.3 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow e^- K^*(892)^0)/\Gamma_{\text{total}}$	$< 5.1 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \mu^- K^*(892)^0)/\Gamma_{\text{total}}$	$< 7.5 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow e^- \bar{K}^*(892)^0)/\Gamma_{\text{total}}$	$< 7.4 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \mu^- \bar{K}^*(892)^0)/\Gamma_{\text{total}}$	$< 7.5 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow e^- \phi)/\Gamma_{\text{total}}$	$< 6.9 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \mu^- \phi)/\Gamma_{\text{total}}$	$< 7.0 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow e^- e^+ e^-)/\Gamma_{\text{total}}$	$< 2.9 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow e^- \mu^+ \mu^-)/\Gamma_{\text{total}}$	$< 1.8 \times 10^{-6}$ , CL = 90%

## TOTAL LEPTON NUMBER

Violation of total lepton number conservation also implies violation of lepton family number conservation.

$\Gamma(Z \rightarrow p e)/\Gamma_{\text{total}}$	$<1.8 \times 10^{-6}$ , CL = 95%
$\Gamma(Z \rightarrow p \mu)/\Gamma_{\text{total}}$	$<1.8 \times 10^{-6}$ , CL = 95%
limit on $\mu^- \rightarrow e^+$ conversion	
$\sigma(\mu^- {}^{32}\text{S} \rightarrow e^+ {}^{32}\text{Si}^*) / \sigma(\mu^- {}^{32}\text{S} \rightarrow \nu_\mu {}^{32}\text{P}^*)$	$<9 \times 10^{-10}$ , CL = 90%
$\sigma(\mu^- {}^{127}\text{I} \rightarrow e^+ {}^{127}\text{Sb}^*) / \sigma(\mu^- {}^{127}\text{I} \rightarrow \text{anything})$	$<3 \times 10^{-10}$ , CL = 90%
$\sigma(\mu^- \text{Ti} \rightarrow e^+ \text{Ca}) / \sigma(\mu^- \text{Ti} \rightarrow \text{capture})$	$<3.6 \times 10^{-11}$ , CL = 90%
$\Gamma(\tau^- \rightarrow e^+ \pi^- \pi^-)/\Gamma_{\text{total}}$	$<1.9 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \mu^+ \pi^- \pi^-)/\Gamma_{\text{total}}$	$<3.4 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow e^+ \pi^- K^-)/\Gamma_{\text{total}}$	$<2.1 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow e^+ K^- K^-)/\Gamma_{\text{total}}$	$<3.8 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \mu^+ \pi^- K^-)/\Gamma_{\text{total}}$	$<7.0 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \mu^+ K^- K^-)/\Gamma_{\text{total}}$	$<6.0 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \bar{p}\gamma)/\Gamma_{\text{total}}$	$<3.5 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \bar{p}\pi^0)/\Gamma_{\text{total}}$	$<1.5 \times 10^{-5}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \bar{p}2\pi^0)/\Gamma_{\text{total}}$	$<3.3 \times 10^{-5}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \bar{p}\eta)/\Gamma_{\text{total}}$	$<8.9 \times 10^{-6}$ , CL = 90%
$\Gamma(\tau^- \rightarrow \bar{p}\pi^0\eta)/\Gamma_{\text{total}}$	$<2.7 \times 10^{-5}$ , CL = 90%
$t_{1/2}( {}^{76}\text{Ge} \rightarrow {}^{76}\text{Se} + 2 e^-)$	$>1.9 \times 10^{25}$ yr, CL = 90%
$\Gamma(\pi^+ \rightarrow \mu^+ \bar{\nu}_e)/\Gamma_{\text{total}}$	[k] $<1.5 \times 10^{-3}$ , CL = 90%
$\Gamma(K^+ \rightarrow \pi^- \mu^+ e^+)/\Gamma_{\text{total}}$	$<5.0 \times 10^{-10}$ , CL = 90%
$\Gamma(K^+ \rightarrow \pi^- e^+ e^+)/\Gamma_{\text{total}}$	$<6.4 \times 10^{-10}$ , CL = 90%
$\Gamma(K^+ \rightarrow \pi^- \mu^+ \mu^+)/\Gamma_{\text{total}}$	[k] $<3.0 \times 10^{-9}$ , CL = 90%
$\Gamma(K^+ \rightarrow \mu^+ \bar{\nu}_e)/\Gamma_{\text{total}}$	[k] $<3.3 \times 10^{-3}$ , CL = 90%
$\Gamma(K^+ \rightarrow \pi^0 e^+ \bar{\nu}_e)/\Gamma_{\text{total}}$	$<3 \times 10^{-3}$ , CL = 90%
$\Gamma(D^+ \rightarrow \pi^- e^+ e^+)/\Gamma_{\text{total}}$	$<9.6 \times 10^{-5}$ , CL = 90%
$\Gamma(D^+ \rightarrow \pi^- \mu^+ \mu^+)/\Gamma_{\text{total}}$	$<4.8 \times 10^{-6}$ , CL = 90%
$\Gamma(D^+ \rightarrow \pi^- e^+ \mu^+)/\Gamma_{\text{total}}$	$<5.0 \times 10^{-5}$ , CL = 90%
$\Gamma(D^+ \rightarrow \rho^- \mu^+ \mu^+)/\Gamma_{\text{total}}$	$<5.6 \times 10^{-4}$ , CL = 90%
$\Gamma(D^+ \rightarrow K^- e^+ e^+)/\Gamma_{\text{total}}$	$<1.2 \times 10^{-4}$ , CL = 90%
$\Gamma(D^+ \rightarrow K^- \mu^+ \mu^+)/\Gamma_{\text{total}}$	$<1.3 \times 10^{-5}$ , CL = 90%
$\Gamma(D^+ \rightarrow K^- e^+ \mu^+)/\Gamma_{\text{total}}$	$<1.3 \times 10^{-4}$ , CL = 90%
$\Gamma(D^+ \rightarrow K^*(892)^- \mu^+ \mu^+)/\Gamma_{\text{total}}$	$<8.5 \times 10^{-4}$ , CL = 90%
$\Gamma(D^0 \rightarrow \pi^- \pi^- e^+ e^+ + \text{c.c.})/\Gamma_{\text{total}}$	$<1.12 \times 10^{-4}$ , CL = 90%
$\Gamma(D^0 \rightarrow \pi^- \pi^- \mu^+ \mu^+ + \text{c.c.})/\Gamma_{\text{total}}$	$<2.9 \times 10^{-5}$ , CL = 90%
$\Gamma(D^0 \rightarrow K^- \pi^- e^+ e^+ + \text{c.c.})/\Gamma_{\text{total}}$	$<2.06 \times 10^{-4}$ , CL = 90%

## Cross section for inverse muon decay

$$\sigma(\nu_\mu e \rightarrow \mu \nu_e) = \sigma_{V-A}(\nu_e e \rightarrow \nu_e e) \left[ 1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu} \right]^2$$

agrees with CHARM II, CCFR data ( $E_\nu \lesssim 600$  GeV)

PW unitarity:  $|\mathcal{M}_J| < 1$

$V - A$  theory:

$$\mathcal{M}_0 = \frac{G_F \cdot 2m_e E_\nu}{\pi \sqrt{2}} \left[ 1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu} \right]$$

satisfies pw unitarity for

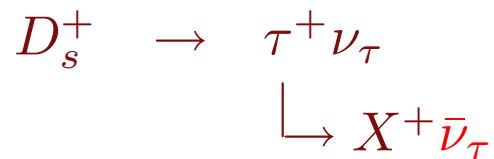
$$E_\nu < \pi/G_F m_e \sqrt{2} \approx 3.7 \times 10^8 \text{ GeV}$$

$\Rightarrow V - A$  theory cannot be complete

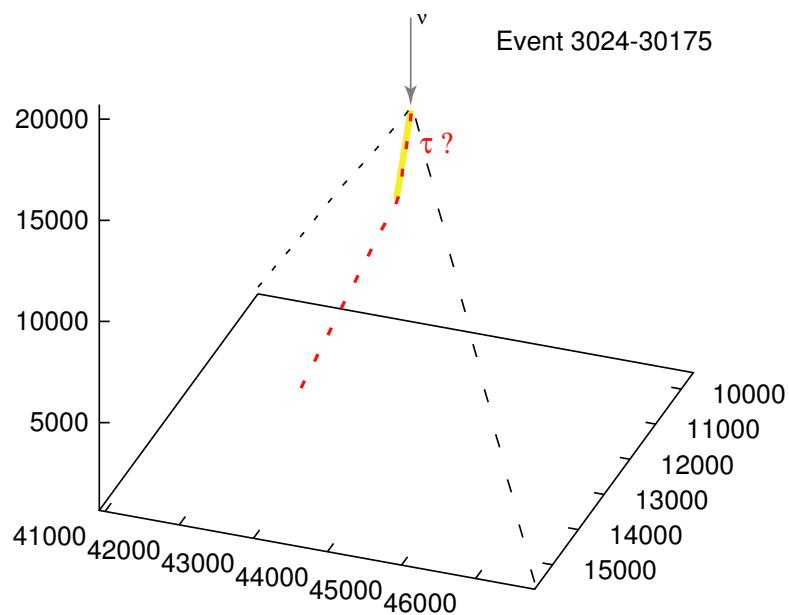
*physics must change before  $\sqrt{s} \approx 600$  GeV*

## 2000: DONuT Three-Neutrino Experiment

- ▷ Prompt (beam-dump)  $\nu_\tau$  beam produced in



- ▷ Try to observe  $\nu_\tau N \rightarrow \tau + \text{anything}$  in emulsion;  
 $\tau$  lifetime is 0.3 ps



Candidate event in ECC1. The three tracks with full emulsion data are shown. The red track shows a 100 mrad kink 4.5mm from the interaction vertex. The scale units are microns.

Kodama, et al., *Phys. Lett. B504*, 218 (2001)

# Leptons are seen as free particles

Table 1: Some properties of the leptons.

Lepton	Mass	Lifetime
$e^-$	$0.510\,998\,92(4) \text{ MeV}/c^2$	$> 4.6 \times 10^{26} \text{ y (90\% CL)}$
$\nu_e$	$< 3 \text{ eV}/c^2$	$\tau/m > 7 \times 10^9 \text{ s/eV}$
$\mu^-$	$105.658\,369(9) \text{ MeV}/c^2$	$2.197\,03(4) \times 10^{-6} \text{ s}$
$\nu_\mu$	$< 0.19 \text{ MeV}/c^2 \text{ (90\% CL)}$	$\tau/m > 15.4 \text{ s/eV}$
$\tau^-$	$1776.99_{-0.26}^{+0.29} \text{ MeV}/c^2$	$290.6 \pm 1.1 \times 10^{-15} \text{ s}$
$\nu_\tau$	$< 18.2 \text{ MeV}/c^2 \text{ (95\% CL)}$	

All spin- $\frac{1}{2}$ , pointlike ( $\lesssim \text{few} \times 10^{-17} \text{ cm}$ )

*kinematically determined*  $\nu$  masses consistent with 0  
( $\nu$  oscillations  $\Rightarrow$  nonzero, unequal masses)

# Universal weak couplings

*Rough and ready test*

Fermi constant from muon decay

$$G_\mu = \left[ \frac{192\pi^3 \hbar}{\tau_\mu m_\mu^5} \right]^{\frac{1}{2}} = 1.1638 \times 10^{-5} \text{ GeV}^{-2}$$

Meticulous analysis yields  $G_\mu = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$

Fermi constant from tau decay

$$G_\tau = \left[ \frac{\Gamma(\tau \rightarrow e\bar{\nu}_e\nu_\tau)}{\Gamma(\tau \rightarrow \text{all})} \frac{192\pi^3 \hbar}{\tau_\tau m_\tau^5} \right]^{\frac{1}{2}} = 1.1642 \times 10^{-5} \text{ GeV}^{-2}$$

Excellent agreement with  $G_\beta = 1.16639(2) \times 10^{-5} \text{ GeV}^{-2}$

Charged currents acting in leptonic and semileptonic interactions are of universal strength;  $\Rightarrow$  *universality of current-current form, or whatever lies behind it*

## Nonleptonic enhancement

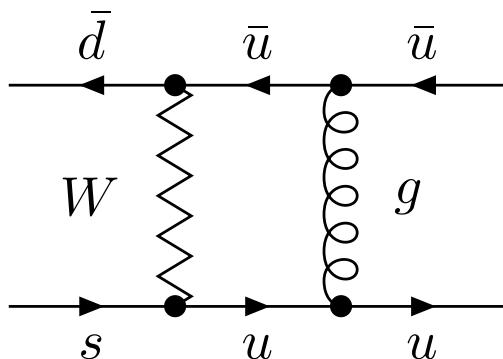
Certain NL transitions are more rapid than universality suggests

$$\underbrace{\Gamma(K_S \rightarrow \pi^+ \pi^-)}_{I=0, 2} \approx 450 \times \underbrace{\Gamma(K^+ \rightarrow \pi^+ \pi^0)}_{I=2}$$

$$A_0 \approx 22 \times A_2$$

$|\Delta I| = \frac{1}{2}$  rule; “octet dominance” (over 27)

Origin of this phenomenological rule is only partly understood. Short-distance (*perturbative*) QCD corrections arise from



... explain  $\approx \sqrt{\text{enhancement}}$

# SYMMETRIES $\Rightarrow$ INTERACTIONS

## Phase Invariance (Symmetry) in Quantum Mechanics

QM STATE: COMPLEX SCHRÖDINGER WAVE  
FUNCTION  $\psi(x)$

OBSERVABLES

$$\langle O \rangle = \int d^n x \psi^* O \psi$$

ARE UNCHANGED

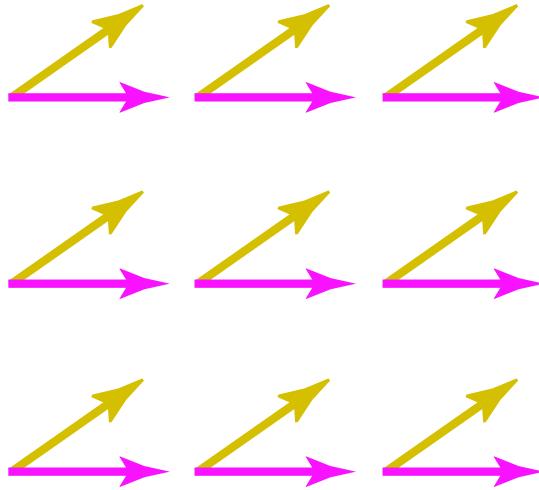
UNDER A GLOBAL PHASE ROTATION

$$\begin{aligned}\psi(x) &\rightarrow e^{i\theta} \psi(x) \\ \psi^*(x) &\rightarrow e^{-i\theta} \psi^*(x)\end{aligned}$$

- Absolute phase of the wave function cannot be measured (is a matter of convention).
- Relative phases (interference experiments) are unaffected by a global phase rotation.

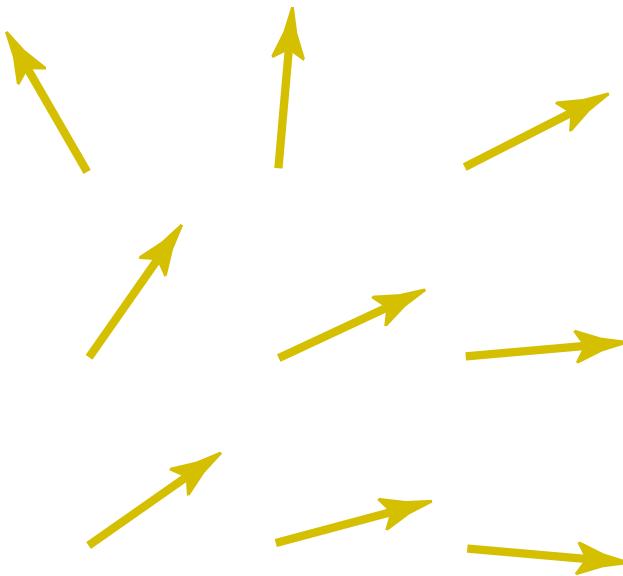


## GLOBAL ROTATION — SAME EVERYWHERE



MIGHT WE CHOOSE ONE PHASE CONVENTION  
IN MIRAMARE AND ANOTHER IN BATAVIA?

A DIFFERENT CONVENTION AT EACH POINT?



$$\psi(x) \rightarrow e^{iq\alpha(x)}\psi(x)$$

## THERE IS A PRICE.

Some variables (e.g., momentum) and the Schrödinger equation itself contain derivatives.  
Under the transformation

$$\psi(x) \rightarrow e^{iq\alpha(x)}\psi(x)$$

the gradient of the wave function transforms as

$$\nabla\psi(x) \rightarrow e^{iq\alpha(x)}[\nabla\psi(x) + iq(\nabla\alpha(x))\psi(x)]$$

The  $\nabla\alpha(x)$  term spoils local phase invariance.

TO RESTORE LOCAL PHASE INVARIANCE ...

Modify the equations of motion and observables.

Replace  $\nabla$  by  $\nabla + iq\vec{A}$

“Gauge-covariant derivative”

If the vector potential  $\vec{A}$  transforms under local phase rotations as

$$\vec{A}(x) \rightarrow \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x),$$

then  $(\nabla + iq\vec{A})\psi \rightarrow e^{iq\alpha(x)}(\nabla + iq\vec{A})\psi$  and  
 $\psi^*(\nabla + iq\vec{A})\psi$  is invariant under local rotations.

## NOTE . . .

- $\vec{A}(x) \rightarrow \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x)$  has the form of a gauge transformation in electrodynamics.
- The replacement  $\nabla \rightarrow (\nabla + iq\vec{A})$  corresponds to  $\vec{p} \rightarrow \vec{p} - q\vec{A}$

FORM OF INTERACTION IS DEDUCED  
FROM LOCAL PHASE INVARIANCE

⇒ MAXWELL'S EQUATIONS

DERIVED

FROM A SYMMETRY PRINCIPLE

QED is the gauge theory based on  
 $U(1)$  phase symmetry

# GENERAL PROCEDURE

- Recognize a symmetry of Nature.
- Build it into the laws of physics.  
(Connection with conservation laws)
- Impose symmetry in stricter (local) form.

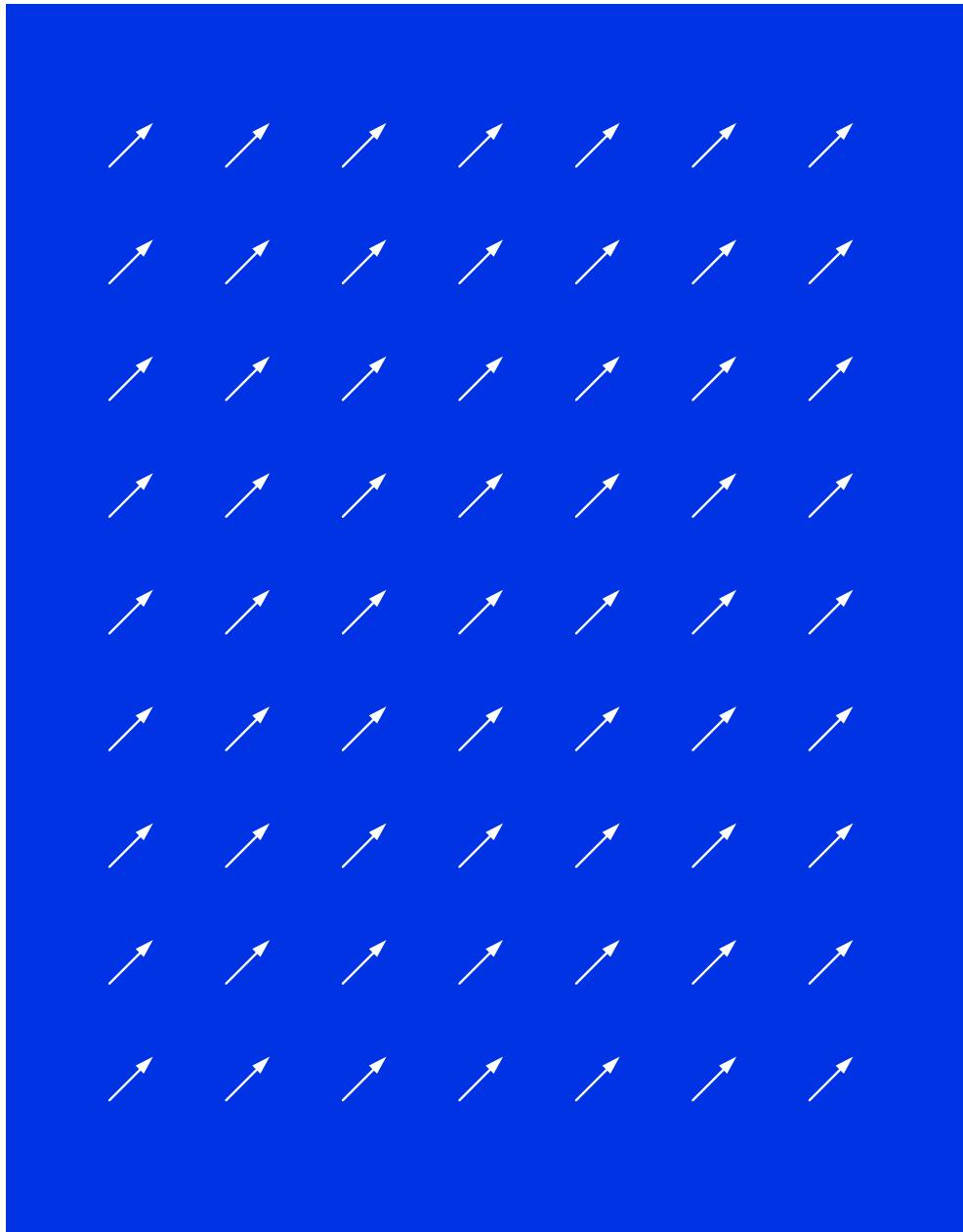
⇒ INTERACTIONS

- Massless vector fields (gauge fields)
- Minimal coupling to the conserved current
- Interactions among the gauge fields, if symmetry is non-Abelian

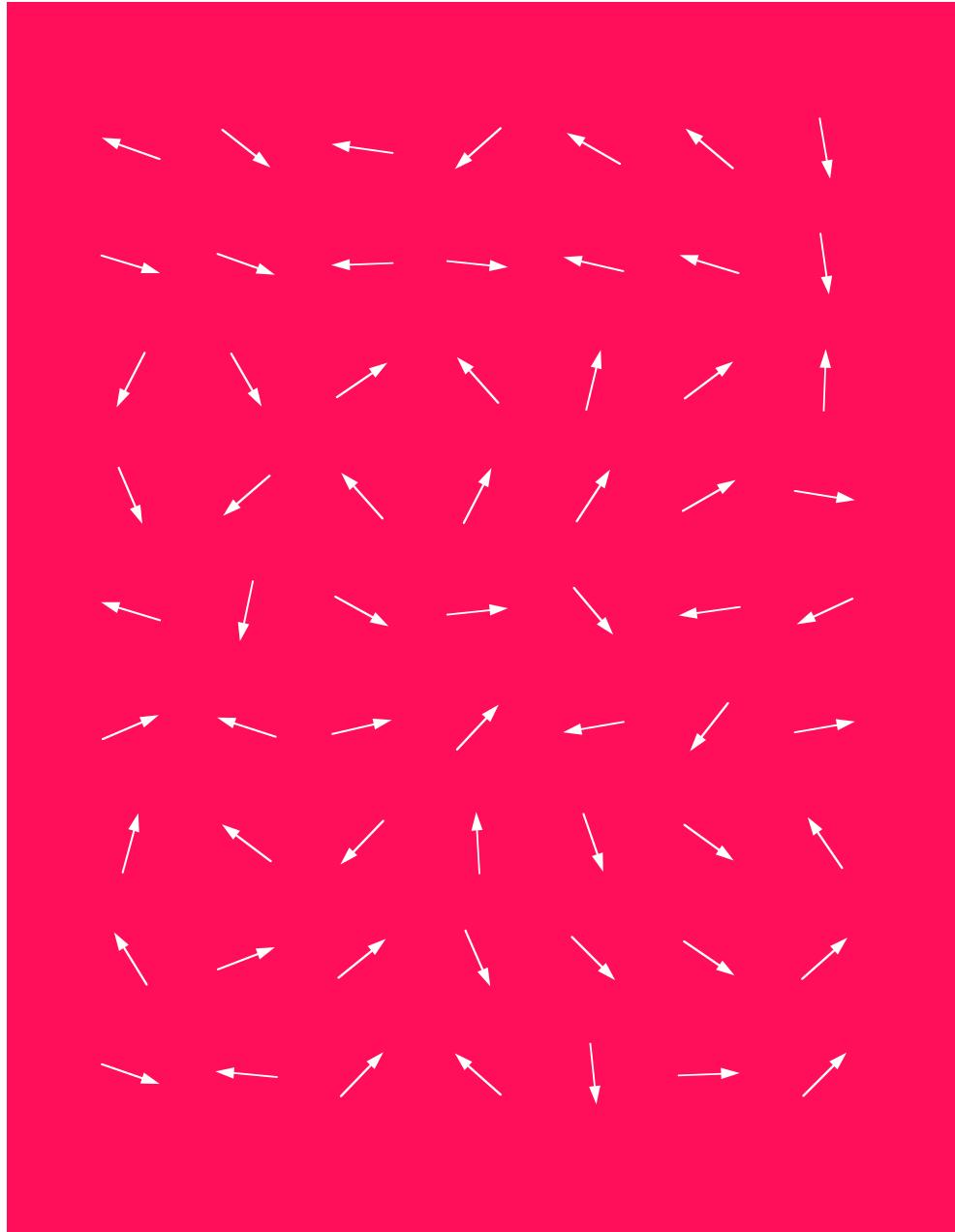
Posed as a problem in mathematics, construction of a gauge theory is always possible (at the level of a classical  $\mathcal{L}$ ; consistent quantum theory may require additional vigilance).

