A Century of Noether's Theorem

Chris Quigg

Münchner Physik Telekolloquium · 4 May 2020

arXiv:1902.01989

Max Lieberman: Felix Klein (1912), Mathematisches Institut Göttingen

Alan Bennett (1995) Variation on the Klein bottle, 1995. Science Museum Group Collection Online.

English translation: arxiv:physics/0503066

Invariante Variationsprobleme.

(F. Klein zum fünfzigjährigen Doktorjubiläum.)

Von

Emmy Noether in Göttingen.

Vorgelegt von F. Klein in der Sitzung vom 26. Juli 1918[1]).

Es handelt sich um Variationsprobleme, die eine kontinuierliche Gruppe (im Lieschen Sinne) gestatten; die daraus sich ergebenden Folgerungen für die zugehörigen Differentialgleichungen finden ihren allgemeinsten Ausdruck in den in § 1 formulierten, in den folgenden Paragraphen bewiesenen Sätzen. Über diese aus Variationsproblemen entspringenden Differentialgleichungen lassen sich viel präzisere Aussagen machen als über beliebige, eine Gruppe gestattende Differentialgleichungen, die den Gegenstand der Lieschen Untersuchungen bilden. Das folgende beruht also auf einer Verbindung der Methoden der formalen Variationsrechnung mit denen der Lieschen Gruppentheorie. Für spezielle Gruppen und Variationsprobleme ist diese Verbindung der Methoden nicht neu; ich erwähne Hamel und Herglotz für spezielle endliche, Lorentz und seine Schüler (z. B. Fokker), Weyl und Klein für spezielle unendliche Gruppen[2]). Insbesondere sind die zweite Kleinsche Note und die vorliegenden Ausführungen gegenseitig durch einander beein-

1) Die endgiltige Fassung des Manuskriptes wurde erst Ende September eingereicht.

2) Hamel: Math. Ann. Bd. 59 und Zeitschrift f. Math. u. Phys. Bd. 50. Herglotz: Ann. d. Phys. (4) Bd. 36, bes. § 9, S. 511. Fokker, Verslag d. Amsterdamer Akad., 27./1. 1917. Für die weitere Litteratur vergl. die zweite Note von Klein: Göttinger Nachrichten 19. Juli 1918.

In einer eben erschienenen Arbeit von Kneser (Math. Zeitschrift Bd. 2) handelt es sich um Aufstellung von Invarianten nach ähnlicher Methode.

Bryn Mawr College Special Collections

Felix Klein (19 July 1918)

XXXII. Über die Differentialgesetze für die Erhaltung von Impuls und Energie in der Einsteinschen Gravitationstheorie.

[Nachrichten der Kgl. Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-physikalische Klasse. (1918.) Vorgelegt in der Sitzung vom 19. Juli 1918[1]).]

Durch Fortsetzung der Untersuchungen, die ich der Gesellschaft der Wissenschaften am 25. Januar dieses Jahres vorlegte[2]), ist es mir gelungen, die verschiedenen Formen der Differentialgesetze für die Erhaltung von Impuls und Energie, wie sie, für die Einsteinsche Gravitationstheorie, von verschiedenen Autoren aufgestellt worden sind[3]), von einem einheitlichen Gesichtspunkte aus abzuleiten und dadurch, wenn ich nicht irre, in deren Bedeutung und wechselseitige Beziehung eine wesentlich verbesserte Einsicht zu gewinnen. Ich habe, wie man sehen wird, bei der im folgenden zu gebenden Darstellung eigentlich überhaupt nicht mehr zu rechnen, sondern nur von den elementarsten Formeln der klassischen Variationsrechnung sinngemäßen Gebrauch zu machen.

Der Kürze wegen knüpfe ich hier gleich, auch in der Bezeichnung, an meine vorige Note an. Als eigentlichen Grund des nunmehrigen Fort-

[1]) Das Manuskript hat erst Mitte September dieses Jahres seine endgültige Form erhalten.

[2]) Siehe das Schlußheft des Jahrgangs 1917 dieser Nachrichten: „Zu Hilberts erster Note über die Grundlagen der Physik". [Abh. XXXI dieser Ausgabe.]

[3]) Von Einstein kommen hier in erster Linie in Betracht die zusammenfassende Schrift von 1916: „Die Grundlagen der allgemeinen Relativitätstheorie" (Leipzig) und die Mitteilung an die Berliner Akademie „Hamiltonsches Prinzip und allgemeine Relativitätstheorie" (Sitzungsbericht vom 26. Oktober 1916), von Hilbert die bereits genannte Note (Göttinger Nachrichten vom 20. November 1915), von Lorentz die vier Artikel, die er auf Grund einer von März bis Juni 1916 in Leiden gehaltenen Vorlesung im Verslag der Amsterdamer Akademie veröffentlicht hat — „over Einsteins theorie der zwaartekracht" —, siehe insbesondere Art. III vom April bzw. September 1916 und Art. IV vom Oktober 1916 bzw. Mai 1917. Ich nenne hier ferner gleich das neuerdings erschienene Buch von Weyl „Raum — Zeit — Materie" (Berlin 1918), auf welches ich mich weiterhin ebenfalls zu beziehen habe. [Weyls Buch liegt bereits in dritter Auflage vor; im Texte wird immer die erste Auflage zitiert.]

