

Gauge Theories & Particle Physics

Physics 539

Spring Semester 1997

Chris Quigg

February 3, 1997

Principal Text: C. Quigg, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions* (Addison-Wesley, Reading, Massachusetts, 1983).

Recommended: I. J. R. Aitchison and A. J. G. Hey, *Gauge Theories in Particle Physics: A Practical Introduction* second edition (Adam Hilger, Bristol, 1989).

L. B. Okun, *Particle Physics: The Quest for the Substance of Substance* (Harwood Academic Press, New York, 1985).

Field Theory Basics: F. Mandl and G. Shaw, *Quantum Field Theory* (Wiley, New York, 1984).

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Recommended Reading:

Okun, Chapter 1.

Gauge Theories, Chapters 1 and 2.

E. L. Hill, *Rev. Mod. Phys.* **23**, 253 (1953).

Cultural Reading:

C. Quigg, "Elementary Particles and Forces," *Sci. Am.* **252**, (April, 1985), p. 84.

H. Harari, "The Structure of Quarks and Leptons," *Sci. Am.* **248**, (April, 1983), p. 56.

J. R. Rees, "The Stanford Linear Collider," *Sci. Am.* **261**, (October, 1989), p. 58.

S. Myers and E. Picasso, "The LEP Collider," *Sci. Am.* **263**, (July, 1990), p. 54.

L. M. Lederman, "The Tevatron," *Sci. Am.* **264**, (March, 1991), p. 48.

J. Huth, "High Energy Physics — The Road Ahead," *Am. Sci.* **82**, 326 (1994)

B. Carithers and P. Grannis, "Discovery of the Top Quark," *SLAC Beam Line* **25**, No. 3, p. 4 (Fall 1995). The entire issue is available in PDF format at

<http://www.slac.stanford.edu/publications/beamline/95iii.pdf>.

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If you do not already own the 1996 *Review of Particle Physics* [*Phys. Rev. D***54**, 1 (1996)] and the *Particle Physics Booklet*, send an e-mail request to pdg@lbl.gov. You may instead write to Particle Data Group, MS 50-308, Lawrence Berkeley Laboratory, Berkeley, CA 94720, to request a copy. There is no charge. The PDG's WWW server is at <http://www-pdg.lbl.gov>.

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Course information, including class schedules and assignments, will be available on the World Wide Web. Point your web client to <http://lutece.fnal.gov/Princeton/>.

Problems (due February 13, 1997)

1. Consider bound states composed of a b -quark and a \bar{b} -antiquark. For $(b\bar{b})$ composites,
(a) Show that a bound state with orbital angular momentum L must have quantum numbers

$$C = (-1)^{L+S} \quad P = (-1)^{L+1},$$

where S is the spin of the composite system.

- (b) Allowing for both orbital and radial excitations, construct a schematic mass spectrum of $(b\bar{b})$ bound states. Label each state with its quantum numbers J^{PC} .

2. Consider bound states composed of color-triplet scalars, or squarks (denoted \tilde{b}) and anti-squarks.

(a) Show that a $(\tilde{b}\tilde{b})$ bound state with angular momentum L (i.e., an orbital excitation) must have quantum numbers

$$C = (-1)^L; \quad P = (-1)^L.$$

(b) Allowing for both orbital and radial excitations, construct a schematic mass spectrum of $(\tilde{b}\tilde{b})$ bound states. Label each state with its quantum numbers J^{PC} . How does the spectrum of squarkonium differ from that of quarkonium?

3. (a) Using the Feynman rules given in Appendix B.5 of *Gauge Theories*, compute the differential cross section $d\sigma/d\Omega$ and the total (integrated) cross section $\sigma \equiv \int d\Omega(d\sigma/d\Omega)$ for the reaction $e^+e^- \rightarrow \mu^+\mu^-$. Work in the center of momentum frame, and in the high-energy limit (where all the masses may be neglected). Assume that the colliding beams are unpolarized, and sum over the spins of the produced muons.

(b) Look up the original evidence for quark-antiquark jets in the inclusive reaction $e^+e^- \rightarrow$ hadrons [G. J. Hanson *et al.*, *Phys. Rev. Lett.* **35**, 1609 (1975)]. Now recompute the differential cross section for the reaction $e^+e^- \rightarrow \mu^+\mu^-$, assuming the initial beams to be transversely polarized. See also R. F. Schwitters *et al.*, *ibid.* **35**, 1320 (1975).

4. Define the requirements for an experiment to measure the gyromagnetic ratio of the tau lepton, taking into account the τ -lifetime and the anticipated result $g_\tau \approx 2$. For background, become acquainted with the methods used to measure the magnetic anomalies of the electron [A. Rich and J. Wesley, *Rev. Mod. Phys.* **44**, 250 (1972); R. S. Van Dyck, Jr., P. B. Schwinberg, and H. G. Dehmelt, *Phys. Rev. Lett.* **38**, 93 (1977); R. S. Van Dyck, Jr., "Anomalous Magnetic Moment of Single Electrons and Positrons: Experiment," in *Quantum Electrodynamics*, edited by T. Kinoshita (World Scientific, Singapore, 1990), p. 322.] and muon [F. Combley, F. J. M. Farley, and E. Picasso, *Phys. Rep.* **68**, 93 (1981); F. J. M. Farley, and E. Picasso, "The Muon $g-2$ Experiments," in Kinoshita's *QED*, p. 479], and the magnetic dipole moments of the nucleons [N. F. Ramsey, *Molecular Beams*, Oxford University Press, Oxford, 1956] and unstable hyperons [L. Schachinger *et al.*, *Phys. Rev. Lett.* **41**, 1348 (1978); L. G. Pondrom, *Phys. Rep.* **122**, 57 (1985). For an interesting new technique, see D. Chen *et al.*, *Phys. Rev. Lett.* **69**, 3286 (1992), and V. V. Baublis, *et al.*, *Nucl. Inst. Meth.* **B90**, 150 (1994)]. Indirect determinations of $(g-2)_\tau$ are discussed by R. Escribano and E. Massó, "Improved Bounds on the Electromagnetic Dipole Moments of the Tau Lepton," (bulletin board: hep-ph/9609423); G. Köpp, D. Schaile, M. Spira, and P. Zerwas, *Z. Phys.* **C65**, 545 (1995); M. A. Samuel and G. Li, *Int. J. Theor. Phys.* **33**, 1471 (1994).

5. For a free particle moving in one dimension, the Lagrangian is $\mathcal{L} = \frac{1}{2}m\dot{x}^2$. Show that the action S corresponding to the classical motion of a free particle from position $x_1 = x(t_1)$ to position $x_2 = x(t_2)$ is

$$S = \frac{m(x_2 - x_1)^2}{2(t_2 - t_1)}.$$

6. For a harmonic oscillator with Lagrangian $\mathcal{L} = \frac{m}{2}(\dot{x}^2 - \omega^2 x^2)$, show that the classical action for a particle that moves from position x_1 at initial time t_1 to position x_2 at final time t_2 is

$$S = \frac{m\omega}{2\sin\omega T} [(x_1^2 + x_2^2) \cos\omega T - 2x_1x_2],$$

where $T \equiv t_2 - t_1$.

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Recommended Reading:

Okun, Chapter 2.

Gauge Theories, Chapter 3.

Howard Georgi, *Weak Interactions and Modern Particle Theory*, (Benjamin, Menlo Park, California, 1984), Chapter 1: "Symmetries in Field Theory."

On the continuation to imaginary time: J. Schwinger, *Proc. Nat. Acad. Sci.* **44**, 956 (1958).