Formalism is no guarantee that the gauge symmetry was chosen wisely.

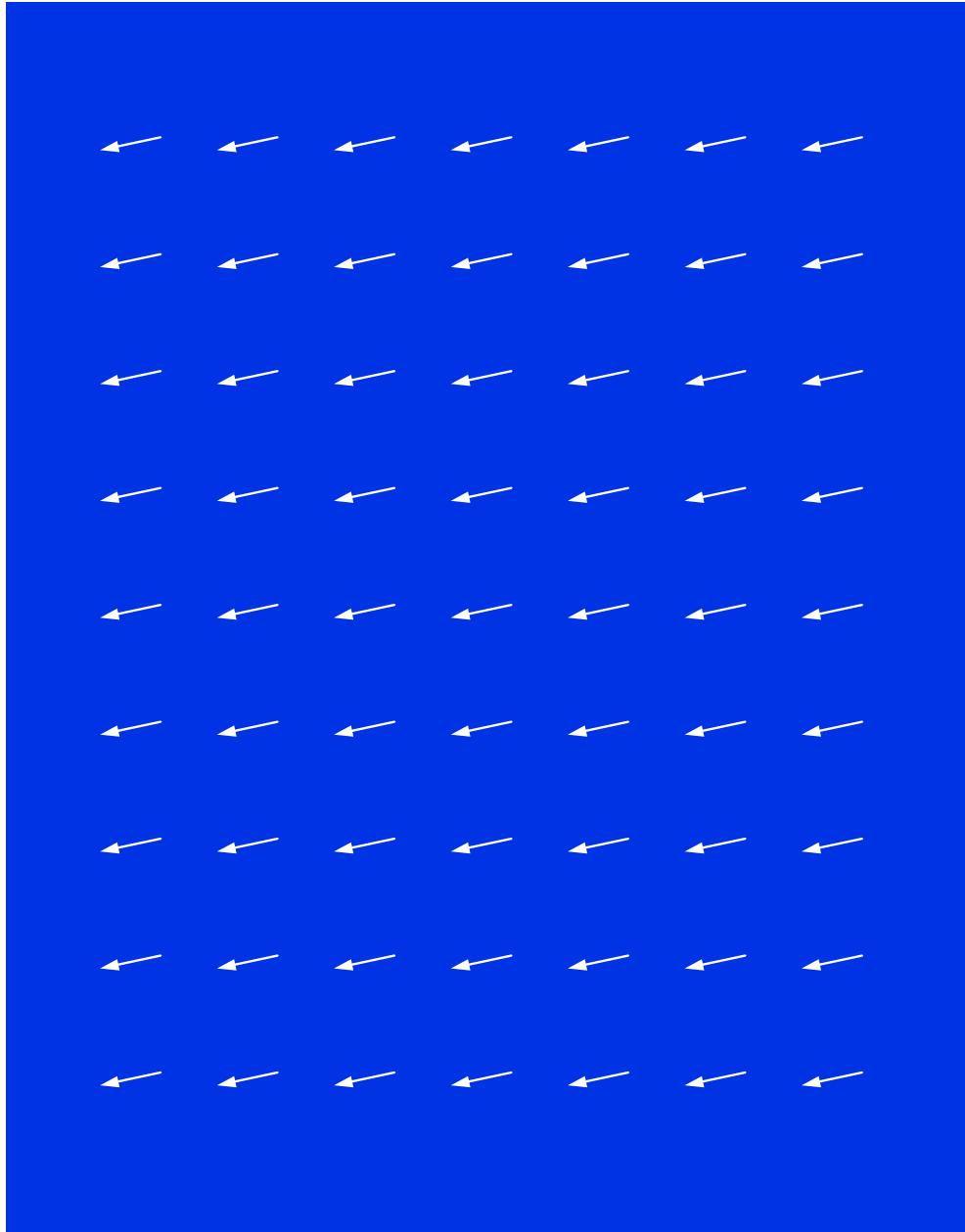
# The Crystal World



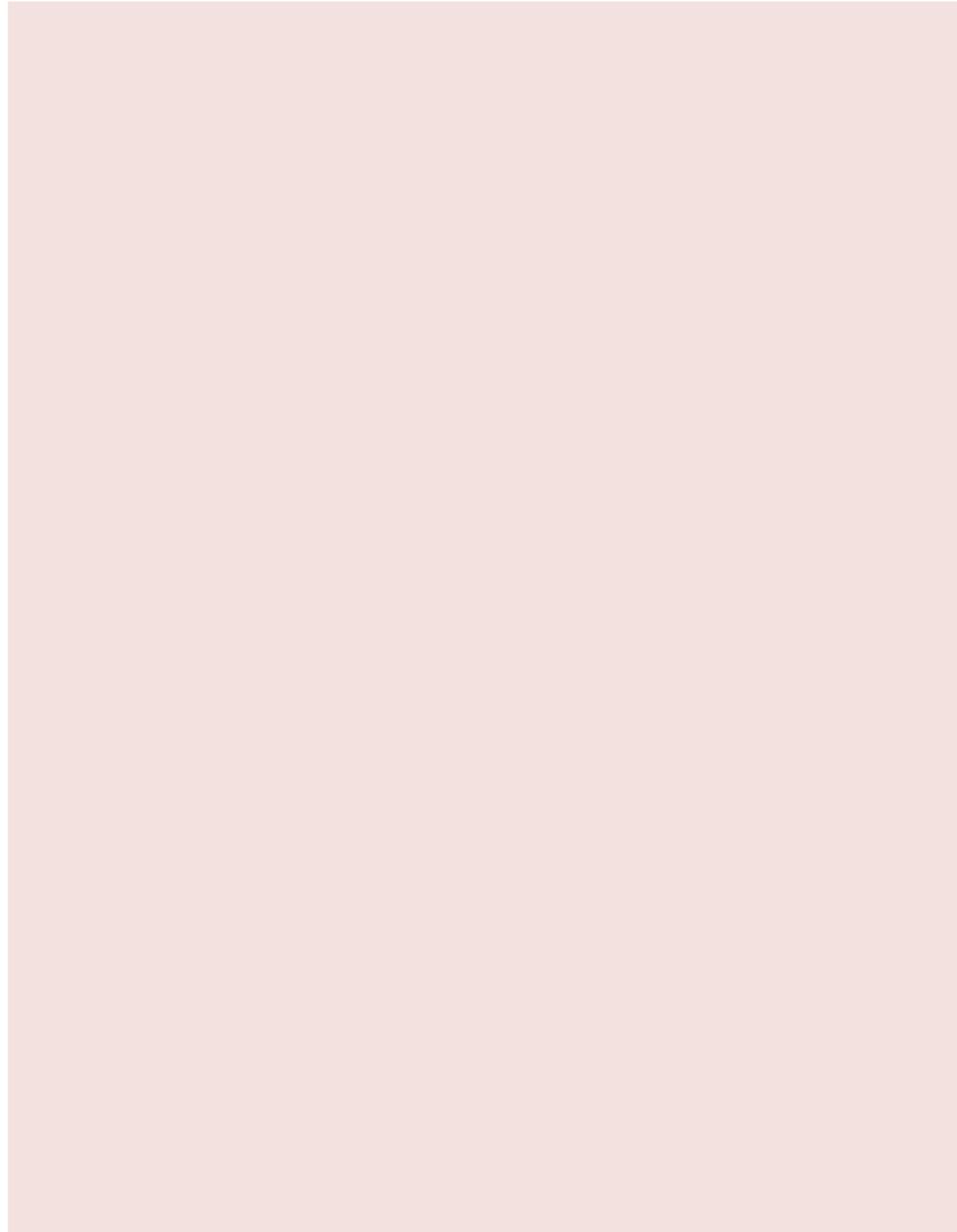
# The Crystal World



# The Crystal World



# The Perfect World



# Massive Photon?

# *Hiding Symmetry*

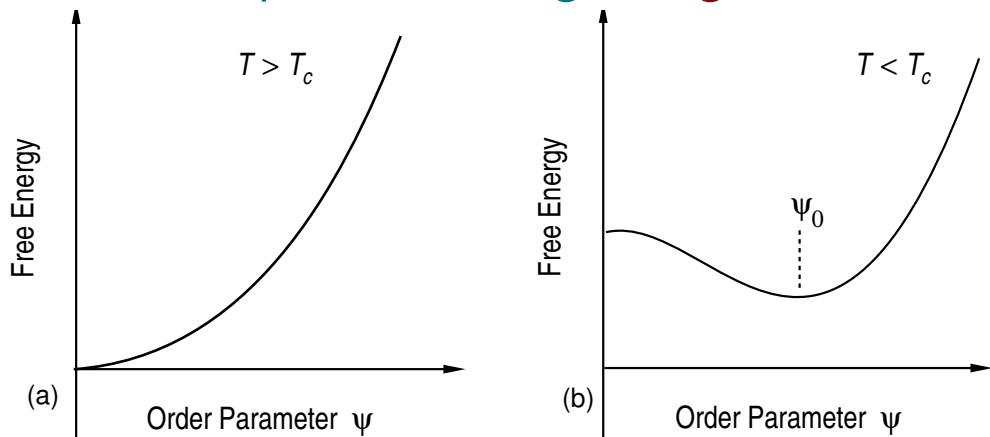
Recall [2] miracles of superconductivity:

- ▷ No resistance
- ▷ Meissner effect (exclusion of  $\mathbf{B}$ )

Ginzburg–Landau Phenomenology  
(not a theory from first principles)

normal, resistive charge carriers . . .

. . . + superconducting charge carriers



$\mathbf{B} = 0$ :

$$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha |\psi|^2 + \beta |\psi|^4$$

$$T > T_c : \quad \alpha > 0 \quad \langle |\psi|^2 \rangle_0 = 0$$

$$T < T_c : \quad \alpha < 0 \quad \langle |\psi|^2 \rangle_0 \neq 0$$

## NONZERO MAGNETIC FIELD

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{8\pi} + \frac{1}{2m^*} \left| -i\hbar\nabla\psi - \frac{e^*}{c}\mathbf{A}\psi \right|^2$$

$$\left. \begin{array}{l} e^* = -2 \\ m^* \end{array} \right\} \text{of superconducting carriers}$$

Weak, slowly varying field

$$\psi \approx \psi_0 \neq 0, \nabla\psi \approx 0$$

Variational analysis  $\implies$

$$\boxed{\nabla^2\mathbf{A} - \frac{4\pi e^*}{m^* c^2} |\psi_0|^2 \mathbf{A} = 0}$$

wave equation of a *massive photon*

Photon—*gauge boson* — acquires mass  
within superconductor

origin of Meissner effect

# Formulate electroweak theory

three crucial clues from experiment:

- ▷ Left-handed weak-isospin doublets,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

and

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L ;$$

- ▷ Universal strength of the (charged-current) weak interactions;
- ▷ Idealization that neutrinos are massless.

First two clues suggest  $SU(2)_L$  gauge symmetry

# A theory of leptons

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad R \equiv e_R$$

weak hypercharges  $Y_L = -1$ ,  $Y_R = -2$

Gell-Mann–Nishijima connection,  $Q = I_3 + \frac{1}{2}Y$

$SU(2)_L \otimes U(1)_Y$  gauge group  $\Rightarrow$  gauge fields:

- ★ weak isovector  $\vec{b}_\mu$ , coupling  $g$
- ★ weak isoscalar  $\mathcal{A}_\mu$ , coupling  $g'/2$

Field-strength tensors

$$F_{\mu\nu}^\ell = \partial_\nu b_\mu^\ell - \partial_\mu b_\nu^\ell + g \varepsilon_{jkl} b_\mu^j b_\nu^k , \text{SU}(2)_L$$

and

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu , U(1)_Y$$

# Interaction Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}} ,$$

with

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}^\ell F^{\ell\mu\nu} - \frac{1}{4}f_{\mu\nu}f^{\mu\nu},$$

and

$$\begin{aligned}\mathcal{L}_{\text{leptons}} &= \bar{R} i\gamma^\mu \left( \partial_\mu + i\frac{g'}{2} \mathcal{A}_\mu Y \right) R \\ &+ \bar{L} i\gamma^\mu \left( \partial_\mu + i\frac{g'}{2} \mathcal{A}_\mu Y + i\frac{g}{2} \vec{\tau} \cdot \vec{b}_\mu \right) L.\end{aligned}$$

Electron mass term

$$\mathcal{L}_e = -m_e (\bar{e}_R e_L + \bar{e}_L e_R) = -m_e \bar{e} e$$

would violate local gauge invariance Theory has four massless gauge bosons

$$\mathcal{A}_\mu \quad b_\mu^1 \quad b_\mu^2 \quad b_\mu^3$$

Nature has but one ( $\gamma$ )

# Hiding EW Symmetry

*Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition*

- ▷ Introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1$$

- ▷ Add to  $\mathcal{L}$  (gauge-invariant) terms for interaction and propagation of the scalars,

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi),$$

where  $\mathcal{D}_\mu = \partial_\mu + i \frac{g'}{2} \mathcal{A}_\mu Y + i \frac{g}{2} \vec{\tau} \cdot \vec{b}_\mu$  and

$$V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2$$

- ▷ Add a Yukawa interaction

$$\mathcal{L}_{\text{Yukawa}} = -\zeta_e [\bar{R}(\phi^\dagger L) + (\bar{L}\phi)R]$$

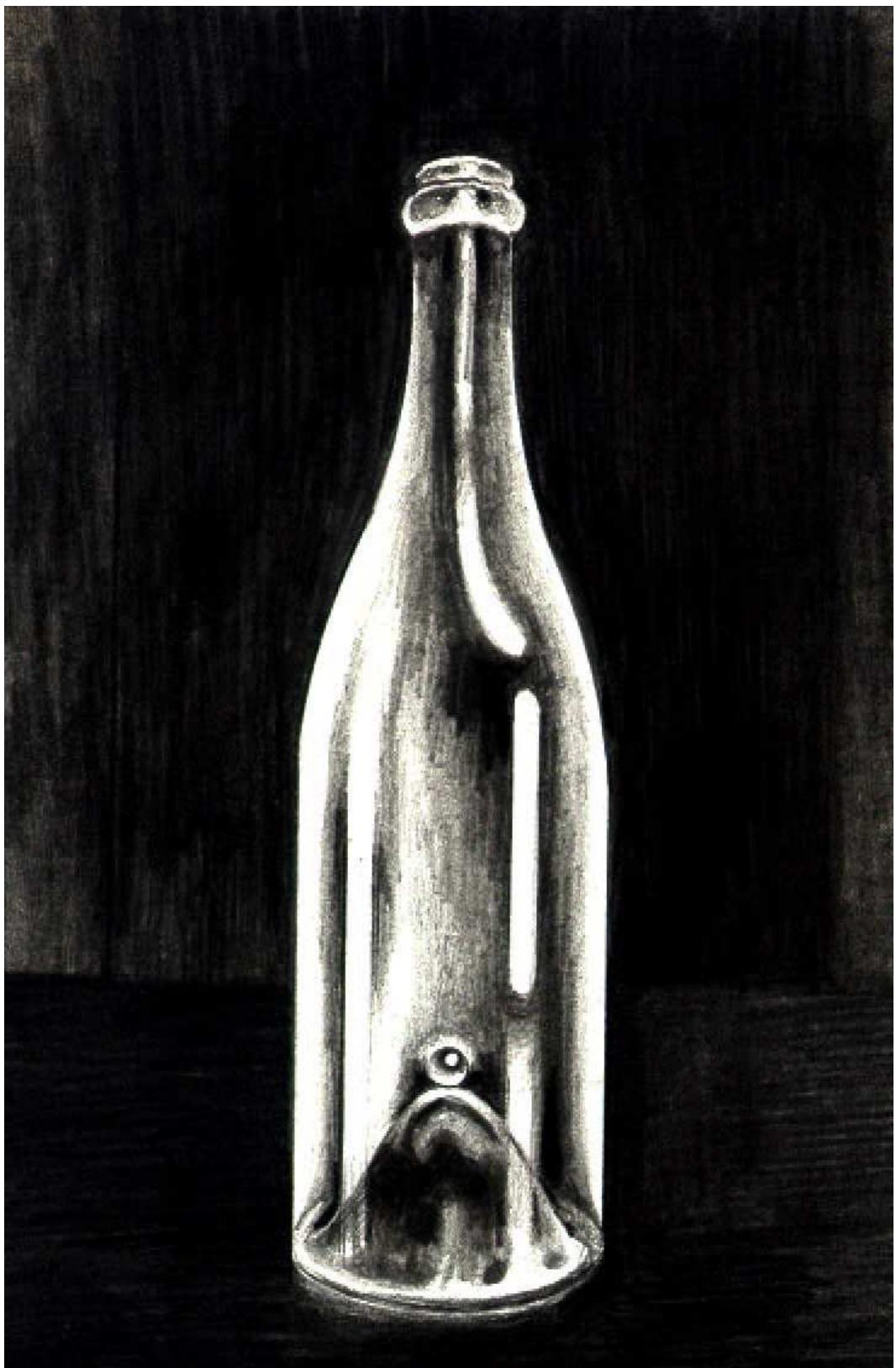
- ▷ Arrange self-interactions so vacuum corresponds to a broken-symmetry solution:  $\mu^2 < 0$   
 Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks)  $SU(2)_L$  and  $U(1)_Y$   
 but preserves  $U(1)_{\text{em}}$  invariance

Invariance under  $\mathcal{G}$  means  $e^{i\alpha\mathcal{G}}\langle \phi \rangle_0 = \langle \phi \rangle_0$ , so  $\mathcal{G}\langle \phi \rangle_0 = 0$

$$\begin{aligned} \tau_1 \langle \phi \rangle_0 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!} \\ \tau_2 \langle \phi \rangle_0 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!} \\ \tau_3 \langle \phi \rangle_0 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!} \\ Y \langle \phi \rangle_0 &= Y_\phi \langle \phi \rangle_0 = +1 \langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!} \end{aligned}$$



Examine electric charge operator  $Q$  on the (electrically neutral) vacuum state

$$\begin{aligned}
 Q\langle\phi\rangle_0 &= \frac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 \\
 &= \frac{1}{2} \begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix} \langle\phi\rangle_0 \\
 &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\
 &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{unbroken!}
 \end{aligned}$$

Four original generators are broken

electric charge is not

▷  $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}}$  (will verify)

▷ Expect massless photon

▷ Expect gauge bosons corresponding to

$$\tau_1, \tau_2, \frac{1}{2}(\tau_3 - Y) \equiv K$$

to acquire masses

## Expand about the vacuum state

Let  $\phi = \begin{pmatrix} 0 \\ (v + \eta)/\sqrt{2} \end{pmatrix}$ ; in *unitary gauge*

$$\begin{aligned}\mathcal{L}_{\text{scalar}} &= \frac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2 \eta^2 \\ &\quad + \frac{v^2}{8}[g^2 |b_1 - ib_2|^2 + (g' \mathcal{A}_\mu - gb_\mu^3)^2] \\ &\quad + \text{interaction terms}\end{aligned}$$

Higgs boson  $\eta$  has acquired (mass)<sup>2</sup>  $M_H^2 = -2\mu^2 > 0$

$$\frac{g^2 v^2}{8}(|W_\mu^+|^2 + |W_\mu^-|^2) \iff M_{W^\pm} = gv/2$$

Now define orthogonal combinations

$$Z_\mu = \frac{-g' \mathcal{A}_\mu + gb_\mu^3}{\sqrt{g^2 + g'^2}} \quad A_\mu = \frac{g \mathcal{A}_\mu + g' b_\mu^3}{\sqrt{g^2 + g'^2}}$$

$$M_{Z^0} = \sqrt{g^2 + g'^2} v/2 = M_W \sqrt{1 + g'^2/g^2}$$

A<sub>μ</sub> remains massless

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= -\zeta_e \frac{(v + \eta)}{\sqrt{2}} (\bar{e}_R e_L + \bar{e}_L e_R) \\ &= -\frac{\zeta_e v}{\sqrt{2}} \bar{e} e - \frac{\zeta_e \eta}{\sqrt{2}} \bar{e} e\end{aligned}$$

electron acquires  $m_e = \zeta_e v / \sqrt{2}$

Higgs coupling to electrons:  $m_e/v$  ( $\propto$  mass)

Desired particle content . . . + Higgs scalar

Values of couplings, electroweak scale  $v$ ?

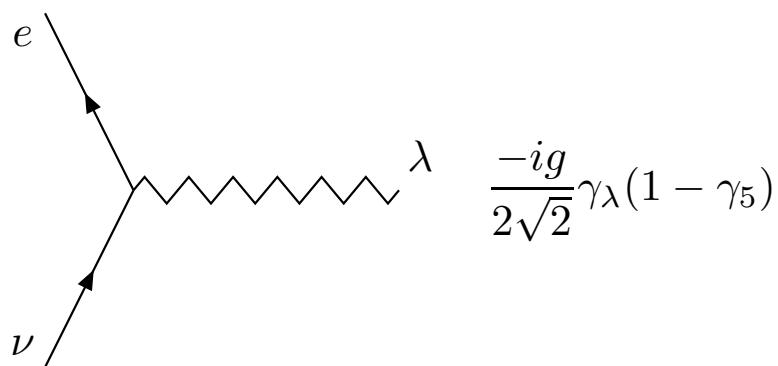
What about interactions?