23. Juli. Frl. Noether, Invariante Variationsprobleme

Im Zusammenhang mit den Untersuchungen über den Hilbertschen Energievektor hat die Referentin folgende allgemeine Sätze aufgestellt:

Gestattet die erste Variation eines Integrals eine endliche kontinuierliche Gruppe von ϱ wesentlichen Parametern, so werden ϱ lineare Verbindungen der Lagrangeschen Ableitungen des Integrals zu Divergenzen. Insbesondere kennt man also im eindimensionalen Falle, wo die Divergenzen zu totalen Differentialquotienten werden, ϱ erste Integrale der durch Nullsetzen der ersten Variation gegebenen Differentialgleichungen.

Gestattet die erste Variation eine unendliche kontinuierliche Gruppe mit ϱ willkürlichen Funktionen, so bestehen zwischen den Lagrangeschen Ableitungen und ihren Differentialquotienten ϱ lineare Beziehungen, so daß ϱ Gleichungen eine Folge der übrigen werden.

Zu beiden Sätzen gilt die Umkehrung.

Jahresbericht der Deutschen Mathematiker-Vereinigung
Mitteilungen und Nachrichten. vol 27, part 2, p. 47 (1918).

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INVARIANT VARIATIONAL PROBLEMS

(For F. Klein, on the occasion of the fiftieth anniversary of his doctorate)

by **Emmy Noether** in Göttingen

Presented by F. Klein at the session of 26 July 1918*

We consider variational problems which are invariant[A] under a continuous group (in the sense of Lie); the consequences that are implied for the associated differential equations find their most general expression in the theorems formulated in §1, which are proven in the subsequent sections. For those differential equations that arise from variational problems, the statements that can be formulated are much more precise than for the arbitrary differential equations that are invariant under a group, which are the subject of Lie's researches. What follows thus depends upon a combination of the methods of the formal calculus of variations and of Lie's theory of groups. For certain groups and variational problems this combination is not new; I shall mention Hamel and Herglotz for certain finite groups, Lorentz and his students (for example, Fokker), Weyl and Klein for certain infinite groups.[1] In particular, Klein's second note and the following developments were mutually influential, and for this reason I take the liberty of referring to the final remarks in Klein's note.

* The definitive version of the manuscript was prepared only at the end of September.

A *gestatten*, to permit, in the sense of admitting [an invariance group] has been translated as "being invariant under [the action of] a group" (Translator's note).

1 Hamel, Math. Ann., vol. 59, and Zeitschrift f. Math. u. Phys., vol. 50. Herglotz, Ann. d. Phys. (4) vol. 36, in particular §9, p. 511. Fokker, Verslag d. Amsterdamer Akad., 27/1 1917. For a more complete bibliography, see Klein's second note, Göttinger Nachrichten, 19 July 1918.

In a paper by Kneser that has just appeared (Math. Zeitschrift, vol. 2), the determination of invariants is dealt with by a similar method.

Y. Kosmann-Schwarzbach, *The Noether Theorems*, Sources and Studies in the History of Mathematics and Physical Sciences, DOI 10.1007/978-0-387-87868-3_1,
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Invariant Variational Problems

I. If the integral $\mathcal{I}$ is invariant under a finite continuous group $\mathcal{G}_\rho$ with ρ parameters, then there are ρ linearly independent combinations among the Lagrangian expressions that become divergences—and conversely, that implies the invariance of $\mathcal{I}$ under a group $\mathcal{G}_\rho$.

I includes all the known theorems in mechanics, etc., concerning first integrals.

II. If the integral $\mathcal{I}$ is invariant under an infinite continuous group $\mathcal{G}^\infty{}_\rho$ depending on ρ arbitrary functions and their derivatives up to order σ, then there are ρ identities among the Lagrangian expressions and their derivatives up to order σ. Here as well the converse is valid.

II can be described as the maximum generalization in group theory of "general relativity."

Elementary Consequences of Theorem I

Translation in space *No preferred location*	Momentum Conservation
Translation in time *No preferred time*	Energy Conservation
Rotational invariance *No preferred direction*	Angular Momentum Conservation
Boost invariance *No preferred frame*	"Center-of-momentum theorem"

All known at some level before Theorem I, of which they are special cases.

Theorem I links a conservation law with every continuous symmetry transformation under which the Lagrangian is invariant in form.

Feza Gürsey (cf. Introduction to Noether's *Collected Works*, 1983):
Before Noether's Theorem, the principle of conservation of energy was shrouded in mystery, leading to the obscure physical systems of Mach and Ostwald. Noether's simple and profound mathematical formulation did much to demystify physics.

Theorem II contains the seeds of gauge theories ("Symmetries dictate interactions") and exhibits the kinship between general relativity (general coordinate invariance) and gauge theories.