Cultural Reading:

Akira Tonomura, "Electron Holography: A New View of the Microscopic," *Phys. Today* **43**, 22 (April, 1990).

Robert Mills, "Gauge Fields," *Am. J. Phys.* **57**, 493 (1989).

On geometrical phases: M. V. Berry, "Anticipations of the Geometric Phase," *Phys. Today* **43**, 34 (December, 1990).

Deep Background:

T. P. Cheng and L.-F. Li, "Resource Letter: GI-1 Gauge Invariance," *Am. J. Phys.* **56**, 586 (1988).

Murray Peshkin and Akira Tonomura, *The Aharonov-Bohm Effect*, (Springer-Verlag, New York, 1989).

Quantum Coherence, Proceedings of the International Conference on Fundamental Aspects of Quantum Theory to Celebrate Thirty Years of the Aharonov-Bohm Effect, edited by J. S. Anandan (World Scientific, Singapore, 1990).

J. W. Zwanzinger, M. Koenig, and A. Pines, "Berry's Phase," *Ann. Rev. Phys. Chem.* **41**, 601 (1990).

Alfred Shapere and Frank Wilczek, *Geometrical Phases in Physics*, (World Scientific, Singapore, 1989).

Problems (due February 20, 1997)

7. Use the requirement that the Lagrangian be invariant under a continuous symmetry to deduce the conserved quantity that corresponds to a particular transformation. Show that invariance under (i) translations in space, (ii) translations in time, (iii) spatial rotations implies conservation of (i) momentum, (ii) energy, (iii) angular momentum.
8. Making use of the Dirac equation, show that the most general parity-conserving form for the electromagnetic current of the proton is

$$J_\mu \sim \bar{u}(p') [\Gamma_1(q^2)\gamma_\mu + \Gamma_2(q^2)i\sigma_{\mu\nu}q^\nu + \Gamma_3(q^2)q_\mu] u(p),$$

where $\sigma_{\mu\nu} \equiv (i/2) [\gamma_\mu, \gamma_\nu]$ and $q \equiv p - p'$. What are the consequences of current conservation, $\partial^\mu J_\mu = 0$?

9. Calculate the differential cross section in the laboratory frame for elastic electron-proton scattering (a) for a structureless proton (*i.e.*, a Dirac particle); (b) for a real proton (using the results of Problem 8).
10. (a) Consider a nonrelativistic particle with charge q moving along the axis of a cylindrical Faraday cage connected to an external generator, which causes the potential $V(t)$ on the cage to vary with time only when the particle is well within the cage. Show that, if the wave

function $\psi^0(\mathbf{x}, t)$ is a solution of the Schrödinger equation for $V(t) \equiv 0$, the solution when the generator is operating will be $\psi(\mathbf{x}, t) = \psi^0(\mathbf{x}, t)e^{iS/\hbar}$, where $S = -\int^t dt' qV(t')$.

(b) Now suppose that a single coherent beam of charged particles is split into two parts, each of which is allowed to pass through its own long cylindrical cage of the kind just described. On emerging from the Faraday cages the beams are recombined and the resulting interference pattern is observed. The beam is chopped into bunches that are long compared with the wavelength of an individual particle but short compared with the Faraday cages. The potentials on the two cages vary independently, but are nonzero only when a bunch is well within the tubes. This ensures that the beam traverses a time-varying potential without experiencing electric or magnetic forces. Describe how the interference pattern depends upon the applied voltages. [Reference: Y. Aharonov and D. Bohm, *Phys. Rev.* **115**, 485 (1959).]

11. If baryon number is absolutely conserved, the conservation law may be a consequence of a global phase symmetry like that of electromagnetism, with the electric charge replaced by baryon number. (a) How would Newton's law of gravitation be modified if the baryonic phase symmetry were a *local* gauge invariance?

(b) In view of the close equality of inertial and gravitational masses imposed by the Eötvös experiment [P. G. Roll, R. Krotkov, and R. H. Dicke, *Ann. Phys. (NY)* **26**, 442 (1967)], what can be said about the strength of a hypothetical gauge interaction coupled to the baryon current? [Reference: T. D. Lee and C. N. Yang, *Phys. Rev.* **98**, 150 (1955).]

(c) How would a gauge boson γ_B coupled to baryon number affect the properties of the $b\bar{b}$ resonances? For part (c) only, allow for the possibility that γ_B has a nonzero mass. [Reference: D. Bailey and S. Davidson, *Phys. Lett.* **B348**, 185 (1995).]

For another recent examination of the possibility that a light $U(1)$ gauge boson is coupled to baryon number, see C. D. Carone and H. Murayama, *Phys. Rev. Lett.* **74**, 3122 (1995). On the possibility of a massless gauge boson coupled to lepton number, see L. B. Okun, *Yad. Fiz.* **10**, 358 (1969) [English translation: *Sov. J. Nucl. Phys.* **10**, 206 (1969)]. For a critical reexamination of the limits, see S. I. Blinnikov, A. D. Dolgov, L. B. Okun, and M. B. Voloshin, *Nucl. Phys.* **B458**, 52 (1996).

12. Outline a “three-neutrino experiment” to establish that a neutral, penetrating beam of ν_τ materializes into τ upon interacting in matter. [For background, look at the first two-neutrino experiment, J. Danby, *et al.*, *Phys. Rev. Lett.* **9**, 36 (1962). See also the Nobel Lectures of “Murder, Inc.,” (as they called themselves), Mel Schwartz, J. Steinberger, and Leon M. Lederman, reprinted in *Rev. Mod. Phys.* **61**, 527, 533, 547 (1989).] What would provide a copious source of ν_τ ? What energy would be advantageous for the detection of the produced τ ? What characteristics would be required of the detector? What are the important backgrounds, and how would you handle them? Some information about a three-neutrino experiment in preparation at Fermilab can be found at <http://fn872.fnal.gov>.

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February 18, 1997

Recommended Reading:*Gauge Theories*, Chapter 4.On (unsuccessful) gauge theories of the strong interaction: J. J. Sakurai, *Ann. Phys. (NY)* **11**, 1 (1960).

Okun, pp. 19–34.

Aitchison and Hey, Chapter 8.

Cultural Reading:

M. M. Nieto, “Past, Present, and Possible Future Limits on the Photon Rest Mass” (bulletin board: hep-ph/9212283).

On limits to the photon mass: J. D. Barrow and R. R. Burman, “New Light on Heavy Light,” *Nature (London)* **307**, 14 (1984).For a limit on M_γ based on Earth’s magnetic field: E. Fischbach, *et al.*, *Phys. Rev. Lett.* **73**, 514 (1994).On prospects for improving the bound on M_γ by an order of magnitude: V. A. Kostelecký and M. M. Nieto, “Bounding the Photon Mass with Solar Probes,” *Phys. Lett.* **B317**, 223 (1993) (bulletin board: hep-ph/9304250).

Problems (due February 27, 1997)

13. Derive the Yang-Mills Lagrangian for a scalar field theory in which the three real scalar fields correspond to the triplet representation of $SU(2)$. The free-particle Lagrangian is

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu \phi)^2 - m^2 \phi^2],$$

with

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}.$$

14. (a) By making the minimal substitution $\partial_\mu \rightarrow \mathcal{D}_\mu$ in the free-particle Lagrangian, construct a theory of the electrodynamics of a massive spin-one boson V^\pm and deduce the Feynman rules for the theory.
 (b) Now compute the differential cross section $d\sigma/d\Omega$ and the total (integrated) cross section $\sigma \equiv \int d\Omega(d\sigma/d\Omega)$ for the reaction $e^+e^- \rightarrow V^+V^-$, for various helicities of the produced particles. Work in the c.m. frame, and in the high-energy limit (where all the masses may be neglected). Assume the colliding beams are unpolarized. Compare the results with the cross sections for $e^+e^- \rightarrow \mu^+\mu^-$ derived in Problem 3. [Reference for Feynman rules: J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics*, McGraw-Hill, New York, 1964, Appendix B.]