# Interactions . . .

$$\mathcal{L}_{W-\ell} = -\frac{g}{2\sqrt{2}} [\bar{\nu}\gamma^\mu(1-\gamma_5)eW_\mu^+ + \bar{e}\gamma^\mu(1-\gamma_5)\nu W_\mu^-]$$

+ similar terms for  $\mu$  and  $\tau$

Feynman rule:



gauge-boson propagator:

$$W \quad \text{wavy line} \quad = \frac{-i(g_{\mu\nu} - k_\mu k_\nu/M_W^2)}{k^2 - M_W^2} .$$

**Compute**  $\nu_\mu e \rightarrow \mu \nu_e$

$$\sigma(\nu_\mu e \rightarrow \mu \nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\begin{aligned} \frac{g^4}{16M_W^4} &= 2G_F^2 \\ \Rightarrow g^4 &= 32(G_F M_W^2)^2 = 64 \left( \frac{G_F M_W^2}{\sqrt{2}} \right)^2 \\ \Rightarrow \frac{g}{2\sqrt{2}} &= \left( \frac{G_F M_W^2}{\sqrt{2}} \right)^{\frac{1}{2}} \end{aligned}$$

Using  $M_W = gv/2$ , determine

$$v = (G_F \sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV}$$

the electroweak scale

$$\Rightarrow \langle \phi^0 \rangle_0 = (G_F \sqrt{8})^{-\frac{1}{2}} \approx 174 \text{ GeV}$$

## **$W$ -propagator modifies HE behavior**

$$\sigma(\nu_\mu e \rightarrow \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

$$\lim_{E_\nu \rightarrow \infty} \sigma(\nu_\mu e \rightarrow \mu\nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\sqrt{2}}$$

independent of energy!

partial-wave unitarity respected for

$$s < M_W^2 [\exp(\pi\sqrt{2}/G_F M_W^2) - 1]$$

# $W$ -boson properties

No prediction yet for  $M_W$  (haven't determined  $g$ )

Leptonic decay  $W^- \rightarrow e^- \bar{\nu}_e$

$$e(p) \quad p \approx \left( \frac{M_W}{2}; \frac{M_W \sin \theta}{2}, 0, \frac{M_W \cos \theta}{2} \right)$$

$$\bar{\nu}_e(q) \quad q \approx \left( \frac{M_W}{2}; -\frac{M_W \sin \theta}{2}, 0, -\frac{M_W \cos \theta}{2} \right)$$

$$\mathcal{M} = -i \left( \frac{G_F M_W^2}{\sqrt{2}} \right)^{\frac{1}{2}} \bar{u}(e, p) \gamma_\mu (1 - \gamma_5) v(\nu, q) \varepsilon^\mu$$

$\varepsilon^\mu = (0; \hat{\varepsilon})$ :  $W$  polarization vector in its rest frame

$$|\mathcal{M}|^2 = \frac{G_F M_W^2}{\sqrt{2}} \text{tr} [\not{q} (1 - \gamma_5) \not{q} (1 + \gamma_5) \not{\varepsilon}^* \not{p}] ;$$

$$\text{tr}[\dots] = [\varepsilon \cdot q \varepsilon^* \cdot p - \varepsilon \cdot \varepsilon^* q \cdot p + \varepsilon \cdot p \varepsilon^* \cdot q + i \epsilon_{\mu\nu\rho\sigma} \varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma]$$

*decay rate* is independent of  $W$  polarization; look first at longitudinal pol.  $\varepsilon^\mu = (0; 0, 0, 1) = \varepsilon^{*\mu}$ , eliminate  $\epsilon_{\mu\nu\rho\sigma}$

$$|\mathcal{M}|^2 = \frac{4 G_F M_W^4}{\sqrt{2}} \sin^2 \theta$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2} \frac{\mathcal{S}_{12}}{M_W^3}$$

$$\mathcal{S}_{12} = \sqrt{[M_W^2 - (m_e + m_\nu)^2][M_W^2 - (m_e - m_\nu)^2]} = M_W^2$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2 \sqrt{2}} \sin^2 \theta$$

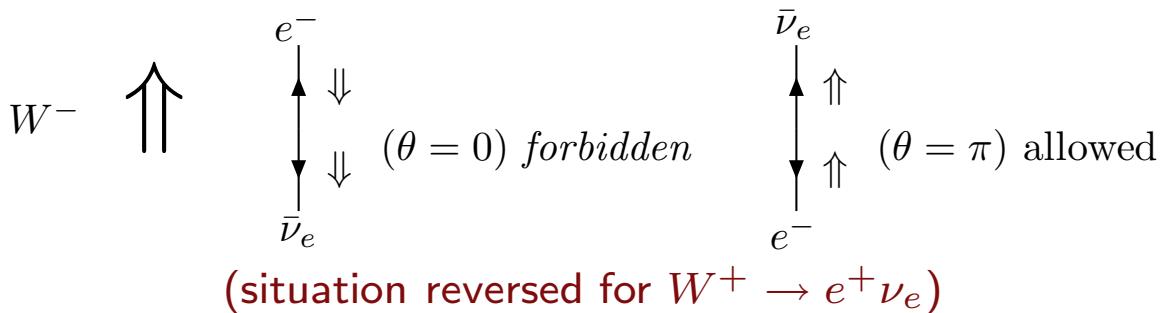
and

$$\boxed{\Gamma(W \rightarrow e\nu) = \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

Other helicities:  $\varepsilon_{\pm 1}^\mu = (0; -1, \mp i, 0)/\sqrt{2}$

$$\frac{d\Gamma_{\pm 1}}{d\Omega} = \frac{G_F M_W^3}{32\pi^2 \sqrt{2}} (1 \mp \cos \theta)^2$$

Extinctions at  $\cos \theta = \pm 1$  are consequences of angular momentum conservation:



$e^+$  follows polarization direction of  $W^+$

$e^-$  avoids polarization direction of  $W^-$

important for discovery of  $W^\pm$  in  $\bar{p}p$  ( $\bar{q}q$ ) C violation

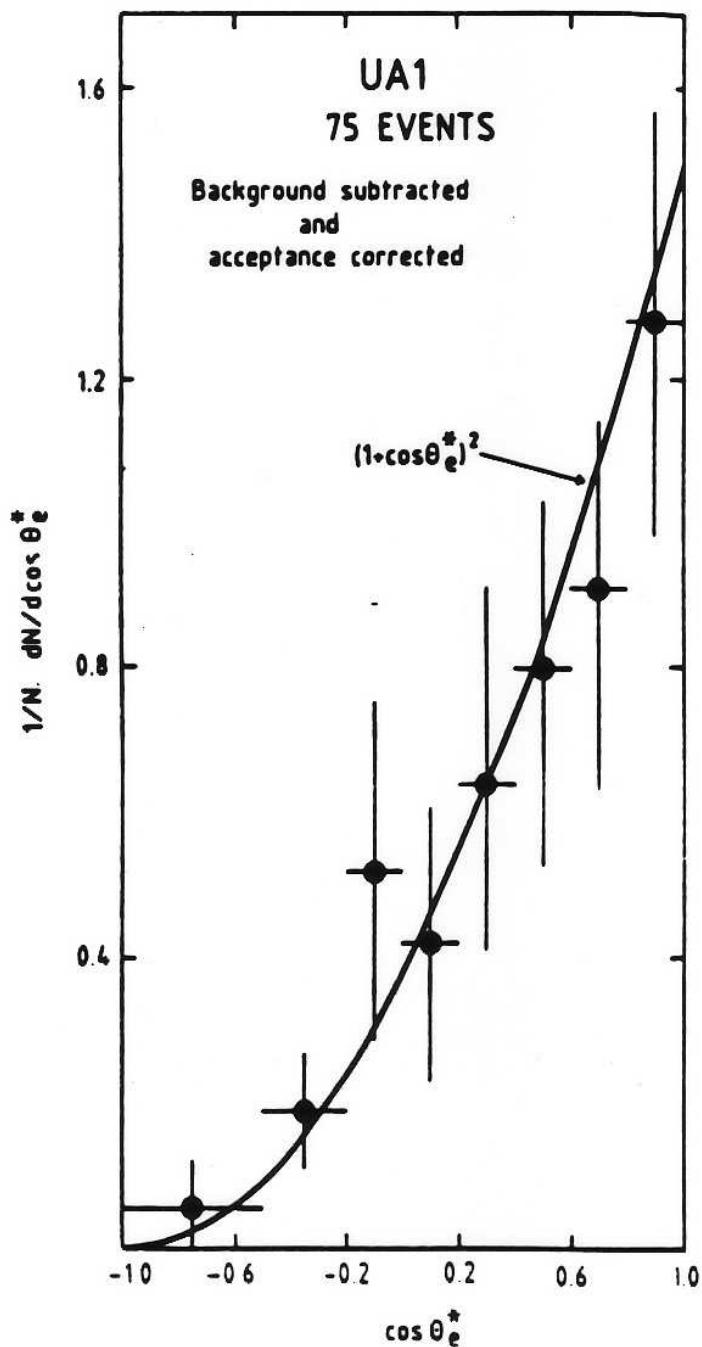


Fig. 2. The W decay angular distribution of the emission angle  $\theta^*$  of the electron (positron) with respect to the proton (anti-proton) direction in the rest frame of the W. Only those events for which the lepton charge and the decay kinematics are well determined have been used. The curve shows the  $(V - A)$  expectation of  $(1 + \cos\theta^*)^2$ .

## Interactions . . .

$$\mathcal{L}_{A-\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e} \gamma^\mu e A_\mu$$

. . . vector interaction;  $\Rightarrow A_\mu$  as  $\gamma$ , provided

$$gg' / \sqrt{g^2 + g'^2} \equiv e$$

Define  $g' = g \tan \theta_W$        $\theta_W$ : weak mixing angle

$$g = e / \sin \theta_W \geq e$$

$$g' = e / \cos \theta_W \geq e$$

$$Z_\mu = b_\mu^3 \cos \theta_W - \mathcal{A}_\mu \sin \theta_W \quad A_\mu = \mathcal{A}_\mu \cos \theta_W + b_\mu^3 \sin \theta_W$$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4 \cos \theta_W} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu Z_\mu$$

$$\mathcal{L}_{Z-e} = \frac{-g}{4 \cos \theta_W} \bar{e} [L_e \gamma^\mu (1 - \gamma_5) + R_e \gamma^\mu (1 + \gamma_5)] e Z_\mu$$

$$L_e = 2 \sin^2 \theta_W - 1 = 2x_W + \tau_3$$

$$R_e = 2 \sin^2 \theta_W = 2x_W$$

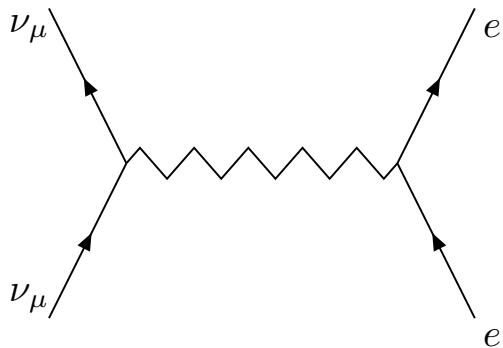
## **$Z$ -boson properties**

Decay calculation analogous to  $W^\pm$

$$\begin{aligned}\Gamma(Z \rightarrow \nu\bar{\nu}) &= \frac{G_F M_Z^3}{12\pi\sqrt{2}} \\ \Gamma(Z \rightarrow e^+e^-) &= \Gamma(Z \rightarrow \nu\bar{\nu}) [L_e^2 + R_e^2]\end{aligned}$$

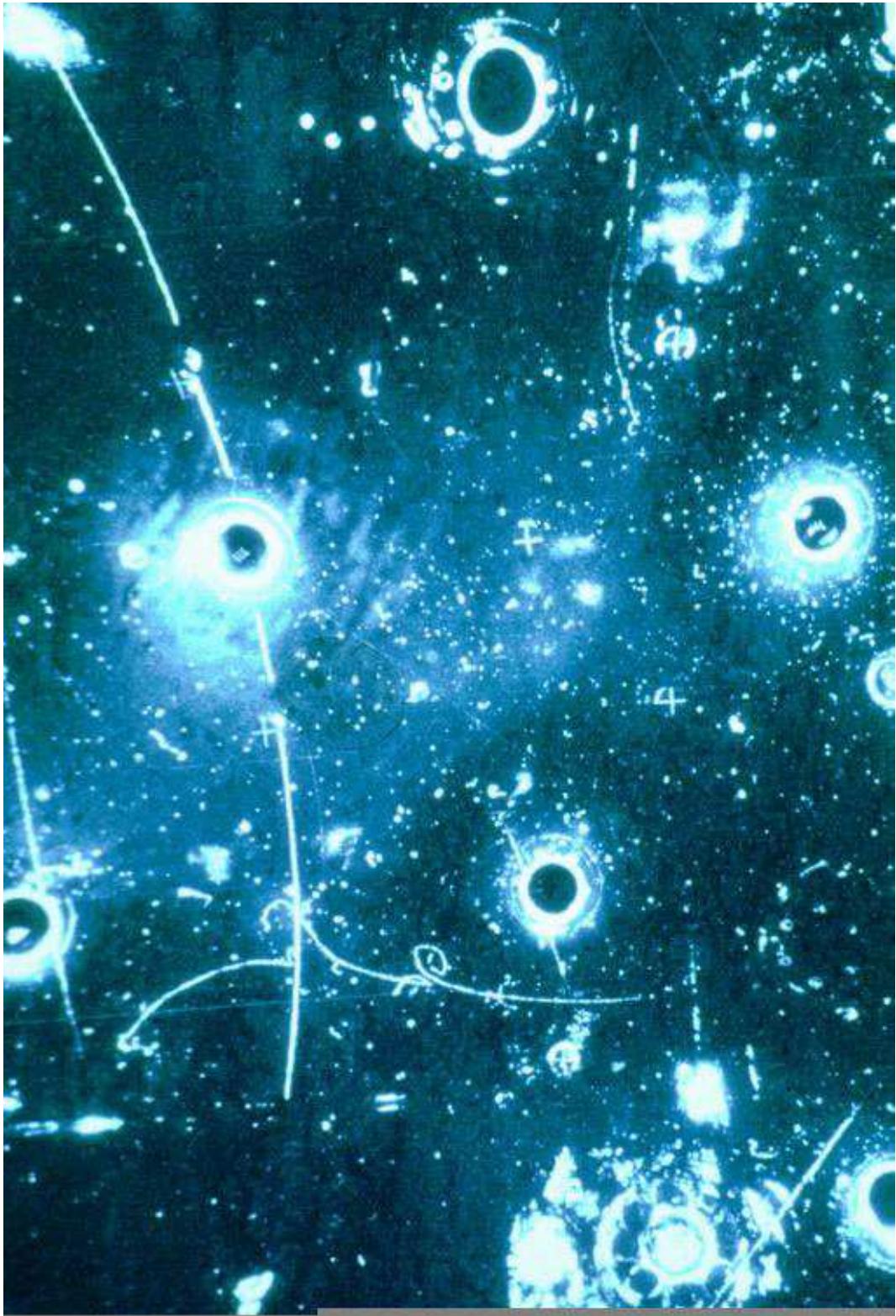
## **Neutral-current interactions**

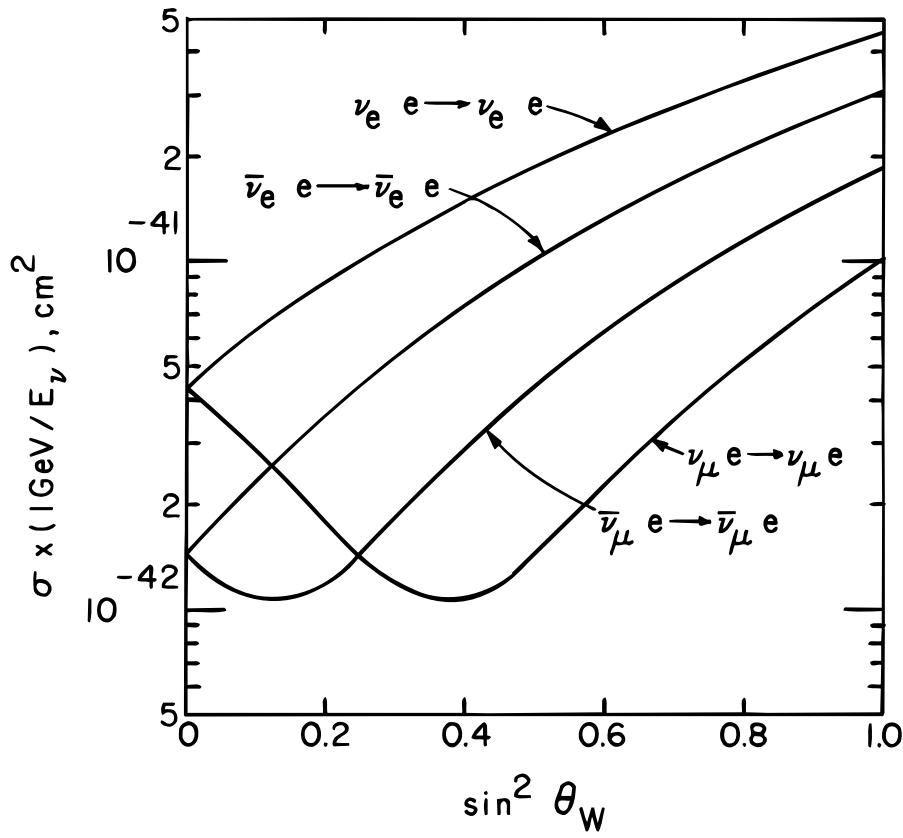
New  $\nu e$  reaction, not present in  $V - A$



$$\begin{aligned}\sigma(\nu_\mu e \rightarrow \nu_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi} [L_e^2 + R_e^2/3] \\ \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi} [L_e^2/3 + R_e^2] \\ \sigma(\nu_e e \rightarrow \nu_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi} [(L_e + 2)^2 + R_e^2/3] \\ \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi} [(L_e + 2)^2/3 + R_e^2]\end{aligned}$$

## Gargamelle $\nu_\mu e$ Event





## “Model-independent” analysis

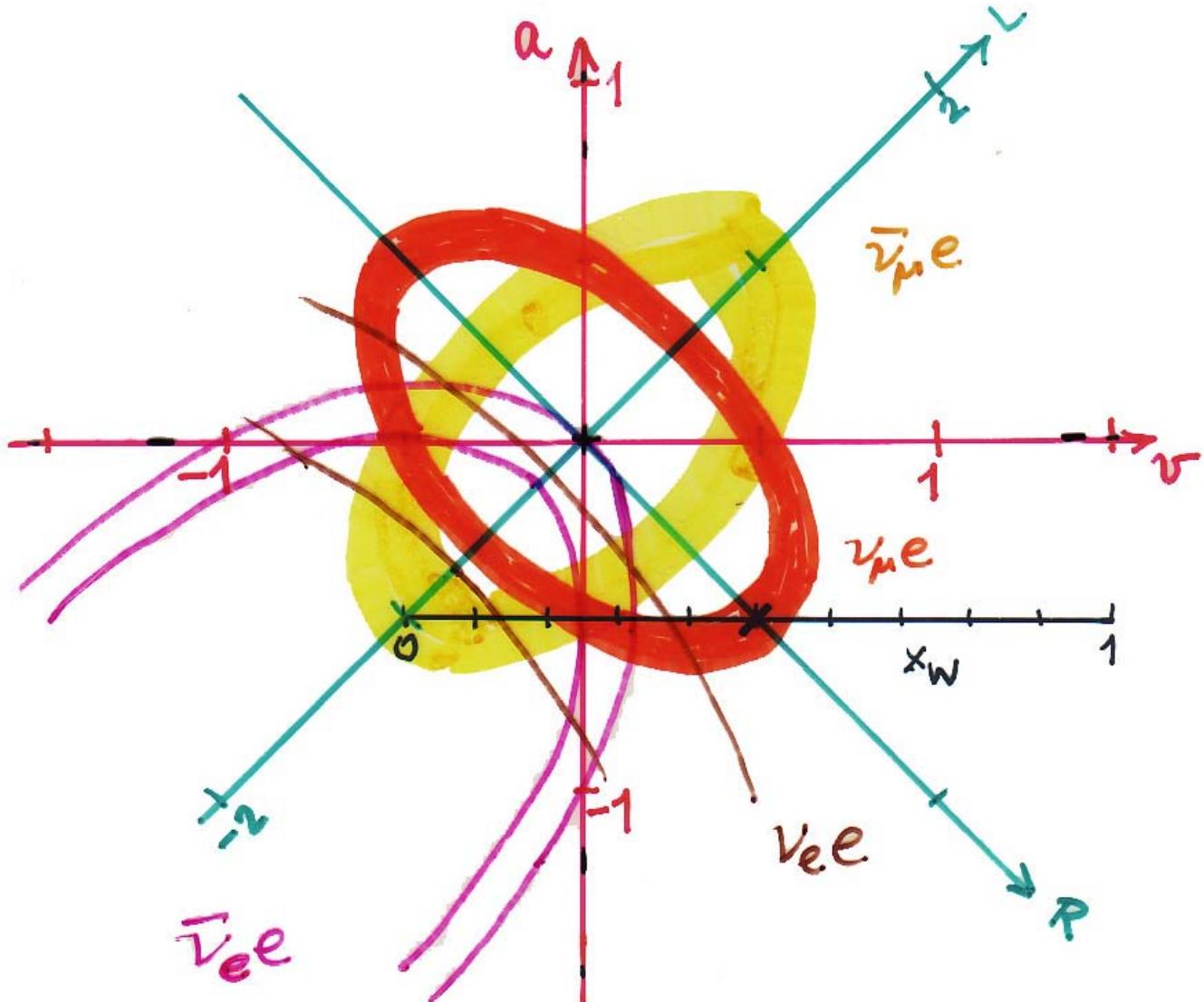
Measure all cross sections to determine chiral couplings  $L_e$  and  $R_e$  or traditional vector and axial couplings  $v$  and  $a$

$$a = \frac{1}{2}(L_e - R_e) \quad v = \frac{1}{2}(L_e + R_e)$$

$$L_e = v + a \quad R_e = v - a$$

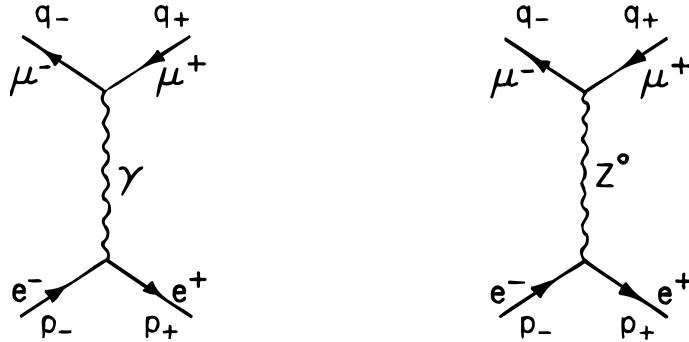
model-independent in  $V, A$  framework

# Neutrino-electron scattering



Twofold ambiguity remains even after measuring all four cross sections: same cross sections result if we interchange  $R_e \leftrightarrow -R_e$  ( $v \leftrightarrow a$ )