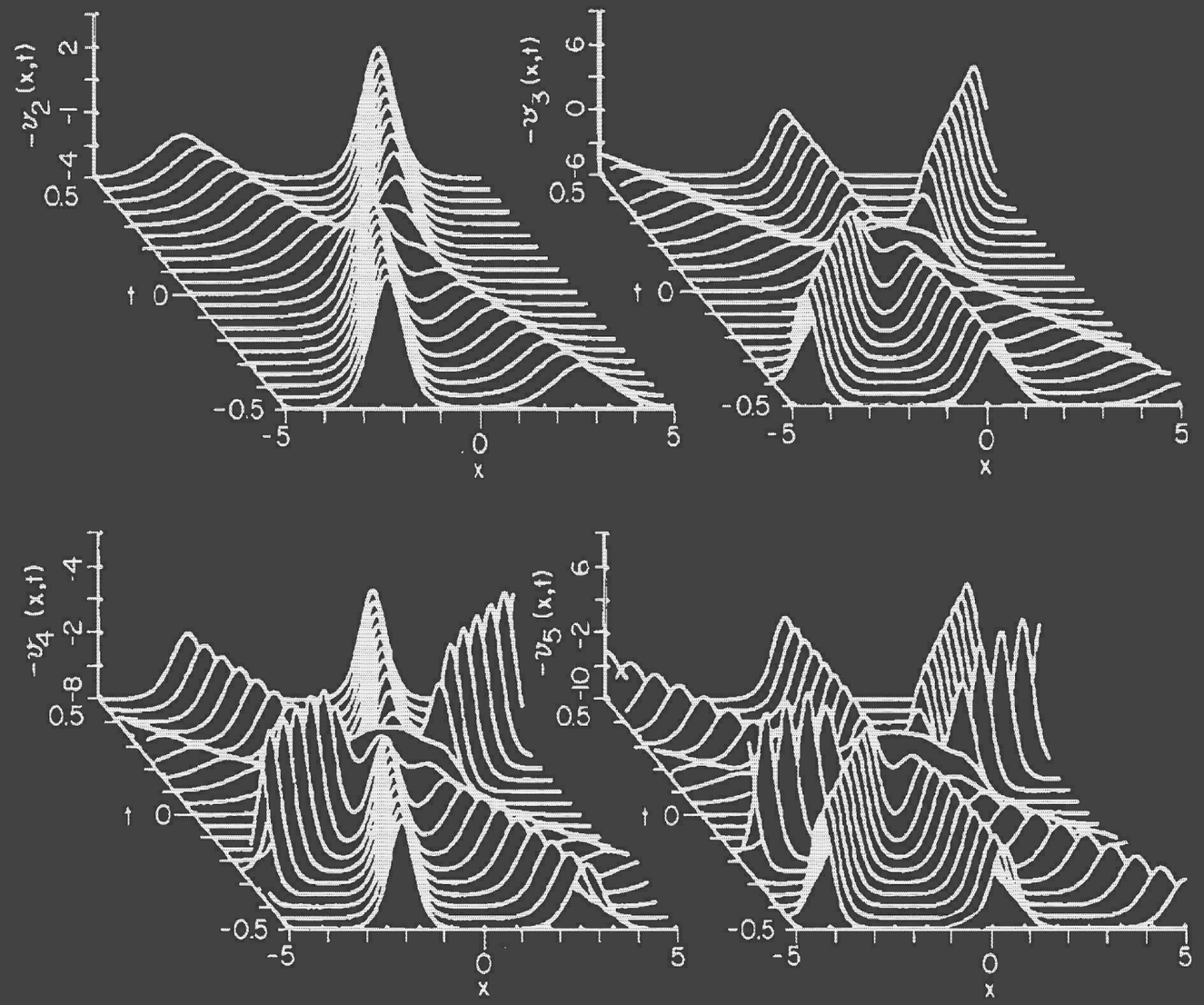
*Clarified Klein & Hilbert's issue about energy conservation in General Relativity.**

Generalized symmetries of Theorem I anticipate conservation laws of solitons, etc.

* K. Brading, "A Note on General Relativity, Energy Conservation, and Noether's Theorems"
Cf. Arnowitt–Deser–Misner (1962)

Colliding Korteweg–de Vries solitons: ∞ hierarchy of conservation laws

Gardner, Greene, Kruskal, Miura, Zabulsky (1960s) $v_t - 6vv_x + v_{xxx} = 0$



General treatment: S. Coleman,
"Classical Lumps and Their Quantum Descendants"

Amalie Emmy Noether ✳ 23 March 1882 · Erlangen

GEORG
SIMON
OHM
1789-1854

Erlangen
Universitätstraße

Father, Max Noether, Professor of Mathematics at Erlangen from 1875

Algebraic geometry / curves on surfaces

Academies of Berlin, Göttingen, Munich, Budapest, Copenhagen, Turin, Accademia dei Lincei, Institut de France, London Mathematical Society

Felix Klein's *Inaugural Address* at Erlangen, 1872 set out a research plan to study geometry from the perspective of group theory.

Commentary: Garrett Birkhoff, M. K. Bennett, *Felix Klein and His "Erlanger Programm"* (1988).