15. Show that the transformations

$$B'_\mu = GB_\mu G^{-1} + \frac{i}{g}(\partial_\mu G)G^{-1} = G \left[B_\mu + \frac{i}{g}G^{-1}(\partial_\mu G) \right] G^{-1}$$

of the gauge fields form a group—that is, under successive transformations on the matter field given by $\psi \rightarrow \psi' = G_1\psi \rightarrow \psi'' = G_2\psi'$ the transformation of the gauge field is characterized by $G = G_2G_1$.

16. Apply the analysis of nucleon-nucleon forces of *Gauge Theories*, §4.3, to the nucleon-antinucleon case. Compare the Yang-Mills “predictions” with the spectrum of nonstrange mesons.

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Typographical errors in the fifth paperbound printing of *Gauge Theories* (identified at the bottom of p. vi by EFGHIJ-MA-89):

$$u_\lambda(p) = \sqrt{E+m} \begin{pmatrix} \chi_\lambda \\ \frac{\lambda|\mathbf{p}|}{E+m}\chi_\lambda \end{pmatrix} \quad (\text{A.4.15})$$

$$\frac{1}{2}(1 \pm \gamma_5)u_\lambda(p) = \frac{1}{2} \left(1 \pm \frac{\lambda|\mathbf{p}|}{E+m} \right) \sqrt{E+m} \begin{pmatrix} \chi_\lambda \\ \pm\chi_\lambda \end{pmatrix} \quad (\text{A.4.20})$$

$$\mathcal{I}_{mn} = (2\pi)^{-4} \int \frac{d^4k (k^2)^{m-2}}{(k^2 - a^2)^n} = \frac{B(m, n-m)}{16\pi^2 i (a^2)^{n-m}} \quad (\text{B.3.12})$$

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February 27, 1997

Recommended Reading:*Gauge Theories*, Chapter 5.M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, Massachusetts, 1995), §11.1 and §20.1.

Okun, Chapter 5.

Aitchison and Hey, Chapter 13.

Cultural Reading:Alan Lightman, *Einstein's Dreams* (Pantheon, New York, 1993).On the analogy between the Ginzburg-Landau picture of the superconducting phase transition and the Higgs mechanism: Robert E. Marshak, *Conceptual Foundations of Modern Particle Physics* (World Scientific, Singapore, 1993), §4.4.S. Weinberg, *The Quantum Theory of Fields*, vol. 2, “Modern Applications” (Cambridge University Press, Cambridge, 1996), §19.1, 19.2, 21.1.

Problems (due March 4, 1997)

17. The two-dimensional Ising model is an instructive cartoon model of a ferromagnet that displays a second-order phase transition—spontaneous magnetization—at a critical (inverse) temperature $\beta_c = 0.4407$. It is easily studied by Monte Carlo simulation. For this exercise, take advantage of Kenji Harada's JAVA applet at <http://fndsys.kuamp.kyoto-u.ac.jp/~harada/Monte-eg.html> or the Ising simulation XISING in Mike Creutz's collection of XTOYS at <http://penguin.phy.bnl.gov/www/xtoys/xtoys.html>.
- (a) Heat the system above the critical temperature and observe the disordered state.
- (b) Cool the system just below the critical temperature to see the formation of domains.
- (c) Cool the system further and watch the evolution of the spontaneous magnetization.
- (d) Beginning from a zero-temperature (ordered) configuration, heat the system and observe the formation of small bubbles of flipped spins. When the temperature exceeds the critical temperature, the bubbles expand and merge, and up-down symmetry is restored.
- Repeat the experiments enough times to develop some intuition about the occurrence of configurations far from the average. [The original paper is E. Ising, *Z. Phys.* **31**, 253 (1925). For more than you want to know, see B. M. McCoy and T. T. Wu, *The Two-Dimensional Ising Model* (Harvard University Press, Cambridge, 1973).]
18. Analyze the spontaneous breakdown of a global $SU(2)$ symmetry. Consider the case of three real scalar fields ϕ_1 , ϕ_2 , and ϕ_3 , which constitute an $SU(2)$ triplet, denoted

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}.$$

The Lagrangian density is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi) \cdot (\partial^\mu \phi) - V(\phi \cdot \phi),$$

where as usual

$$V(\phi \cdot \phi) = \frac{1}{2}\mu^2 \phi \cdot \phi + \frac{1}{4}|\lambda|(\phi \cdot \phi)^2.$$

Assume that for $\mu^2 < 0$ the potential has a minimum at

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}.$$

Then show that (a) the vacuum remains invariant under the action of the generator T_3 , but not under T_1 or T_2 ; (b) the particles associated with T_1 and T_2 become massless (Goldstone) particles; (c) the particle associated with T_3 acquires a mass $= \sqrt{-2\mu^2}$.

19. Generalize the preceding example to a Lagrangian that describes the interactions of n scalar fields and is invariant under global transformations under the group $O(n)$. After spontaneous symmetry breaking and the choice of a vacuum state, show that the vacuum is invariant under the group $O(n-1)$. Verify that the number of Goldstone bosons corresponds to the number of broken generators of the original symmetry group—*i.e.*, to the difference between the number of generators of $O(n)$ and $O(n-1)$.
20. The Ginzburg–Landau theory of superconductivity provides a phenomenological understanding of the Meissner effect: the observation that an external magnetic field does not penetrate the superconductor. Ginzburg and Landau introduce an “order parameter” ψ , such that $|\psi|^2$ is related to the density of superconducting electrons. In the absence of an impressed magnetic field, expand the free energy of the superconductor as

$$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha |\psi|^2 + \beta |\psi|^4,$$

where α and β are phenomenological parameters.

(a) Minimize $G_{\text{super}}(0)$ with respect to the order parameter and discuss the circumstances under which spontaneous symmetry breaking occurs. Compute $\langle |\psi|^2 \rangle_0$, the value at which $G_{\text{super}}(0)$ is minimized.

(b) In the presence of an external magnetic field \mathbf{B} , a gauge-invariant expression for the free energy is

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{2} + \frac{1}{2m^*} \psi^* (-i\nabla - e^* \mathbf{A})^2 \psi.$$

(The effective charge e^* turns out to be $2e$, because $|\psi|^2$ represents the density of Cooper pairs.) Derive the field equations that follow from minimizing $G_{\text{super}}(\mathbf{B})$ with respect to ψ and \mathbf{A} . Show that in the weak-field approximation ($\nabla\psi \approx 0$, $\psi \approx \langle \psi \rangle_0$) the photon acquires a mass within the superconductor. [Reference: V. L. Ginzburg and L. D. Landau, *Zh. Eksp. Teor. Fiz.* **20**, 1064 (1950); English translation: see *Men of Physics: Landau*, Vol. II, edited by D. ter Haar, Pergamon, New York, 1965. For further information, see §21.6 of Weinberg.]

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Typographical errors in the fifth paperbound printing of *Gauge Theories* (identified at the bottom of p. vi by EFGHIJ-MA-89):

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (5.2.4)$$

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \quad (5.3.2)$$

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March 4, 1997

Recommended Reading:*Gauge Theories*, §6.1 – 6.4.

Aitchison and Hey, §14-1 – 14-5.

