Neutrinos in the Electroweak Theory

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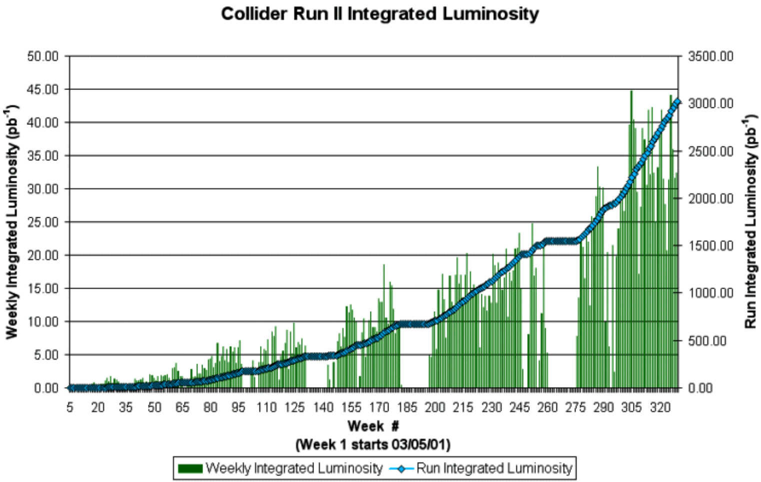
A Decade of Discovery Past . . .

- EW theory → law of nature [Z, e^+e^-, $\bar{p}p$, νN, $(g-2)_\mu, \ldots$]
- Higgs-boson influence in the vacuum [EW experiments]
- ν oscillations: $\nu_\mu \to \nu_\tau$, $\nu_e \to \nu_\mu/\nu_\tau$ [$\nu_\odot$, ν_{atm}, reactors]
- Understanding QCD [heavy flavor, Z^0, $\bar{p}p$, νN, ep, ions, lattice]
- Discovery of top quark [$\bar{p}p$]
- Direct $\mathcal{CP}$ violation in $K \to \pi\pi$ [fixed-target]
- B-meson decays violate $\mathcal{CP}$ [$e^+e^- \to B\bar{B}$]
- Flat universe: dark matter, energy [SN Ia, CMB, LSS]
- Detection of ν_τ interactions [fixed-target]
- Quarks, leptons structureless at 1 TeV scale [mostly colliders]

Tevatron Collider is breaking new ground in sensitivity


Collider Run II Integrated Luminosity
Weekly Integrated Luminosity (pb-1)
Run Integrated Luminosity (pb-1)
50.00
45.00
40.00
35.00
30.00
25.00
20.00
15.00
10.00
5.00
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2000.00
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1000.00
500.00
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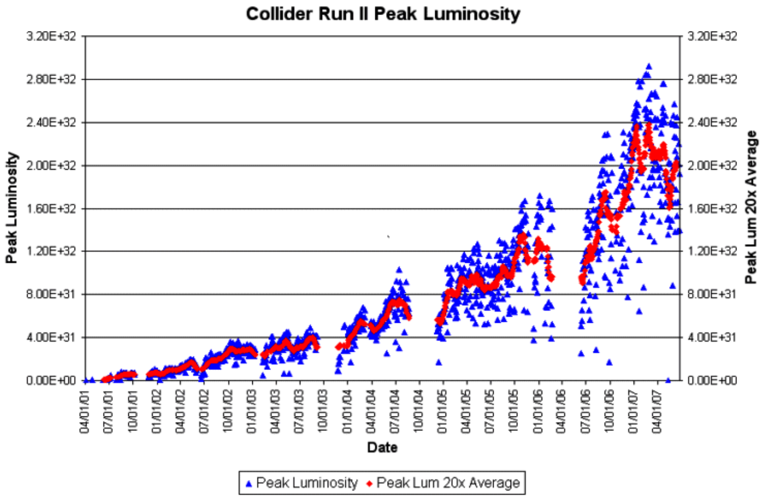
5 20 35 50 65 80 95 110 125 140 155 170 185 200 215 230 245 260 275 290 305 320
Week #
(Week 1 starts 03/05/01)
Weekly Integrated Luminosity
Run Integrated Luminosity


Collider Run II Peak Luminosity
Peak Luminosity
Peak Lum 20x Average
Date
3.20E+32
2.80E+32
2.40E+32
2.00E+32
1.60E+32
1.20E+32
8.00E+31
4.00E+31
0.00E+00
04/01/01
07/01/01
10/01/01
01/01/02
04/01/02
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01/01/07
04/01/07

Tevatron Collider in a Nutshell

980-GeV protons, antiprotons (2π km)

frequency of revolution $\approx 45\,000\ \mathrm{s}^{-1}$

392 ns between crossings

($36 \otimes 36$ bunches)

collision rate $= \mathcal{L} \cdot \sigma_{\text{inelastic}} \approx 10^7\ \mathrm{s}^{-1}$

$c \approx 10^9$ km/h; $\quad v_p \approx c - 495$ km/h

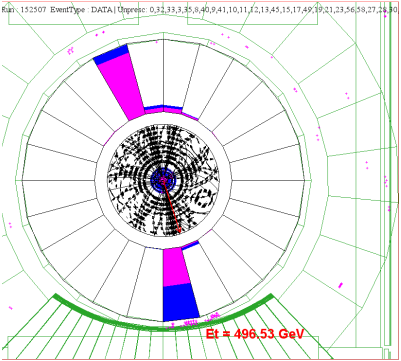
Record $\mathcal{L}_{\text{init}} = 2.85 \times 10^{32}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$

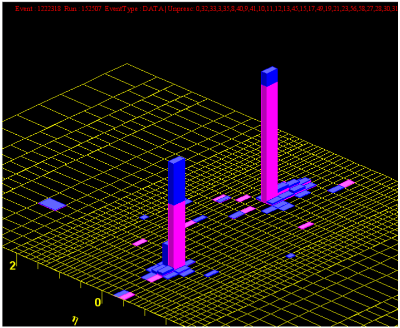
[CERN ISR: pp, $1.4 \times 10^{32}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$]

Goal: $\approx 8\ \mathrm{fb}^{-1}$ by 10.2009

The World's Most Powerful Microscopes

nanonanophysics

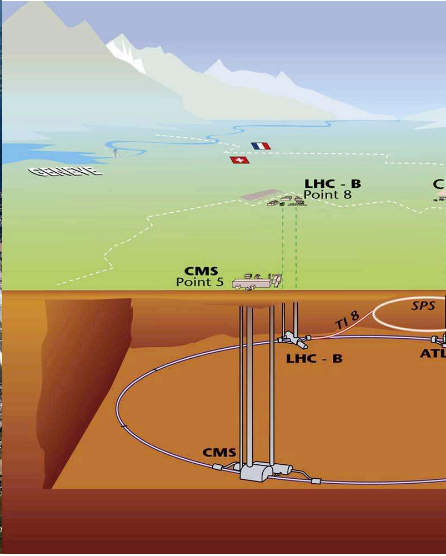




CDF dijet event ($\sqrt{s} = 1.96$ TeV): $E_T = 1.364$ TeV

$$\boxed{q\bar{q} \longrightarrow \text{jet} + \text{jet}}$$

LHC will operate *soon*, breaking new ground in E & $\mathcal{L}$



LHC in a nutshell

7-TeV protons on protons (27 km); $v_p \approx c - 10$ km/h

Novel two-in-one dipoles (≈ 9 teslas)

First collisions at $E_{cm} = 14$ TeV: May 2008

First physics run! Goal of $\gtrsim 1\ \mathrm{fb}^{-1}$ by end 2008

Eventual: $\mathcal{L} \gtrsim 10^{34}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$: $100\ \mathrm{fb}^{-1}/\mathrm{year}$

Why the LHC is so exciting (I)

- Even low luminosity opens vast new realm:
  10 pb^{-1} *(few days at initial $\mathcal{L}$)* yields
  8000 top quarks, 10^5 W-bosons,
  100 QCD dijets beyond Tevatron kinematic limit
  Supersymmetry hints recorded in a few weeks ?
- Essential first step: rediscover the standard model
- The antithesis of a one-experiment machine;
  enormous scope and versatility beyond high-$p_\perp$
- $\mathcal{L}$ upgrade extends $\gtrsim$10-year program . . .

The importance of the 1-TeV scale

EW theory does not predict Higgs-boson mass

▷ Conditional *upper bound* from Unitarity

Compute amplitudes $\mathcal{M}$ for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta)$$

Most channels decouple – pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies) – $\forall M_H$.

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0/\sqrt{2} \quad HH/\sqrt{2} \quad HZ_L^0$$

L: longitudinal, $1/\sqrt{2}$ for identical particles

In HE limit,[1] s-wave amplitudes $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2} (a_0) \rightarrow \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect pw unitarity condition $|a_0| \leq 1$

$$\Longrightarrow \quad M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F} \right)^{1/2} = 1\ \mathrm{TeV}/c^2$$

condition for perturbative unitarity

[1] Convenient to calculate using *Goldstone-boson equivalence theorem*, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by $\mathcal{L}_{\mathrm{int}} = -\lambda v h(2w^+w^- + z^2 + h^2) - (\lambda/4)(2w^+w^- + z^2 + h^2)^2$, with $1/v^2 = G_F\sqrt{2}$ and $\lambda = G_F M_H^2/\sqrt{2}$.

- If the bound is respected
  - weak interactions remain weak at all energies
  - perturbation theory is everywhere reliable

- If the bound is violated
  - perturbation theory breaks down
  - weak interactions among $W^{\pm}$, Z, H become strong on 1-TeV scale

$\Rightarrow$ features of *strong* interactions at GeV energies will characterize *electroweak* gauge boson interactions at TeV energies

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

Threshold behavior of the pw amplitudes a_{IJ} follows from chiral symmetry

$$\begin{array}{lll} a_{00} \approx & G_F s/8\pi\sqrt{2} & \text{attractive} \\ a_{11} \approx & G_F s/48\pi\sqrt{2} & \text{attractive} \\ a_{20} \approx & -G_F s/16\pi\sqrt{2} & \text{repulsive} \end{array}$$

Lee, Quigg, Thacker, *Phys. Rev. D* **16,** 1519 (1977)

What the LHC *is not really* for . . .

- Find the Higgs boson,
  the Holy Grail of particle physics,
  the source of all mass in the Universe.
- Celebrate.
- Then particle physics will be over.

We are not ticking off items on a shopping list . . .

We are exploring a vast new terrain
. . . and reaching the Fermi scale

The Origins of Mass

(masses of nuclei "understood")

$p, [\pi], \rho$ understood: QCD

confinement energy is the source

"Mass without mass" Wilczek, *Phys. Today* (November 1999)

We understand the visible mass of the Universe

... without the Higgs mechanism

W, Z electroweak symmetry breaking

$$M_W^2 = \tfrac{1}{2} g^2 v^2 = \pi\alpha / G_F \sqrt{2} \sin^2\theta_W$$
$$M_Z^2 = M_W^2 / \cos^2\theta_W$$

$q, \ell^{\mp}$ EWSB + Yukawa couplings

ν_ℓ EWSB + Yukawa couplings; new physics?

All fermion masses $\Leftrightarrow$ physics beyond standard model

H ?? fifth force ??

Challenge: Understanding the Everyday

- Why are there atoms?
- Why chemistry?
- Why stable structures?
- What makes life possible?

What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?

Searching for the mechanism of electroweak symmetry breaking, we seek to understand

why the world is the way it is.

This is one of the deepest questions humans have ever pursued, and

it is coming within the reach of particle physics.

Our picture of matter

Pointlike constituents ($r < 10^{-18}$ m)

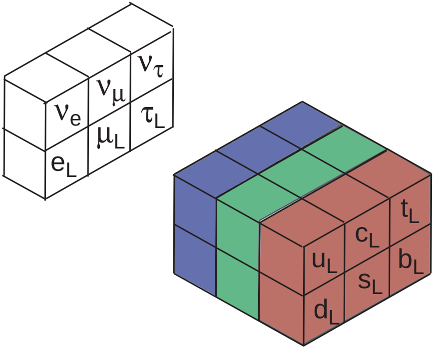
$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

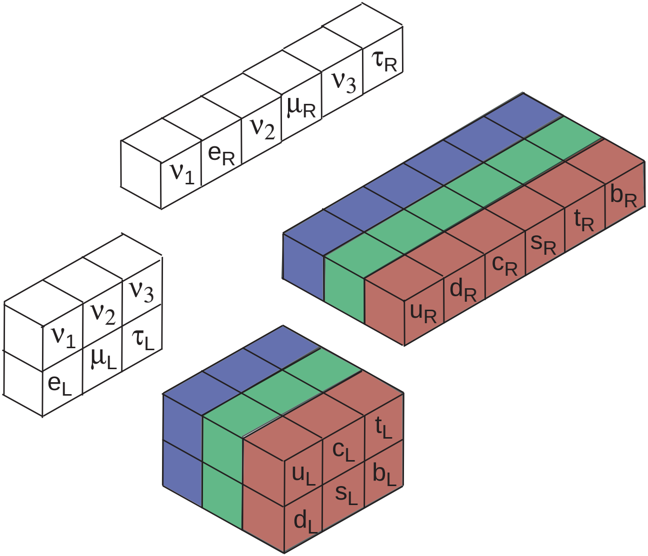
Few fundamental forces, derived from gauge symmetries

$$\mathrm{SU(3)_c} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$$

Electroweak symmetry breaking: Higgs mechanism?



ν_e
ν_μ
ν_τ
e_L
μ_L
τ_L
u_L
c_L
t_L
d_L
s_L
b_L


ν1
eR
ν2
μR
ν3
τR
ν1
ν2
ν3
eL
μL
τL
uR
dR
cR
sR
tR
bR
uL
dL
cL
sL
tL
bL

Formulate electroweak theory

Three crucial clues from experiment:

- Left-handed weak-isospin doublets,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \; ;$$

- Universal strength of the (charged-current) weak interactions;
- Idealization that neutrinos are massless.

$\boxed{\text{First two clues suggest } SU(2)_L \text{ gauge symmetry}}$

Parity violation in weak decays

1956 Wu *et al.*: correlation between spin vector $\vec{J}$ of polarized ^{60}Co and direction $\hat{p}_e$ of outgoing β particle

Parity leaves spin (axial vector) unchanged $\boxed{\mathcal{P} : \vec{J} \to \vec{J}}$

Parity reverses electron direction $\boxed{\mathcal{P} : \hat{p}_e \to -\hat{p}_e}$

Correlation $\vec{J} \cdot \hat{p}_e$ is *parity violating*

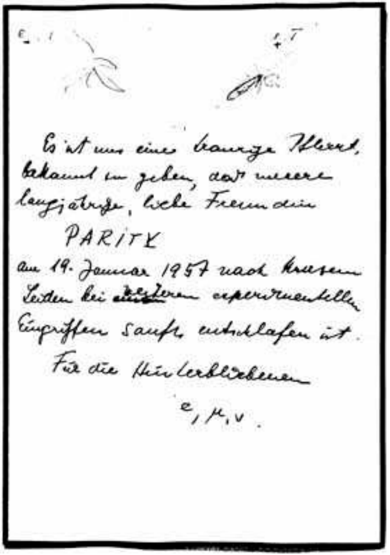
Late 1950s: (charged-current) weak interactions are left-handed

Parity links left-handed, right-handed ν,

$$\nu_L \xrightarrow{\Lleftarrow} \mathcal{P} \xleftarrow{\Lleftarrow} \cancel{\nu}_R$$

$\Rightarrow$ build a manifestly parity-violating theory with only ν_L.