Alfred Clebsch
Max Noether collaborator

44. Clebsch-Gordan Coefficients, Spherical Harmonics, and d Functions

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

	J	J	…
M	M	…	
m_1	m_2		Coefficients
m_1	m_2		
⋮	⋮		

1/2×1/2

m_1	m_2	$J=1$, $M=+1$
+1/2	+1/2	1

m_1	m_2	$J=1$, $M=0$	$J=0$, $M=0$
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	$J=1$, $M=-1$
−1/2	−1/2	1

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

2×1/2

m_1	m_2	$J=5/2$, $M=+5/2$
+2	+1/2	1

m_1	m_2	$J=5/2$, $M=+3/2$	$J=3/2$, $M=+3/2$
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

m_1	m_2	$J=5/2$, $M=+1/2$	$J=3/2$, $M=+1/2$
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	$J=5/2$, $M=-1/2$	$J=3/2$, $M=-1/2$
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	$J=5/2$, $M=-3/2$	$J=3/2$, $M=-3/2$
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

m_1	m_2	$J=5/2$, $M=-5/2$
−2	−1/2	1

1×1/2

m_1	m_2	$J=3/2$, $M=+3/2$
+1	+1/2	1

m_1	m_2	$J=3/2$, $M=+1/2$	$J=1/2$, $M=+1/2$
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	$J=3/2$, $M=-1/2$	$J=1/2$, $M=-1/2$
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	$J=3/2$, $M=-3/2$
−1	−1/2	1

3/2×1/2

m_1	m_2	$J=2$, $M=+2$
+3/2	+1/2	1

m_1	m_2	$J=2$, $M=+1$	$J=1$, $M=+1$
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

m_1	m_2	$J=2$, $M=0$	$J=1$, $M=0$
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	$J=2$, $M=-1$	$J=1$, $M=-1$
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	$J=2$, $M=-2$
−3/2	−1/2	1

2×1

m_1	m_2	$J=3$, $M=+3$
+2	+1	1

m_1	m_2	$J=3$, $M=+2$	$J=2$, $M=+2$
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	$J=3$, $M=+1$	$J=2$, $M=+1$	$J=1$, $M=+1$
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

m_1	m_2	$J=3$, $M=0$	$J=2$, $M=0$	$J=1$, $M=0$
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	$J=3$, $M=-1$	$J=2$, $M=-1$	$J=1$, $M=-1$
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	$J=3$, $M=-2$	$J=2$, $M=-2$
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	$J=3$, $M=-3$
−2	−1	1

1×1

m_1	m_2	$J=2$, $M=+2$
+1	+1	1

m_1	m_2	$J=2$, $M=+1$	$J=1$, $M=+1$
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	$J=2$, $M=0$	$J=1$, $M=0$	$J=0$, $M=0$
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	$J=2$, $M=-1$	$J=1$, $M=-1$
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	$J=2$, $M=-2$
−1	−1	1

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

3/2×3/2

m_1	m_2	$J=3$, $M=+3$
+3/2	+3/2	1

m_1	m_2	$J=3$, $M=+2$	$J=2$, $M=+2$
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

m_1	m_2	$J=3$, $M=+1$	$J=2$, $M=+1$	$J=1$, $M=+1$
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

m_1	m_2	$J=3$, $M=0$	$J=2$, $M=0$	$J=1$, $M=0$	$J=0$, $M=0$
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

m_1	m_2	$J=3$, $M=-1$	$J=2$, $M=-1$	$J=1$, $M=-1$
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

m_1	m_2	$J=3$, $M=-2$	$J=2$, $M=-2$
−1/2	−3/2	1/2	1/2
−3/2	−1/2	1/2	−1/2

m_1	m_2	$J=3$, $M=-3$
−3/2	−3/2	1

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^1_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

2×3/2

m_1	m_2	$J=7/2$, $M=+7/2$
+2	+3/2	1

m_1	m_2	$J=7/2$, $M=+5/2$	$J=5/2$, $M=+5/2$
+2	+1/2	3/7	4/7
+1	+3/2	4/7	−3/7

m_1	m_2	$J=7/2$, $M=+3/2$	$J=5/2$, $M=+3/2$	$J=3/2$, $M=+3/2$
+2	−1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	−2/5
0	+3/2	2/7	−18/35	1/5

m_1	m_2	$J=7/2$, $M=+1/2$	$J=5/2$, $M=+1/2$	$J=3/2$, $M=+1/2$	$J=1/2$, $M=+1/2$
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10

m_1	m_2	$J=7/2$, $M=-1/2$	$J=5/2$, $M=-1/2$	$J=3/2$, $M=-1/2$	$J=1/2$, $M=-1/2$
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5

m_1	m_2	$J=7/2$, $M=-3/2$	$J=5/2$, $M=-3/2$	$J=3/2$, $M=-3/2$
0	−3/2	2/7	18/35	1/5
−1	−1/2	4/7	−1/35	−2/5
−2	+1/2	1/7	−16/35	2/5

m_1	m_2	$J=7/2$, $M=-5/2$	$J=5/2$, $M=-5/2$
−1	−3/2	4/7	3/7
−2	−1/2	3/7	−4/7

m_1	m_2	$J=7/2$, $M=-7/2$
−2	−3/2	1

2×2

m_1	m_2	$J=4$, $M=+4$
+2	+2	1

m_1	m_2	$J=4$, $M=+3$	$J=3$, $M=+3$
+2	+1	1/2	1/2
+1	+2	1/2	−1/2

m_1	m_2	$J=4$, $M=+2$	$J=3$, $M=+2$	$J=2$, $M=+2$
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	−3/7
0	+2	3/14	−1/2	2/7