On the σ Model: J. F. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the Standard Model*, (Cambridge University Press, Cambridge, 1992), pp. 10–12, 21–22.K. Gottfried and V. F. Weisskopf, *Concepts of Particle Physics* vol. II (Oxford University Press, New York, 1986): on the choice of the electroweak gauge group, §VI.B.2 (pp. 491-496); on the analogy with superconductivity, §VI.C.1 (pp. 509-516).**Cultural Reading:**To celebrate the hundredth anniversary of the discovery of the electron (and the beginning of particle physics), read the articles by Abraham Pais, Steven Weinberg, and Chris Quigg in the Spring 1997 issue of the SLAC *Beam Line*, available in PDF format at <http://www.slac.stanford.edu/pubs/beamline/97i.pdf>.Robert N. Cahn and Gerson Goldhaber, *The Experimental Foundations of Particle Physics* (Cambridge University Press, Cambridge and New York, 1989), chapter 12.On spontaneous symmetry breaking: Mary M. Crone and Marc Sher, “The environmental impact of vacuum decay,” *Am. J. Phys.* **59**, 25 (1991).

Problems (due March 13, 1997)

21. Derive the equation of motion for the photon field A_ν in the Abelian Higgs model and show that it amounts to a relativistic generalization of the Ginzburg–Landau description of a superconductor.
22. (a) Compute the decay rate for the disintegration of the tau lepton, $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$, neglecting the electron mass, and compare with the familiar rate for muon decay.
(b) How would the rate change if, by virtue of an unconventional lepton number assignment, $\nu_\tau \equiv \bar{\nu}_e$?
23. (a) Compute the differential and total cross sections for $\nu_e e$ and $\bar{\nu}_e e$ elastic scattering in the Weinberg–Salam model. Work in the limit of large M_W and M_Z , and neglect the electron mass with respect to large energies. The computation is done most gracefully by Fierz reordering one of the graphs, as indicated in §6.4 of *Gauge Theories*. [Reference: G. 't Hooft, *Phys. Lett.* **37B**, 195 (1971).]
(b) Without calculating the cross section, compute the $\bar{\nu}_e$ energy for which W^- is formed as an s -channel resonance.
24. (a) Compute the cross section for the reaction

$$e^+ e^- \rightarrow H \rightarrow f \bar{f} \quad ,$$

where f is a massive fermion.

(b) How does the result differ for leptons and quarks?

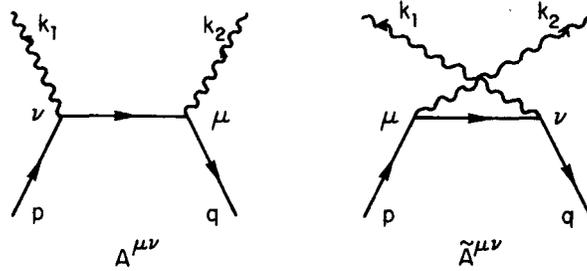
(c) Write the cross section for the reaction $e^+ e^- \rightarrow \text{all}$ in the form

$$\sigma(e^+ e^- \rightarrow H) = \frac{4\pi\Gamma(H \rightarrow e^+ e^-)\Gamma(H \rightarrow \text{all})}{(s - M_H^2)^2 + M_H^2\Gamma(H \rightarrow \text{all})^2} \quad .$$

How does the result change if the colliding particles are muons, rather than electrons?

(d) What integrated luminosity would be required to discover a 120-GeV Higgs boson in the reaction $\mu^+ \mu^- \rightarrow H$, if the only decay products are $e\bar{e}$, $\mu\bar{\mu}$, $\tau\bar{\tau}$, $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$, and $b\bar{b}$ pairs?

25. Consider the process $e^+e^- \rightarrow \gamma\gamma$, which is described, in QED, by the two Feynman diagrams



and for which the amplitude may be written $\mathcal{M} = \epsilon_{1\nu}^* \epsilon_{2\mu}^* (A^{\mu\nu} + \tilde{A}^{\mu\nu})$, where ϵ_i is the polarization vector of a photon and $A^{\mu\nu}(\tilde{A}^{\mu\nu})$ corresponds to the first (second) diagram.

(a) Calculate the tensors $A^{\mu\nu}$ and $\tilde{A}^{\mu\nu}$.

(b) Show that gauge invariance requires that

$$k_{1\nu}(A^{\mu\nu} + \tilde{A}^{\mu\nu}) = 0 = k_{2\mu}(A^{\mu\nu} + \tilde{A}^{\mu\nu}).$$

(c) Verify that these conditions are met, although the quantities $k_{1\nu}A^{\mu\nu}$, $k_{1\nu}\tilde{A}^{\mu\nu}$, $k_{2\mu}A^{\mu\nu}$, and $k_{2\mu}\tilde{A}^{\mu\nu}$ are all different from zero.

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I strongly recommend that you work through the calculations in §6.1 of *Gauge Theories*. So that we may cover many topics during the semester, I will not do all of these calculations in class. The calculations are done very explicitly in the book, so you should have no trouble in reproducing them. It might also be a useful experience to try doing the calculations at the blackboard, explaining steps to classmates who are free to object, offer advice, or peek at the answer.

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Comment on Problem 12: Because the interaction rate must be very high in a multi-TeV hadron collider, the continuous fluxes of neutrinos may be high enough to allow very high-energy neutrino experiments, including a three-neutrino experiment. The possibilities are worked out in some detail by A. De Rújula, E. Fernández, and J. J. Gómez-Cadenas, *Nucl. Phys.* **B405**, 80 (1993).

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Typographical errors in the fifth paperbound printing of *Gauge Theories* (identified at the bottom of p. vi by EFGHIJ-MA-89):

p. 140, Problem 6-3, below (A.4.29):

. . . where $\Gamma_i = (1, \gamma_\mu, \sigma_{\mu\nu}, i\gamma_\mu\gamma_5, \gamma_5) \dots$

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March 13, 1997

Recommended Reading:*Gauge Theories*, §6.5 – 7.2.

Aitchison and Hey, §14-1 – 14-5.

On parity violation in atomic physics: E. D. Commins, “Atomic Parity Violation Experiments,” *Physica Scripta* **36**, 468 (1987); M.-A. Bouchiat, “Neutral Currents in Atoms, Experiments in Cesium and Implications,” in *Neutral Currents: Twenty Years Later*, edited by U. Nguyen-Khac and A. M. Lutz (World Scientific, Singapore, 1994), p. 247.

E. D. Commins, “Resource Letter ETDSTS-1: Experimental Tests of the Discrete Space-Time Symmetries,” *Am. J. Phys.* **61**, 778 (1993).

Cultural Reading:

D. B. Cline, C. Rubbia, and S. van der Meer, “The Search for Intermediate Vector Bosons,” *Sci. Am.* **246**, (March, 1982), p. 48.

S. van der Meer, “Stochastic cooling and the accumulation of antiprotons,” *Rev. Mod. Phys.* **57**, 689 (1985).

C. Rubbia, “Experimental observation of the intermediate vector bosons W^+ , W^- , and Z^0 ,” *Rev. Mod. Phys.* **57**, 699 (1985).

M.-A. Bouchiat and L. Pottier, “An Atomic Preference between Left and Right,” *Sci. Am.* **250**, (June, 1984), p. 100.

Deep Background:

M. J. Musolf, *et al.*, “Intermediate-energy semileptonic probes of the hadronic neutral current,” *Phys. Rep.* **239**, 1 (1994), gives the nuclear-physicist’s perspective on characterizing the neutral current.