Pauli's Reaction to the Downfall of Parity



Es ist uns eine traurige Pflicht,
bekannt zu geben, daß unsere
langjährige, liebe Freundin
PARITY
am 19. Januar 1957 nach kurzem
Leiden bei weiteren experimentellen
Eingriffen sanft entschlafen ist.
Für die Hinterbliebenen
e, μ, ν.

Pauli's Reaction to the Downfall of Parity

Es ist uns eine traurige Pflicht, bekannt zu geben, daß unsere langjährige ewige Freundin

PARITY

den 19. Januar 1957 nach kurzen Leiden bei weiteren experimentellen Eingriffen sanfte entschlafen ist.

Für die hinterbliebenen

$e \quad \mu \quad \nu$

It is our sad duty to announce that our loyal friend of many years

PARITY

went peacefully to her eternal rest on the nineteenth of January 1957, after a short period of suffering in the face of further experimental interventions.

For those who survive her,

$e \quad \mu \quad \nu$

How do we know ν is left-handed?

▷ $\boxed{\nu_\mu}$ Measure μ^+ helicity in (spin-zero) $\pi^+ \to \mu^+\nu_\mu$

$$\nu_\mu \xleftarrow{\;\Rightarrow\;} \pi^+ \xrightarrow{\;\Leftarrow\;} \mu^+$$

$$\boxed{h(\nu_\mu) = h(\mu^+)}$$

Bardon, PRL **7,** 23 (1961); Possoz, PL **70B,** 265 (1977)

μ^+ forced to have "wrong" helicity

...inhibits decay, and inhibits $\pi^+ \to e^+\nu_e$ more

$$\Gamma(\pi^+ \to e^+\nu_e)/\Gamma(\pi^+ \to \mu^+\nu_\mu) = 1.23 \times 10^{-4}$$

▷ Longitudinal pol. of recoil nucleus in $\mu^{-12}\mathrm{C}(J=0) \to {}^{12}\mathrm{B}(J=1)\nu_\mu$

$$\boxed{\text{Infer } h(\nu_\mu) \text{ by angular momentum conservation}}$$

Roesch, *Am. J. Phys.* **50**, 931 (1981)

▷ $\boxed{\nu_e}$ Measure longitudinal polarization of recoil nucleus in

$$e^- \, {}^{152}\mathrm{Eu}^{\mathrm{m}}(J=0) \;\rightarrow\; {}^{152}\mathrm{Sm}^*(J=1)\,\nu_e$$
$$\hookrightarrow \gamma\, {}^{152}\mathrm{Sm}$$

$\boxed{\text{Infer } h(\nu_e) \text{ from } \gamma \text{ polarization}}$

Goldhaber, *Phys. Rev.* **109**, 1015 (1958)

▷ $\boxed{\nu_\tau}$ Variety of determinations in $\tau \rightarrow \pi\nu_\tau$, $\tau \rightarrow \rho\nu_\tau$, etc.

e.g., Abe, *et al.* (SLD), *Phys. Rev. Lett.* **78**, 4691 (1997)

Charge conjugation is also violated . . .

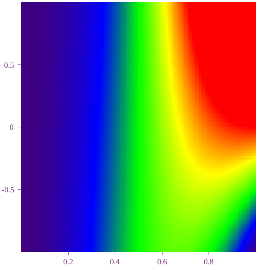
$$\nu_L \xrightarrow{\Leftarrow} \mathcal{C} \xrightarrow{\Leftarrow} \cancel{\bar{\nu}_L}$$

$\mu^\pm$ decay: angular distributions of $e^\pm$ reversed

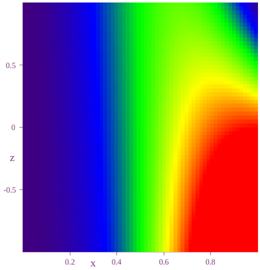
$$\frac{dN(\mu^\pm \to e^\pm + \ldots)}{dxdz} = x^2(3-2x)\left[1 \pm z\frac{(2x-1)}{(3-2x)}\right]$$

$$x \equiv p_e/p_e^{\max},\; z \equiv \hat{s}_\mu \cdot \hat{p}_e$$

e^+ follows μ^+ spin



e^- avoids μ^- spin



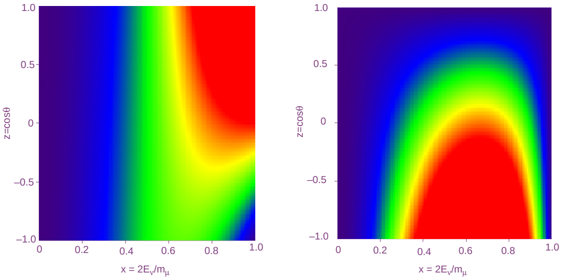
Consequences for neutrino factory

$$\mu^+ \to e^+ \bar{\nu}_\mu \nu_e$$

$$\frac{d^2 N_{\bar{\nu}_\mu}}{dx dz} = x^2[(3-2x) - (1-2x)z]\,, \quad x \equiv p_\nu / p_\nu^{\max},\; z \equiv \hat{p}_\nu \cdot \hat{s}_\mu$$

$$\mu^+ \to e^+ \bar{\nu}_\mu \nu_e$$

$$\frac{d^2 N_{\nu_e}}{dx dz} = 6x^2[(1-x)(1-z)]$$



$\bar{\nu}_\mu$ ν_e

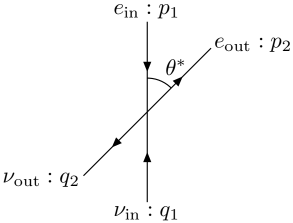
Effective Lagrangian . . .

Late 1950s: current-current interaction

$$\mathcal{L}_{V-A} = \frac{-G_F}{\sqrt{2}} \bar{\nu}\gamma_\mu(1-\gamma_5)e \; \bar{e}\gamma^\mu(1-\gamma_5)\nu + \text{h.c.}$$

$$G_F = 1.16632 \times 10^{-5}\ \text{GeV}^{-2}$$

Compute $\bar{\nu}e$ scattering amplitude:



$$\begin{aligned} \mathcal{M} \;\; = \;\; & -\frac{iG_F}{\sqrt{2}} \bar{v}(\nu, q_1)\gamma_\mu(1-\gamma_5)u(e, p_1) \\ & \cdot \bar{u}(e, p_2)\gamma^\mu(1-\gamma_5)v(\nu, q_2) \end{aligned}$$

$\boxed{\bar{\nu}e \to \bar{\nu}e}$

$$\frac{d\sigma_{V-A}(\bar{\nu}e \to \bar{\nu}e)}{d\Omega_{\text{cm}}} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s} = \frac{G_F^2 \cdot 2mE_\nu(1-z)^2}{16\pi^2} \quad z = \cos\theta^*$$

$$\begin{aligned}\sigma_{V-A}(\bar{\nu}e \to \bar{\nu}e) &= \frac{G_F^2 \cdot 2mE_\nu}{3\pi} \\ &\approx 0.574 \times 10^{-41}\ \text{cm}^2 \left(\frac{E_\nu}{1\ \text{GeV}}\right)\end{aligned}$$

Small! $\approx 10^{-14}\ \sigma(pp)$ at 100 GeV

$\boxed{\nu e \to \nu e}$

$$\frac{d\sigma_{V-A}(\nu e \to \nu e)}{d\Omega_{\text{cm}}} = \frac{G_F^2 \cdot 2mE_\nu}{4\pi^2}$$

$$\begin{aligned}\sigma_{V-A}(\nu e \to \nu e) &= \frac{G_F^2 \cdot 2mE_\nu}{\pi} \\ &\approx 1.72 \times 10^{-41}\ \text{cm}^2 \left(\frac{E_\nu}{1\ \text{GeV}}\right)\end{aligned}$$

Why 3× difference?

incoming $\begin{array}{c} e \\ \downarrow \;\Uparrow \\ \uparrow \;\Downarrow \\ \nu \end{array}$ $J_z = 0$ outgoing, $z = +1$ $\begin{array}{c} e \\ \uparrow \;\Downarrow \\ \downarrow \;\Uparrow \\ \nu \end{array}$ $J_z = 0$

allowed at all angles

incoming $\begin{array}{c} e \\ \downarrow \;\Uparrow \\ \uparrow \;\Uparrow \\ \bar{\nu} \end{array}$ $J_z = +1$ outgoing, $z = +1$ $\begin{array}{c} e \\ \uparrow \;\Downarrow \\ \downarrow \;\Downarrow \\ \bar{\nu} \end{array}$ $J_z = -1$

forbidden (angular momentum) at $z = +1$

1962: Lederman, Schwartz, Steinberger $\nu_\mu \neq \nu_e$

- ▷ Make HE $\pi \to \mu\nu$ beam
- ▷ Observe $\nu N \to \mu + \text{anything}$
- ▷ Don't observe $\nu N \to e + \text{anything}$

Danby, *et al.*, *Phys. Rev. Lett.* **9**, 36 (1962)

Suggests family structure

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$$

$\approx$ no interactions known to cross boundaries

Generalize effective (current-current) Lagrangian:

$$\mathcal{L}_{V-A}^{(e\mu)} = \frac{-G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu \; \bar{e}\gamma^\mu(1-\gamma_5)\nu_e + \text{h.c.}\,,$$

Compute muon decay rate

$$\Gamma(\mu \to e\bar{\nu}_e\nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

accounts for the 2.2-μs muon lifetime

Cross section for inverse muon decay

$$\sigma(\nu_\mu e \to \mu\nu_e) = \sigma_{V-A}(\nu_e e \to \nu_e e)\left[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu\right]^2$$

agrees with CHARM II, CCFR data ($E_\nu \lesssim 600$ GeV)

$\boxed{\text{PW unitarity: } |\mathcal{M}_J| < 1}$

$$V - A \text{ theory:} \quad \mathcal{M}_0 = \frac{G_F \cdot 2m_e E_\nu}{\pi\sqrt{2}}\left[1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu}\right]$$

satisfies pw unitarity for

$$E_\nu < \pi/G_F m_e\sqrt{2} \approx 3.7 \times 10^8 \text{ GeV}$$

$\boxed{\Rightarrow V - A \text{ theory cannot be complete}}$

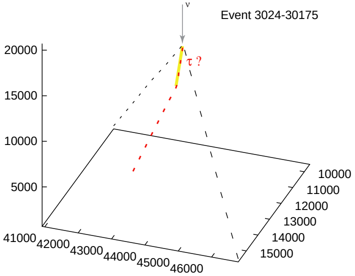
Physics must change below $\sqrt{s} \approx 600$ GeV

2000: DONuT Three-Neutrino Experiment

- ▷ Prompt (beam-dump) ν_τ beam produced in

$$D_s^+ \rightarrow \tau^+ \nu_\tau \\ \hookrightarrow X^+ \bar{\nu}_\tau$$

- ▷ Observe $\nu_\tau N \rightarrow \tau +$ anything in emulsion; τ lifetime is 0.3 ps



Candidate event in ECC1. The three tracks with full emulsion data are shown. The red track shows a 100 mrad kink 4.5mm from the interaction vertex. The scale units are microns.

Kodama, *et al.*, *Phys. Lett.* **B504**, 218 (2001)

Leptons are seen as free particles

Table: Some properties of the leptons.

Lepton	Mass	Lifetime
ν_e	< 2 eV	
e^-	0.510 998 918(44) MeV	$> 4.6 \times 10^{26}$ y (90% CL)
ν_μ	< 0.19 MeV (90% CL)	
μ^-	105.658 369 2(94) MeV	$2.197\,03(4) \times 10^{-6}$ s
ν_τ	< 18.2 MeV (95% CL)	
τ^-	1776.90 ± 0.20 MeV	$290.6 \pm 1.0 \times 10^{-15}$ s

All spin-$\frac{1}{2}$, pointlike ($\lesssim$ few $\times\, 10^{-17}$ cm)

kinematically determined ν masses consistent with 0
(ν oscillations $\Rightarrow$ nonzero, unequal masses)

Universal weak couplings: *Rough and ready test*

Fermi constant from muon decay

$$G_\mu = \left[\frac{192\pi^3\hbar}{\tau_\mu m_\mu^5}\right]^{\frac{1}{2}} = 1.1638 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Meticulous analysis yields $G_\mu = 1.16637(1) \times 10^{-5}\ \mathrm{GeV}^{-2}$

Fermi constant from tau decay

$$G_\tau = \left[\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)}{\Gamma(\tau \to \mathrm{all})}\frac{192\pi^3\hbar}{\tau_\tau m_\tau^5}\right]^{\frac{1}{2}} = 1.1642 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Excellent agreement with $G_\beta = 1.16639(2) \times 10^{-5}\ \mathrm{GeV}^{-2}$

Charged currents acting in leptonic and semileptonic interactions are of universal strength; $\Rightarrow$ *universality of current-current form, or whatever lies behind it*

Nonleptonic enhancement

Certain NL transitions are more rapid than universality suggests

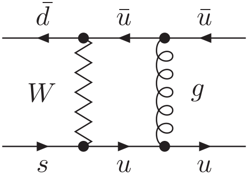
$$\underbrace{\Gamma(K_S \to \pi^+\pi^-)}_{I=0,2} \approx 450 \times \underbrace{\Gamma(K^+ \to \pi^+\pi^0)}_{I=2}$$

$$\boxed{A_0 \approx 22 \times A_2}$$

$|\Delta I| = \frac{1}{2}$ rule; "octet dominance" (over **27**)

Origin of this phenomenological rule is only partly understood.