m_1	m_2	$J=4$, $M=+1$	$J=3$, $M=+1$	$J=2$, $M=+1$	$J=1$, $M=+1$
+2	−1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	−1/14	−3/10
0	+1	3/7	−1/5	−1/14	3/10
−1	+2	1/14	−3/10	3/7	−1/5

m_1	m_2	$J=4$, $M=0$	$J=3$, $M=0$	$J=2$, $M=0$	$J=1$, $M=0$	$J=0$, $M=0$
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5

m_1	m_2	$J=4$, $M=-1$	$J=3$, $M=-1$	$J=2$, $M=-1$	$J=1$, $M=-1$
+1	−2	1/14	3/10	3/7	1/5
0	−1	3/7	1/5	−1/14	−3/10
−1	0	3/7	−1/5	−1/14	3/10
−2	+1	1/14	−3/10	3/7	−1/5

m_1	m_2	$J=4$, $M=-2$	$J=3$, $M=-2$	$J=2$, $M=-2$
0	−2	3/14	1/2	2/7
−1	−1	4/7	0	−3/7
−2	0	3/14	−1/2	2/7

m_1	m_2	$J=4$, $M=-3$	$J=3$, $M=-3$
−1	−2	1/2	1/2
−2	−1	1/2	−1/2

m_1	m_2	$J=4$, $M=-4$
−2	−2	1

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,2} = \Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2} \qquad d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta \qquad d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta - 1)$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta - 1}{2}\cos\frac{\theta}{2} \qquad d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta + 1}{2}\sin\frac{\theta}{2} \qquad d^2_{2,-2} = \Big(\frac{1-\cos\theta}{2}\Big)^2 \qquad d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta + 1) \qquad d^2_{0,0} = \Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

Figure 44.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).

Paul Gordan
Erlangen Professor, 1874–1912
“Ein Algorithmiker”

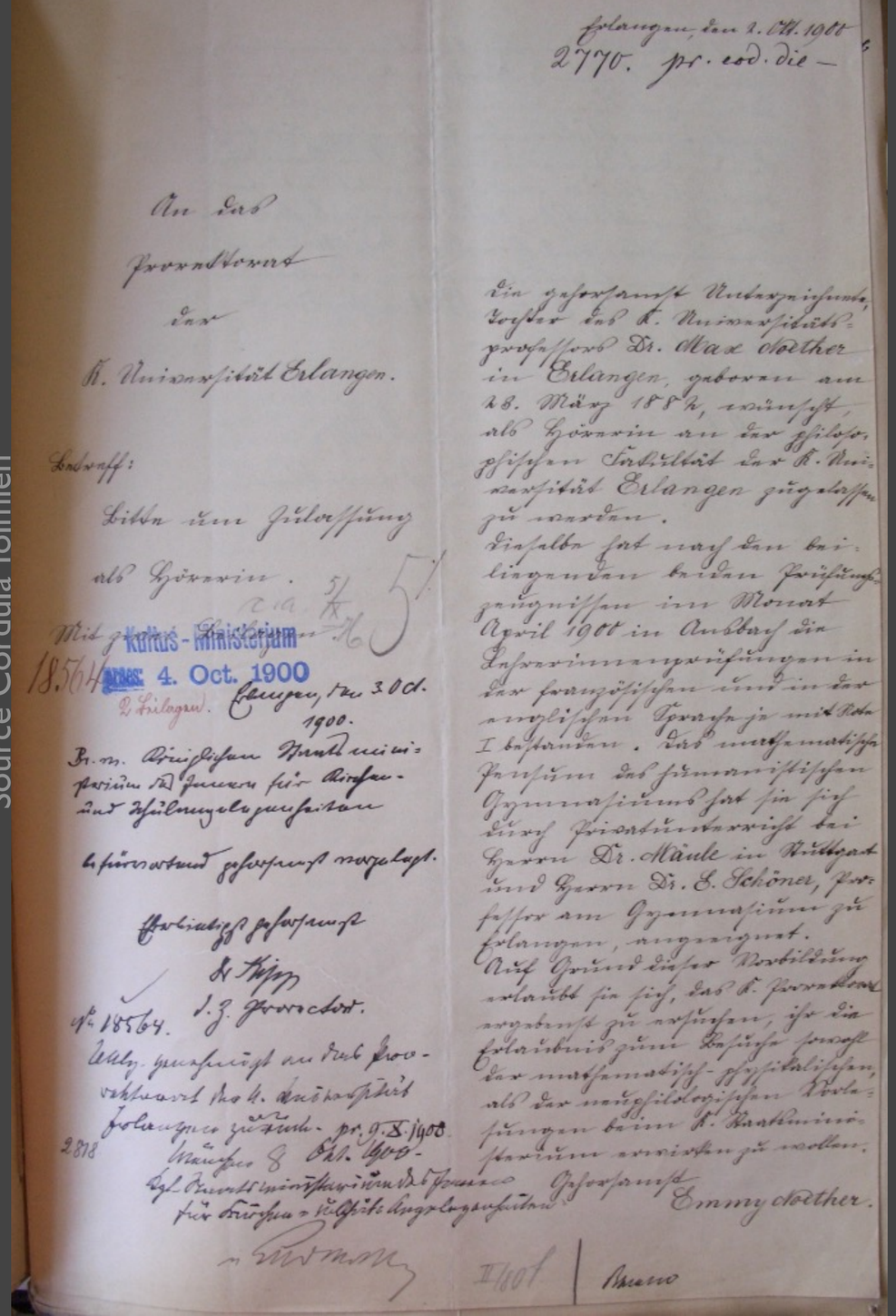
1889–1897: Städtische Höheren Töchterschule