Problems (due March 27, 1997)

26. At very low momentum transfers, as in atomic physics applications, the nucleon appears elementary. If the effective Lagrangian for nucleon β -decay can be written in the limit of zero momentum transfer as

$$\mathcal{L}_\beta = -\frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-\gamma_5)\nu\bar{p}\gamma^\lambda(1-g_A\gamma_5)n,$$

where $g_A = 1.2601 \pm 0.0025$ is the axial charge of the nucleon (renormalized from unity by the strong interactions), show that the eN neutral-current interactions may be represented by

$$\mathcal{L}_{ep} = \frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-4x_W-\gamma_5)e\bar{p}\gamma^\lambda(1-4x_W-g_A\gamma_5)p$$

and

$$\mathcal{L}_{en} = -\frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-4x_W-\gamma_5)e\bar{n}\gamma^\lambda(1-g_A\gamma_5)n.$$

Regard the nucleus as a noninteracting collection of Z protons and N neutrons. Now perform the nonrelativistic reduction of the implied nuclear matrix elements. Show that, for a heavy nucleus, the dominant parity-violating contribution to the electron–nucleus amplitude will be of the form

$$\mathcal{M}_{p.v.} = -\frac{iG_F}{2\sqrt{2}}Q^W\bar{e}\rho_N(\mathbf{r})\gamma_5e,$$

where ρ_N is the nucleon density as a function of the electron coordinate \mathbf{r} , and the weak charge is

$$Q^W = Z(1 - 4x_W) - N.$$

27. The weak charge of $^{205}_{81}\text{Tl}$ has been measured by a group at the University of Washington [P. A. Vetter, D. M. Meekhof, P. K. Majumder, S. K. Lamoreaux, and E. N. Fortson, *Phys. Rev. Lett.* **74**, 2658 (1995)] as

$$Q^W = -114.2 \pm 1.3 \pm 3.4 ,$$

where the first error is experimental and the second refers to the uncertainty in atomic theory. Using the first-order expression derived in Problem 26, deduce the weak mixing parameter $\sin^2 \theta_W$ and its uncertainties.

28. LEP measurements on the Z^0 were summarized by Joachim Mnich, *Phys. Rep.* **271**, 181 (1996), as follows:

M_Z	$91.1884 \pm 0.0022 \text{ GeV}/c^2$
Γ_Z	$2.4963 \pm 0.0032 \text{ GeV}$
$\Gamma(Z \rightarrow \text{hadrons})$	$1744.8 \pm 3.0 \text{ MeV}$
$\sigma_{\text{had}}(\text{pole})$	$41.488 \pm 0.078 \text{ nb}$
$\Gamma(Z \rightarrow e^+e^-)$	$83.92 \pm 0.17 \text{ MeV}$
$\Gamma(Z \rightarrow \mu^+\mu^-)$	$83.92 \pm 0.23 \text{ MeV}$
$\Gamma(Z \rightarrow \tau^+\tau^-)$	$83.85 \pm 0.29 \text{ MeV}$
$\Gamma_{\text{leptonic}} = \Gamma(Z \rightarrow \ell^+\ell^-)$	$83.93 \pm 0.14 \text{ MeV}$
$\Gamma_{\text{invisible}}$	$499.9 \pm 2.5 \text{ MeV}$
a_e	-0.5011 ± 0.0004
v_e	-0.0380 ± 0.0007

Let us confront these results with the lowest-order predictions of the Weinberg–Salam model.

- Use the value of the Z^0 mass to determine $x_W \equiv \sin^2 \theta_W$.
- Using your value of x_W , predict the W^\pm mass and compare it with the current world average: $M_W = 80.38 \pm 0.09 \text{ GeV}/c^2$.
- Use the measured value of the Z^0 mass to predict the partial width $\Gamma(Z \rightarrow \nu\bar{\nu})$. By comparing with $\Gamma_{\text{invisible}}$ measured at LEP, determine the number of light neutrino species.
- Use your value of the weak mixing parameter x_W to predict $\Gamma_{\text{leptonic}} = \Gamma(Z \rightarrow \ell^+\ell^-)$, $\Gamma(Z \rightarrow \text{hadrons})$, a_e , and v_e . Neglect the masses of all fermions but the top quark.

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Supplement to references on inverse muon decay, p. 95 of *Gauge Theories*:

¶ S. R. Mishra, *et al.*, *Phys. Lett.* **252B**, 170 (1990).

$$\sigma(\nu_{\mu}e \rightarrow \mu^{-}\nu_e)/\sigma_{\text{th}} = 0.981 \pm 0.049 \text{ (stat.)} \pm 0.030 \text{ (syst.)}$$

for neutrino energies up to 600 GeV.

¶ P. Vilain, *et al.* (CHARM II Collaboration), *Phys. Lett.* **364B**, 121 (1995).

$$\sigma(\nu_{\mu}e \rightarrow \mu^{-}\nu_e)/\sigma_{\text{th}} = 0.929 \pm 0.019 \pm 0.048$$

in the SPS wideband beam ($\langle E_{\nu} \rangle = 23.8$ GeV, neutrino energies up to 150 GeV).

¶ J. Dorenbosch, *et al.* (CHARM Collaboration), *Z. Phys.* **C41**, 567 (1989).

$$\sigma(\nu_{\mu}e \rightarrow \mu^{-}\nu_e)/\sigma_{\text{th}} = 1.10 \pm 0.08 \text{ (stat.)} \pm 0.13 \text{ (syst.)}$$

in the SPS wideband beam.

Additional references on $\nu_{\mu}e$ neutral current reactions:

¶ N. J. Baker, *et al.*, *Phys. Rev.* **D40**, 2753 (1989), *ibid.* **41**, 1715E (1990).

¶ L. A. Ahrens, *et al.*, *Phys. Rev.* **D41**, 3297 (1990), Brookhaven experiment E-734.

¶ P. Vilain, *et al.* (CHARM II Collaboration), *Phys. Lett.* **335B**, 246 (1994).

¶ J. Dorenbosch, *et al.* (CHARM Collaboration), *Z. Phys.* **C41**, 567 (1989).

Detection of $\bar{\nu}_e e$ scattering:

F. Reines, H. S. Gurr, and H. W. Sobel, *Phys. Rev. Lett.* **37**, 315 (1976).

Detection of $\nu_e e$ scattering:

R. C. Allen, *et al.*, *Phys. Rev.* **D47**, 11 (1993).

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First class after spring break: Monday, March 24.

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Course information, including class schedules and assignments, is available on the World Wide Web. Point your browser to <http://lutece.fnal.gov/Princeton/>.

March 25, 1997

Recommended Reading:*Gauge Theories*, §7.3 – 7.5.

Aitchison and Hey, Chapter 7.

On the mixing of quark flavors: F. J. Gilman, K. Kleinknecht, and B. Renk, “The Cabibbo–Kobayashi–Maskawa Mixing Matrix,” in Particle Data Group, *Phys. Rev. D* **54**, 1 (1996); see pp. 94–97.

Cultural Reading:

For an excellent overview of the analysis of LEP data, see J. Mnich, *Phys. Rep.* **271**, 181 (1996).

On physics opportunities at $\mu^+\mu^-$ colliders, see V. Barger, *et al.*, (bulletin board: hep-ph/9604334).

Background Reading:

F. E. Close, *An Introduction to Quarks and Partons*, Chapters 9 and 11.

On lower bounds to the Higgs-boson mass deduced from the requirement of vacuum stability of electroweak potentials, see M. Sher, *Phys. Rep.* **179**, 273 (1989). For useful updates in light of the large mass of the top quark ($m_t \approx 175 \text{ GeV}/c^2$), see G. Altarelli and G. Isidori, *Phys. Lett.* **B337**, 141 (1994); J. Espinosa and M. Quirós, *Phys. Lett.* **B353**, 257 (1995).