Short-distance *(perturbative)* QCD corrections arise from



... explain $\approx \sqrt{\text{enhancement}}$

A theory of leptons

$$\mathsf{L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \mathsf{R} \equiv e_R$$

weak hypercharges $Y_L = -1$, $Y_R = -2$

Gell-Mann–Nishijima connection, $Q = I_3 + \frac{1}{2}Y$

$\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$ gauge group $\Rightarrow$ gauge fields:

- weak isovector $\vec{b}_\mu$, coupling g $\qquad \boxed{b^\ell_\mu = b^\ell_\mu - \varepsilon_{jk\ell}\alpha^j b^k_\mu - (1/g)\partial_\mu \alpha^\ell}$
- weak isoscalar $\mathcal{A}_\mu$, coupling $g'/2$ $\qquad \boxed{\mathcal{A}_\mu \to \mathcal{A}_\mu - \partial_\mu \alpha}$

Field-strength tensors

$$F^\ell_{\mu\nu} = \partial_\nu b^\ell_\mu - \partial_\mu b^\ell_\nu + g\varepsilon_{jk\ell} b^j_\mu b^k_\nu \;, SU(2)_L$$

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu \;, U(1)_Y$$

Interaction Lagrangian

$$\boxed{\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}}}$$

$$\mathcal{L}_{\text{gauge}} = -\tfrac{1}{4} F^{\ell}_{\mu\nu} F^{\ell\mu\nu} - \tfrac{1}{4} f_{\mu\nu} f^{\mu\nu},$$

$$\begin{aligned}\mathcal{L}_{\text{leptons}} &= \overline{\mathsf{R}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu} Y\right)\mathsf{R} \\ &+ \overline{\mathsf{L}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu} Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_{\mu}\right)\mathsf{L}.\end{aligned}$$

Mass term $\mathcal{L}_e = -m_e(\bar{e}_{\mathsf{R}} e_{\mathsf{L}} + \bar{e}_{\mathsf{L}} e_{\mathsf{R}}) = -m_e \bar{e} e$ violates local gauge inv.

Theory: 4 massless gauge bosons ($\mathcal{A}_{\mu} \quad b^1_{\mu} \quad b^2_{\mu} \quad b^3_{\mu}$); Nature: 1 ($\gamma$)

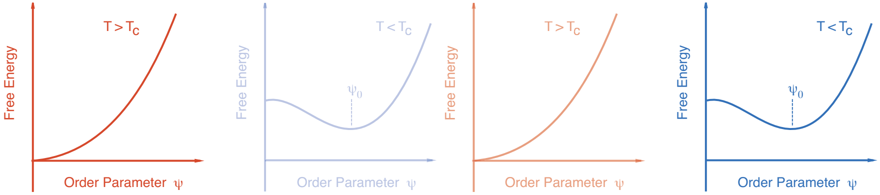
Massive Photon? *Hiding Symmetry*

Recall $\boxed{2}$ miracles of superconductivity:

- No resistance . . . . . . Meissner effect (exclusion of **B**)

Ginzburg–Landau Phenomenology (not a theory from first principles)

normal, resistive charge carriers . . . . . . + superconducting charge carriers



$$\mathbf{B} = 0: \qquad G_{\mathrm{super}}(0) = G_{\mathrm{normal}}(0) + \alpha\,|\psi|^2 + \beta\,|\psi|^4$$

$$T > T_c: \quad \alpha > 0 \quad \langle|\psi|^2\rangle_0 = 0$$

$$T < T_c: \quad \alpha < 0 \quad \langle|\psi|^2\rangle_0 \neq 0$$

In a nonzero magnetic field ...

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{8\pi} + \frac{1}{2m^*}\left|-i\hbar\nabla\psi - \frac{e^*}{c}\mathbf{A}\psi\right|^2$$

$$\left.\begin{array}{c} e^* = -2 \\ m^* \end{array}\right\} \text{of superconducting carriers}$$

Weak, slowly varying field: $\psi \approx \psi_0 \neq 0$, $\nabla\psi \approx 0$

Variational analysis $\rightsquigarrow$

$$\boxed{\nabla^2\mathbf{A} - \frac{4\pi e^{*2}}{m^* c^2}\left|\psi_0\right|^2 \mathbf{A} = 0}$$

wave equation of a *massive photon*

Photon – *gauge boson* – acquires mass
within superconductor

origin of Meissner effect

Meissner effect levitates Leon Lederman (Snowmass 2001)

Hiding EW Symmetry

Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition

- Introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1$$

- Add to $\mathcal{L}$ (gauge-invariant) terms for interaction and propagation of the scalars,

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi),$$

where $\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau} \cdot \vec{b}_\mu$ and

$$\boxed{V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + |\lambda| \, (\phi^\dagger \phi)^2}$$

- Add a Yukawa interaction $\mathcal{L}_{\text{Yukawa}} = -\zeta_e \left[\overline{\mathsf{R}}(\phi^\dagger \mathsf{L}) + (\overline{\mathsf{L}}\phi)\mathsf{R}\right]$

- Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^2 < 0$

  Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks) $SU(2)_L$ and $U(1)_Y$

but preserves $U(1)_{\text{em}}$ invariance

Invariance under $\mathcal{G}$ means $e^{i\alpha\mathcal{G}}\langle\phi\rangle_0 = \langle\phi\rangle_0$, so $\mathcal{G}\langle\phi\rangle_0 = 0$

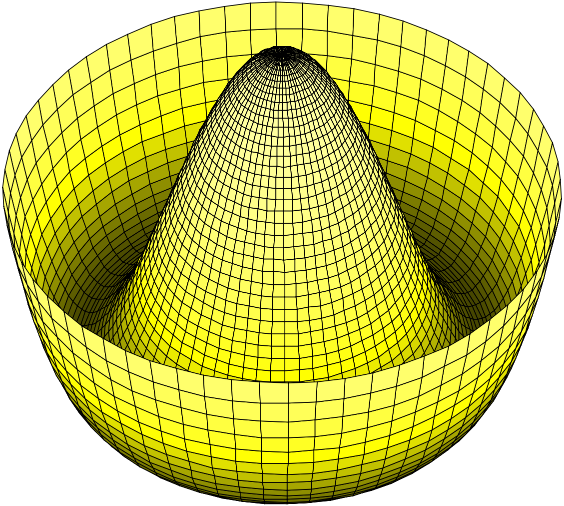
$$\tau_1\langle\phi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

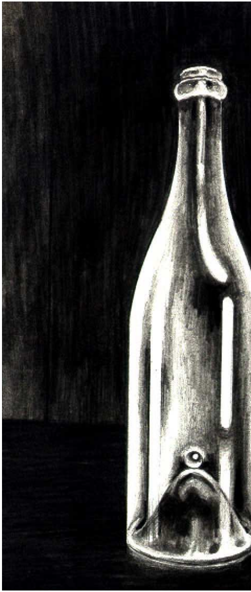
$$\tau_2\langle\phi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_3\langle\phi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$Y\langle\phi\rangle_0 = Y_\phi\langle\phi\rangle_0 = +1\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

Symmetry of laws $\not\Rightarrow$ symmetry of outcomes





Examine electric charge operator Q on the (neutral) vacuum

$$\begin{aligned}
Q\langle\phi\rangle_0 &= \tfrac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 \\
&= \tfrac{1}{2}\begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix}\langle\phi\rangle_0 \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \textit{unbroken!}
\end{aligned}$$

Four original generators are broken, *electric charge is not*

- $\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_Y \rightarrow U(1)_{\mathrm{em}}$ (will verify)
- Expect massless photon
- Expect gauge bosons corresponding to

$$\tau_1,\ \tau_2,\ \tfrac{1}{2}(\tau_3 - Y) \equiv K \quad \text{to acquire masses}$$

- Electromagnetism is mediated by a massless photon, coupled to the electric charge;
- Mediator of charged-current weak interaction acquires a mass $M_W^2 = \pi\alpha/G_F\sqrt{2}\sin^2\theta_W$,
- Mediator of (new!) neutral-current weak interaction acquires mass $M_Z^2 = M_W^2/\cos^2\theta_W$;
- Massive neutral scalar particle, the Higgs boson, appears, but its mass is not predicted;
- Fermions can acquire mass—values not predicted.

Expand about the vacuum state

Let $\phi = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}$; in *unitary gauge*

$$\begin{aligned} \mathcal{L}_{\text{scalar}} &= \tfrac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2\eta^2 \\ &+ \frac{v^2}{8}[g^2 \left|b_\mu^1 - ib_\mu^2\right|^2 + (g'\mathcal{A}_\mu - gb_\mu^3)^2] \\ &+ \text{interaction terms} \end{aligned}$$

"Higgs boson" η has acquired (mass)^2 $M_H^2 = -2\mu^2 > 0$

$$\text{Define } W_\mu^\pm = \frac{b_\mu^1 \mp ib_\mu^2}{\sqrt{2}}$$

$$\frac{g^2v^2}{8}(\left|W_\mu^+\right|^2 + \left|W_\mu^-\right|^2) \Longleftrightarrow M_{W^\pm} = gv/2$$

$(v^2/8)(g'\mathcal{A}_\mu - g b_\mu^3)^2 \ldots$

Now define orthogonal combinations

$$Z_\mu = \frac{-g'\mathcal{A}_\mu + g b_\mu^3}{\sqrt{g^2 + g'^2}} \qquad A_\mu = \frac{g\mathcal{A}_\mu + g' b_\mu^3}{\sqrt{g^2 + g'^2}}$$

$$M_{Z^0} = \sqrt{g^2 + g'^2}\, v/2 = M_W \sqrt{1 + g'^2/g^2}$$

$\boxed{A_\mu \text{ remains massless}}$

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\zeta_e \frac{(v+\eta)}{\sqrt{2}}(\bar{e}_R e_L + \bar{e}_L e_R) \\
&= -\frac{\zeta_e v}{\sqrt{2}}\bar{e}e - \frac{\zeta_e \eta}{\sqrt{2}}\bar{e}e
\end{aligned}
$$

electron acquires $m_e = \zeta_e v/\sqrt{2}$

Higgs-boson coupling to electrons: m_e/v ($\propto$ mass)

Desired particle content . . . plus a Higgs scalar

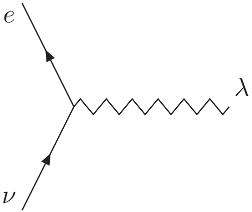
Values of couplings, electroweak scale v?

What about interactions?

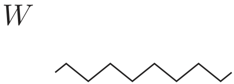
Interactions . . .

$$\mathcal{L}_{W\text{-}\ell} = -\frac{g}{2\sqrt{2}}[\bar{\nu}\gamma^{\mu}(1-\gamma_5)eW_{\mu}^{+} + \bar{e}\gamma^{\mu}(1-\gamma_5)\nu W_{\mu}^{-}]$$

$+$ similar terms for μ and τ



$$\frac{-ig}{2\sqrt{2}}\gamma_{\lambda}(1-\gamma_5)$$



$$= \frac{-i(g_{\mu\nu} - k_{\mu}k_{\nu}/M_W^2)}{k^2 - M_W^2} \, .$$

Compute $\nu_\mu e \to \mu\nu_e$

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\frac{g^4}{16 M_W^4} = 2G_F^2 \Rightarrow \frac{g}{2\sqrt{2}} = \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}}$$

Using $M_W = gv/2$, determine the electroweak scale

$$\boxed{v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246\text{ GeV}}$$

$$\Rightarrow \langle\phi^0\rangle_0 = (G_F\sqrt{8})^{-\frac{1}{2}} \approx 174\text{ GeV}$$

W-propagator modifies HE behavior

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

$$\lim_{E_\nu \to \infty} \sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\sqrt{2}}$$

independent of energy!

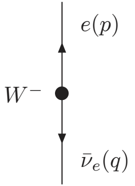
Partial-wave unitarity respected for

$$s < M_W^2[\exp\left(\pi\sqrt{2}/G_F M_W^2\right) - 1]$$

W-boson properties

No prediction yet for M_W (haven't determined g)

Leptonic decay $W^- \to\ e^-\nu_e$



$$p \approx \left(\frac{M_W}{2};\frac{M_W\sin\theta}{2},0,\frac{M_W\cos\theta}{2}\right)$$

$$q \approx \left(\frac{M_W}{2};-\frac{M_W\sin\theta}{2},0,-\frac{M_W\cos\theta}{2}\right)$$

$$\mathcal{M} = -i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}}\bar{u}(e,p)\gamma_\mu(1-\gamma_5)v(\nu,q)\,\varepsilon^\mu$$

$\varepsilon^\mu = (0;\hat{\varepsilon})$: W polarization vector in its rest frame

$$\begin{aligned} |\mathcal{M}|^2 &= \frac{G_F M_W^2}{\sqrt{2}}\mathrm{tr}\left[\not\varepsilon(1-\gamma_5)\not q(1+\gamma_5)\not\varepsilon^*\not p\right]\ ; \\ \mathrm{tr}[\cdots] &= \left[\varepsilon\cdot q\,\varepsilon^*\cdot p - \varepsilon\cdot\varepsilon^*\,q\cdot p + \varepsilon\cdot p\,\varepsilon^*\cdot q + i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu q^\nu\varepsilon^{*\rho}p^\sigma\right] \end{aligned}$$

$$\mathrm{tr}[\cdots] \;=\; [\varepsilon\cdot q\,\varepsilon^*\cdot p - \varepsilon\cdot\varepsilon^*\,q\cdot p + \varepsilon\cdot p\,\varepsilon^*\cdot q + i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma]$$

decay rate is independent of W polarization; look first at longitudinal pol.
$\varepsilon^\mu = (0;0,0,1) = \varepsilon^{*\mu}$, eliminate $\epsilon_{\mu\nu\rho\sigma}$

$$|\mathcal{M}|^2 = \frac{4G_F M_W^4}{\sqrt{2}}\sin^2\theta$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2}\,\frac{\mathcal{S}_{12}}{M_W^3}$$

$$\mathcal{S}_{12} = \sqrt{[M_W^2 - (m_e + m_\nu)^2][M_W^2 - (m_e - m_\nu)^2]} = M_W^2$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2\sqrt{2}}\sin^2\theta \qquad \boxed{\Gamma(W\to e\nu) = \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

Other helicities: $\varepsilon^{\mu}_{\pm 1} = (0; -1, \mp i, 0)/\sqrt{2}$

$$\frac{d\Gamma_{\pm 1}}{d\Omega} = \frac{G_F M_W^3}{32\pi^2\sqrt{2}}(1 \mp \cos\theta)^2$$

Extinctions at $\cos\theta = \pm 1$ are consequences of angular momentum conservation:

W^- $\Uparrow$

e^- $\Downarrow$ $\bar{\nu}_e$ $\Downarrow$ $(\theta = 0)$ *forbidden*

$\bar{\nu}_e$ $\Uparrow$ e^- $\Uparrow$ $(\theta = \pi)$ allowed

(situation reversed for $W^+ \to e^+\nu_e$)

e^+ follows polarization direction of W^+

e^- avoids polarization direction of W^-

important for discovery of $W^\pm$ in $\bar{p}p$ $(\bar{q}q)$ $\mathcal{C}$ violation

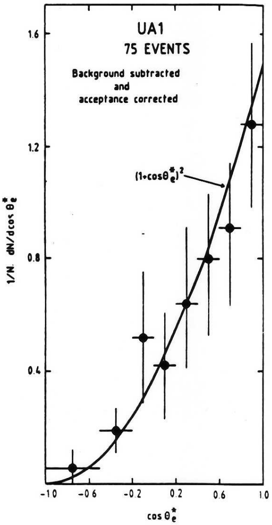


Fig. 2. The W decay angular distribution of the emission angle θ^* of the electron (positron) with respect to the proton (antiproton) direction in the rest frame of the W. Only those events for which the lepton charge and the decay kinematics are well determined have been used. The curve shows the $(V - A)$ expectation of $(1 + \cos\theta^*)^2$.