1900: Bavarian State Exam for teachers of French and English

1900: Could not enroll in University of Erlangen, permitted to audit

Admission of women would overthrow all academic order

—Erlangen Academic Senate, 1898



Erlangen, den 2. Oct[ober] 1900
An das Prorektorat der K[öniglichen] Universität Erlangen
Betreff: Bitte um Zulassung als Hörerin

Die gehorsamst Unterzeichnete Tochter des k[öniglichen] Universitätsprofessors Dr. Max Noether in Erlangen, geboren am 23. März 1882, wünscht, als Hörerin an der philosophischen Fakultät der K[öniglichen] Universität Erlangen zugelassen zu werden.

Dieselbe hat nach den beiliegenden beiden Prüfungszeugnissen im Monat April 1900 in Ansbach die Lehrerinnenprüfungen in der französischen und in der englischen Sprache mit Note I bestanden. Das mathematische Pensum des humanistischen Gymnasiums hat sie sich durch Privatunterricht bei Herrn Dr. Mäule in Stuttgart und Herrn Dr. E[rnst] Schöner, Professor am Gymnasium zu Erlangen, angeeignet.

Auf Grund dieser Vorbildung erlaubt sie sich, das K[öniglichen] Prorektorat ergebenst zu ersuchen, ihr die Erlaubnis zum Besuche sowohl der mathematisch-physikalischen, als der neuphilologischen Vorlesungen beim K[öniglichen] Staatsministerium erwirken zu wollen.

Gehorsamst Emmy Noether

Detailed account: C. Tollmien, Mathematik und Gender 5 (2016) 1-12

Source Cordula Tolmien

1903: Passed *Reifeprüfung* (university qualification), enrolled in University of Göttingen. *Lecture courses given by Karl Schwarzschild, Hermann Minkowski, Felix Klein, David Hilbert, …*

1904: Admitted to Uni-Erlangen as student of mathematics.

1907: *D. Phil.* summa cum laude under Paul Gordan (his only student)
Über die Bildung des Formensystems der ternären biquadratischen Form
(On the construction of the system of forms of a ternary quartic form)
Computation of all 331 invariants of a homogeneous polynomial

Mist!

(First woman math Ph.D. in Europe: Sofia Kovalevskaya [Göttingen, 1874, with Karl Weierstraß]; Full Prof. Stockholm 1889, d. 1891)

1908–1915: Unpaid member of Erlangen Mathematical Institute
influence of Ernest Fischer

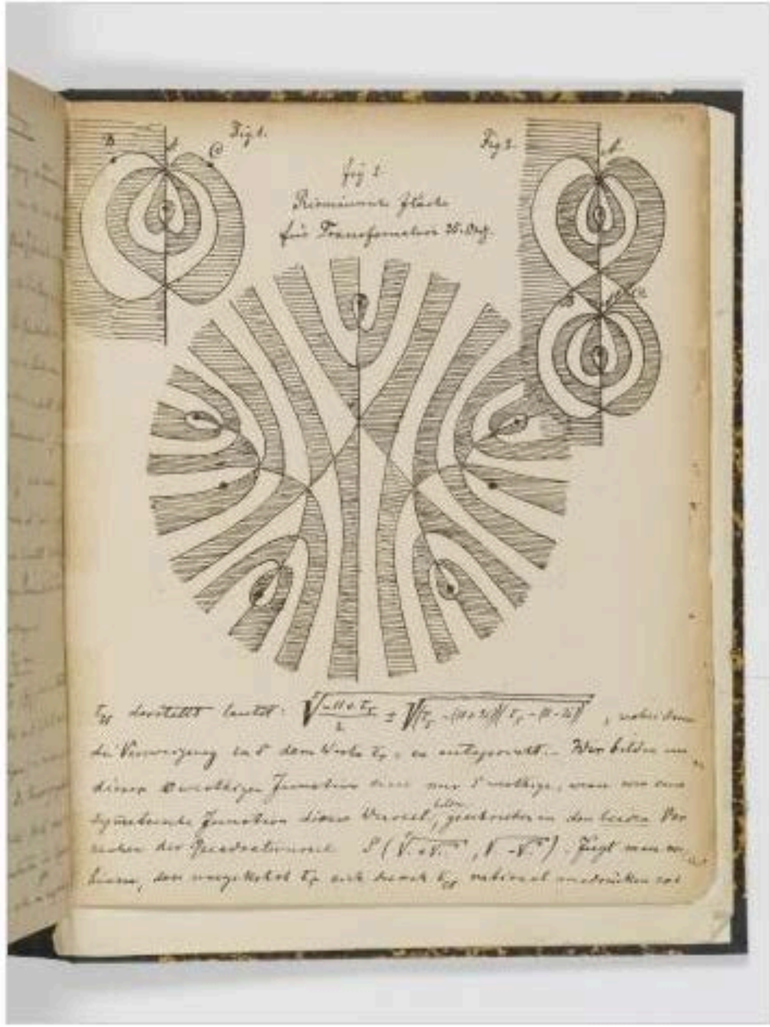
1915: Invited to Göttingen (Mount Olympus) by Klein & Hilbert