Consider  $e^+e^- \rightarrow \mu^+\mu^-$



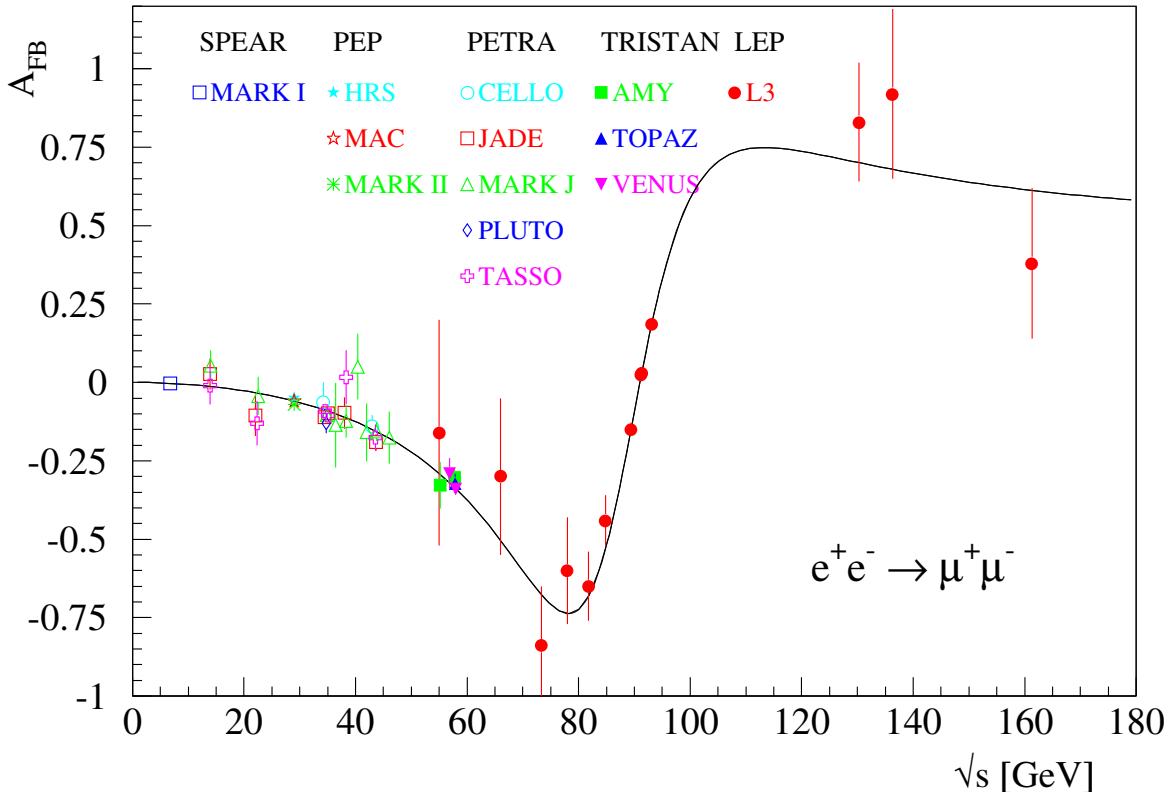
$$\begin{aligned} \mathcal{M} = & -ie^2 \bar{u}(\mu, q_-) \gamma_\lambda Q_\mu v(\mu, q_+) \frac{g^{\lambda\nu}}{s} \bar{v}(e, p_+) \gamma_\nu u(e.p_-) \\ & + \frac{i}{2} \left( \frac{G_F M_Z^2}{\sqrt{2}} \right) \bar{u}(\mu, q_-) \gamma_\lambda [R_\mu (1 + \gamma_5) + L_\mu (1 - \gamma_5)] v(\mu, q_+) \\ & \times \frac{g^{\lambda\nu}}{s - M_Z^2} \bar{v}(e, p_+) \gamma_\nu [R_e (1 + \gamma_5) + L_e (1 - \gamma_5)] u(e.p_-) \end{aligned}$$

muon charge  $Q_\mu = -1$

$$\begin{aligned} \frac{d\sigma}{dz} = & \frac{\pi \alpha^2 Q_\mu^2}{2s} (1 + z^2) \\ & - \frac{\alpha Q_\mu G_F M_Z^2 (s - M_Z^2)}{8\sqrt{2}[(s - M_Z^2)^2 + M_Z^2 \Gamma^2]} \\ & \times [(R_e + L_e)(R_\mu + L_\mu)(1 + z^2) + 2(R_e - L_e)(R_\mu - L_\mu)z] \\ & + \frac{G_F^2 M_Z^4 s}{64\pi[(s - M_Z^2)^2 + M_Z^2 \Gamma^2]} \\ & \times [(R_e^2 + L_e^2)(R_\mu^2 + L_\mu^2)(1 + z^2) + 2(R_e^2 - L_e^2)(R_\mu^2 - L_\mu^2)z] \end{aligned}$$

$$\text{F-B asymmetry } A \equiv \frac{\int_0^1 dz d\sigma/dz - \int_{-1}^0 dz d\sigma/dz}{\int_{-1}^1 dz d\sigma/dz}$$

$$\begin{aligned}\lim_{s/M_Z^2 \ll 1} A &= \frac{3G_F s}{16\pi\alpha Q_\mu \sqrt{2}} (R_e - L_e)(R_\mu - L_\mu) \\ &\approx -6.7 \times 10^{-5} \left( \frac{s}{1 \text{ GeV}^2} \right) (R_e - L_e)(R_\mu - L_\mu) \\ &= -3G_F s a^2 / 4\pi\alpha\sqrt{2}\end{aligned}$$

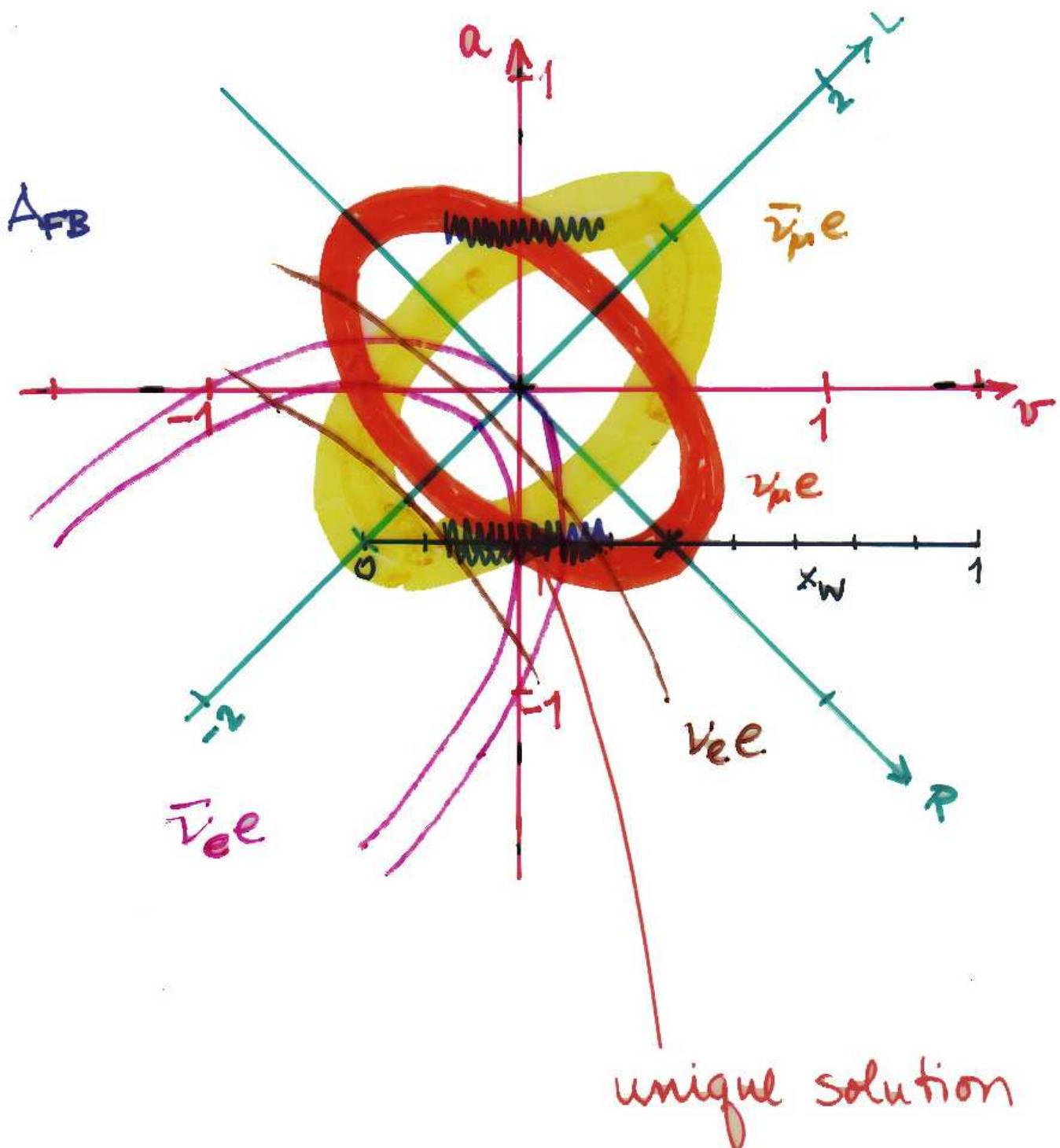


J. Mnich *Phys. Rep.* **271**, 181-266 (1996)

Measuring  $A$  resolves ambiguity

Validate EW theory, measure  $\sin^2 \theta_W$

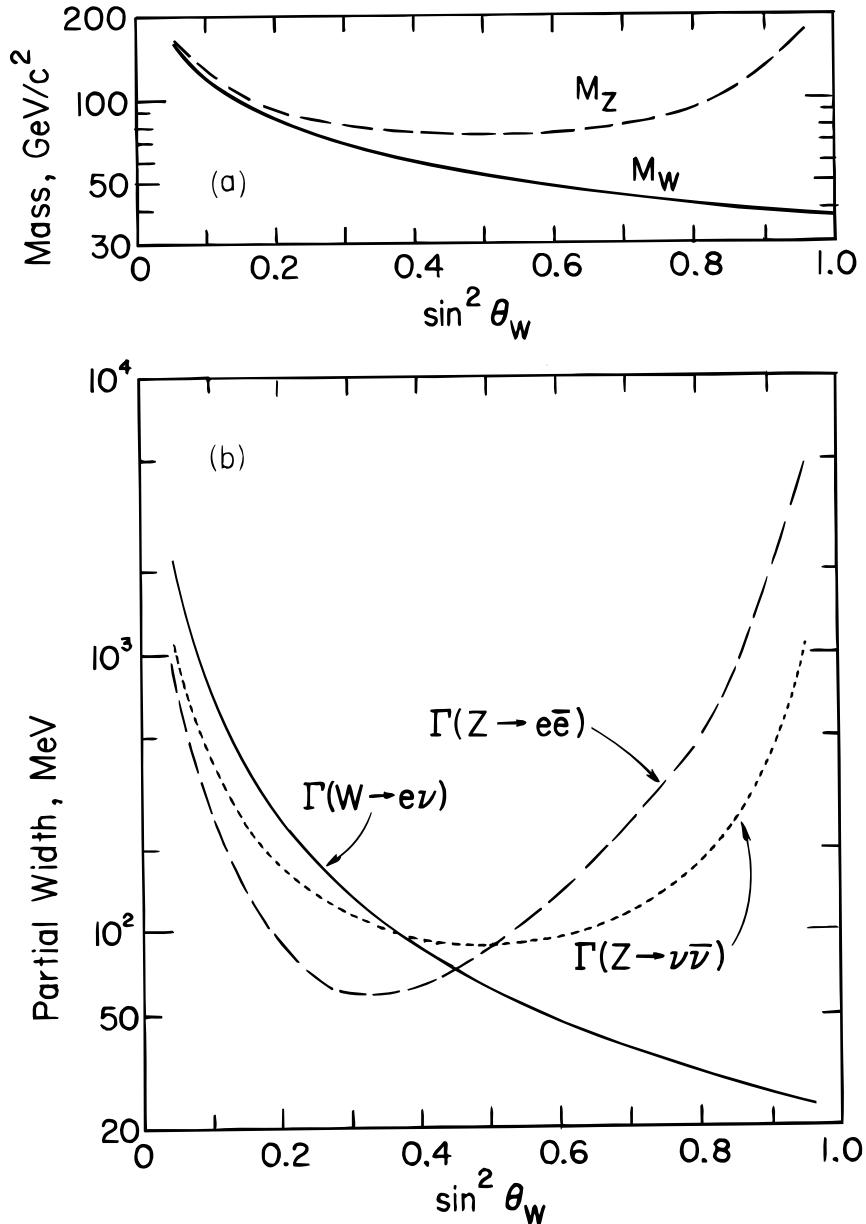
# Neutrino-electron scattering



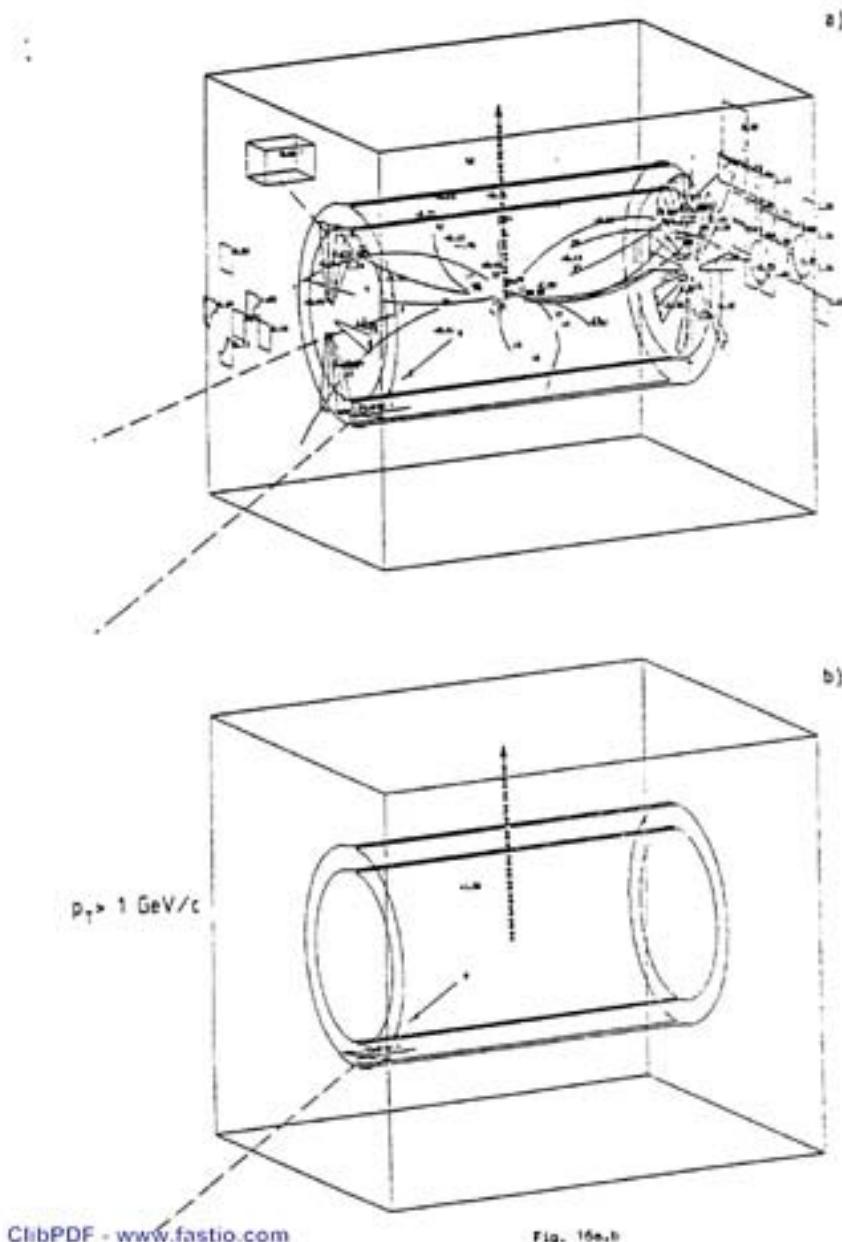
With a measurement of  $\sin^2 \theta_W$ , predict

$$M_W^2 = g^2 v^2 / 4 = e^2 / 4G_F \sqrt{2} \sin^2 \theta_W \approx (37.3 \text{ GeV}/c^2)^2 / \sin^2 \theta_W$$

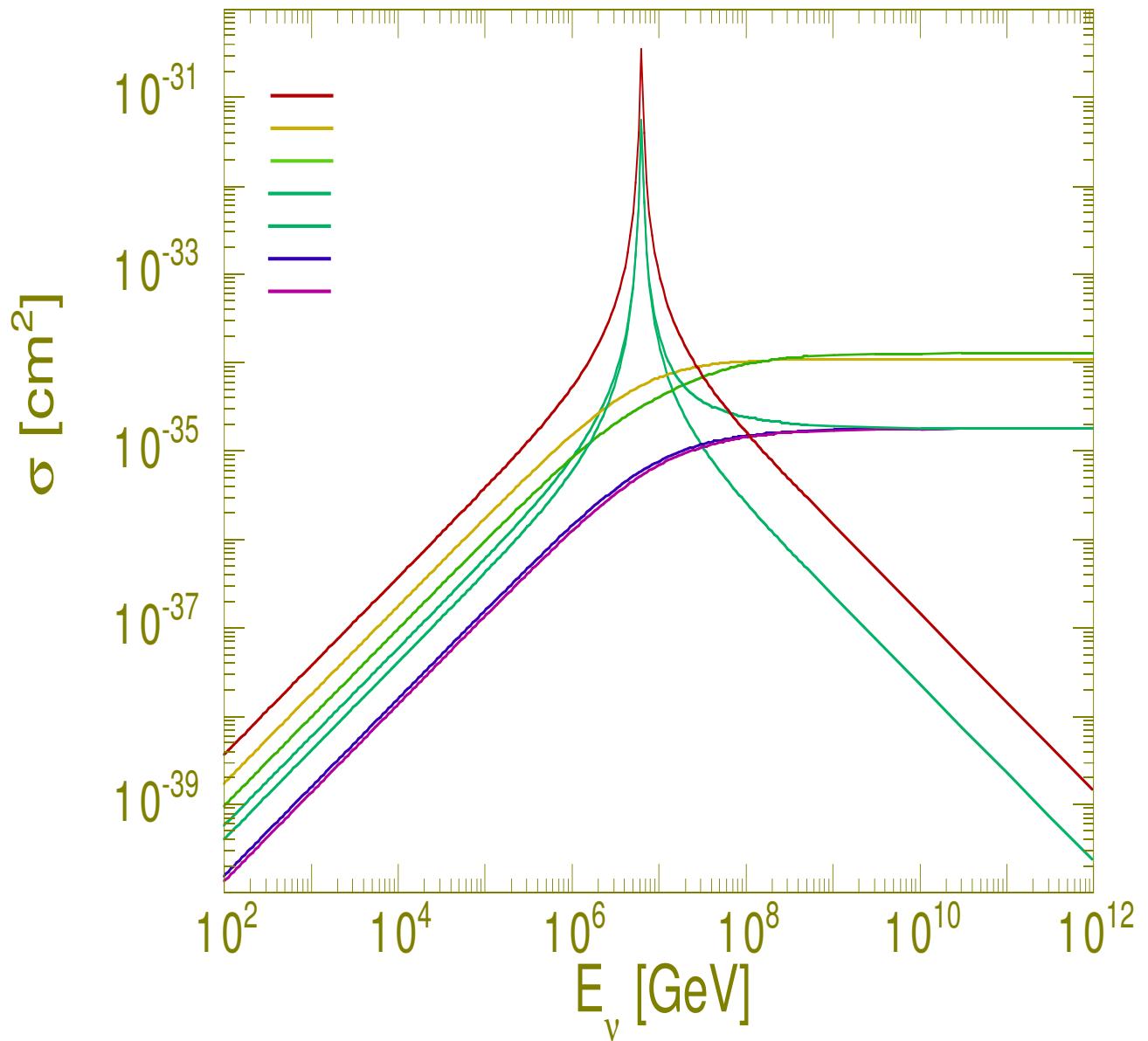
$$M_Z^2 = M_W^2 / \cos^2 \theta_W$$



568 Intermediate Vector Bosons  $W^+$ ,  $W^-$ , and  $Z^0$



UA1



At low energies:  $\sigma(\bar{\nu}_e e \rightarrow \text{hadrons}) > \sigma(\nu_\mu e \rightarrow \mu\nu_e) >$   
 $\sigma(\nu_e e \rightarrow \nu_e e) > \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) >$   
 $\sigma(\nu_\mu e \rightarrow \nu_\mu e) > \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e)$

# EW interactions of quarks

- ▷ Left-handed doublet

$$I_3 \quad Q \quad Y = 2(Q - I_3)$$

$$\mathsf{L}_q = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{matrix} \frac{1}{2} \\ -\frac{1}{2} \end{matrix} \quad \begin{matrix} +\frac{2}{3} \\ -\frac{1}{3} \end{matrix} \quad \frac{1}{3}$$

- ▷ two right-handed singlets

$$I_3 \quad Q \quad Y = 2(Q - I_3)$$

$$\mathsf{R}_u = u_R \quad 0 \quad +\frac{2}{3} \quad +\frac{4}{3}$$

$$\mathsf{R}_d = d_R \quad 0 \quad -\frac{1}{3} \quad -\frac{2}{3}$$

- ▷ CC interaction

$$\mathcal{L}_{W-q} = \frac{-g}{2\sqrt{2}} [\bar{u}_e \gamma^\mu (1 - \gamma_5) d W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u W_\mu^-]$$

identical in form to  $\mathcal{L}_{W-\ell}$ : universality  $\Leftrightarrow$  weak isospin

- ▷ NC interaction

$$\mathcal{L}_{Z-q} = \frac{-g}{4 \cos \theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu [L_i(1 - \gamma_5) + R_i(1 + \gamma_5)] q_i Z_\mu$$

$$L_i = \tau_3 - 2Q_i \sin^2 \theta_W \quad R_i = -2Q_i \sin^2 \theta_W$$

equivalent in form (not numbers) to  $\mathcal{L}_{Z-\ell}$

# Trouble in Paradise

Universal  $u \leftrightarrow d$ ,  $\nu_e \leftrightarrow e$  not quite right

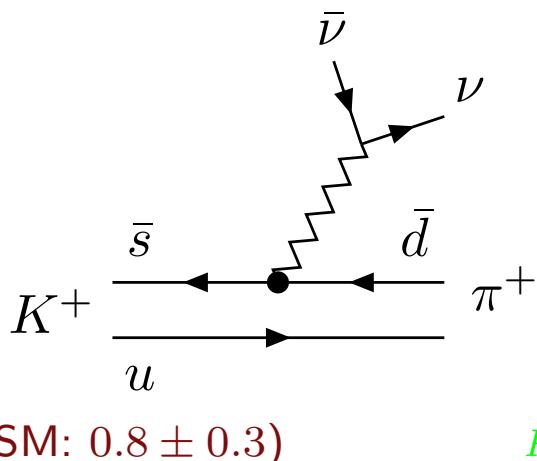
$$\text{Good: } \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow \text{Better: } \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

$$d_\theta \equiv d \cos \theta_C + s \sin \theta_C \quad \cos \theta_C = 0.9736 \pm 0.0010$$

“Cabibbo-rotated” doublet perfects CC interaction (up to small third-generation effects) but  $\Rightarrow$  serious trouble for NC

$$\begin{aligned} \mathcal{L}_{Z-q} = & \frac{-g}{4 \cos \theta_W} Z_\mu \{ \bar{u} \gamma^\mu [L_u(1 - \gamma_5) + R_u(1 + \gamma_5)] u \\ & + \bar{d} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] d \cos^2 \theta_C \\ & + \bar{s} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] s \sin^2 \theta_C \\ & + \bar{d} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] s \sin \theta_C \cos \theta_C \\ & + \bar{s} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] d \sin \theta_C \cos \theta_C \} \end{aligned}$$

Strangeness-changing NC interactions highly suppressed!



BNL E-787/E-949 has three  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  candidates, with  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1.47^{+1.30}_{-0.89} \times 10^{-10}$

*Phys. Rev. Lett.* **93**, 031801 (2004)

# Glashow–Iliopoulos–Maiani

two left-handed doublets

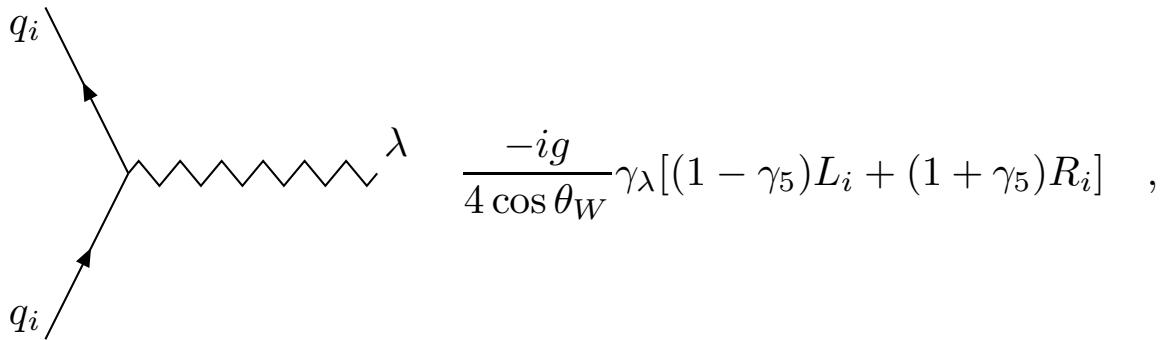
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \quad \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$$

$$(s_\theta = s \cos \theta_C - d \sin \theta_C)$$

+ right-handed singlets,  $e_R, \mu_R, u_R, d_R, c_R, s_R$

Required new charmed quark,  $c$

Cross terms vanish in  $\mathcal{L}_{Z-q}$ ,



$$L_i = \tau_3 - 2Q_i \sin^2 \theta_W \quad R_i = -2Q_i \sin^2 \theta_W$$

flavor-diagonal interaction!