Interactions . . .

$$\mathcal{L}_{A\text{-}\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e}\gamma^\mu e A_\mu$$

. . . vector interaction; $\Rightarrow A_\mu$ as γ, provided we identify

$$\boxed{gg'/\sqrt{g^2 + g'^2} \equiv e}$$

Define $g' = g \tan\theta_W$ θ_W: weak mixing angle

$$\begin{aligned} g &= e/\sin\theta_W \geq e \\ g' &= e/\cos\theta_W \geq e \end{aligned}$$

$$Z_\mu = b^3_\mu \cos\theta_W - \mathcal{A}_\mu \sin\theta_W \quad A_\mu = \mathcal{A}_\mu \cos\theta_W + b^3_\mu \sin\theta_W$$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4\cos\theta_W} \bar{\nu}\gamma^\mu (1 - \gamma_5)\nu \, Z_\mu$$

Purely left-handed!

Interactions . . .

$$\mathcal{L}_{Z-e} = \frac{-g}{4\cos\theta_W}\bar{e}\left[L_e\gamma^\mu(1-\gamma_5) + R_e\gamma^\mu(1+\gamma_5)\right]e\,Z_\mu$$

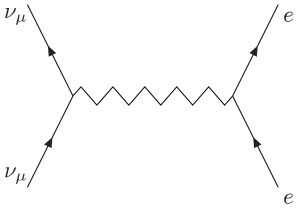
$$\begin{aligned} L_e &= 2\sin^2\theta_W - 1 = 2x_W + \tau_3 \\ R_e &= 2\sin^2\theta_W = 2x_W \end{aligned}$$

Z-decay calculation analogous to $W^\pm$

$$\begin{aligned} \Gamma(Z \to \nu\bar{\nu}) &= \frac{G_F M_Z^3}{12\pi\sqrt{2}} \\ \Gamma(Z \to e^+e^-) &= \Gamma(Z \to \nu\bar{\nu})\left[L_e^2 + R_e^2\right] \end{aligned}$$

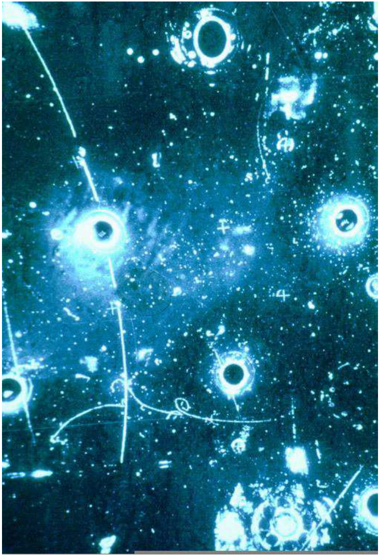
Neutral-current interactions

New νe reaction, not present in $V - A$



$$\begin{aligned}
\sigma(\nu_\mu e \to \nu_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2 + R_e^2/3\right] \\
\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2/3 + R_e^2\right] \\
\sigma(\nu_e e \to \nu_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2 + R_e^2/3\right] \\
\sigma(\bar{\nu}_e e \to \bar{\nu}_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2/3 + R_e^2\right]
\end{aligned}$$

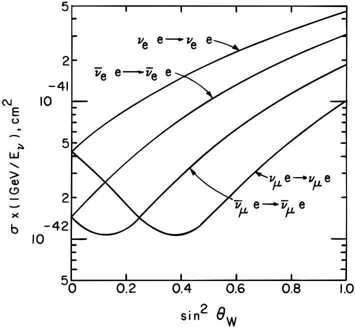
Gargamelle $\bar{\nu}_\mu e$ event (1973)



- Electromagnetism is mediated by a massless photon, coupled to the electric charge;
- Mediator of charged-current weak interaction acquires a mass $M_W^2 = \pi\alpha/G_F\sqrt{2}\sin^2\theta_W$,
- Mediator of (new!) neutral-current weak interaction acquires mass $M_Z^2 = M_W^2/\cos^2\theta_W$;
- Massive neutral scalar particle, the Higgs boson, appears, but its mass is not predicted;
- Fermions can acquire mass—values not predicted.

Determine $\sin^2\theta_W$ to predict M_W, M_Z

“Model-independent” analysis



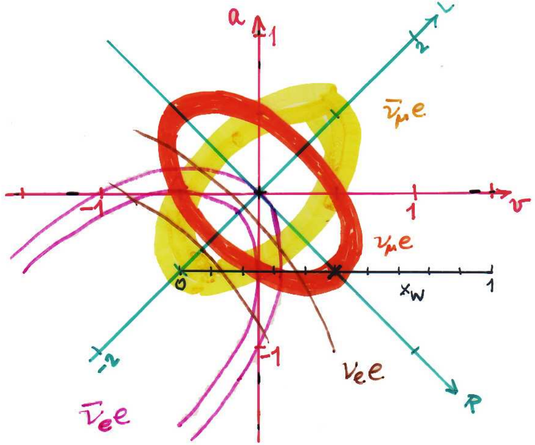
Measure all cross sections to determine chiral couplings L_e and R_e or traditional vector and axial couplings v and a

$$a = \tfrac{1}{2}(L_e - R_e) \qquad v = \tfrac{1}{2}(L_e - R_e)$$

$$L_e = v + a \qquad R_e = v - a$$

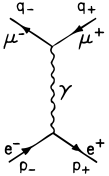
model-independent in V, A framework

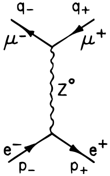
Neutrino–electron scattering



Twofold ambiguity remains even after measuring all four cross sections: same cross sections result if we interchange $R_e \leftrightarrow -R_e$ ($v \leftrightarrow a$)

Consider $e^+e^- \to \mu^+\mu^-$





$$
\begin{aligned}
\mathcal{M} \quad = \quad & -ie^2\bar{u}(\mu, q_-)\gamma_\lambda Q_\mu v(\mu, q_+)\frac{g^{\lambda\nu}}{s}\bar{v}(e, p_+)\gamma_\nu u(e.p_-) \\
& +\frac{i}{2}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)\bar{u}(\mu, q_-)\gamma_\lambda[R_\mu(1+\gamma_5) + L_\mu(1-\gamma_5)]v(\mu, q_+) \\
& \times\frac{g^{\lambda\nu}}{s - M_Z^2}\bar{v}(e, p_+)\gamma_\nu[R_e(1+\gamma_5) + L_e(1-\gamma_5)]u(e, p_-)
\end{aligned}
$$

muon charge $Q_\mu = -1$

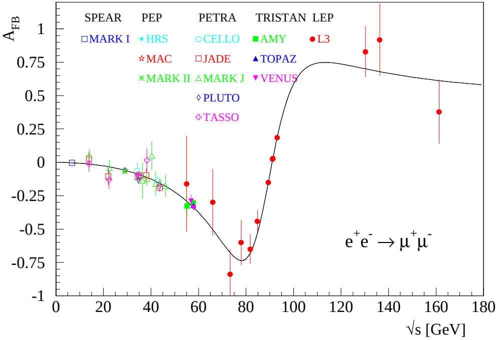
$e^+e^- \longrightarrow \ \mu^+\mu^- \ \ldots$

$$
\begin{aligned}
\frac{d\sigma}{dz} \;=\;& \frac{\pi\alpha^2 Q_\mu^2}{2s}(1+z^2) \\
& -\frac{\alpha Q_\mu G_F M_Z^2 (s-M_Z^2)}{8\sqrt{2}[(s-M_Z^2)^2 + M_Z^2\Gamma^2]} \\
& \times[(R_e+L_e)(R_\mu+L_\mu)(1+z^2) + 2(R_e-L_e)(R_\mu-L_\mu)z] \\
& +\frac{G_F^2 M_Z^4 s}{64\pi[(s-M_Z^2)^2 + M_Z^2\Gamma^2]} \\
& \times[(R_e^2+L_e^2)(R_\mu^2+L_\mu^2)(1+z^2) + 2(R_e^2-L_e^2)(R_\mu^2-L_\mu^2)z]
\end{aligned}
$$

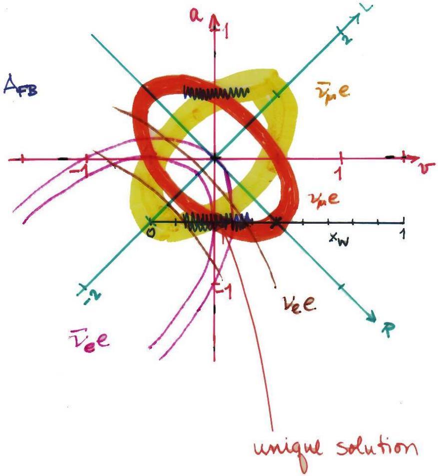
Measuring A resolves ambiguity

$$\text{Forward-backward asymmetry } A \equiv \frac{\int_0^1 \, dz\, d\sigma/dz - \int_{-1}^0 \, dz\, d\sigma/dz}{\int_{-1}^1 \, dz\, d\sigma/dz}$$

$$\begin{aligned}\lim_{s/M_Z^2 \ll 1} A &= \frac{3G_F s}{16\pi\alpha Q_\mu \sqrt{2}}(R_e - L_e)(R_\mu - L_\mu) \\ &\approx -6.7 \times 10^{-5} \left(\frac{s}{1\ \mathrm{GeV}^2}\right)(R_e - L_e)(R_\mu - L_\mu) = -3G_F s a^2/4\pi\alpha\sqrt{2}\end{aligned}$$



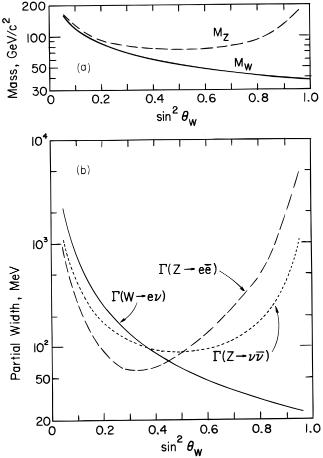
Neutrino–electron scattering $e^+e^- \to \mu^+\mu^-$



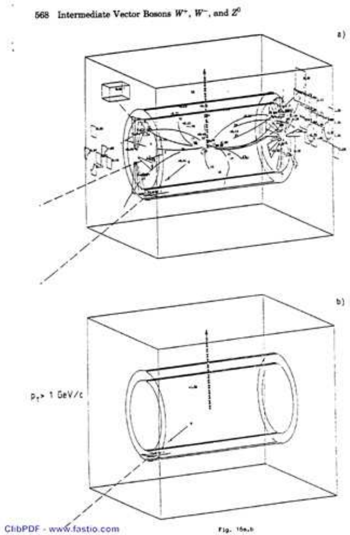
Validate EW theory, measure $\sin^2\theta_W$

With a measurement of $\sin^2\theta_W$, predict

$M_W^2 = \pi\alpha / G_F\sqrt{2}\sin^2\theta_W \approx (37.28\ \mathrm{GeV}/c^2)^2/\sin^2\theta_W$ $\quad$ $M_Z^2 = M_W^2/\cos^2\theta_W$



First Z from UA1



Electroweak interactions of quarks (one generation)

- Left-handed doublet

$$
\begin{array}{cccc}
 & I_3 & Q & Y = 2(Q - I_3) \\
\mathsf{L}_q = \begin{pmatrix} u \\ d \end{pmatrix}_L & \begin{array}{c} \frac{1}{2} \\ -\frac{1}{2} \end{array} & \begin{array}{c} +\frac{2}{3} \\ -\frac{1}{3} \end{array} & \frac{1}{3}
\end{array}
$$

- two right-handed singlets

$$
\begin{array}{cccc}
 & I_3 & Q & Y = 2(Q - I_3) \\
\mathsf{R}_u = u_R & 0 & +\frac{2}{3} & +\frac{4}{3} \\
\mathsf{R}_d = d_R & 0 & -\frac{1}{3} & -\frac{2}{3}
\end{array}
$$

Electroweak interactions of quarks

- CC interaction

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{u}\gamma^{\mu}(1-\gamma_5)d\, W_{\mu}^{+} + \bar{d}\gamma^{\mu}(1-\gamma_5)u\, W_{\mu}^{-} \right]$$

identical in form to $\mathcal{L}_{W\text{-}\ell}$: universality $\Leftrightarrow$ weak isospin

- NC interaction

$$\mathcal{L}_{Z\text{-}q} = \frac{-g}{4\cos\theta_W} \sum_{i=u,d} \bar{q}_i\gamma^{\mu}\left[L_i(1-\gamma_5) + R_i(1+\gamma_5)\right] q_i Z_{\mu}$$

$$L_i = \tau_3 - 2Q_i\sin^2\theta_W \quad R_i = -2Q_i\sin^2\theta_W$$

equivalent in form (not numbers) to $\mathcal{L}_{Z\text{-}\ell}$

Trouble in Paradise

Universal $u \leftrightarrow d$, $\nu_e \leftrightarrow e$ *not quite right*

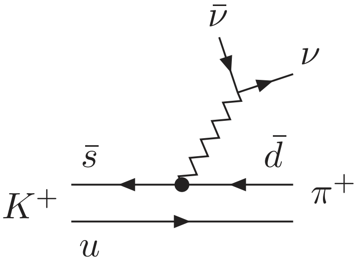
$$\text{Good:} \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow \text{Better:} \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

$$d_\theta \equiv d \cos\theta_C + s \sin\theta_C \qquad \cos\theta_C = 0.9736 \pm 0.0010$$

"Cabibbo-rotated" doublet perfects CC interaction (up to small third-generation effects) but $\Rightarrow$ serious trouble for NC

$$\begin{aligned}
\mathcal{L}_{Z\text{-}q} \;=\; & \frac{-g}{4\cos\theta_W} Z_\mu \{\bar{u}\gamma^\mu [L_u(1-\gamma_5) + R_u(1+\gamma_5)]\, u \\
& + \bar{d}\gamma^\mu [L_d(1-\gamma_5) + R_d(1+\gamma_5)]\, d \cos^2\theta_C \\
& + \bar{s}\gamma^\mu [L_d(1-\gamma_5) + R_d(1+\gamma_5)]\, s \sin^2\theta_C \\
& + \bar{d}\gamma^\mu [L_d(1-\gamma_5) + R_d(1+\gamma_5)]\, s \sin\theta_C \cos\theta_C \\
& + \bar{s}\gamma^\mu [L_d(1-\gamma_5) + R_d(1+\gamma_5)]\, d \sin\theta_C \cos\theta_C \}
\end{aligned}$$

Strangeness-changing NC interactions highly suppressed!