Historische Persönlichkeiten Göttingens in der Mathematik

- Bernstein, Felix (1878-1956)
- Carathédory, Constantin (1873-1950)
- Clebsch, Alfred (1833-1872)
- Cohn-Vossen, Stefan (1902-1936)
- Courant, Richard (1888-1972)
- Deuring, Max (1907-1984)
- Dirichlet (Lejeune-), Peter Gustav (1805-1859)
- Gauß, Carl Friedrich (1777-1855)
- Hecke, Erich (1887-1947)
- Herglotz, Gustav (1881-1953)
- Hilbert, David (1862-1943)
- Kästner, Abraham Gotthelf (1719-1800)
- Klein, Felix (1849-1925)
- Landau, Edmund (1877-1938)
- Minkowski, Hermann (1864-1909)
- Noether, Amalie Emmy (1882-1935)
- Reidemeister, Kurt (1893-1971)
- Rellich, Franz (1906-1955)
- Riemann, Bernhard (1826-1866)
- Runge, Carl David Tolmé (1856-1927)
- Schwarz, Hermann Amandus (1843-1921)
- Siegel, Carl-Ludwig (1896-1981)
- Weyl, Hermann (1885-1955)

Carl Friedrich Gauß (1777-1855) war seit 1807 Direktor der Sternwarte in Göttingen. Er gilt bis heute als einer der bedeutendsten Mathematiker aller Zeiten.

Klein Protokolle



"Two plain shelves in Göttingen, in the entrance room of the *Mathematisches Institut* library, hold an astounding collection of mathematical manuscripts and rare books …

The collection runs from early handwritten lectures by Dirichlet, Riemann, and Clebsch, through almost 100 volumes by Hilbert, to volumes of Minkowski, Hasse, and Siegel on number theory, Noether on algebra, and Max Born on quantum mechanics. But the largest and most impressive of its centerpieces is the *Seminar-Protokolle* of Felix Klein: a detailed handwritten registry, spanning over 8,000 pages in 29 volumes, of forty years of seminar lectures by him, his colleagues and students, and distinguished visitors. These *Protocols*, previously unpublished, are now available digitally, as part of a project sponsored by the Clay Mathematics Institute. They constitute one of the richest records of mathematical activity in modern times."

From *The Felix Klein Protocols* by Eugene Chislenko and Yuri Tschinkel, Notices of the AMS, September 2007, pp 961–970

This image is from Volume I, part 2, page 113, seminar of Sunday, February 14, 1880.

1915: Habilitation lecture to become *Privatdozent* in Göttingen, with unanimous support of the Math / Science Department of the Philosophical Faculty

"I have had up to now uniformly unsatisfactory experiences with female students and I hold that the female brain is unsuited to mathematical production. Miss Noether seems to be a rare exception."

—Göttingen Mathematician Edmund Landau in his referee report for the Habilitation of Emmy Noether, 1915