Problems (due April 3, 1997)

29. Carry out the computation of the amplitudes for the reaction $e^+e^- \rightarrow W^+W^-$ described in §6.5 of *Gauge Theories*, retaining the electron mass. Verify the role of the Higgs boson in the cancellation of divergences.
30. Because the most serious high-energy divergences of a spontaneously broken gauge theory are associated with the longitudinal degrees of freedom of the gauge bosons, which arise from auxiliary scalars, it is instructive to study the Higgs sector in isolation. Consider, therefore, the Lagrangian for the Higgs sector of the Weinberg–Salam model before the gauge couplings are turned on,

$$\mathcal{L}_{\text{scalar}} = (\partial^\mu \phi)^\dagger (\partial_\mu \phi) - \mu^2 (\phi^\dagger \phi) - |\lambda| (\phi^\dagger \phi)^2.$$

(a) Choosing $\mu^2 < 0$, investigate the effect of spontaneous symmetry breaking. Show that the theory describes three massless scalars (w^+, w^-, z^0) and one massive neutral scalar (h), which interact according to

$$\begin{aligned} \mathcal{L}_{\text{int}} = & -|\lambda| v h (2w^+ w^- + z^2 + h^2) \\ & - (|\lambda|/4) (2w^+ w^- + z^2 + h^2)^2, \end{aligned}$$

where $v^2 = -\mu^2/|\lambda|$. In the language of the full Weinberg–Salam theory, $1/v^2 = G_F \sqrt{2}$ and $\lambda = G_F M_H^2 / \sqrt{2}$. Note the resemblance to the σ -model Lagrangian discussed in class on March 3 and 4. The correspondence is $\pi^\pm \leftrightarrow w^\pm$, $\pi^0 \leftrightarrow z^0$, $h \leftrightarrow \sigma$. In the σ model, $v = F_\pi$ and $|\lambda| = M_\sigma^2 / 2F_\pi^2$.

(b) Deduce the Feynman rules for interactions and compute the lowest-order (tree diagram) amplitude for the reaction $hz \rightarrow hz$.

(c) Compute the $J = 0$ partial-wave amplitude in the high-energy limit and show that it respects partial-wave unitarity only if $M_H^2 < 8\pi\sqrt{2}/G_F$. [Reference: B. W. Lee, C. Quigg, and H. B. Thacker, *Phys. Rev. D* **16**, 1519 (1977).]

31. The parton model leads to many informative sum rules about structure functions. Some of these are true quite generally (up to perturbative QCD corrections), while some depend on additional assumptions.

(a) Show that in the quark-parton model, the integral

$$\mathcal{I}_G = \int_0^1 dx \frac{[F_2^{\mu p}(x, Q^2) - F_2^{\mu n}(x, Q^2)]}{x}$$

can be expressed as

$$\mathcal{I}_G = \frac{1}{3} + \frac{2}{3} \int_0^1 dx [\bar{u}(x, Q^2) - \bar{d}(x, Q^2)] \quad .$$

(b) Derive the Gottfried sum rule [K. Gottfried, *Phys. Rev. Lett.* **18**, 1174 (1967)], $\mathcal{I}_G = \frac{1}{3}$, from the assumption that the sea is up-down symmetric.

(c) How do you interpret the observation that $\mathcal{I}_G(Q^2 = 4 \text{ GeV}/c^2) = 0.2281 \pm 0.0065$? [See M. Arneodo, *et al.* (New Muon Collaboration), *Nucl. Phys.* **B487**, 3 (1997).]

(d) What other experiments might be sensitive to a difference between the sea distributions of up and down quarks?

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You can follow the progress of the LEP experiments on their Web pages.

- ALEPH, <http://alephwww.cern.ch>
- DELPHI, <http://delinfo.cern.ch/Delphi/Welcome.html>
- L3, <http://hpl3sn02.cern.ch/welcome.html>
- OPAL, <http://www.cern.ch/Opal/>

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Course information, including class schedules and assignments, is available on the World Wide Web. Point your browser to <http://lutece.fnal.gov/Princeton/>.

April 3, 1997

Recommended Reading:

On tests of lepton-number conservation, see the minireview, “Tests of Conservation Laws,” in the 1996 *Review of Particle Physics* [Particle Data Group, *Phys. Rev. D* **54**, 1 (1996)], pp. 59–63.

On neutrino mass and mixing, see the minireviews, “Neutrinos” (pp. 275–280) and “Searches for Massive Neutrinos and Lepton Mixing” (pp. 287–301), also in the 1996 *RPP*.

On neutrino oscillations in matter, see T. K. Kuo and J. Pantaleone, *Rev. Mod. Phys.* **61**, 937 (1989).

Cultural Reading:

F. Boehm and P. Vogel, *Physics of Massive Neutrinos*, second edition (Cambridge University Press, Cambridge, 1992)

B. Kayser, F. Gibrat-Débu, and F. Perrier, *The Physics of Massive Neutrinos* (World Scientific, Singapore, 1989).

S. M. Bilenky and S. T. Petcov, “Massive neutrinos and neutrino oscillations,” *Rev. Mod. Phys.* **59**, 671 (1987).

T. J. Bowles and V. N. Gavrin, “The Status of the Solar Neutrino Problem,” *Ann. Rev. Nucl. Part. Sci.* **43**, 117 (1993).

J. Bahcall, *Neutrino Astrophysics* (Cambridge University Press, Cambridge, 1989).

M. Moe and P. Vogel, “Double Beta Decay,” *Ann. Rev. Nucl. Part. Sci.* **44**, 247 (1994).

Escapist Literature for Those Afflicted by Generals:

Ettore Majorana vanished without a trace from a ferry between Sicily and the Italian mainland. Leonardo Sciascia has written a fictional account that Majorana’s colleagues denounced as scurrilous fantasy. Firestone Library has the Italian original, *La Scomparsa di Majorana*, as well as an English translation, which is in *The Moro Affair and the Mystery of Majorana*.

Problems (due April 10, 1997)

32. Because the neutrino-nucleon cross section increases with energy, the Earth is opaque to ultrahigh-energy neutrinos. From what you know of the measured cross section for charged-current νN interactions [Particle Data Group, *Phys. Rev. D* **45**, S1 (1992); see p. III.82], compute the “water equivalent” interaction length, $\mathcal{L}_{\text{int}} = 1/[\sigma_{\nu N}(E_\nu)N_A]$, where N_A is Avogadro’s number. If the diameter of the Earth is 11 kilotonnes/cm², *i.e.*, 1.1×10^{10} cmwe (centimeters of water equivalent), at what energy does the charged-current interaction length become smaller than one Earth diameter? How does the W -boson modify your conclusions? [A. DeRújula, S. L. Glashow, R. R. Wilson, and G. Charpak, *Phys. Rep.* **99**, 341 (1983), §8; R. Gandhi, C. Quigg, M. H. Reno, and I. Sarcevic, *Astroparticle Physics* **5**, 81 (1996).]
33. Compute the rate for the dominant decay rate of the top quark, $t \rightarrow bW^+$, neglecting the mass of the b quark. You will need to sum over the possible polarizations of the W -boson. Show that it takes the form

$$\Gamma(t \rightarrow bW^+) = \frac{G_F m_t^3}{8\pi\sqrt{2}} |V_{tb}|^2 \left(1 - \frac{M_W^2}{m_t^2}\right)^2 \left(1 + \frac{2M_W^2}{m_t^2}\right),$$

where V_{tb} measures the strength of the $t \rightarrow bW^+$ coupling. What fraction of the decays produce W -bosons with longitudinal polarization? [I. Bigi, Yu. L. Dokshitzer, V. Khoze, J. Kühn, and P. Zerwas, *Phys. Lett.* **B181**, 157 (1986).]