Straightforward generalization to  $n$  quark doublets

$$\mathcal{L}_{W-q} = \frac{-g}{2\sqrt{2}} [\bar{\Psi} \gamma^\mu (1 - \gamma_5) \mathcal{O} \Psi W_\mu^+ + \text{h.c.}]$$

composite  $\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix}$

flavor structure  $\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$

$U$ : unitary quark mixing matrix

Weak-isospin part:  $\mathcal{L}_{Z-q}^{\text{iso}} = \frac{-g}{4 \cos \theta_W} \bar{\Psi} \gamma^\mu (1 - \gamma_5) [\mathcal{O}, \mathcal{O}^\dagger] \Psi$

Since  $[\mathcal{O}, \mathcal{O}^\dagger] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

$\Rightarrow$  NC interaction is flavor-diagonal

General  $n \times n$  quark-mixing matrix  $U$ :

$n(n-1)/2$  real  $\angle$ ,  $(n-1)(n-2)/2$  complex phases

$3 \times 3$  (Cabibbo–Kobayashi–Maskawa):  $3 \angle + 1$  phase  
 $\Rightarrow$  CP violation

## Qualitative successes of $SU(2)_L \otimes U(1)_Y$ theory:

- ▷ neutral-current interactions
- ▷ necessity of charm
- ▷ existence and properties of  $W^\pm$  and  $Z^0$

## Decade of precision tests EW (one-per-mille)

---

$M_Z$	$91\,187.6 \pm 2.1$ MeV/c <sup>2</sup>
$\Gamma_Z$	$2495.2 \pm 2.3$ MeV
$\sigma_{\text{hadronic}}^0$	$41.541 \pm 0.037$ nb
$\Gamma_{\text{hadronic}}$	$1744.4 \pm 2.0$ MeV
$\Gamma_{\text{leptonic}}$	$83.984 \pm 0.086$ MeV
$\Gamma_{\text{invisible}}$	$499.0 \pm 1.5$ MeV

---

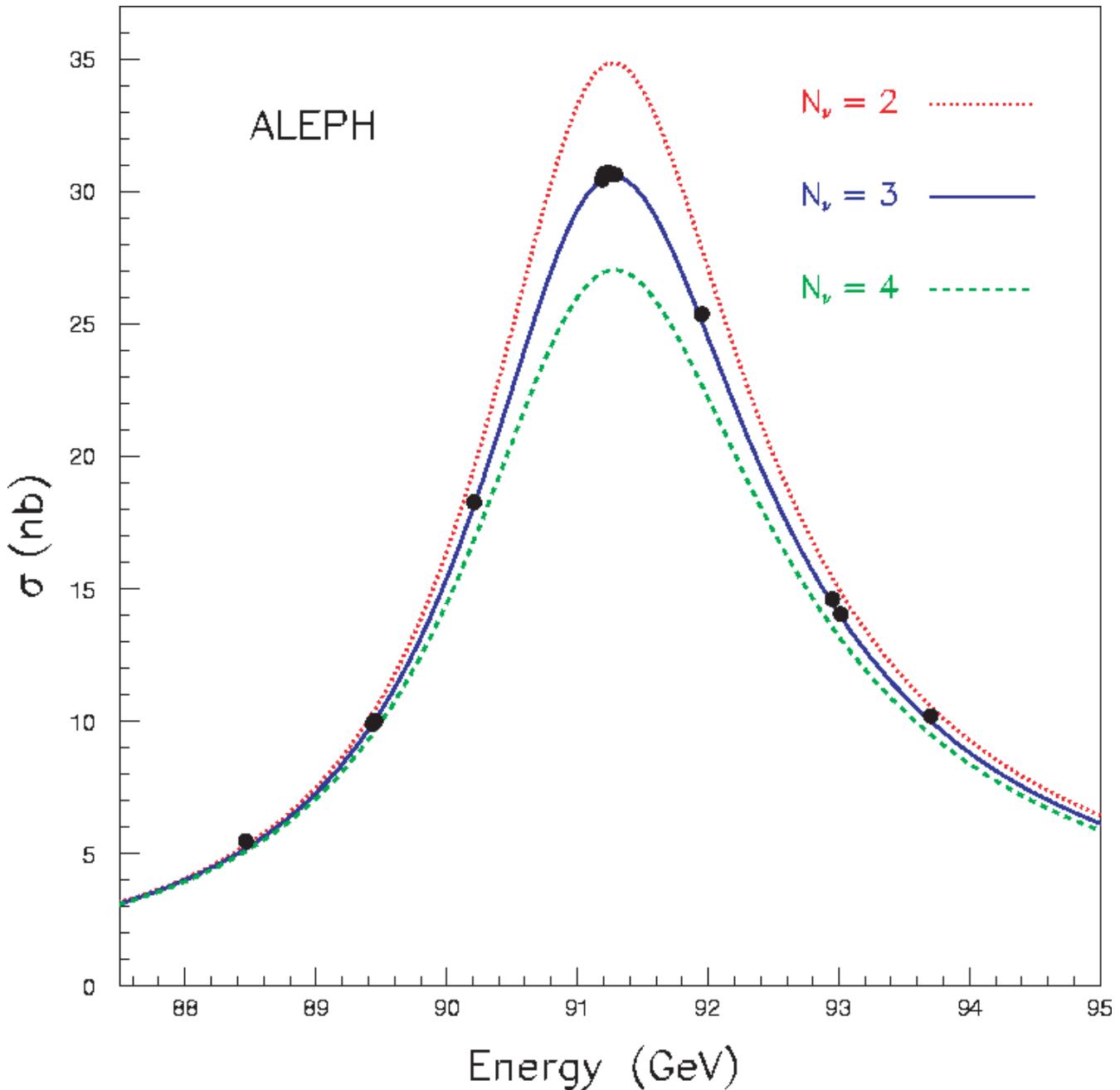
where  $\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$

light neutrinos  $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \rightarrow \nu_i \bar{\nu}_i)$

Current value:  $N_\nu = 2.994 \pm 0.012$

. . . excellent agreement with  $\nu_e$ ,  $\nu_\mu$ , and  $\nu_\tau$

# Three light neutrinos



## The top quark must exist

- ▷ Two families

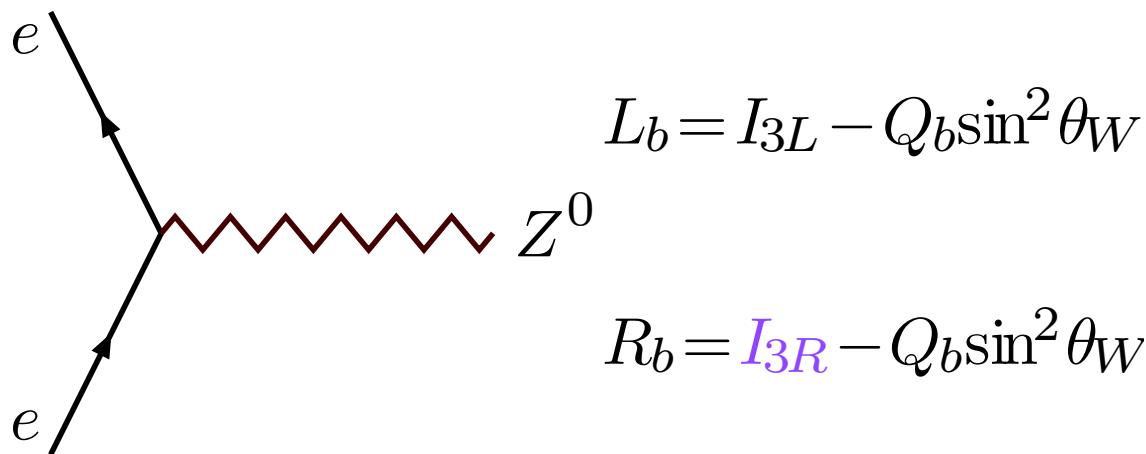
$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L$$

don't account for CP violation. Need a third family . . . or another answer.

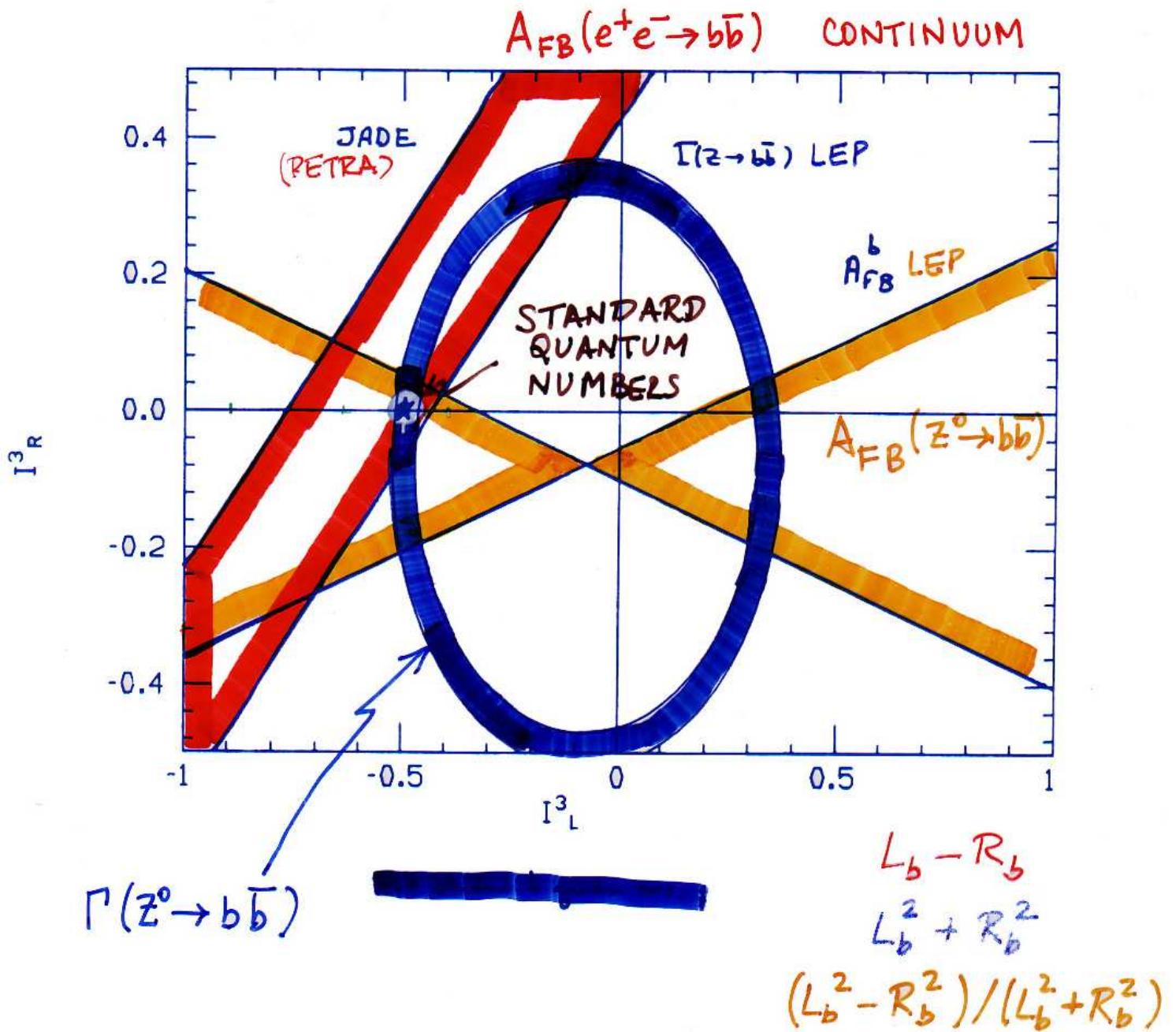
Given the existence of  $b$ , ( $\tau$ )

- ▷ top is needed for an anomaly-free EW theory
- ▷ absence of FCNC in  $b$  decay ( $b \not\rightarrow s\ell^+\ell^-$ , etc.)
- ▷  $b$  has weak isospin  $I_{3L} = -\frac{1}{2}$ ; needs partner

$$\begin{pmatrix} t \\ b \end{pmatrix}_L$$



Measure  $I_{3L}^{(b)} = -0.490^{+0.015}_{-0.012}$   $I_{3R}^{(b)} = -0.028 \pm 0.056$



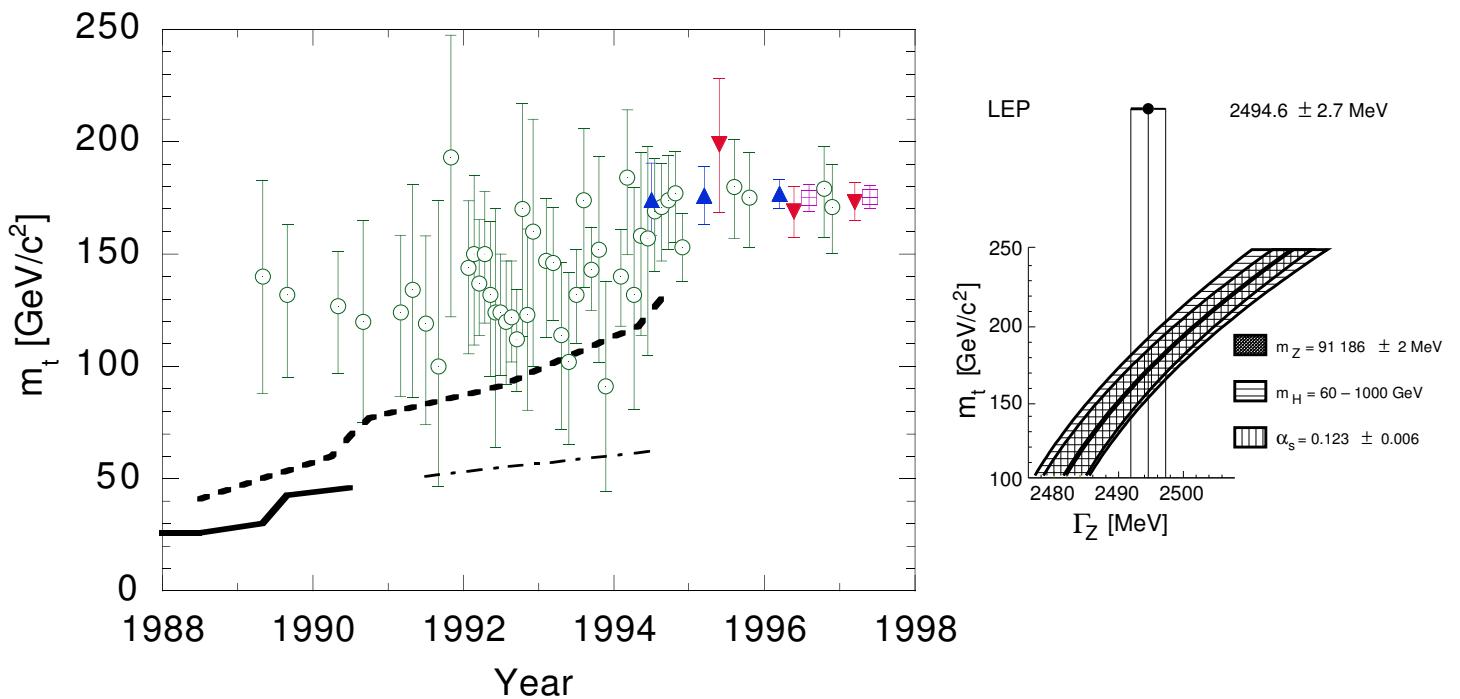
Needed: top with  $I_{3L} = +\frac{1}{2}$

D. Schaile & P. Zerwas, *Phys. Rev. D45*, 3262 (1992)

## Global fits . . .

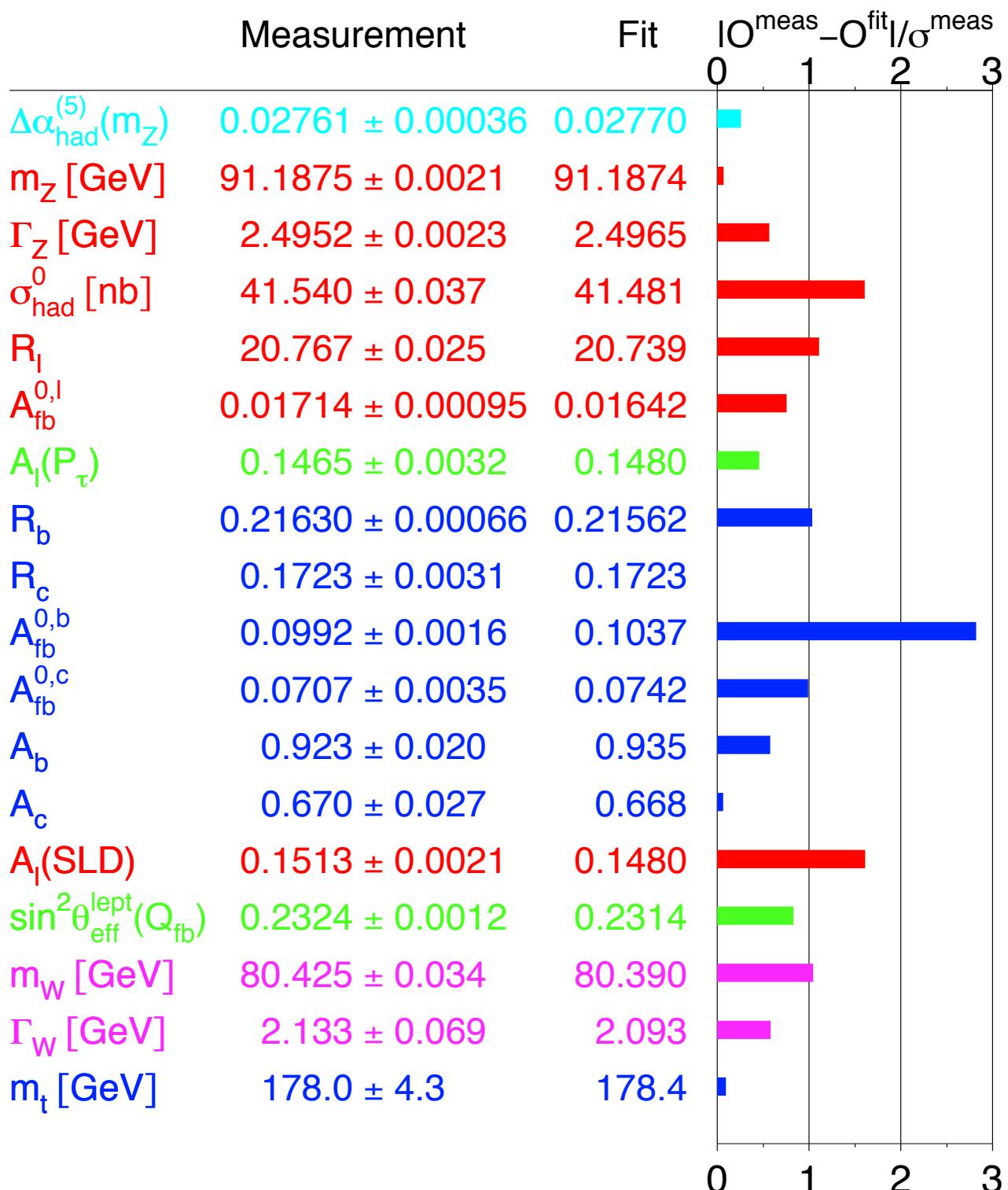
to precision EW measurements:

- ▷ precision improves with time
- ▷ calculations improve with time



11.94, LEPEWWG:  $m_t = 178 \pm 11^{+18}_{-19}$  GeV/c $^2$

Direct measurements:  $m_t = 174.3 \pm 5.1$  GeV/c $^2$



LEP Electroweak Working Group, Winter 2005

# Parity violation in atoms

Nucleon appears elementary at very low  $Q^2$ ; effective Lagrangian for nucleon  $\beta$ -decay

$$\mathcal{L}_\beta = - \frac{G_F}{\sqrt{2}} \bar{e} \gamma_\lambda (1 - \gamma_5) \nu \bar{p} \gamma^\lambda (1 - g_A \gamma_5) n$$

$g_A \approx 1.26$ : axial charge

NC interactions ( $x_W \equiv \sin^2 \theta_W$ ):

$$\begin{aligned} \mathcal{L}_{ep} &= \frac{G_F}{2\sqrt{2}} \bar{e} \gamma_\lambda (1 - 4x_W - \gamma_5) e \bar{p} \gamma^\lambda (1 - 4x_W - \gamma_5) p , \\ \mathcal{L}_{en} &= \frac{G_F}{2\sqrt{2}} \bar{e} \gamma_\lambda (1 - 4x_W - \gamma_5) e \bar{n} \gamma^\lambda (1 - \gamma_5) n \end{aligned}$$

▷ Regard nucleus as a **noninteracting collection** of  $Z$  protons and  $N$  neutrons    ▷ Perform NR reduction; nucleons contribute coherently to  $A_e V_N$  coupling, so dominant P-violating contribution to  $eN$  amplitude is

$$\mathcal{M}_{\text{pv}} = \frac{-iG_F}{2\sqrt{2}} Q^W \bar{e} \rho_N(\mathbf{r}) \gamma_5 e$$

$\rho_N(\mathbf{r})$ : nucleon density at  $e^-$  coordinate  $\mathbf{r}$

$Q^W \equiv Z(1 - 4x_W) - N$ : weak charge

Bennett & Wieman (Boulder) determined weak charge of Cesium by measuring 6S-7S transition polarizability

$$Q_W(\text{Cs}) = -72.06 \pm 0.28 \text{ (expt)} \pm 0.34 \text{ (theory)}$$

about  $2.5\sigma$  above SM prediction

# The vacuum energy problem

Higgs potential  $V(\varphi^\dagger \varphi) = \mu^2 (\varphi^\dagger \varphi) + |\lambda| (\varphi^\dagger \varphi)^2$

At the minimum,

$$V(\langle \varphi^\dagger \varphi \rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda| v^4}{4} < 0.$$