BNL E-787/E-949 has three $K^+ \to \pi^+ \nu\bar{\nu}$ candidates, with $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) = 1.5^{+1.3}_{-0.9} \times 10^{-10}$ *Phys. Rev. Lett.* **93**, 031801 (2004)

(SM: 0.78 ± 0.11: U. Haisch, hep-ph/0605170)

Glashow–Iliopoulos–Maiani

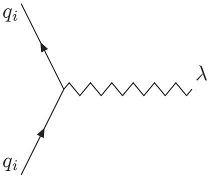
two LH doublets: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \quad \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$

$$(s_\theta = s\cos\theta_C - d\sin\theta_C)$$

$+$ right-handed singlets, e_R, μ_R, u_R, d_R, c_R, s_R

Required new charmed quark, c

Cross terms vanish in $\mathcal{L}_{Z\text{-}q}$,



$$\frac{-ig}{4\cos\theta_W}\gamma_\lambda[(1-\gamma_5)L_i + (1+\gamma_5)R_i] \quad ,$$

$$L_i = \tau_3 - 2Q_i\sin^2\theta_W \quad R_i = -2Q_i\sin^2\theta_W$$

flavor-diagonal interaction!

Straightforward generalization to n quark doublets

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{\Psi}\gamma^{\mu}(1-\gamma_5)\mathcal{O}\Psi\, W_{\mu}^{+} + \text{h.c.}\right]$$

composite $\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix}$ flavor structure $\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$

U: unitary quark mixing matrix

Weak-isospin part: $\mathcal{L}_{Z\text{-}q}^{\text{iso}} = \dfrac{-g}{4\cos\theta_W}\bar{\Psi}\gamma^{\mu}(1-\gamma_5)\left[\mathcal{O},\mathcal{O}^{\dagger}\right]\Psi$

Since $\left[\mathcal{O},\mathcal{O}^{\dagger}\right] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

$\boxed{\Rightarrow \text{NC interaction is flavor-diagonal}}$

General $n \times n$ mixing matrix U: $n(n-1)/2$ real $\angle$, $(n-1)(n-2)/2$ complex phases

3×3 (Cabibbo–Kobayashi-Maskawa): $3\ \angle + 1$ phase $\Rightarrow \mathcal{CP}$ violation

Successful predictions of $SU(2)_L \otimes U(1)_Y$ theory:

- neutral-current interactions
- necessity of charm
- existence and properties of $W^{\pm}$ and Z^0

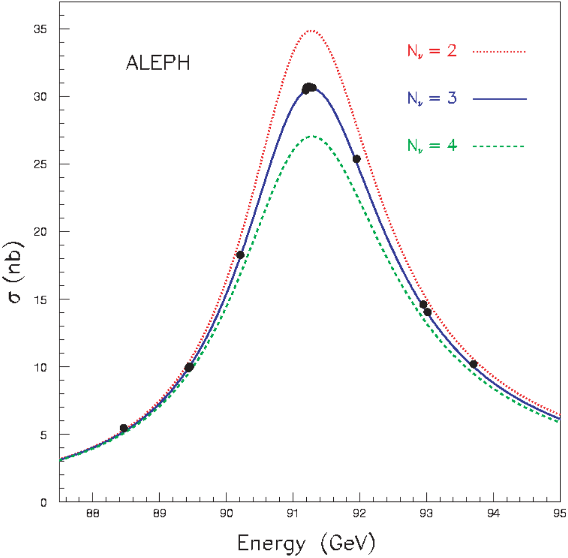
$+$ a decade of precision EW tests (one-per-mille)

M_W	$80\,398 \pm 25$ MeV
Γ_W	$2\,140 \pm 60$ MeV
M_Z	$91\,187.6 \pm 2.1$ MeV
Γ_Z	2495.2 ± 2.3 MeV
$\sigma^0_{\text{hadronic}}$	41.541 ± 0.037 nb
Γ_{hadronic}	1744.4 ± 2.0 MeV
Γ_{leptonic}	83.984 ± 0.086 MeV
$\Gamma_{\text{invisible}}$	499.0 ± 1.5 MeV

$$\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$$

light ν : $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \to \nu_i \bar{\nu}_i) = 2.994 \pm 0.012 \quad (\nu_e, \nu_\mu, \nu_\tau)$

Three light neutrinos



$\nu_\mu N \rightarrow \mu^- +$ anything: *influence of W propagator*

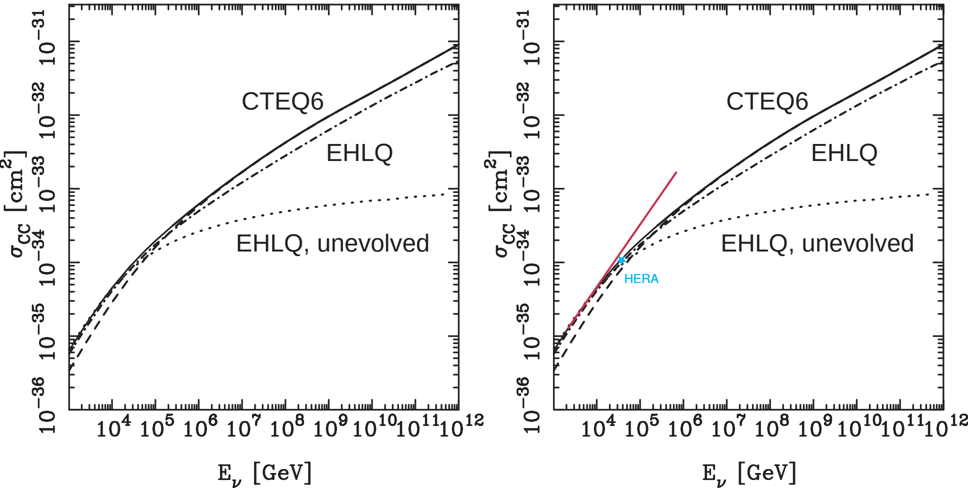
$$\frac{d^2\sigma}{dxdy} = \frac{2G_F^2 M E_\nu}{\pi}\left(\frac{M_W^2}{Q^2 + M_W^2}\right)^2 \left[xq(x, Q^2) + x\bar{q}(x, Q^2)(1-y)^2\right]$$

$$\begin{aligned} q(x, Q^2) &= \frac{u_v(x, Q^2) + d_v(x, Q^2)}{2} + \frac{u_s(x, Q^2) + d_s(x, Q^2)}{2} \\ &\quad + s_s(x, Q^2) + b_s(x, Q^2) \\ \bar{q}(x, Q^2) &= \frac{u_s(x, Q^2) + d_s(x, Q^2)}{2} + c_s(x, Q^2) + t_s(x, Q^2), \end{aligned}$$

. . . isoscalar nucleon

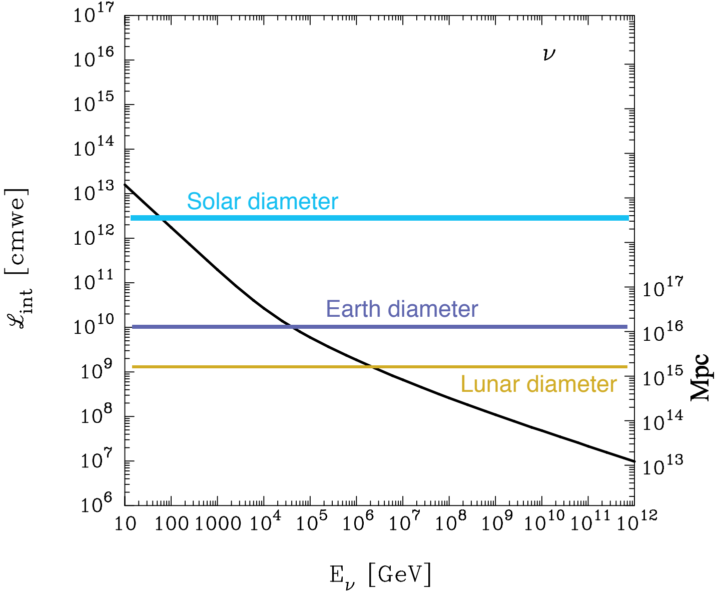
$$x = Q^2/2M\nu \qquad y \equiv \nu/E_\nu \qquad \nu \equiv E_\nu - E_\mu$$

$\nu N \to \mu + \ldots$ Cross Sections

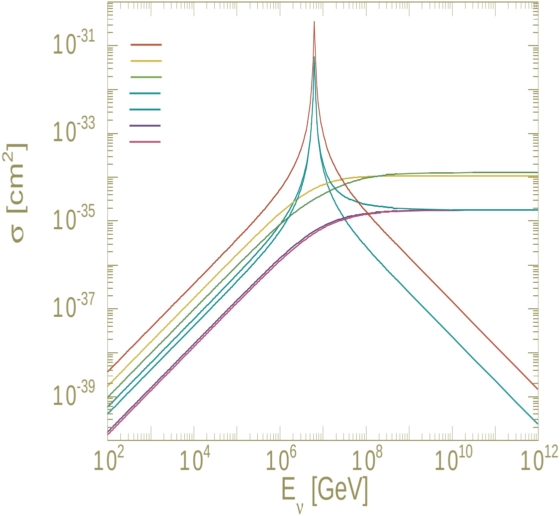


M. H. Reno

$\nu N \to \mu + \ldots$ Interaction Lengths

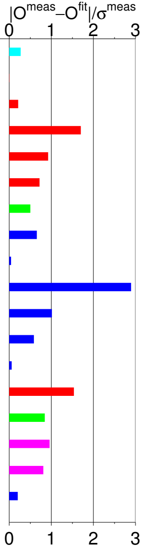


νe cross sections . . .



At low energies: $\sigma(\bar{\nu}_e e \to \text{ hadrons}) > \sigma(\nu_\mu e \to \mu\nu_e) > \sigma(\nu_e e \to \nu_e e) >$ $\sigma(\bar{(}\nu_e e \to \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \to \bar{\nu}_e e) > \sigma(\nu_\mu e \to \nu_\mu e) > \sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ_{had}^{0} [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21586
R_c	0.1721 ± 0.0030	0.1722
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.398 ± 0.025	80.374
Γ_W [GeV]	2.140 ± 0.060	2.091
m_t [GeV]	170.9 ± 1.8	171.3

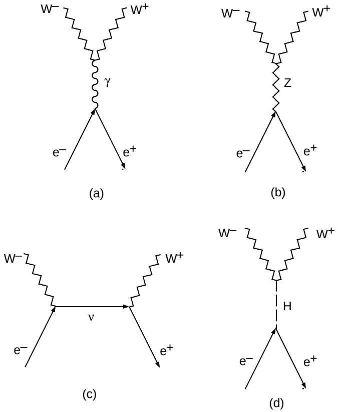


LEP Electroweak Working Group, Winter 2007

Why a Higgs boson must exist

▷ Role in canceling high-energy divergences

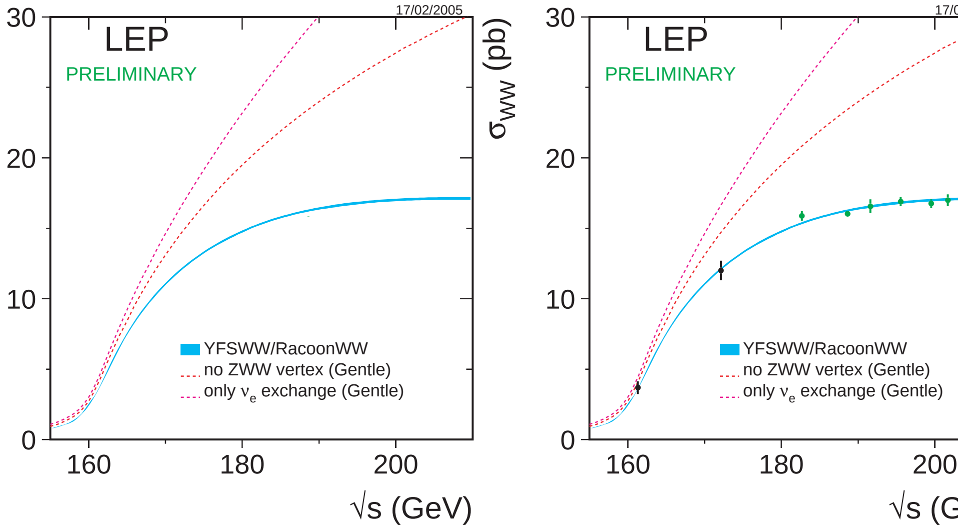
S-matrix analysis of $e^+e^- \to W^+W^-$



Individual $J = 1$ partial-wave amplitudes $\mathcal{M}_\gamma^{(1)}$, $\mathcal{M}_Z^{(1)}$, $\mathcal{M}_\nu^{(1)}$ have unacceptable high-energy behavior ($\propto s$)

. . . But sum is well-behaved

“Gauge cancellation” observed at LEP2 (Tevatron)



$J = 0$ amplitude exists because electrons have mass, and can be found in "wrong" helicity state

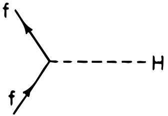
$$\mathcal{M}_\nu^{(0)} \propto s^{\frac{1}{2}} \text{ : unacceptable HE behavior}$$

(no contributions from γ and Z)

This divergence is canceled by the Higgs-boson contribution

$$\boxed{\Rightarrow He\bar{e} \text{ coupling must be } \propto m_e,}$$

because "wrong-helicity" amplitudes $\propto m_e$



$$\frac{-im_f}{v} = -im_f(G_F\sqrt{2})^{1/2}$$

If the Higgs boson did not exist, something else would have to cure divergent behavior

If gauge symmetry were unbroken . . .

- no Higgs boson
- no longitudinal gauge bosons
- no extreme divergences
- no wrong-helicity amplitudes

. . . and no viable low-energy phenomenology

In spontaneously broken theory . . .

- gauge structure of couplings eliminates the most severe divergences
- lesser—but potentially fatal—divergence arises because the electron has mass . . . due to the Higgs mechanism
- SSB provides its own cure—the Higgs boson

Similar interplay & compensation *must exist* in any acceptable theory

EWSB: another path?

Modeled EWSB on Ginzburg–Landau description of superconducting phase transition;

. . . had to introduce new, elementary scalars

GL is not the last word on superconductivity:

dynamical Bardeen–Cooper–Schrieffer theory

The elementary fermions – electrons – and gauge interactions – QED – needed to generate the scalar bound states are already present in the case of superconductivity.

Could a scheme of similar economy account for EWSB?

$\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ + massless u and d

(treat $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ as perturbation)

$m_u = m_d = 0$:
QCD has exact $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ chiral symmetry.

At an energy scale $\sim \Lambda_{\mathrm{QCD}}$, strong interactions become strong, fermion condensates appear, and

$$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$$

$\rightsquigarrow$ 3 Goldstone bosons, one for each broken generator:
3 massless pions (Nambu)

Broken generators: 3 axial currents; couplings to π measured by pion decay constant f_π.