34. Consider the one-loop modifications to Coulomb scattering in the limit of low momentum transfer $-q^2 \ll m^2$. Beginning from Equation (8.2.31) in *Gauge Theories*, show that the amplitude is modified by a factor

$$\left[1 - \frac{\alpha_R}{15\pi} \frac{q^2}{m^2} + O(\alpha_R^2) \right].$$

Show that this corresponds, in position space, to an additional interaction of the form

$$\frac{4}{15} \frac{\alpha_R^2}{m^2} \delta^3(\mathbf{x}),$$

and estimate the first-order shift in the energy levels of the hydrogen atom. [E. A. Uehling, *Phys. Rev.* **48**, 55 (1935); for an early application, see R. Serber, *ibid.*, p. 49.]

35. The model of M.-Y. Han and Y. Nambu [*Phys. Rev.* **149B**, 1006 (1965)] is an integer-charge alternative to the fractional-charge quark model, with charges assigned as

Color	Flavor		
	<i>u</i>	<i>d</i>	<i>s</i>
Red	0	-1	-1
Green	1	0	0
Blue	1	0	0

- (a) Show that below the threshold for color liberation, the ratio

$$R \equiv \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$$

is $R = 2$, as in the fractional-charge model, and that $R = 4$ if color can be liberated.

- (b) Consider the reaction $\gamma\gamma \rightarrow \text{hadrons}$, viewed as $\gamma\gamma \rightarrow q\bar{q}$. Show that with fractionally charged quarks,

$$\sigma(\gamma\gamma \rightarrow \text{hadrons}) \propto \sum_i e_i^4 = \frac{2}{3},$$

and that in the Han-Nambu model,

$$\sigma(\gamma\gamma \rightarrow \text{hadrons}) \propto \sum_i e_i^4 = \begin{cases} 2 & \text{below color threshold} \\ 4 & \text{above color threshold} \end{cases}$$

[References: F. E. Close, *An Introduction to Quarks and Partons*, (Academic Press, New York, 1979), Chapter 8; H. J. Lipkin, *Nucl. Phys.* **B155**, 104 (1979); M. S. Chanowitz, "Color and Experimental Physics," in *Particles and Fields-1975*, edited by H. J. Lubatti and P. M. Mockett (University of Washington, Seattle, 1975), p. 448.]

36. Show that the $n \times n$ unitary matrix U introduced to describe the mixing of quarks of different flavors [cf. equations (7.1.27-29) in *Gauge Theories*] can be parametrized in terms of $n(n-1)/2$ real mixing angles and $(n-1)(n-2)/2$ complex phases, after the freedom to redefine quark fields has been taken into account. For the specific case of three quark doublets ($n = 3$), show that U may be written in the form

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \cdot \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix},$$

where $s_i = \sin \theta_i$ and $c_i = \cos \theta_i$. Discuss the implications of the phase δ for CP invariance. [M. Kobayashi and T. Maskawa, *Prog. Theor. Phys. (Kyoto)* **49**, 652 (1973).]

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Update on parity violation in atomic physics. In Problem 27, I should have cited a second determination of the weak charge of ${}^{205}_{81}\text{Tl}$, derived from measurements of the parity nonconserving optical rotation near the $6P_{1/2} \rightarrow 6P_{3/2}$ magnetic dipole transition in atomic thallium. The University of Oxford group [N. H. Edwards, S. J. Phipp, P. E. G. Baird, and S. Nakayama, *Phys. Rev. Lett.* **74**, 2654 (1995)] report $Q_W({}^{205}\text{Tl}) = -121.9 \pm 3.5 \pm 3.4$ (my interpretation).

The most incisive of the parity-violation experiments in atoms is the measurement of weak charge of ${}^{133}_{55}\text{Cs}$ by a group from the University of Colorado [M. C. Noecker, B. P. Masterson, and C. E. Wiemann, *Phys. Rev. Lett.* **61**, 310 (1988), atomic structure calculations revised by S. A. Blundell, W. R. Johnson, and J. Sapirstein, *ibid.* **65**, 1411 (1990)]. Their value is $Q^W({}^{133}_{55}\text{Cs}) = -71.04 \pm 1.58 \pm 0.88$, where the first error is experimental and the second refers to the uncertainty in atomic theory.

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Typographical errors in the fifth paperbound printing of *Gauge Theories* (identified at the bottom of p. vi by EFGHIJ-MA-89): In equation (7.1.13), the coefficient of $x_W Q$ should be 4, not 2. This appears twice.

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Course information, including class schedules and assignments, is available on the World Wide Web. Point your browser to <http://lutece.fnal.gov/Princeton/>.

April 8, 1997

Recommended Reading:

On the importance of the 1-TeV scale, and experimental approaches to it: E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg, "Supercollider Physics", *Rev. Mod. Phys.* **56**, 579 (1984).
 R. S. Chivukula, M. J. Dugan, M. Golden, and E. H. Simmons, *Ann. Rev. Nucl. Part. Sci.* **45**, 255 (1995).

H. Murayama and M. Peskin, *Ann. Rev. Nucl. Part. Sci.* **46**, 533 (1996).

On Quantum Chromodynamics: *Gauge Theories*, §8.1 – 8.5.

Aitchison and Hey, Chapter 9.

M. Gell-Mann, *Acta Physica Austriaca Suppl.* **IV**, 773 (1972).

Cultural Reading:

On the physics of a supernova: H. A. Bethe and G. Brown, *Sci. Am.* **252**, (May, 1985), p. 60.

On the $U(1)$ axial anomaly: J. F. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the Standard Model*, (Cambridge University Press, Cambridge, 1992), §III-3.

On gauge theories of the forces between elementary particles, G. 't Hooft, *Sci. Am.* **242**, (June, 1980), p. 104.

Deep Background:

T. Muta, *Foundations of Quantum Chromodynamics* (World Scientific, Singapore, 1987).

On QCD corrections to the reaction $e^+e^- \rightarrow$ hadrons: L. Surguladze and M. Samuel, *Phys. Rev. Lett.* **66**, 560 (1991).

Problems (due April 17, 1997)

37. Consider neutrino oscillations in vacuum, in a world with N flavors of neutrinos. The flavor eigenstates $\nu_e, \nu_\mu, \nu_\tau, \dots$, denoted by Greek labels as $|\nu_\alpha\rangle$, can be expressed as linear combinations of the mass eigenstates $\nu_1, \nu_2, \dots, \nu_N$, denoted by roman indices as $|\nu_i\rangle$. The evolution of a state that was initially a flavor eigenstate can be written as

$$|\nu_\alpha\rangle_t = \sum_i U_{\alpha i} \exp(-iE_i t) |\nu_i\rangle,$$

where U is a real unitary matrix, providing that CP is conserved.

(a) Ignoring oscillatory terms, show that the probability that an electron neutrino remain an electron neutrino after a time t is

$$P_{e \leftarrow e} = |\langle \nu_e | \nu_e \rangle|^2 = \sum_i |U_{ei}|^4.$$

(b) By minimizing $P_{e \leftarrow e}$ subject to the constraint that $\sum_i |U_{ei}|^2 = 1$, show that the minimum probability is attained when all of the $|U_{ei}|^2$ are equal, *i.e.*, when

$$|U_{ei}|^2 = \frac{1}{N}.$$

Use the method of Lagrange multipliers to impose the constraint.