Identify  $M_H^2 = -2\mu^2$

contributes field-independent vacuum energy density

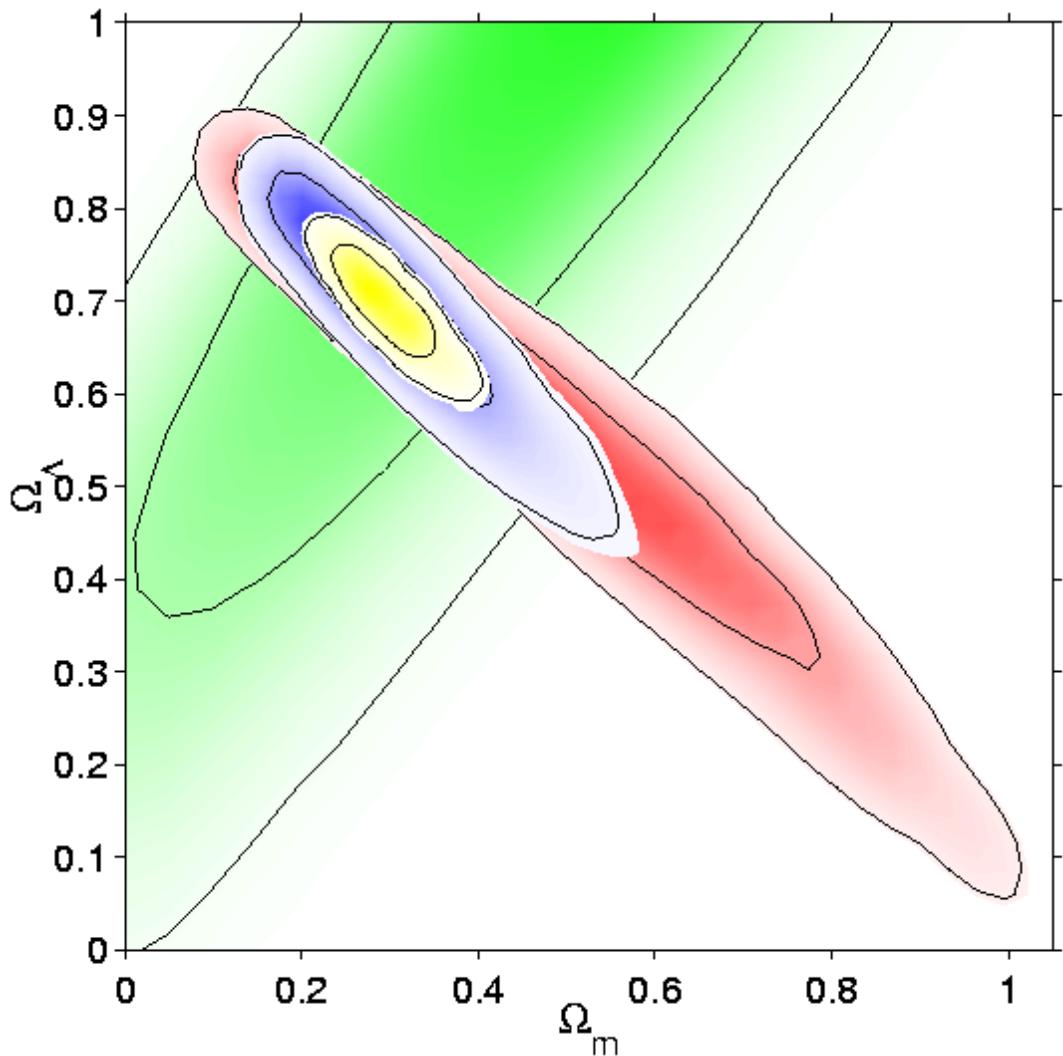
$$\varrho_H \equiv \frac{M_H^2 v^2}{8}$$

Adding vacuum energy density  $\varrho_{\text{vac}}$   $\Leftrightarrow$  adding cosmological constant  $\Lambda$  to Einstein's equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$$\Lambda = \frac{8\pi G_N}{c^4} \varrho_{\text{vac}}$$

observed vacuum energy density  $\varrho_{\text{vac}} \lesssim 10^{-46} \text{ GeV}^4$



Lewis & Bridle, astro-ph/0205436

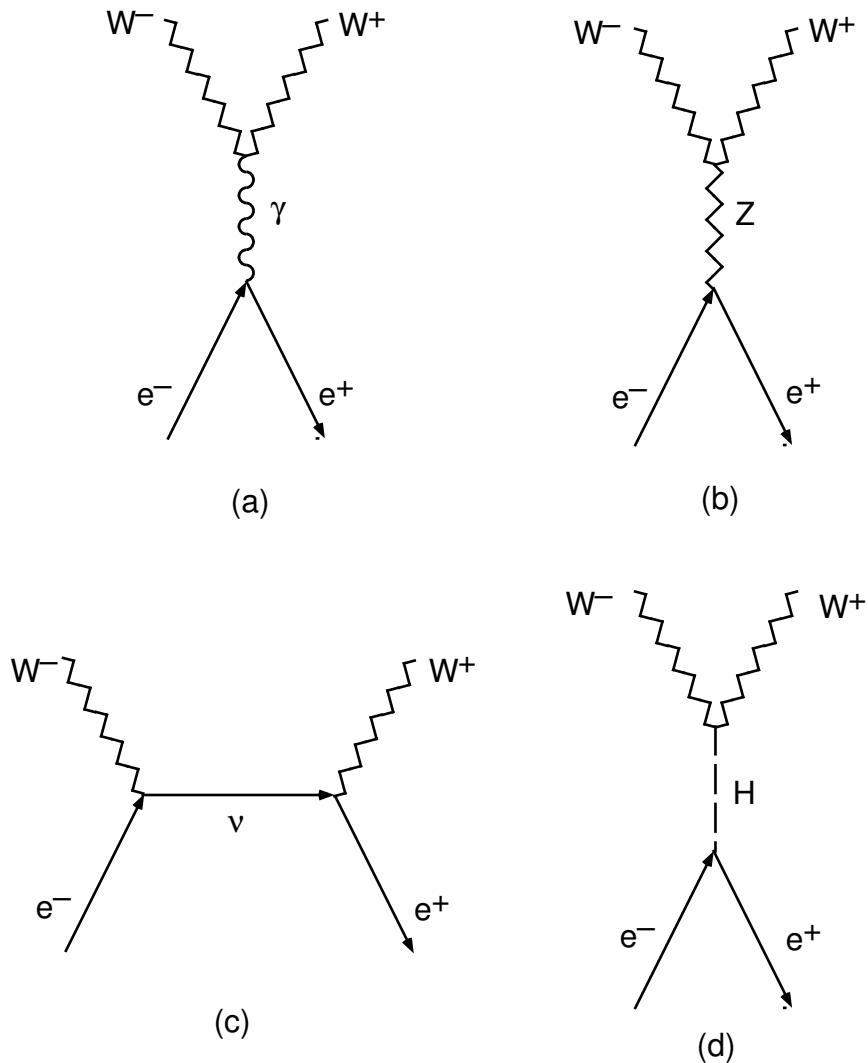
But  $M_H \gtrsim 114 \text{ GeV}/c^2 \Rightarrow$

$$\varrho_H \gtrsim 10^8 \text{ GeV}^4$$

MISMATCH BY 54 ORDERS OR MAGNITUDE

# Why a Higgs Boson Must Exist

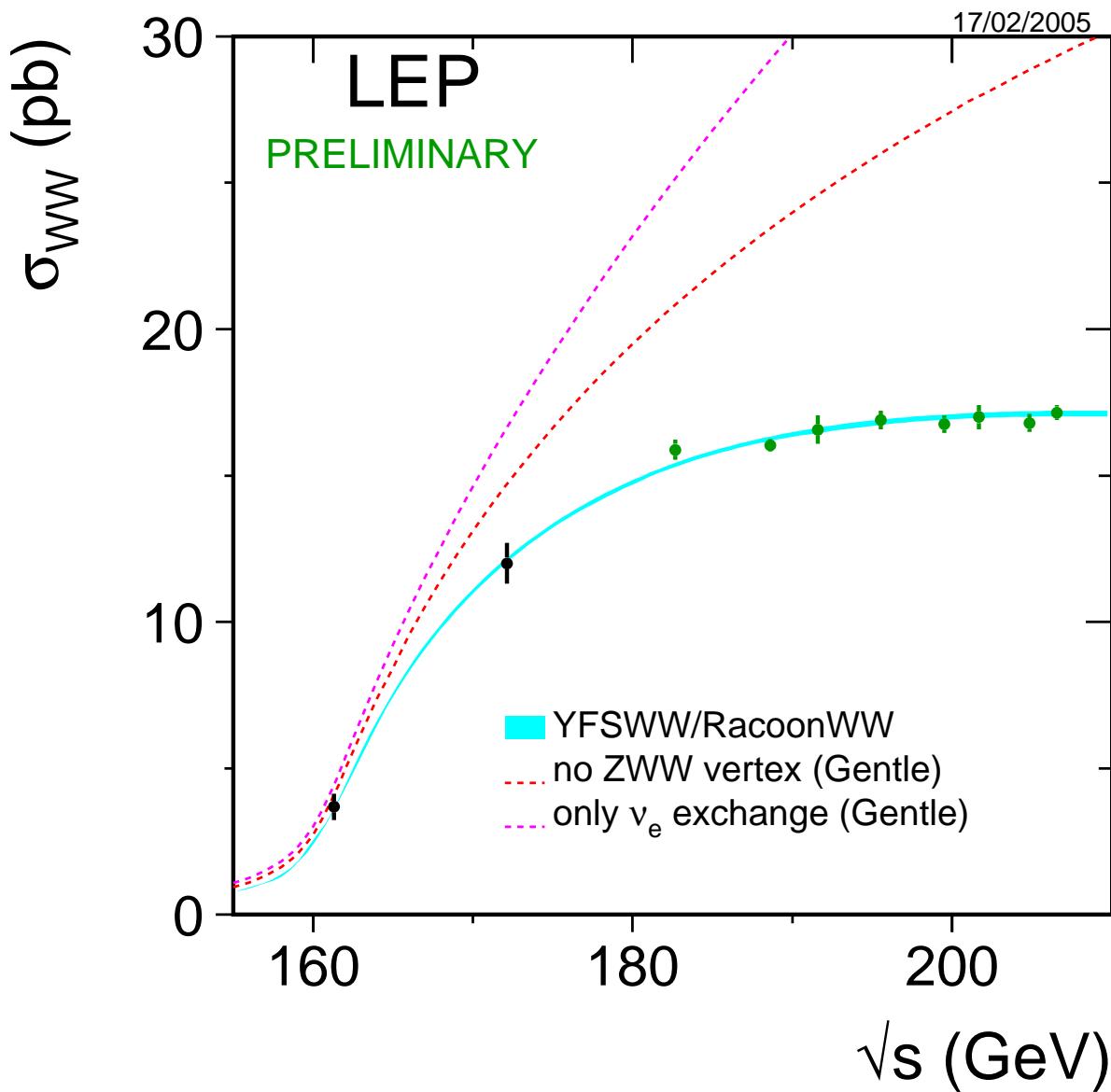
- ▷ Role in canceling high-energy divergences  
*S-matrix analysis of  $e^+e^- \rightarrow W^+W^-$*



$J = 1$  partial-wave amplitudes  $\mathcal{M}_\gamma^{(1)}$ ,  $\mathcal{M}_Z^{(1)}$ ,  $\mathcal{M}_\nu^{(1)}$   
have—individually—unacceptable high-energy  
behavior ( $\propto s$ )

... But sum is well-behaved

"Gauge cancellation" observed at LEP2, Tevatron



$J = 0$  amplitude exists because electrons have mass, and can be found in “wrong” helicity state

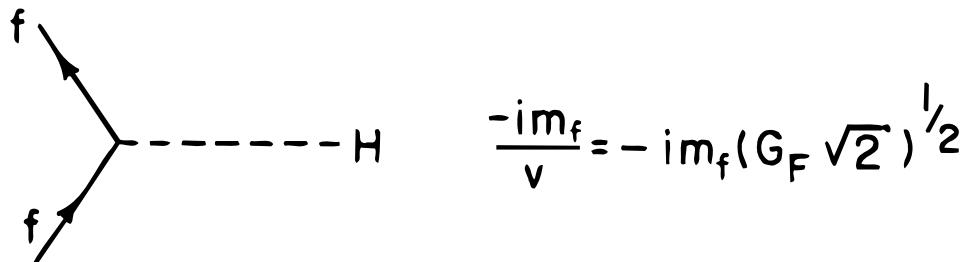
$$\mathcal{M}_\nu^{(0)} \propto s^{\frac{1}{2}} : \text{unacceptable HE behavior}$$

(no contributions from  $\gamma$  and  $Z$ )

*This divergence is canceled by the Higgs-boson contribution*

$\Rightarrow He\bar{e}$  coupling must be  $\propto m_e$ ,

because “wrong-helicity” amplitudes  $\propto m_e$



If the Higgs boson did not exist, *something else* would have to cure divergent behavior

## IF gauge symmetry were unbroken . . .

- ▷ no Higgs boson
- ▷ no longitudinal gauge bosons
- ▷ no extreme divergences
- ▷ no wrong-helicity amplitudes

    . . . and no viable low-energy phenomenology

## In spontaneously broken theory . . .

- ▷ gauge structure of couplings eliminates the most severe divergences
- ▷ lesser—but potentially fatal—divergence arises because the electron has mass  
        . . . due to the Higgs mechanism
- ▷ SSB provides its own cure—the Higgs boson

A similar interplay and compensation *must exist* in any acceptable theory

## Bounds on $M_H$

EW theory does not predict Higgs-boson mass

Self-consistency  $\Rightarrow$  plausible lower and upper bounds

▷ Conditional *upper bound* from Unitarity

Compute amplitudes  $\mathcal{M}$  for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s, t) = 16\pi \sum_J (2J + 1) a_J(s) P_J(\cos \theta)$$

Most channels decouple—pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies)—for any  $M_H$ .

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0 / \sqrt{2} \quad HH / \sqrt{2} \quad HZ_L^0$$

$L$ : longitudinal,  $1/\sqrt{2}$  for identical particles

In HE limit,<sup>a</sup>  $s$ -wave amplitudes  $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2} (a_0) \rightarrow \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect the partial-wave unitarity condition  $|a_0| \leq 1$

$$\implies M_H \leq \left( \frac{8\pi\sqrt{2}}{3G_F} \right)^{1/2} = 1 \text{ TeV}/c^2$$

condition for perturbative unitarity

---

<sup>a</sup>Convenient to calculate using *Goldstone-boson equivalence theorem*, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by  $\mathcal{L}_{\text{int}} = -\lambda v h(2w^+w^- + z^2 + h^2) - (\lambda/4)(2w^+w^- + z^2 + h^2)^2$ , with  $1/v^2 = G_F\sqrt{2}$  and  $\lambda = G_F M_H^2 / \sqrt{2}$ .

- ▷ If the bound is respected
  - ★ weak interactions remain weak at all energies
  - ★ perturbation theory is everywhere reliable
  
- ▷ If the bound is violated
  - ★ perturbation theory breaks down
  - ★ weak interactions among  $W^\pm$ ,  $Z$ , and  $H$  become strong on the 1-TeV scale
    - ⇒ features of *strong* interactions at GeV energies will characterize *electroweak* gauge boson interactions at TeV energies

Threshold behavior of the pw amplitudes  $a_{IJ}$  follows from chiral symmetry

$$\begin{aligned}
 a_{00} &\approx G_F s / 8\pi\sqrt{2} && \text{attractive} \\
 a_{11} &\approx G_F s / 48\pi\sqrt{2} && \text{attractive} \\
 a_{20} &\approx -G_F s / 16\pi\sqrt{2} && \text{repulsive}
 \end{aligned}$$

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

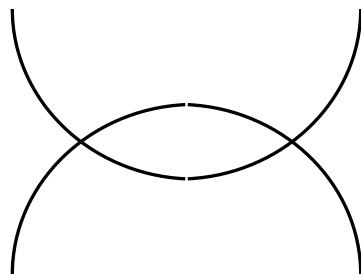
## ▷ Triviality of scalar field theory

Only *noninteracting* scalar field theories make sense on all energy scales

Quantum field theory vacuum is a dielectric medium that screens charge  $\Rightarrow$  *effective charge* is a function of the distance or, equivalently, of the energy scale

running coupling constant

In  $\lambda\phi^4$  theory, it is easy to calculate the variation of the coupling constant  $\lambda$  in perturbation theory by summing bubble graphs



$\lambda(\mu)$  is related to a higher scale  $\Lambda$  by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log(\Lambda/\mu)$$

(Perturbation theory reliable only when  $\lambda$  is small, lattice field theory treats strong-coupling regime)

For stable Higgs potential (*i.e.*, for vacuum energy not to race off to  $-\infty$ ), *require*  $\lambda(\Lambda) \geq 0$

Rewrite RGE as an inequality

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log(\Lambda/\mu) .$$

implies an *upper bound*

$$\lambda(\mu) \leq 2\pi^2/3 \log(\Lambda/\mu)$$

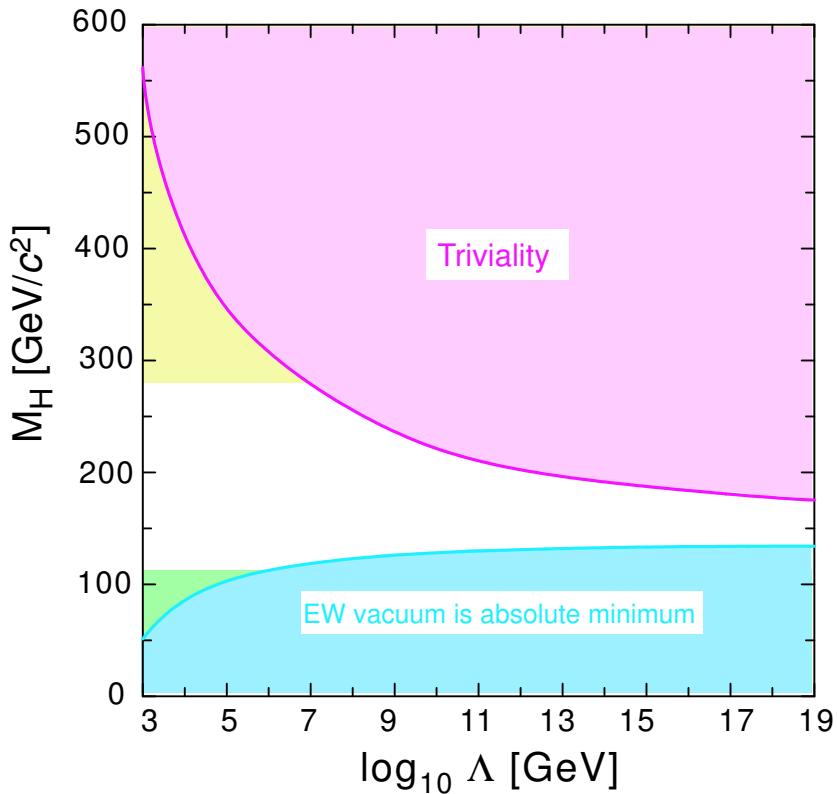
If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit  $\Lambda \rightarrow \infty$  while holding  $\mu$  fixed at some reasonable physical scale. In this limit, the bound forces  $\lambda(\mu)$  to zero.  $\longrightarrow$  free field theory “trivial”

Rewrite as bound on  $M_H$ :

$$\Lambda \leq \mu \exp\left(\frac{2\pi^2}{3\lambda(\mu)}\right)$$

Choose  $\mu = M_H$ , and recall  $M_H^2 = 2\lambda(M_H)v^2$

$$\Lambda \leq M_H \exp\left(4\pi^2 v^2/3M_H^2\right)$$



**Moral:** For any  $M_H$ , there is a *maximum energy scale*  $\Lambda^*$  at which the theory ceases to make sense.

The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies

Perturbative analysis breaks down when  $M_H \rightarrow 1 \text{ TeV}/c^2$  and interactions become strong

Lattice analyses  $\implies M_H \lesssim 710 \pm 60 \text{ GeV}/c^2$  if theory describes physics to a few percent up to a few TeV

If  $M_H \rightarrow 1 \text{ TeV}$  EW theory lives on brink of instability

- ▷ *Lower bound* by requiring EWSB vacuum  
 $V(v) < V(0)$

Requiring that  $\langle \phi \rangle_0 \neq 0$  be an absolute minimum of the one-loop potential up to a scale  $\Lambda$  yields the vacuum-stability condition

$$M_H^2 > \frac{3G_F\sqrt{2}}{8\pi^2} (2M_W^4 + M_Z^4 - 4m_t^4) \log(\Lambda^2/v^2)$$

... for  $m_t \lesssim M_W$

(No illuminating analytic form for heavy  $m_t$ )

If the Higgs boson is relatively light—which would itself require explanation—then the theory can be self-consistent up to very high energies

If EW theory is to make sense all the way up to a unification scale  $\Lambda^* = 10^{16}$  GeV, then

$$134 \text{ GeV}/c^2 \lesssim M_H \lesssim 177 \text{ GeV}/c^2$$

# Higgs-Boson Properties

$$\Gamma(H \rightarrow f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

$\propto M_H$  in the limit of large Higgs mass

$$\Gamma(H \rightarrow W^+W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}} (1-x)^{1/2} (4-4x+3x^2)$$

$$x \equiv 4M_W^2/M_H^2$$

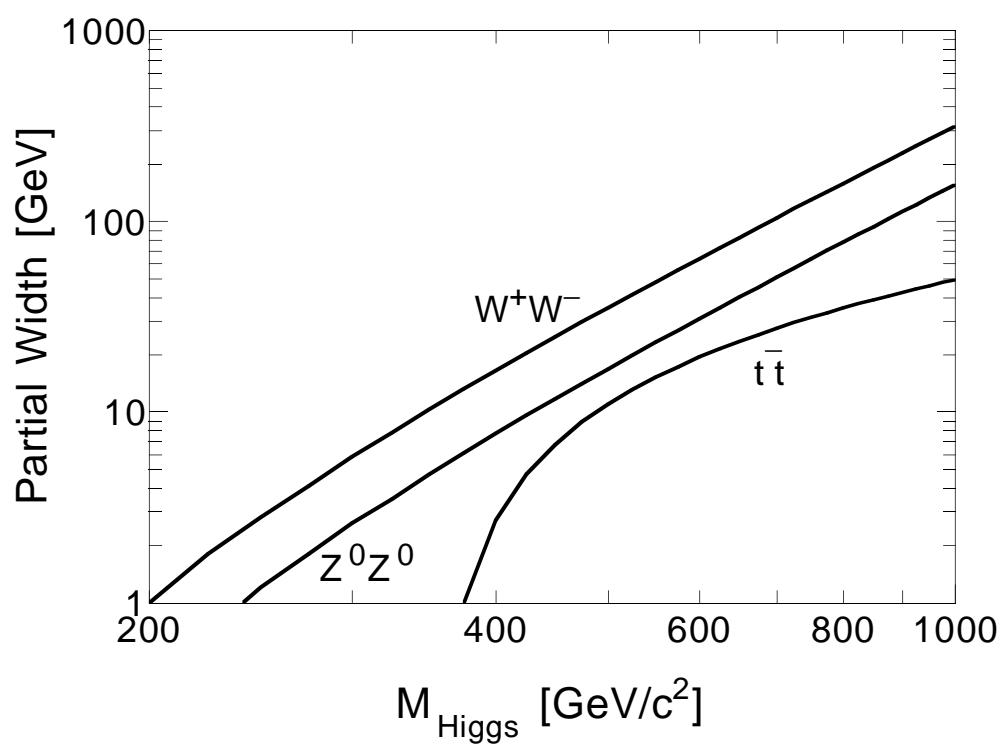
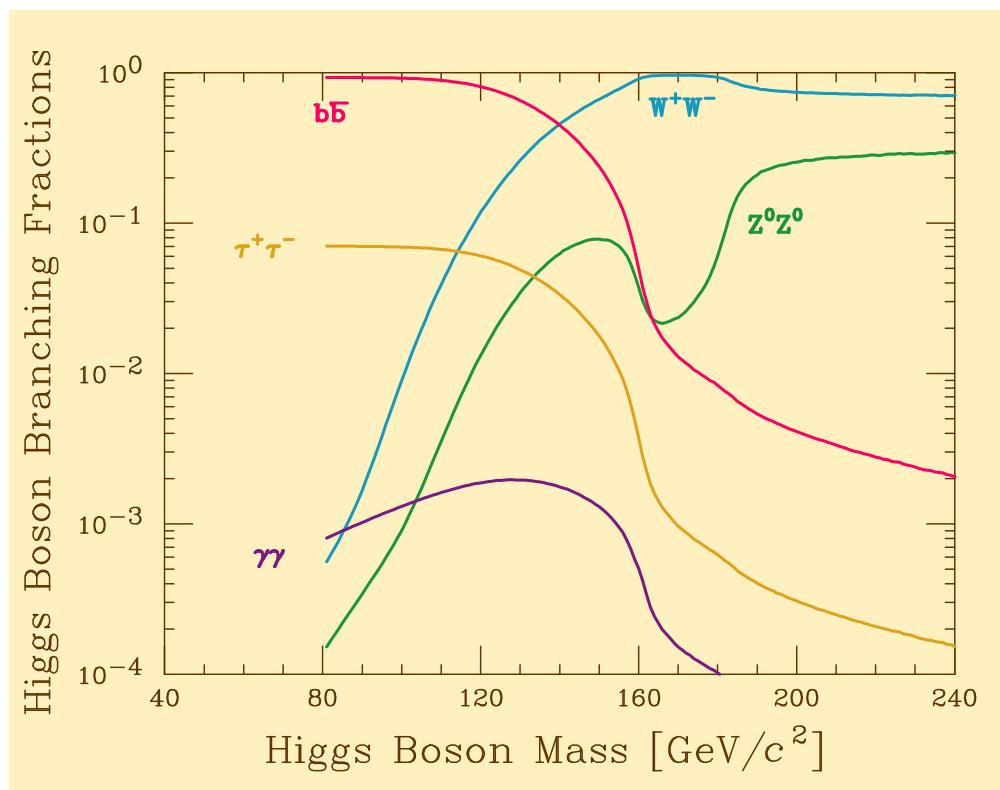
$$\Gamma(H \rightarrow Z^0Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}} (1-x')^{1/2} (4-4x'+3x'^2)$$