Turn on $\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$: EW gauge bosons couple to axial currents, acquire masses of order $\sim g f_\pi$.

$$\mathcal{M}^2 = \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & gg' \\ 0 & 0 & gg' & g'^2 \end{pmatrix} \frac{f_\pi^2}{4} \quad (W^+, W^-, W_3, \mathcal{A})$$

same structure as standard EW theory.

Diagonalize: $M_W^2 = g^2 f_\pi^2/4$, $M_Z^2 = (g^2 + g'^2) f_\pi^2/4$, $M_A^2 = 0$, so

$$\frac{M_Z^2}{M_W^2} = \frac{(g^2 + g'^2)}{g^2} = \frac{1}{\cos^2\theta_W}$$

Massless pions disappear from physical spectrum, to become longitudinal components of weak bosons. $\boxed{M_W \approx 30\ \mathrm{MeV}/c^2}$ No fermion masses . . .

Origin of fermion masses: quarks & charged leptons

$$\mathcal{L}_{\text{Yukawa}} = -\zeta_d^{ij}(\bar{\mathsf{L}}_i\phi)d_{\mathsf{R}j} - \zeta_u^{ij}(\bar{\mathsf{L}}_i\bar{\phi})u_{\mathsf{R}j} + \text{h.c.} \; ,$$

$\zeta_{u,d}$: 3×3 complex matrices $\qquad$ i, j: generation indices

L_i: LH quark doublets $\qquad$ $u_{\mathsf{R}j}, d_{\mathsf{R}j}$: RH quark singlets

Mass eigenstates: $\zeta_f^{\text{diag}} = U_{\mathsf{L}}^f \zeta_f U_{\mathsf{R}}^{f\dagger}$

$f = u, d$: up-like or down-like quarks; $U_{\mathsf{L,R}}^f$: unitary matrices

Define $\mathbf{u}_{\mathsf{L}} = (u_{\mathsf{L}}, c_{\mathsf{L}}, t_{\mathsf{L}}) \qquad \mathbf{d}_{\mathsf{L}} = (d_{\mathsf{L}}, s_{\mathsf{L}}, b_{\mathsf{L}})$

$$\mathcal{L}_{\mathsf{CC}}^{(q)} = -\frac{g}{\sqrt{2}}\bar{\mathbf{u}}_{\mathsf{L}}\,\gamma^{\mu} V \mathbf{d}_{\mathsf{L}} W_{\mu}^{+} + \text{ h.c.}$$

V: quark mixing (Cabibbo–Kobayashi–Maskawa) matrix

$$V \equiv U_{\mathrm{L}}^{u} U_{\mathrm{L}}^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

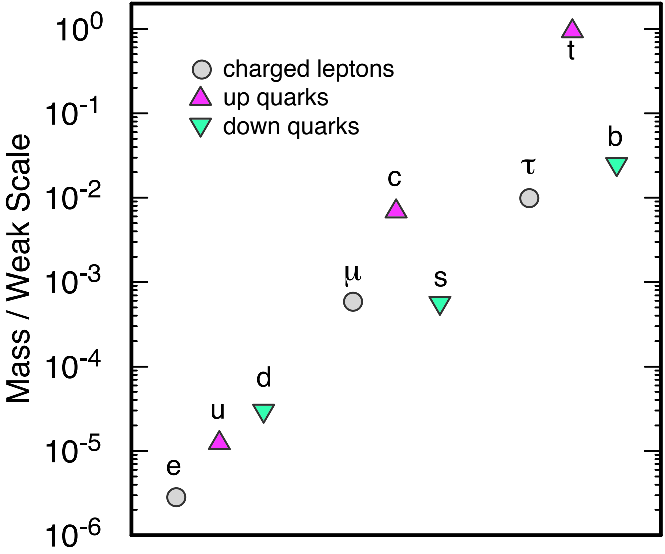
Experiment tells us ...

$$|V| \equiv \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.974 & 0.227 & 0.004 \\ 0.227 & 0.973 & 0.042 \\ 0.008 & 0.042 & 0.999 \end{pmatrix}$$

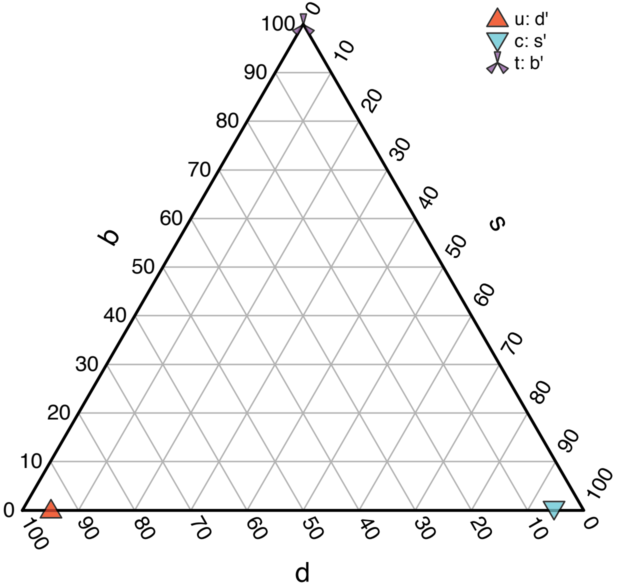
The Higgs scalar is the only element of the standard model that distinguishes among the generations.

Veltman: *It knows something we do not know.*

Yukawa couplings (mass eigenstates) ζ_f^{diag}



Quark Mixing



Components: $|V_{u\alpha}|^2$, etc.

Origin of fermion masses: neutrinos

Define $\boldsymbol{\nu} = (\nu_1, \nu_2, \nu_3) \quad \boldsymbol{\ell}_\mathsf{L} = (e_\mathsf{L}, \mu_\mathsf{L}, \tau_\mathsf{L})$

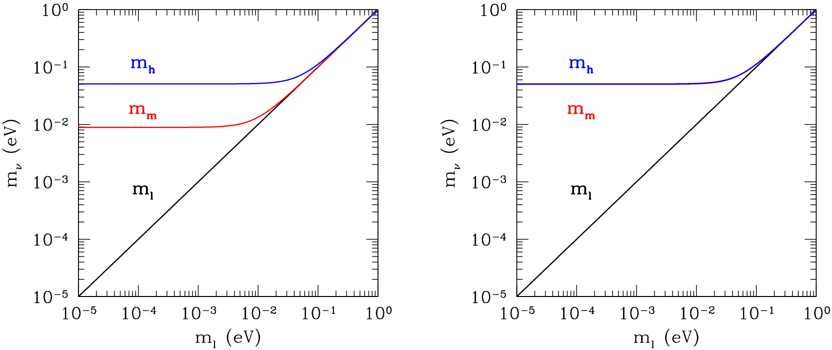
$$\mathcal{L}^{(q)}_{\mathsf{CC}} = -\frac{g}{\sqrt{2}}\,\bar{\boldsymbol{\nu}}\,\gamma^\mu \mathcal{V}^\dagger \boldsymbol{\ell}_\mathsf{L} W^+_\mu + \text{ h.c. },$$

$\mathcal{V}$: ν mixing (Pontecorvo–Maki–Nakagawa–Sakata) matrix

$$\mathcal{V} = \begin{pmatrix} \mathcal{V}_{e1} & \mathcal{V}_{e2} & \mathcal{V}_{e3} \\ \mathcal{V}_{\mu 1} & \mathcal{V}_{\mu 2} & \mathcal{V}_{\mu 3} \\ \mathcal{V}_{\tau 1} & \mathcal{V}_{\tau 2} & \mathcal{V}_{\tau 3} \end{pmatrix}$$

Convention: ν_1, ν_2: solar pair, $m_1 < m_2$
ν_3 separated by $\Delta m^2_{\mathsf{atm}}$; above or below?

Absolute scale of neutrino masses is not yet known



Normal spectrum   Inverted spectrum

$m_2^2 - m_1^2 = \Delta m_\odot^2 = 7.9 \times 10^{-5}\ \mathrm{eV}^2 \quad m_3^2 - m_1^2 = \Delta m_{\mathrm{atm}}^2 = 2.5 \times 10^{-3}\ \mathrm{eV}^2$

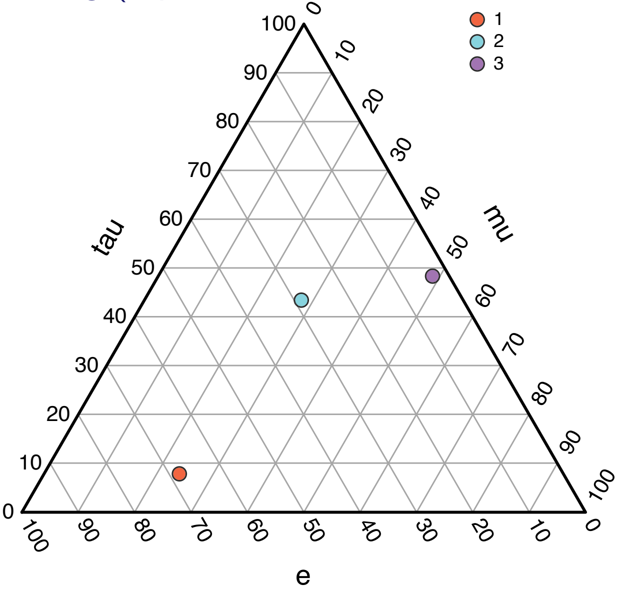
Astronomical data: $\sum_i m_{\nu_i} < 0.62$ eV

If Dirac masses, Yukawa couplings $\lesssim 10^{-11}$

Experiment tells us . . .

$$|\mathcal{V}| = \begin{pmatrix} 0.79-0.88 & 0.47-0.61 & <0.20 \\ 0.19-0.52 & 0.42-0.73 & 0.58-0.82 \\ 0.20-0.53 & 0.44-0.74 & 0.56-0.81 \end{pmatrix}$$

Neutrino Mixing (representative values, $\theta_{13} = 10^\circ$)



Application to cosmic ν: Barenboim & CQ, *Phys. Rev. D* **67**, 073024 (2003)

Varieties of neutrino mass: Dirac mass

Chiral decomposition of Dirac spinor:

$$\psi = \tfrac{1}{2}(1 - \gamma_5)\psi + \tfrac{1}{2}(1 + \gamma_5)\psi \equiv \psi_\mathsf{L} + \psi_\mathsf{R}$$

Dirac mass connects LH, RH components of *same field*

$$\mathcal{L}_D = -D(\bar{\psi}_\mathsf{L}\psi_\mathsf{R} + \bar{\psi}_\mathsf{R}\psi_\mathsf{L}) = -D\bar{\psi}\psi$$

$\Longrightarrow$ mass eigenstate $\psi = \psi_\mathsf{L} + \psi_\mathsf{R}$

(invariant under global phase rotation $\nu \to e^{i\theta}\nu$, $\ell \to e^{i\theta}\ell$, so that lepton number is conserved)

Varieties of neutrino mass: Dirac mass

Add RH neutrino N_R to the standard-model spectrum

N_R: $\mathrm{SU(2)_L}$ singlet with $Y = 0$, so sterile

$$\mathcal{L}_\mathrm{D}^{(\nu)} = -\zeta_\nu \left[(\bar{\mathsf{L}}_\ell \bar{\phi}) N_\mathrm{R} + \bar{N}_\mathrm{R} (\bar{\phi}^\dagger \mathsf{L}_\ell) \right] \to -m_\mathrm{D} \left(\bar{\nu}_\mathrm{L} N_\mathrm{R} + \bar{N}_\mathrm{R} \nu_\mathrm{L} \right)$$

$$m_\mathrm{D} = \zeta_\nu v / \sqrt{2}$$

Some argue that $\zeta_\nu \lesssim 10^{-11}$ is unnatural, while the range $\zeta_t \approx 1$ to $\zeta_e \approx$ few $\times 10^{-6}$ is merely puzzling. All Dirac masses involve physics beyond the standard model.

Varieties of neutrino mass: Majorana mass

Neutrinos: no color, no Q: their own antiparticles?

Majorana[2] fermions

Charge conjugate of RH field is LH: $\psi_{\mathsf{L}}^{c} \equiv (\psi^{c})_{\mathsf{L}} = (\psi_{\mathsf{R}})^{c}$

Majorana joins LH, RH components of *conjugate fields*

$$
\begin{aligned}
-\mathcal{L}_{\mathsf{MA}} &= A(\bar{\nu}_{\mathsf{R}}^{c}\nu_{\mathsf{L}} + \bar{\nu}_{\mathsf{L}}\nu_{\mathsf{R}}^{c}) = A\bar{\chi}\chi \\
-\mathcal{L}_{\mathsf{MB}} &= B(\bar{\nu}_{\mathsf{L}}^{c}\nu_{\mathsf{R}} + \bar{\nu}_{\mathsf{R}}\nu_{\mathsf{L}}^{c}) = B\bar{\omega}\omega
\end{aligned}
$$

for which the mass eigenstates are

$$
\begin{aligned}
\chi &\equiv \nu_{\mathsf{L}} + \nu_{\mathsf{R}}^{c} = \chi^{c} = \nu_{\mathsf{L}} + (\nu_{\mathsf{L}})^{c} \\
\omega &\equiv \nu_{\mathsf{R}} + \nu_{\mathsf{L}}^{c} = \omega^{c} = \nu_{\mathsf{R}} + (\nu_{\mathsf{R}})^{c}
\end{aligned}
$$

[2] Escapist Literature: Ettore Majorana vanished without a trace from a ferry between Sicily and the Italian mainland. Leonardo Sciascia has written a fictional account that Majoranas colleagues denounced as scurrilous fantasy. Italian original: *La Scomparsa di Majorana*, English translation: *The Moro Affair and the Mystery of Majorana.*

Lepton number violation

Majorana ν: no conserved additive quantum number

$\mathcal{L}_{\mathsf{M}}$ violates lepton number by two units

$\Rightarrow$ Majorana ν can mediate $\beta\beta_{0\nu}$ decays

$$(Z, A) \rightarrow (Z + 2, A) + e^- + e^-$$

Detecting $\beta\beta_{0\nu}$ would offer decisive evidence for Majorana nature of ν

Active ν_{L} mass generated by $I = 1$ Higgs with vev or effective operator containing two $I = \frac{1}{2}$ Higgs combined to transform as $I = 1$.

Assessment

$\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$: 25 years of confirmations

- neutral currents;
- $W^\pm$, Z^0
- charm

(+ experimental guidance)

- τ, ν_τ
- b, t

+ experimental surprises

- narrowness of ψ, ψ'
- long B lifetime; large B^0–$\bar{B}^0$ mixing
- large B^0–$\bar{B}^0$ mixing
- heavy top
- neutrino oscillations

10 years precise measurements: no significant deviations

Quantum corrections tested at $\pm 10^{-3}$

No "new physics" ... yet!