38. Compute the one-loop charge renormalization in QED using the method of dimensional regularization.
39. Modify the calculation (Problem 25) of the amplitude for the process $e^+e^- \rightarrow \gamma\gamma$ to describe the reaction $q\bar{q} \rightarrow gg$ in quantum chromodynamics. In this case, the two diagrams shown in Problem 24 are not by themselves gauge invariant.
- Show that in QCD the quantities $k_{1\nu}(A^{\mu\nu} + \tilde{A}^{\mu\nu})$ and $k_{2\mu}(A^{\mu\nu} + \tilde{A}^{\mu\nu})$ are proportional to $[\lambda^a, \lambda^b]$, where a and b are the $SU(3)$ color indices of the two gluons.
 - What is the resolution of this noninvariance?
 - For the full gauge-invariant amplitude described by $\epsilon_{1\nu}^* \epsilon_{2\mu}^* T^{\mu\nu}$, under what conditions are the requirements $k_{1\nu} T^{\mu\nu} = 0 = k_{2\mu} T^{\mu\nu}$ fulfilled?
40. Consider a gauge theory of the strong interactions based on the color symmetry group $SO(3)$, in which both quarks and gluons are assigned to the adjoint representation. By appropriately modifying the color factors entering Equation (8.3.27) of *Gauge Theories*, evaluate the running coupling constant in one-loop order. What is the condition for asymptotic freedom in this theory?

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April 17, 1997

Recommended Reading:*Gauge Theories*, §8.6 – 8.8.

Aitchison and Hey, §15.1.

J. F. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the Standard Model*, (Cambridge University Press, Cambridge, 1992), §II-2.R. K. Ellis, W. J. Stirling, and B. Webber, *QCD and Collider Physics*, (Cambridge University Press, Cambridge, 1996), Chapters 1 and 2.**Cultural Reading:**

On precision electroweak measurements: J. L. Rosner, “New Developments in Precision Electroweak Physics,” EFI-97-18 (electronic archive: hep-ph/9704331).

On massive neutrinos: G. G. Raffelt, “Neutrino Masses in Astrophysics and Cosmology” (electronic archive: hep-ph/9704315).

Deep Background:T.-P. Cheng and L.-F. Li, *Gauge Theory of Elementary Particles* (Oxford University Press, New York, 1984), Chapter 10.On new calculational techniques: R. Gastmans and T. T. Wu, *The Ubiquitous Photon: the helicity method for QED and QCD*, (Clarendon Press, Oxford, 1990), Chapters 1 – 3.M. L. Mangano and S. J. Parke, *Phys. Rep.* **200**, 301 (1991).Z. Bern, L. Dixon, and D. A. Kosower, *Ann. Rev. Nucl. Part. Sci.* **46**, 109 (1996).

Problems (due April 24, 1997)

41. Compute the differential cross section $d\sigma/dt$ for the scattering of unlike quarks in QCD, *i.e.*, for the elementary reaction $ud \rightarrow ud$.

(a) Show that

$$d\sigma(ud \rightarrow ud)/dt = \frac{4\pi\alpha_s^2}{9s^2} \frac{s^2 + u^2}{t^2},$$

where s, t, u are the usual Mandelstam invariants.(b) Compare with the cross section for μ^-e^- scattering in QED.

(c) How would the result change if the quarks were color sextets, instead of color triplets?

42. Using the Feynman rules for the Faddeev–Popov ghost given in Figure 8-15 and §8.3 of *Gauge Theories*, verify that the modification to the gluon propagator due to one ghost loop as shown in Figure 8-16 is given by

$$\bar{I}_{\text{ghosts}}^{\mu\nu}(q^2) = \frac{-ig^2}{16\pi^2} f^{acd} f^{bcd} \left(\frac{1}{6} q^\mu q^\nu + \frac{1}{12} q^2 g^{\mu\nu} \right) \log \left(\frac{-q^2}{\mu^2} \right).$$

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Updated update on parity violation in atomic physics. The Boulder group [C. S. Wood, *et al.*, *Science* **275**, 1759 (1997)] has published a new measurement of the weak charge of ^{133}Cs : $Q^W(^{133}\text{Cs}) = -72.11 \pm 0.27 \pm 0.88$, where the first error is experimental (greatly improved) and the second refers to the uncertainty in atomic theory (unchanged).

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Course information, including class schedules and assignments, is available on the World Wide Web. Point your browser to <http://lutece.fnal.gov/Princeton/>.

May 1, 1997

Recommended Reading:*Gauge Theories*, Chapter 9.

Aitchison and Hey, §15.1–15.2.

K. Nakamura, “Note on Nucleon Decay,” in 1994 Review of Particle Properties, *Phys. Rev. D* **50**, 1173 (1994), pp. 1673–1675.**Cultural Reading:**

P. Langacker, “Proton Decay” (electronic archive: hep-ph/9210238).

Deep Background:G. G. Ross, *Grand Unified Theories* (Addison-Wesley, Reading, Massachusetts, 1984).W. McKay and J. Patera, *Tables of Dimensions, Indices, and Branching Rules for Representations of Simple Algebras* (Dekker, New York, 1981).

Problems (due May 6, 1997)

43. Let us investigate the coupling-constant unification predicted by the $SU(5)$ unified theory.
- (a) Using your favorite values of the fine structure constant and the strong coupling constant at the Z^0 mass [for example, $1/\alpha(M_Z^2) = 129.89 \pm 0.09$ from H. Burkhardt and B. Pietrzyk, *Phys. Lett.* **B356**, 398 (1995) or S. Eidelman and F. Jegerlehner, *Z. Phys.* **C67**, 1585 (1995), and $\alpha_s(M_Z^2) = 0.118 \pm 0.003$ from the 1996 Review of Particle Physics], calculate the unification energy u and the value $1/\alpha_u$ of the $SU(5)$ coupling constant at the unification scale.
- (b) Using the value of the unification scale you found in (a), compute the weak mixing parameter $x_W \equiv \sin^2 \theta_W$ at the Z^0 mass and compare with the value that you found in Problem 28 using the LEP data.
- (c) Now use the connection between the measured value of x_W , α_2 , and α to determine $1/\alpha_2(M_Z^2)$. Compute $1/\alpha_Y(M_Z^2)$ and, from it, $1/\alpha_1(M_Z^2)$. Evolve $1/\alpha_1$, $1/\alpha_2$, and $1/\alpha_3$ to high energies and test whether they meet at a single unification point.
44. Consider a unified theory based on the gauge group $SO(10)$.
- (a) By referring to the $SU(5) \otimes U(1)$ decomposition of the representations of $SO(10)$, show that each fermion generation can be accommodated in an irreducible **16**-dimensional representation, which also has a place for a left-handed antineutrino.
- (b) Show that the adjoint **45** representation contains the gauge bosons of the $SU(5)$ theory.
- (c) Now examine the branching of $SO(10)$ into $SU(4) \otimes SU(2) \otimes SU(2)$, and the subsequent branching of $SU(4)$ into $SU(3) \otimes U(1)$. Use the $SU(3) \otimes SU(2) \otimes SU(2)$ decomposition of the fermion representation to show that $SO(10)$ contains the left-right-symmetric electroweak group as a subgroup.
- (d) Give the transformation properties of the **45** gauge bosons under $SU(3) \otimes SU(2) \otimes SU(2)$, and identify the $SU(5)$ gauge bosons among them. [Reference: H. Fritzsch and P. Minkowski, *Ann. Phys. (NY)* **93**, 193 (1975).]

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