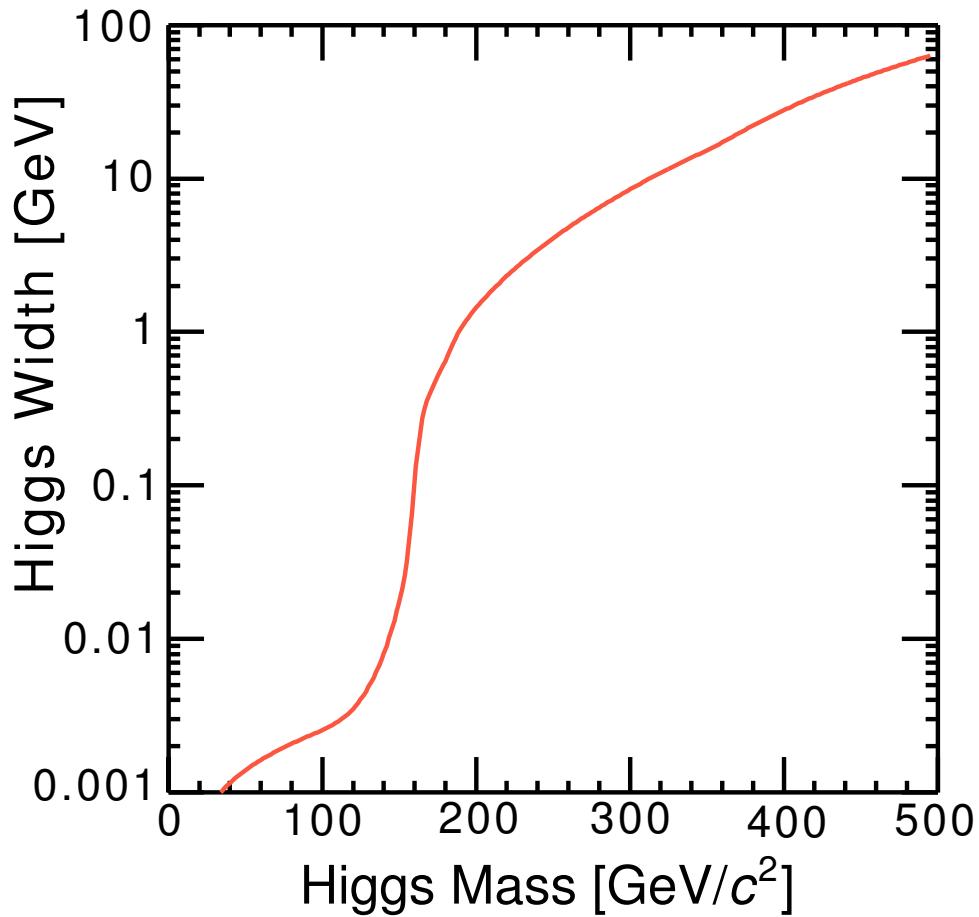
$$x' \equiv 4M_Z^2/M_H^2$$

asymptotically  $\propto M_H^3$  and  $\frac{1}{2}M_H^3$ , respectively  
( $\frac{1}{2}$  from weak isospin)

$2x^2$  and  $2x'^2$  terms  $\Leftrightarrow$  decays into transversely polarized gauge bosons

Dominant decays for large  $M_H$  into pairs of longitudinally polarized weak bosons





Below  $W^+W^-$  threshold,  $\Gamma_H \lesssim 1$  GeV

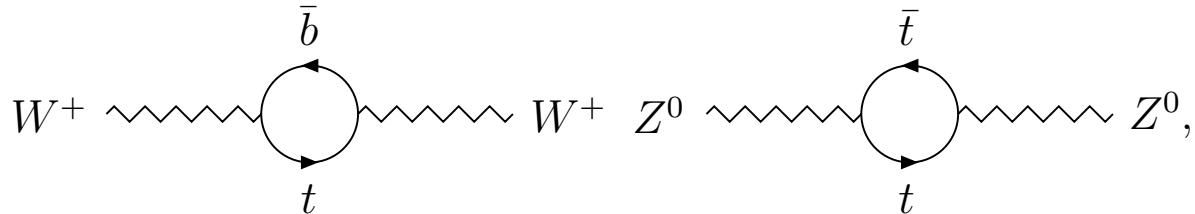
Far above  $W^+W^-$  threshold,  $\Gamma_H \propto M_H^3$

For  $M_H \rightarrow 1$  TeV/ $c^2$ , Higgs boson is an *ephemeron*, with a perturbative width approaching its mass.

## Clues to the Higgs-boson mass

Sensitivity of EW observables to  $m_t$  gave early indications for massive top

quantum corrections to SM predictions for  $M_W$  and  $M_Z$  arise from different quark loops



... alter link between the  $M_W$  and  $M_Z$ :

$$M_W^2 = M_Z^2 (1 - \sin^2 \theta_W) (1 + \Delta\rho)$$

where  $\Delta\rho \approx \Delta\rho^{(\text{quarks})} = 3G_F m_t^2 / 8\pi^2 \sqrt{2}$

strong dependence on  $m_t^2$  accounts for precision of  $m_t$  estimates derived from EW observables

$m_t$  known to  $\pm 3\%$  from Tevatron ...

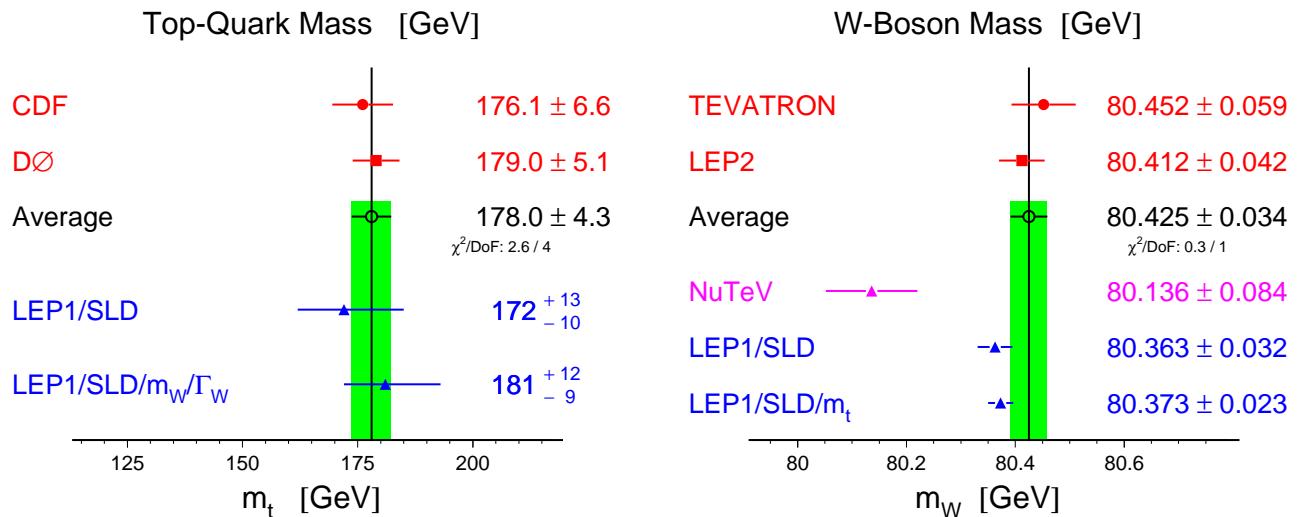
⇒ look beyond the quark loops to next most important quantum corrections:

Higgs-boson effects

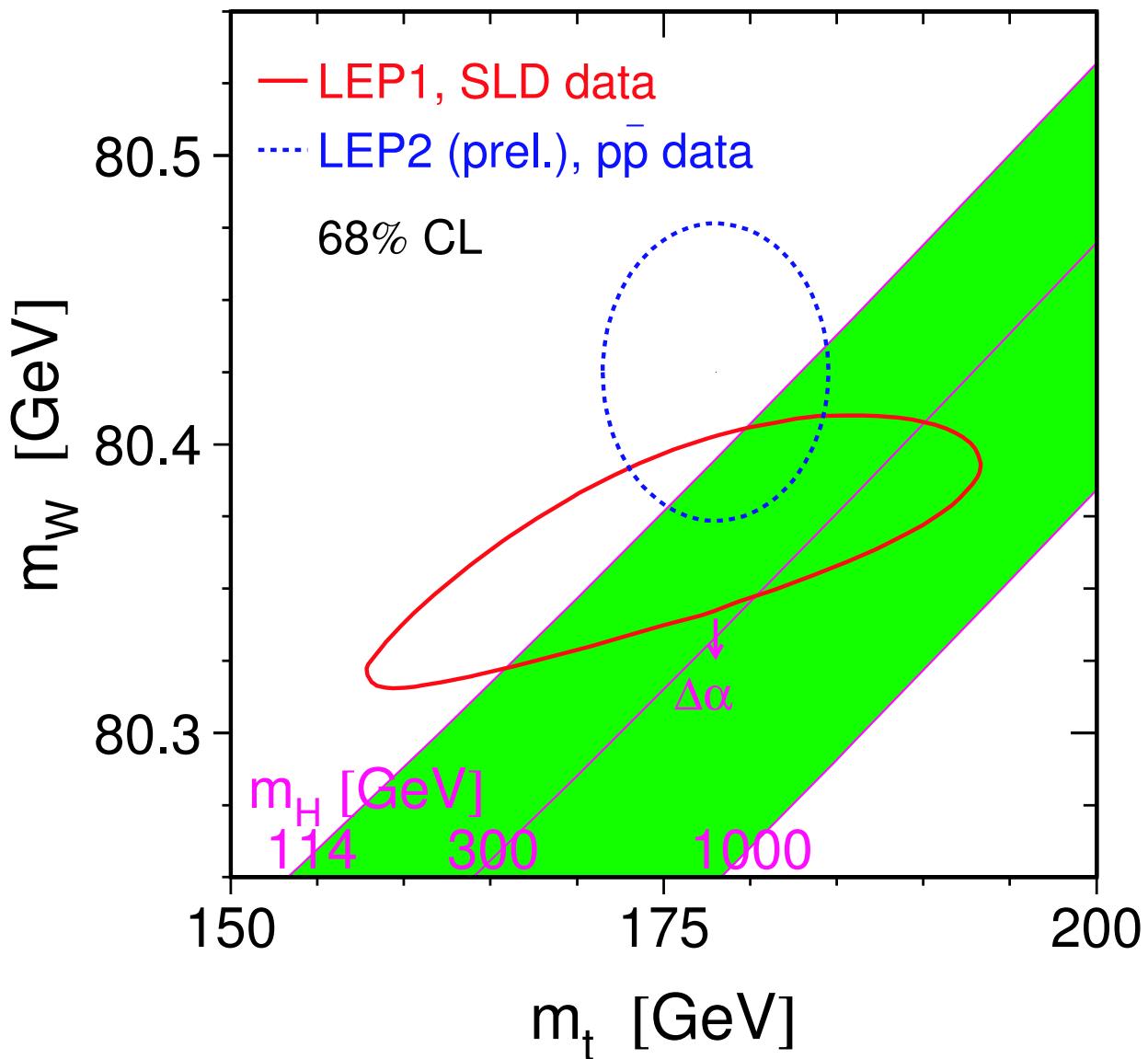
$H$  quantum corrections smaller than  $t$  corrections, exhibit more subtle dependence on  $M_H$  than the  $m_t^2$  dependence of the top-quark corrections

$$\Delta\rho^{(\text{Higgs})} = \mathcal{C} \cdot \ln\left(\frac{M_H}{v}\right)$$

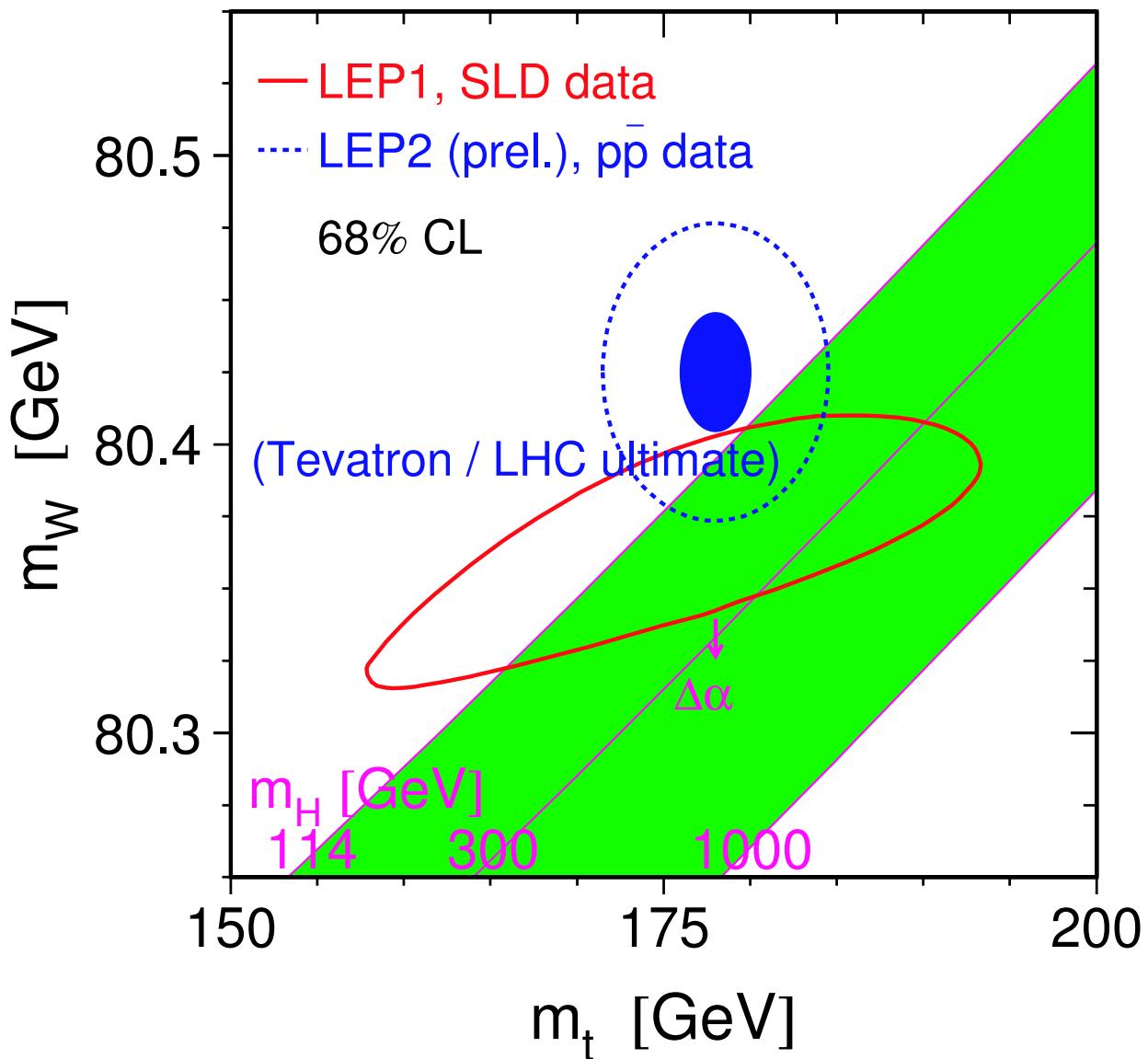
$M_Z$  known to 23 ppm,  $m_t$  and  $M_W$  well measured



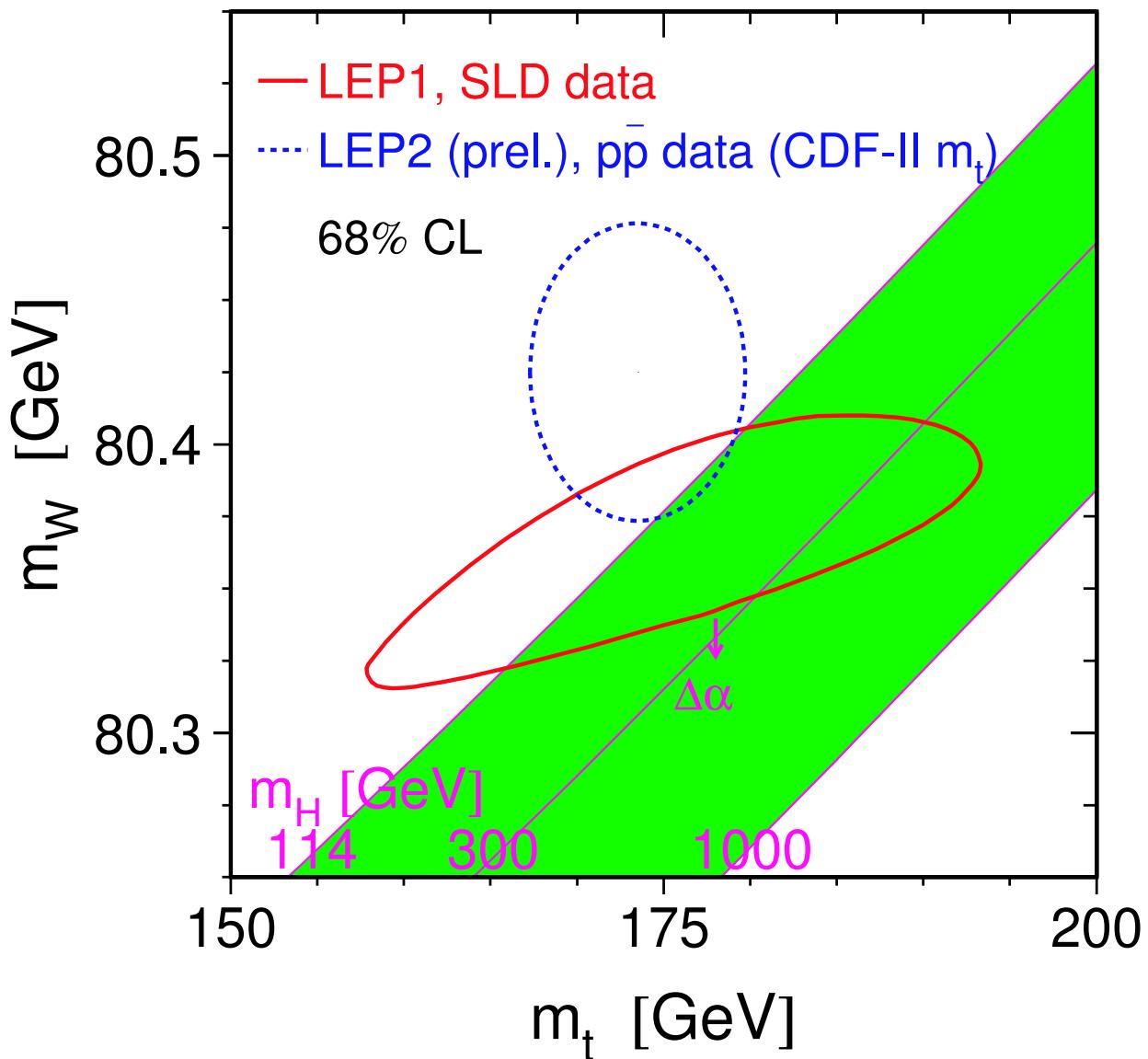
so examine dependence of  $M_W$  upon  $m_t$  and  $M_H$



Direct, indirect determinations agree reasonably  
 Both favor a light Higgs boson,  
*within framework of SM analysis.*