Theory tested at distances from 10^{-17} cm to $\sim 10^{22}$ cm

origin	Coulomb's law (tabletop experiments)
smaller	$\begin{cases} \text{Atomic physics} \rightarrow \text{QED} \\ \text{high-energy expts.} \rightarrow \text{EW theory} \end{cases}$
larger	$M_\gamma \approx 0$ in planetary ... measurements

Is EW theory true? Is it complete ??

Challenge: Understanding the Everyday (bis)

What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?

Consider the effects of all the $\mathrm{SU}(3)_{\mathrm{c}} \otimes \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_{Y}$ *interactions!*

With no Higgs mechanism . . .

- Quarks and leptons would remain massless
- QCD would confine the quarks in color-singlet hadrons
- *N mass little changed,* but p outweighs n
- QCD breaks EW to EM, gives $(1/2500\times$observed$)$ masses to W, Z, so weak-isospin force doesn't confine
- Rapid! β-decay $\Rightarrow$ lightest nucleus is n; no H atom
- Some light elements in BBN (?), but ∞ Bohr radius
- No atoms (as we know them) means no chemistry, no stable composite structures like solids and liquids

. . . the character of the physical world would be profoundly changed

Parameters of the Standard Model

3	coupling parameters $\alpha_s, \alpha_{\mathrm{EM}}, \sin^2\theta_W$
2	parameters of the Higgs potential
1	vacuum phase (QCD)
6	quark masses
3	quark mixing angles
1	CP-violating phase
3	charged-lepton masses
3	neutrino masses
3	leptonic mixing angles
1	leptonic CP-violating phase (+ Majorana . . .)
26^+	arbitrary parameters

parameter count not improved by unification

The EW scale and beyond

EWSB scale, $v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246$ GeV, sets

$$M_W^2 = g^2 v^2/2 \qquad M_Z^2 = M_W^2/\cos^2\theta_W$$

But it is not the only scale of physical interest

quasi-certain: $M_{\text{Planck}} = 1.22 \times 10^{19}$ GeV

probable: $\mathrm{SU(3)_c} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$ unification scale $\sim 10^{15-16}$ GeV

somewhere: flavor scale

How to keep the distant scales from mixing in the face of quantum corrections?

OR

How to stabilize the mass of the Higgs boson on the electroweak scale?

OR

Why is the electroweak scale small?

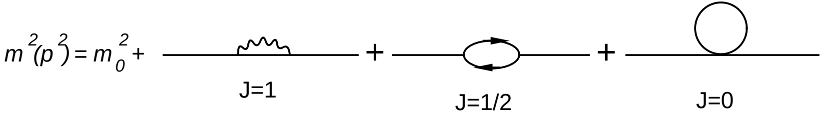
$\boxed{\text{“The hierarchy problem”}}$

Higgs potential $V(\phi^\dagger\phi) = \mu^2(\phi^\dagger\phi) + |\lambda|\,(\phi^\dagger\phi)^2$

$\mu^2 < 0$: $\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y \to U(1)_{\mathrm{em}}$, as

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ \sqrt{-\mu^2/2|\lambda|} \end{pmatrix} \equiv \begin{pmatrix} 0 \\ \underbrace{(G_F\sqrt{8})^{-1/2}}_{175\ \mathrm{GeV}} \end{pmatrix}$$

Beyond classical approximation, quantum corrections to scalar mass parameters:



Loop integrals are potentially divergent

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \cdots$$

Λ: reference scale at which m^2 is known

g: coupling constant of the theory

C: coefficient calculable in specific theory

For mass shifts induced by radiative corrections to remain under control (not greatly exceed the value measured on the laboratory scale), *either*

- Λ must be small, *or*
- New Physics must intervene to cut off integral

But natural reference scale for Λ is

$$\Lambda \approx M_{\mathrm{Planck}} = \left(\frac{\hbar c}{G_{\mathrm{Newton}}}\right)^{1/2} \approx 1.22 \times 10^{19}\ \mathrm{GeV}$$

for $\mathrm{SU(3)_c} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$

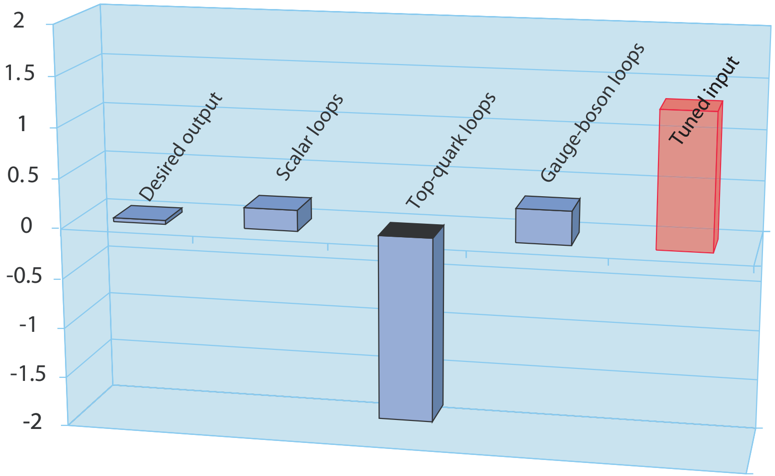
or

$$\Lambda \approx M_U \approx 10^{15}\text{-}10^{16}\ \mathrm{GeV}$$ for unified theory

Both $\gg v/\sqrt{2} \approx 175$ GeV $\Longrightarrow$

$\boxed{\text{New Physics at } E \lesssim 1 \text{ TeV}}$

δM_H^2 for 5-TeV cutoff



$$\delta M_H^2 = \frac{G_F \Lambda^2}{4\pi^2\sqrt{2}}(6M_W^2 + 3M_Z^2 + M_H^2 - 12m_t^2)$$

Only a few distinct scenarios . . .

- Supersymmetry: balance contributions of fermion loops (-1) and boson loops $(+1)$
  Exact supersymmetry,

$$\sum_{i=\substack{\text{fermions}\\+\text{bosons}}} C_i \int dk^2 = 0$$

  Broken supersymmetry, shifts acceptably small if superpartner mass splittings are not too large

$$g^2 \Delta M^2 \text{ “small enough”} \Rightarrow \widetilde{M} \lesssim 1 \text{ TeV}/c^2$$

Only a few distinct scenarios . . .

- Composite scalars (technicolor): New physics arises on scale of composite Higgs-boson binding,

$$\Lambda_{\mathrm{TC}} \simeq O(1\ \mathrm{TeV})$$

"Form factor" cuts effective range of integration

Only a few distinct scenarios . . .

- Strongly interacting gauge sector: WW resonances, multiple W production, probably scalar bound state "quasiHiggs" with $M < 1$ TeV
- Spontaneously broken global symmetries $\rightsquigarrow$ pseudo-Nambu–Goldstone bosons
  - Higgs boson would be massless if Nambu–Goldstone boson protected against large quantum corrections
  - Global symmetry must be large enough that left-over NGBs remain after some provide longitudinal components of massive gauge bosons
  - Explicit symmetry breaking needed for $M_H \neq 0$
    Extra gauge bosons ($M_{W'} \approx 4\pi M_H$), enlarged fermion multiplets cancel quadratic divergences in δM_H^2

"Little Higgs" effective theories, $\Lambda \approx (4\pi)^2 M_H$

Only a few distinct scenarios . . .

- Or maybe the problem is with (our understanding of) *gravity*, not with the electroweak theory?

Large extra dimensions

Arkani-Hamed, Dimopoulos, Dvali, *Phys. Lett.* **429**, 263 (1998)

Universal extra dimensions

Appelquist, Cheng, Dobrescu, *Phys. Rev. D***64**, 035002 (2001)

Warped extra dimensions

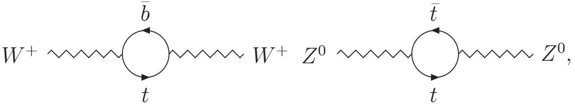
Randall & Sundrum, *Phys. Rev. Lett.* **83**, 3370, 4690 (1999)

Higgs boson: the missing element of electroweak theory

Experimental clues to the Higgs-boson mass

Sensitivity of EW observables to m_t gave early indications for massive top

Quantum corrections to SM predictions for M_W and M_Z arise from different quark loops



$$\ldots\text{alter the link}\quad \underbrace{M_W^2}_{(80.398 \pm 0.025\ \text{GeV})^2} \;=\; \underbrace{M_Z^2\left(1-\sin^2\theta_W\right)}_{(80.939\ \text{GeV})^2}(1-\Delta\rho)$$

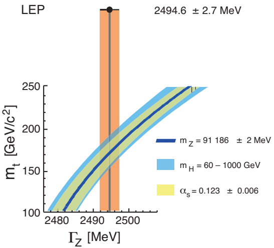
where $\Delta\rho \approx \Delta\rho^{(\text{quarks})} = 3G_F m_t^2/8\pi^2\sqrt{2}$

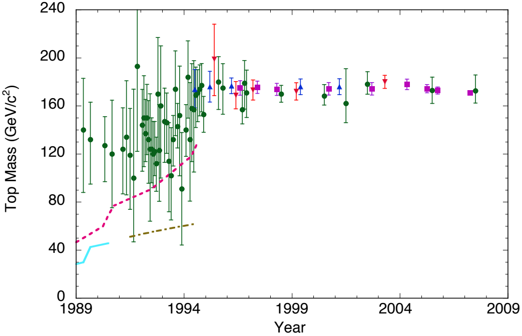
Strong dependence on m_t^2 accounts for precision of m_t estimates derived from EW observables

Tevatron: $\delta m_t/m_t \approx 1.28\%$... Look beyond quark loops to next most important quantum corrections: $\boxed{\textit{Higgs-boson effects}}$

Global fits to precision EW measurements

- precision improves with time / calculations improve with time





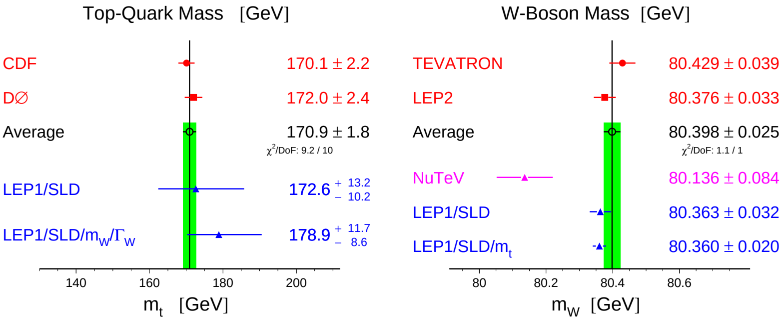
11.94, LEPEWWG: $m_t = 178 \pm 11^{+18}_{-19}$ GeV/c^2

Direct measurements: $m_t = 170.9 \pm 1.8$ GeV/c^2

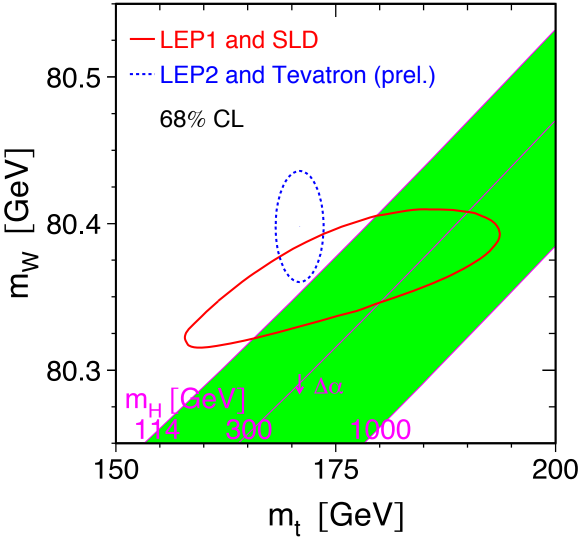
H quantum corrections smaller than t corrections, exhibit more subtle dependence on M_H than the m_t^2 dependence of the top-quark corrections

$$\Delta\rho^{(\text{Higgs})} = \mathcal{C} \cdot \ln\left(\frac{M_H}{v}\right)$$

M_Z known to 23 ppm, m_t and M_W well measured



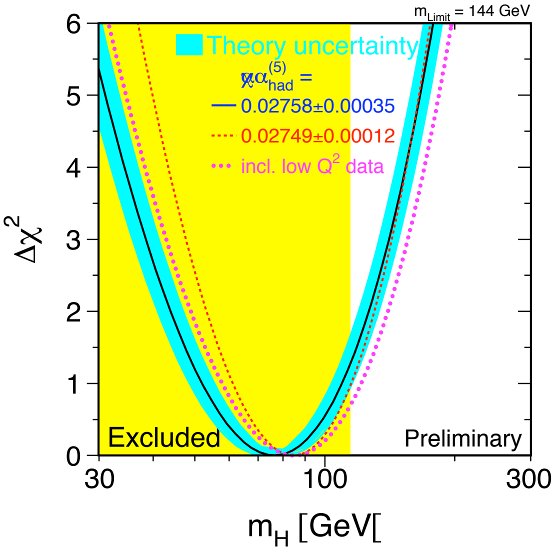
…so examine dependence of M_W upon m_t and M_H



Direct, indirect determinations agree reasonably

Both favor a light Higgs boson, . . . *within framework of SM analysis.*

Fit to a universe of data



Standard-Model $M_H \lesssim 182$ GeV at 95% CL

Fit to a universe of data . . .

- *Within SM,* LEP EWWG deduce a 95% CL upper limit, $M_H \lesssim 182$ GeV/c^2
- Direct searches at LEP $\Rightarrow M_H > 114.4$ GeV/c^2, excluding much of the favored region
- Either the Higgs boson is just around the corner, or SM analysis is misleading

Things will soon be popping!

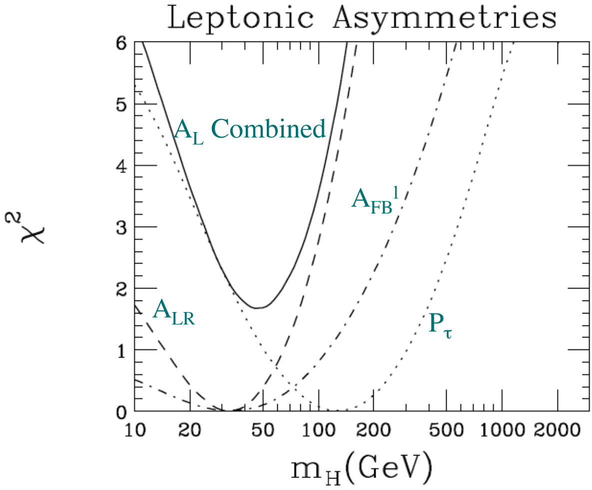
A Cautionary Note

- A_{FB}^{b}, which exerts the greatest "pull" on the global fit [slide 83], is most responsible for raising M_H above the range excluded by direct searches [slide 122].
- Leptonic and hadronic observables point to different best-fit values of M_H
- Many subtleties in experimental and theoretical analyses

M. Chanowitz, *Phys. Rev. Lett.* **87**, 231802 (2001); *Phys. Rev. D***66**, 073002 (2002); hep-ph/0304199;
http://phyweb.lbl.gov/∼chanowitz/rpm-10-06.pdf

Introduction to global analyses: J. L. Rosner, hep-ph/0108195; hep-ph/0206176

χ^2 Distributions: Leptonic Asymmetries



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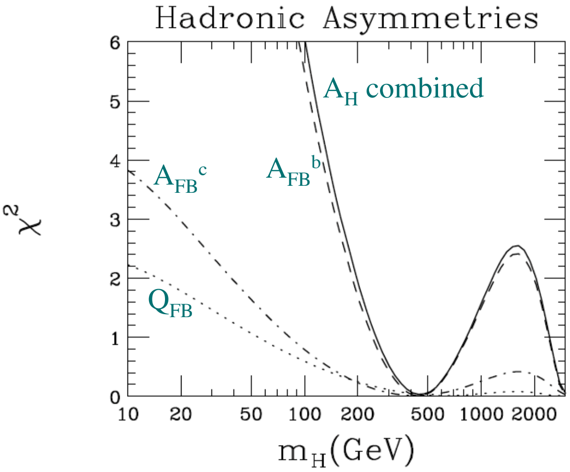
χ^2 Distributions: Hadronic Asymmetries



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- Tevatron, LHC measurements will determine m_t within 1 or 2 GeV
  ... and improve δM_W to about 15 MeV
- As the Tevatron's integrated luminosity approaches 10 fb^{-1}, CDF and DØ will explore the region of M_H not excluded by LEP
- ATLAS and CMS will carry on the exploration of the Higgs sector at the LHC;
  could require a few years, at low mass;
  full range accessible, $\gamma\gamma, \ell\ell\nu\nu, b\bar{b}, \ell^+\ell^-\ell^+\ell^-, \ell\nu jj, \tau\tau$ channels.

Electroweak theory confronts experiment

- G. Altarelli and M. Grünewald, "Precision Electroweak Tests of the SM," hep-ph/0404165.
- F. Teubert, "Precision tests of the electroweak interactions," *Int. J. Mod. Phys. A* **20**, 5174 (2005).
- S. de Jong, "Tests of the Electroweak Sector of the Standard Model," *PoS* **HEP2005**, 397 (2006) [hep-ph/0512043].

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Details of Electroweak Theory . . .

For more, see my

- " Spontaneous symmetry breaking as a basis of particle mass," *Rep. Prog. Phys.* **70,** 1019 (2007), arXiv:0704.2232.
- "The Electroweak Theory," in *Flavor Physics for the Millennium: TASI 2000,* edited by Jonathan L. Rosner (World Scientific, Singapore, 2001), pp. 367; hep-ph/0204104.
- *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions* (Westview Press, 1997)

  http://www.perseusbooksgroup.com/perseus/book_detail.jsp?isbn=0201328321

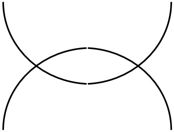
Bounding M_H from above . . .

Triviality of scalar field theory

- Only *noninteracting* scalar field theories make sense on all energy scales
- Quantum field theory vacuum is a dielectric medium that screens charge
- $\Rightarrow$ *effective charge* is a function of the distance or, equivalently, of the energy scale

running coupling constant

In $\lambda\phi^4$ theory, calculate variation of coupling constant λ in perturbation theory by summing bubble graphs



$\lambda(\mu)$ is related to a higher scale Λ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log\left(\Lambda/\mu\right)$$

(Perturbation theory reliable only when λ is small, lattice field theory treats strong-coupling regime)

For stable Higgs potential (*i.e.*, for vacuum energy not to race off to $-\infty$), *require* $\lambda(\Lambda) \geq 0$

Rewrite RGE as an inequality

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log\left(\Lambda/\mu\right)$$

... implies an *upper bound*

$$\boxed{\lambda(\mu) \leq 2\pi^2/3 \log\left(\Lambda/\mu\right)}$$

If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit $\Lambda \to \infty$ while holding μ fixed at some reasonable physical scale. In this limit, the bound forces $\lambda(\mu)$ to zero.

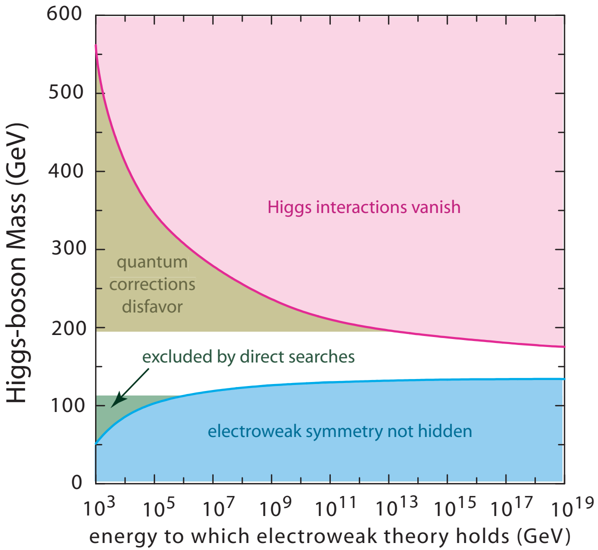
$\longrightarrow$ free field theory "trivial"

Rewrite as bound on M_H:

$$\Lambda \leq \mu \exp\left(\frac{2\pi^2}{3\lambda(\mu)}\right)$$

Choose $\mu = M_H$, and recall $M_H^2 = 2\lambda(M_H)v^2$

$$\boxed{\Lambda \leq M_H \exp\left(4\pi^2 v^2 / 3M_H^2\right)}$$


Higgs interactions vanish
quantum corrections disfavor
excluded by direct searches
electroweak symmetry not hidden
Higgs-boson Mass (GeV)
600
500
400
300
200
100
0
10^3 10^5 10^7 10^9 10^{11} 10^{13} 10^{15} 10^{17} 10^{19}
energy to which electroweak theory holds (GeV)

Moral: For any M_H, there is a *maximum energy scale* $\Lambda^\star$ at which the theory ceases to make sense.

The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies

Perturbative analysis breaks down when $M_H \to 1\ \mathrm{TeV}/c^2$ and interactions become strong

Lattice analyses $\Longrightarrow M_H \lesssim 710 \pm 60\ \mathrm{GeV}/c^2$ if theory describes physics to a few percent up to a few TeV

$\boxed{\text{If } M_H \to 1 \text{ TeV EW theory lives on brink of instability}}$

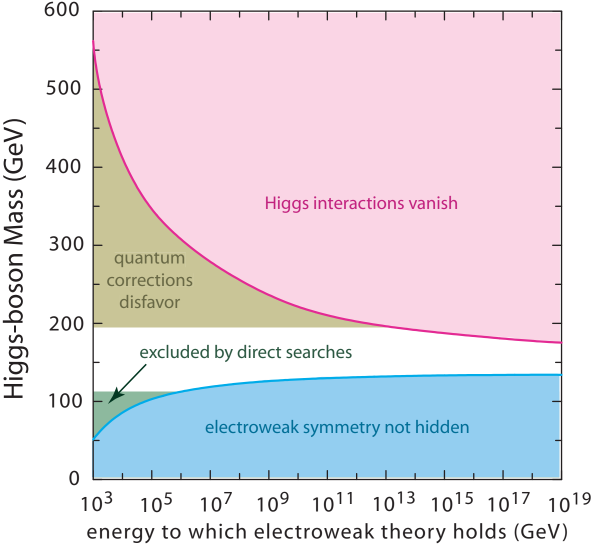
Lower bound by requiring EWSB vacuum $V(v) < V(0)$

Requiring that $\langle \phi \rangle_0 \neq 0$ be an absolute minimum of the one-loop potential up to a scale Λ yields the vacuum-stability condition . . . (for $m_t \lesssim M_W$)

$$M_H^2 > \frac{3G_F\sqrt{2}}{8\pi^2}(2M_W^4 + M_Z^4 - 4m_t^4)\log(\Lambda^2/v^2)$$

(No illuminating analytic form for heavy m_t)

If the Higgs boson is relatively light (which would require explanation) then the theory can be self-consistent up to very high energies



If EW theory is to make sense all the way up to a unification scale $\Lambda^\star = 10^{16}$ GeV, then $134\ \mathrm{GeV}/c^2 \lesssim M_H \lesssim 177$ GeV

Higgs-Boson Properties

$$\Gamma(H \to f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

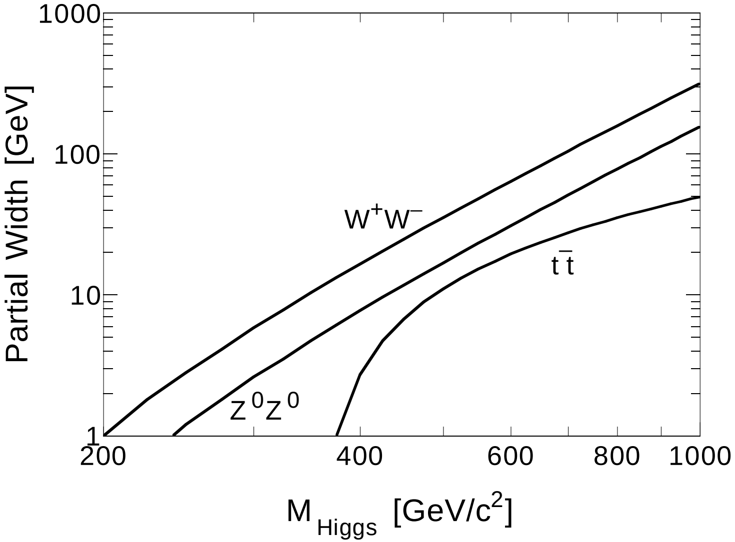
$\propto M_H$ in the limit of large Higgs mass; $\quad \propto \beta^3$ for scalar

$$\Gamma(H \to W^+W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}}(1-x)^{1/2}(4 - 4x + 3x^2) \quad x \equiv 4M_W^2/M_H^2$$

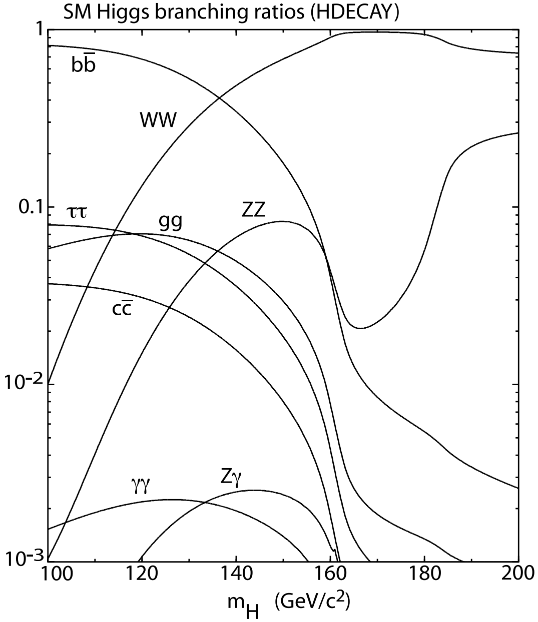
$$\Gamma(H \to Z^0Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}}(1-x')^{1/2}(4 - 4x' + 3x'^2) \quad x' \equiv 4M_Z^2/M_H^2$$

asymptotically $\propto M_H^3$ and $\frac{1}{2}M_H^3$, respectively $\qquad$ ($\frac{1}{2}$ from weak isospin)

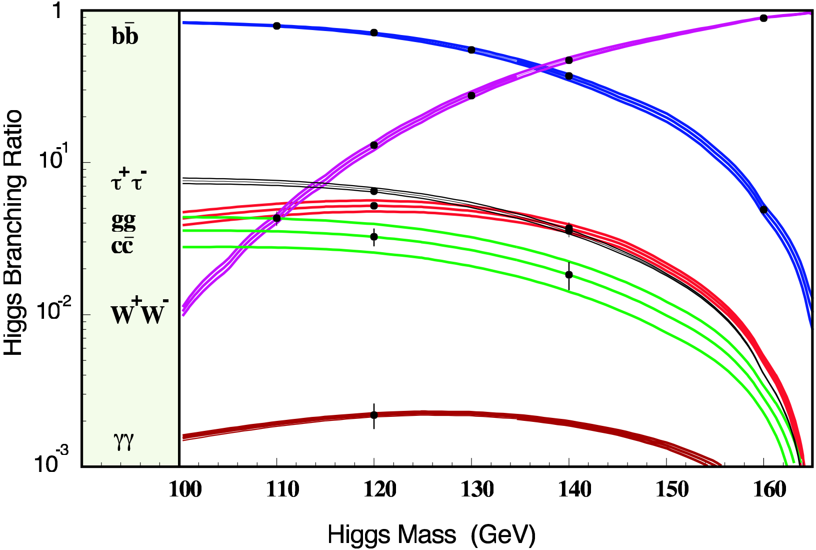
$2x^2$ and $2x'^2$ terms $\Leftrightarrow$ decays into transverse gauge bosons
Dominant decays for large M_H: pairs of longitudinal weak bosons



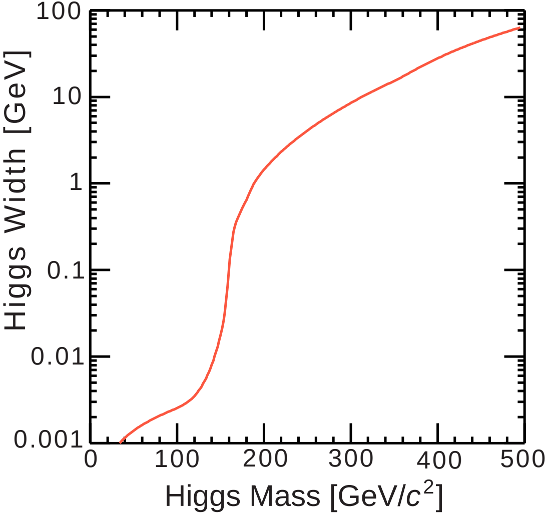
For $M_H \to 1$ TeV, Higgs boson is *ephemeral:* $\Gamma_H \to M_H$.


SM Higgs branching ratios (HDECAY)
1
0.1
10-2
10-3
bb̄
WW
ττ
gg
ZZ
cc̄
γγ
Zγ
100
120
140
160
180
200
mH (GeV/c2)

ILC would measure light Higgs-boson couplings precisely



Points: 500 fb^{-1} @ 350 GeV   Bands: theory uncertainty (m_b)



Below W^+W^- threshold, $\Gamma_H \lesssim 1$ GeV

Far above W^+W^- threshold, $\Gamma_H \propto M_H^3$