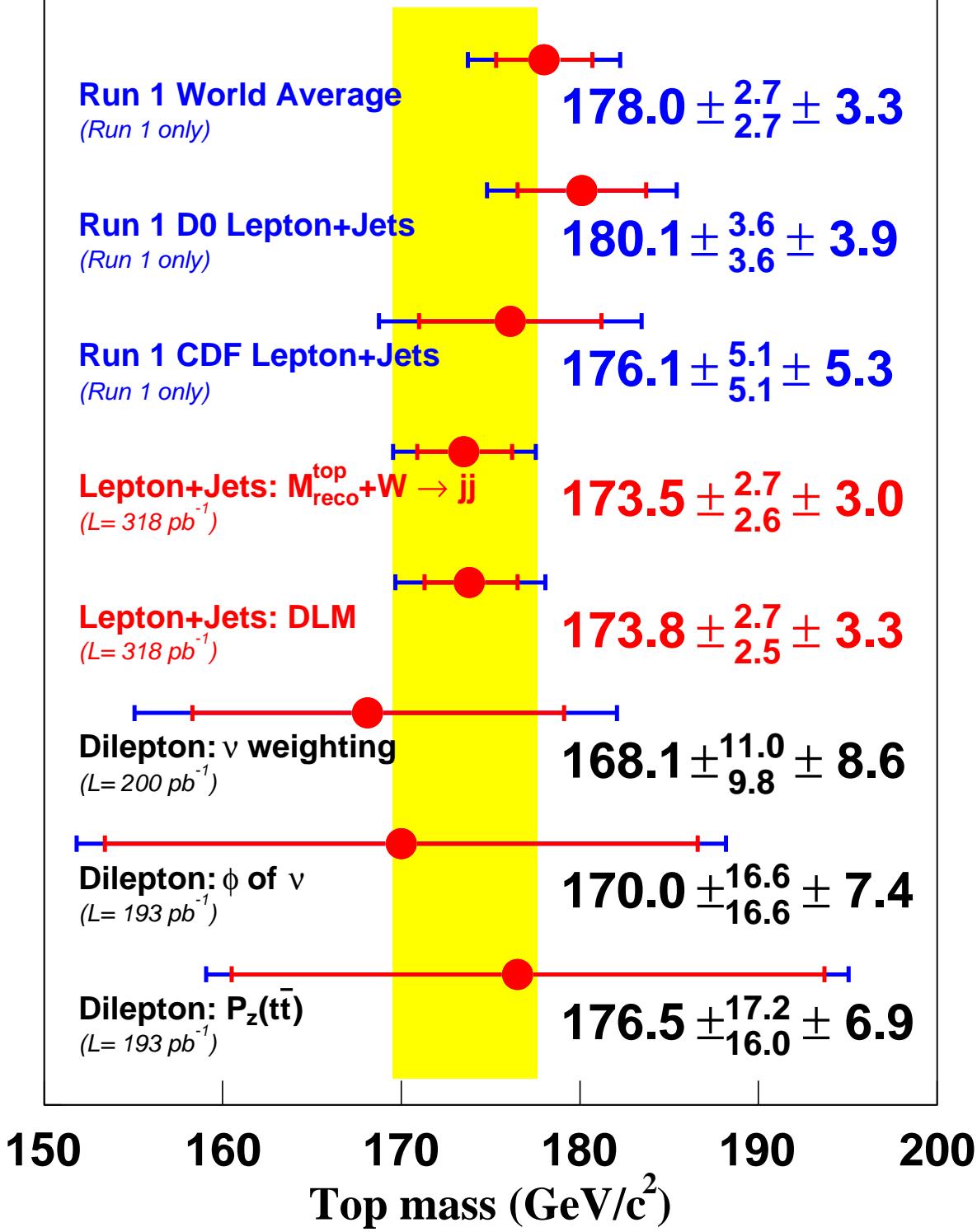


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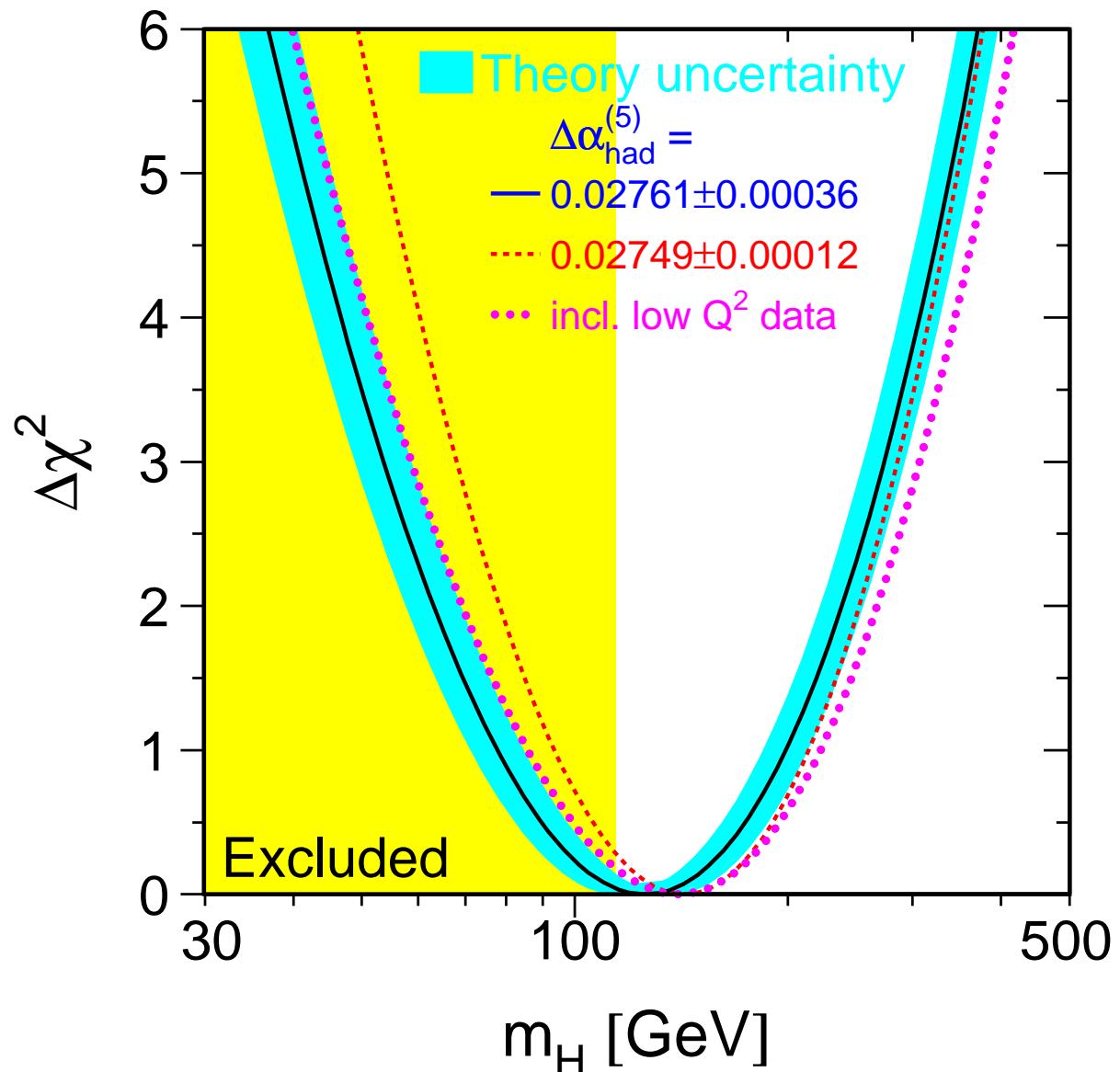


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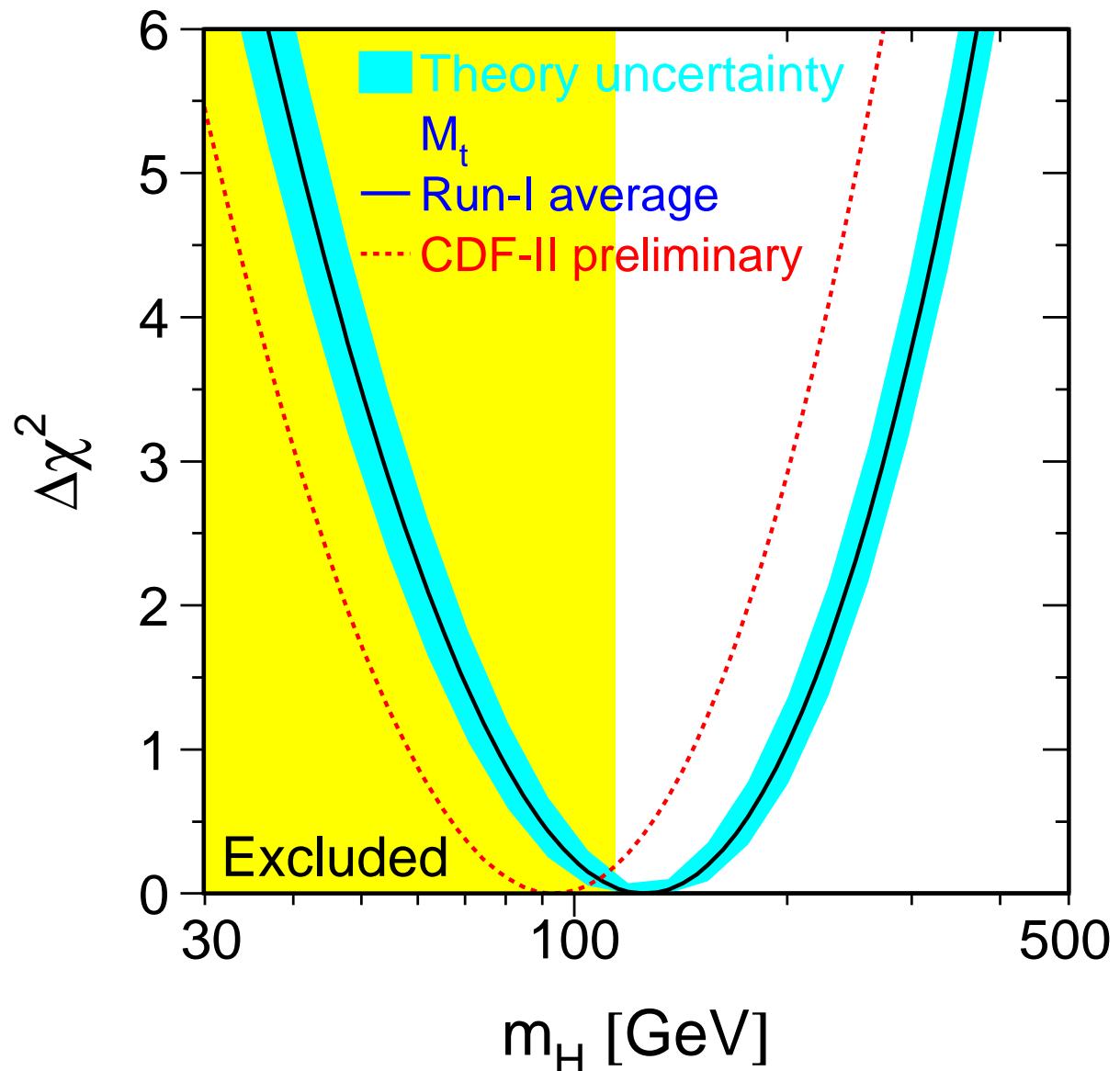
## CDF Run 2 Preliminary (June 2 2005)



## Fit to a universe of data



## Fit to a universe of data



*Within SM, LEPEWWG deduce a 95% CL upper limit,  $M_H \lesssim 280 \text{ GeV}/c^2$ .*

*Direct searches at LEP  $\Rightarrow M_H > 114.4 \text{ GeV}/c^2$ , excluding much of the favored region*

*either the Higgs boson is just around the corner, or SM analysis is misleading*

**Things will soon be popping!**

**Expect progress from  $M_W$ - $m_t$ - $M_H$  correlation**

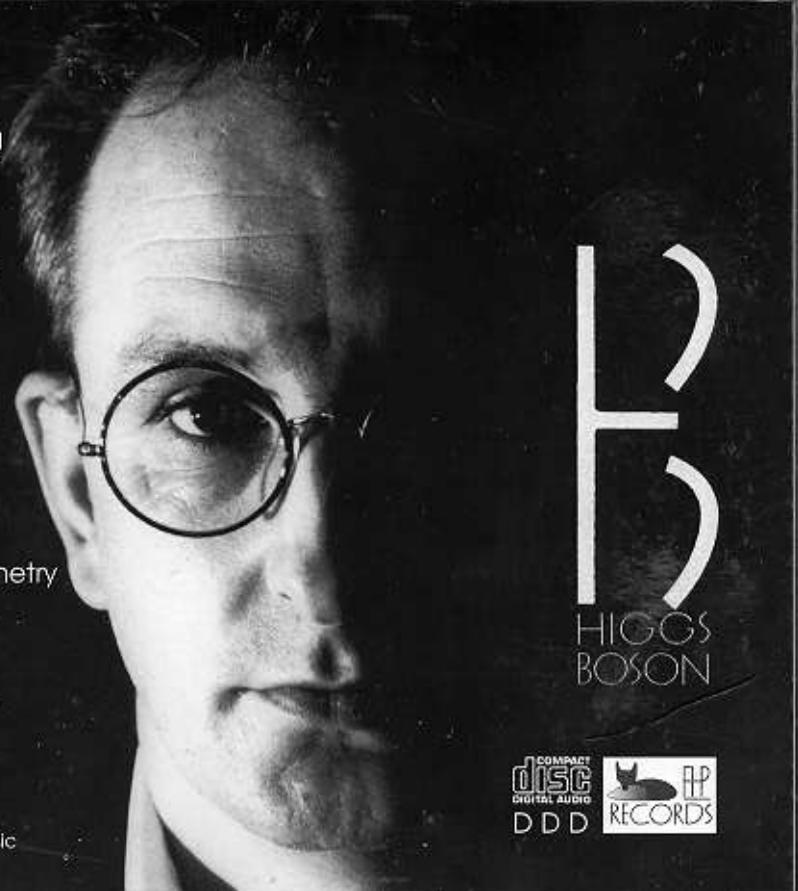
- ▷ Tevatron and LHC measurements will determine  $m_t$  within 1 or 2  $\text{GeV}/c^2$
- ▷ ... and improve  $\delta M_W$  to about 15  $\text{MeV}/c^2$
- ▷ As the Tevatron's integrated luminosity approaches  $10 \text{ fb}^{-1}$ , CDF and DØ will begin to explore the region of  $M_H$  not excluded by LEP
- ▷ ATLAS and CMS will carry on the exploration of the Higgs sector at the LHC

# HIGGS BOSON

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Penumbra

Synthetic Spring

Neptune

Big Technology

I'm With You

Cooled

Faith (Yourself)

Travel

Perpetual Symmetry

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HIGGS  
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# Assessment

## 25 YEARS OF CONFIRMATIONS OF $SU(2)_L \otimes U(1)_Y$

★ neutral currents

★  $W^\pm, Z^0$

★ charm

(+ experimental guidance)

★  $\tau, \nu_\tau$

★  $b, t$

+ experimental surprises

★ narrowness of  $\psi, \psi'$

★ long  $B$  lifetime

★ large  $B^0 - \bar{B}^0$  mixing

★ heavy top

★ neutrino oscillations

10 YEARS OF PRECISION MEASUREMENTS...  
... FIND NO SIGNIFICANT DEVIATIONS

QUANTUM CORRECTIONS TESTED AT  $\pm 10^{-3}$

NO “NEW” PHYSICS ... YET!

Theory tested at distances  
from  $10^{-17}$  cm  
to  $\sim 10^{22}$  cm

origin	Coulomb's law (tabletop experiments)
smaller	$\left\{ \begin{array}{l} \text{Atomic physics} \rightarrow \text{QED} \\ \text{high-energy experiments} \rightarrow \text{EW theory} \end{array} \right.$
larger	$M_\gamma \approx 0$ in planetary ... measurements

IS EW THEORY TRUE ?  
COMPLETE ??

## EWSB: another path?

Modeled EWSB on Ginzburg–Landau description of SC phase transition

had to introduce new, elementary scalars

GL is not the last word on superconductivity:  
*dynamical* Bardeen–Cooper–Schrieffer theory

The elementary fermions—electrons—and gauge interactions—QED—needed to generate the scalar bound states are already present in the case of superconductivity. Could a scheme of similar economy account for EWSB?

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y + \text{massless } u \text{ and } d$$

Treat  $SU(2)_L \otimes U(1)_Y$  as perturbation

$m_u = m_d = 0$ : QCD has exact  $SU(2)_L \otimes SU(2)_R$  chiral symmetry. At an energy scale  $\sim \Lambda_{\text{QCD}}$ , strong interactions become strong, fermion condensates appear, and  $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$   
⇒ 3 Goldstone bosons, one for each broken generator: 3 massless pions (Nambu)

Broken generators: 3 axial currents; couplings to  $\pi$   
measured by pion decay constant  $f_\pi$

Turn on  $SU(2)_L \otimes U(1)_Y$ : EW gauge bosons couple  
to axial currents, acquire masses of order  $\sim g f_\pi$

$$\mathcal{M}^2 = \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & gg' \\ 0 & 0 & gg' & g'^2 \end{pmatrix} \frac{f_\pi^2}{4},$$

$$(W^+, W^-, W_3, \mathcal{A})$$

same structure as standard EW theory. Diagonalize:  
 $M_W^2 = g^2 f_\pi^2 / 4$ ,  $M_Z^2 = (g^2 + g'^2) f_\pi^2 / 4$ ,  $M_A^2 = 0$ , so

$$\frac{M_Z^2}{M_W^2} = \frac{(g^2 + g'^2)}{g^2} = \frac{1}{\cos^2 \theta_W}$$

Massless pions disappear from physical spectrum, to  
become longitudinal components of weak bosons

$$M_W \approx 30 \text{ MeV}/c^2$$

## *With no Higgs mechanism . . .*

- ▷ Quarks and leptons would remain massless
- ▷ QCD would confine them in color-singlet hadrons
- ▷ *Nucleon mass would be little changed*, but proton outweighs neutron
- ▷ QCD breaks EW symmetry, gives  $(1/2500 \times \text{observed})$  masses to  $W$ ,  $Z$ , so weak-isospin force doesn't confine
- ▷ Rapid!  $\beta$ -decay  $\Rightarrow$  lightest nucleus is one neutron; no hydrogen atom
- ▷ Probably some light elements in BBN, but  $\infty$  Bohr radius
- ▷ No atoms (as we know them) means no chemistry, no stable composite structures like the solids and liquids we know

*. . . the character of the physical world would be profoundly changed*

## In a decade or two, we can hope to ...

Understand electroweak symmetry breaking	<i>Detect neutrinos from the universe</i>
<i>Observe the Higgs boson</i>	Learn how to quantize gravity
Measure neutrino masses and mixings	<i>Learn why empty space is nearly weightless</i>
<i>Establish Majorana neutrinos (<math>\beta\beta_{0\nu}</math>)</i>	Test the inflation hypothesis
Thoroughly explore CP violation in $B$ decays	<i>Understand discrete symmetry violation</i>
<i>Exploit rare decays (<math>K, D, \dots</math>)</i>	Resolve the hierarchy problem
Observe neutron EDM, pursue electron EDM	<i>Discover new gauge forces</i>
<i>Use top as a tool</i>	Directly detect dark-matter particles
Observe new phases of matter	<i>Explore extra spatial dimensions</i>
<i>Understand hadron structure quantitatively</i>	Understand the origin of large-scale structure
Uncover QCD's full implications	<i>Observe gravitational radiation</i>
<i>Observe proton decay</i>	Solve the strong CP problem
Understand the baryon excess	<i>Learn whether supersymmetry is TeV-scale</i>
<i>Catalogue matter and energy of the universe</i>	Seek TeV-scale dynamical symmetry breaking
Measure dark energy equation of state	<i>Search for new strong dynamics</i>
<i>Search for new macroscopic forces</i>	Explain the highest-energy cosmic rays
Determine GUT symmetry	<i>Formulate problem of identity</i>
	....

... and learn to ask the right questions

## Appendix: The EW scale and beyond

EWSB scale,  $v = (G_F \sqrt{2})^{-\frac{1}{2}} \approx 246$  GeV, sets

$$M_W^2 = g^2 v^2 / 2 \quad M_Z^2 = M_W^2 / \cos^2 \theta_W$$

But it is not the only scale of physical interest

**quasi-certain:**  $M_{\text{Planck}} = 1.22 \times 10^{19}$  GeV

**probable:**  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$  unification scale  
 $\sim 10^{15-16}$  GeV

**somewhere:** flavor scale

How to keep the distant scales from mixing in the face of quantum corrections?

*OR*

How to stabilize the mass of the Higgs boson on the electroweak scale?

*OR*

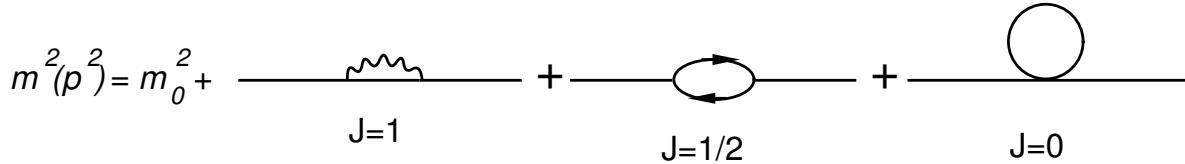
Why is the electroweak scale small?

Higgs potential     $V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2$

$\mu^2 < 0$ :  $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}}$ , as

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ \sqrt{-\mu^2/2|\lambda|} \end{pmatrix} \equiv \begin{pmatrix} 0 \\ \underbrace{(G_F \sqrt{8})^{-1/2}}_{175 \text{ GeV}} \end{pmatrix}$$

*Beyond classical approximation*, quantum corrections to scalar mass parameters:



Loop integrals are potentially divergent.

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \dots$$

$\Lambda$ : reference scale at which  $m^2$  is known

$g$ : coupling constant of the theory

$C$ : coefficient calculable in specific theory

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \dots$$

For the mass shifts induced by radiative corrections to remain under control (not greatly exceed the value measured on the laboratory scale), *either*

- ▷  $\Lambda$  must be small, or
- ▷ new physics must intervene to cut off the integral

BUT natural reference scale for  $\Lambda$  is

$$\Lambda \sim M_{\text{Planck}} = \left( \frac{\hbar c}{G_{\text{Newton}}} \right)^{1/2} \approx 1.22 \times 10^{19} \text{ GeV}$$

for  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$   
OR

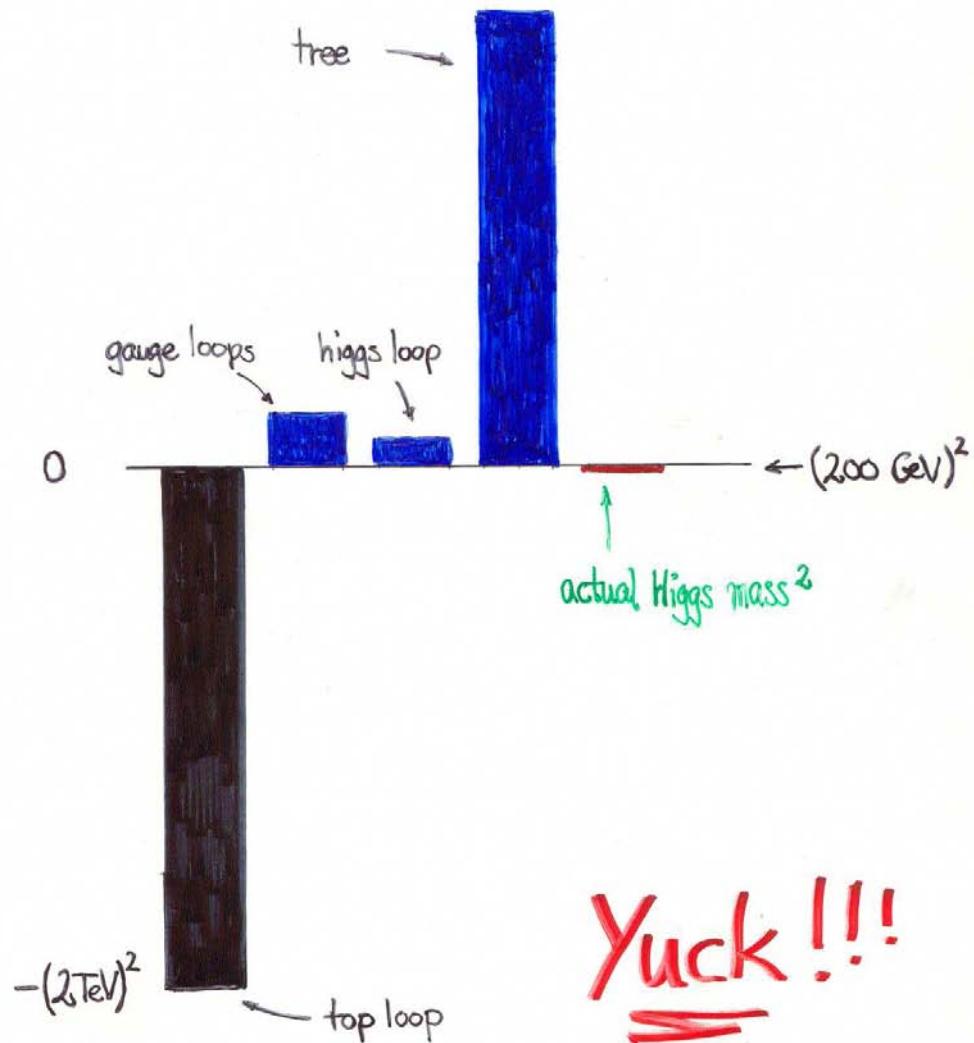
$$\Lambda \sim M_U \approx 10^{15}\text{-}10^{16} \text{ GeV}$$

for unified theory

Both  $\gg v/\sqrt{2} \approx 175$  GeV  $\implies$

New Physics at  $E \lesssim 1$  TeV

# Fine tuning the Higgs

$$\Lambda = 10 \text{ TeV}$$


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# Only a few distinct scenarios . . .

- ▷ Supersymmetry: balance contributions of fermion loops ( $-1$ ) and boson loops ( $+1$ )

*Exact supersymmetry,*

$$\sum_{\substack{i= \\ \text{fermions} \\ + \text{bosons}}} C_i \int dk^2 = 0$$

*Broken supersymmetry*, shifts acceptably small if superpartner mass splittings are not too large

$$g^2 \Delta M^2 \text{ “small enough”} \Rightarrow \widetilde{M} \lesssim 1 \text{ TeV}/c^2$$

- ▷ Composite scalars (technicolor): New physics arises on scale of composite Higgs-boson binding,

$$\Lambda_{\text{TC}} \simeq O(1 \text{ TeV})$$

“Form factor” cuts effective range of integration

- ▷ Strongly interacting gauge sector:  $WW$  resonances, multiple  $W$  production, probably scalar bound state “quasiHiggs” with  $M < 1 \text{ TeV}$
- ▷ Extra spacetime dimensions: pseudo-Nambu–Goldstone bosons, extra particles to cancel integrand, . . .