Historical-Philological Department *opposed*

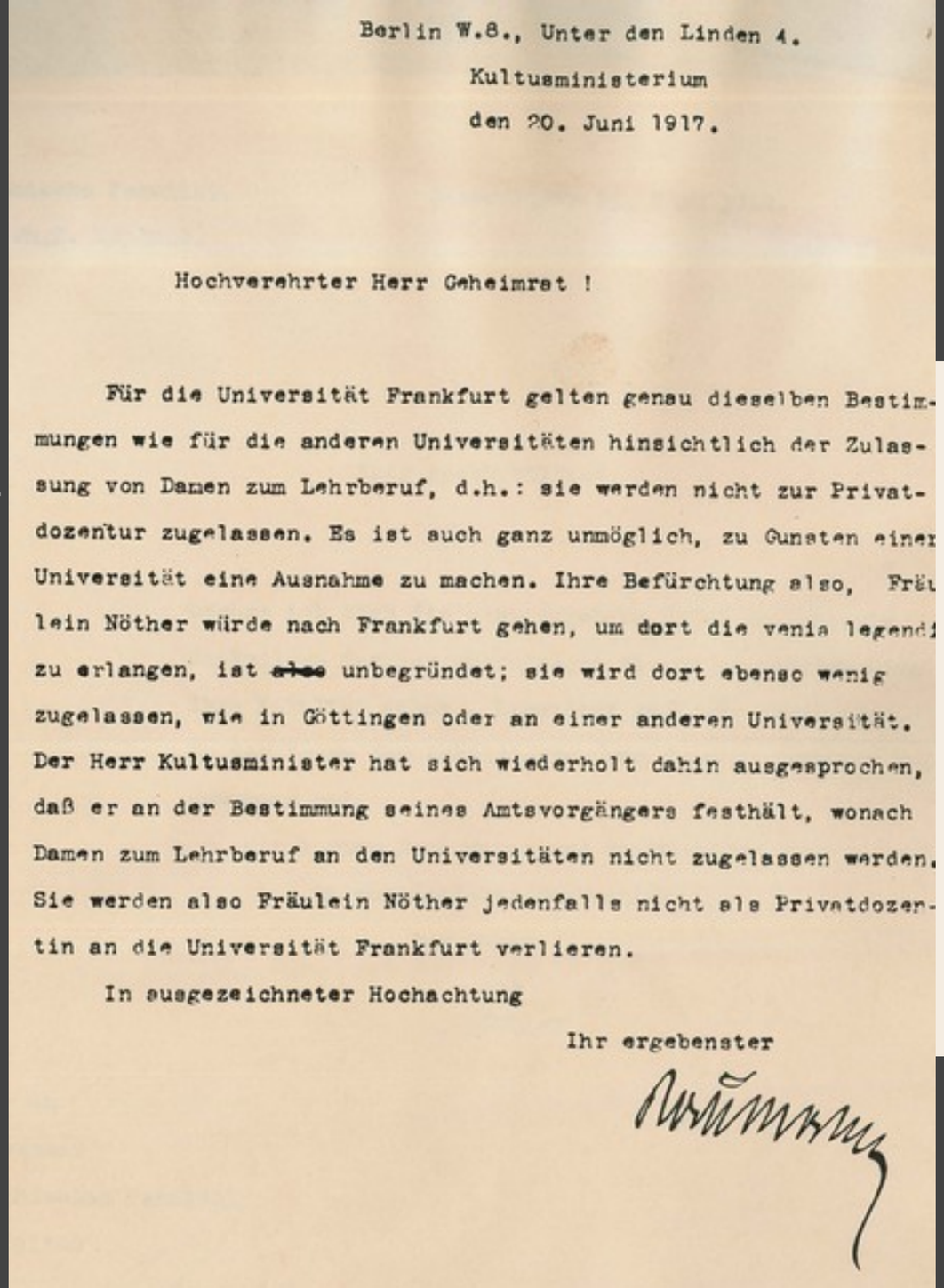
Concern that seeing a female organism might be distracting to the students.

Special vote against the Habilitation of Emmy Noether, 19 November 1915

Not approved; EN permitted to lecture under Hilbert's name

Full German texts in C. Tollmien, "Weibliches Genie: Frau und Mathematiker: Emmy Noether"

1917: Another failed try



Berlin W.8., Unter den Linden 4.
Kultusministerium
den 20. Juni 1917.

Hochverehrter Herr Geheimrat !

Für die Universität Frankfurt gelten genau dieselben Bestimmungen wie für die anderen Universitäten hinsichtlich der Zulassung von Damen zum Lehrberuf, d.h.: sie werden nicht zur Privatdozentur zugelassen. Es ist auch ganz unmöglich, zu Gunsten einer Universität eine Ausnahme zu machen. Ihre Befürchtung also, Fräulein Nöther würde nach Frankfurt gehen, um dort die venia legendi zu erlangen, ist ~~also~~ unbegründet; sie wird dort ebenso wenig zugelassen, wie in Göttingen oder an einer anderen Universität. Der Herr Kultusminister hat sich wiederholt dahin ausgesprochen, daß er an der Bestimmung seines Amtsvorgängers festhält, wonach Damen zum Lehrberuf an den Universitäten nicht zugelassen werden. Sie werden also Fräulein Nöther jedenfalls nicht als Privatdozentin an die Universität Frankfurt verlieren.

In ausgezeichneter Hochachtung

Ihr ergebenster

With regard to accepting women to teaching positions, the regulations of Frankfurt University are identical to those of all the universities: women are not allowed to be appointed to positions of external lecturers. It is completely impossible to make an exception to the rule in one university. Therefore, your concern that Miss. Noether will leave, move to Frankfurt and receive a position there is completely unfounded: she will not be given the right to teach there, just as she will not receive such a thing in Göttingen or in any other university. The Minister of Education has expressed this time and time again and emphasized that he supports his predecessor's instructions, and therefore women will not be permitted to receive teaching positions in universities.

Therefore, there is no concern that you will lose Miss Noether as an external lecturer in Frankfurt University.

Edelstein Collection, National Library of Israel

1919: Legal status of women improved after War of 1914–1918 (Weimar Republic)

Habilitation granted on the basis of *Invariante Variationsprobleme*

1918: Hermann Weyl speculated on a scale-symmetry–based unified theory of electromagnetism and gravitation. *Failed!*

1929: After invention of quantum mechanics, succeeded in deriving electrodynamics from a Noetherian symmetry principle: invariance under local variations in the convention for the phase of a QM wave function: U(1) symmetry.

1931: Dirac invents QED, discusses "non-integrable phase."

1959: Aharonov & Bohm establish that potentials contain too much information, fields too little, path-dependent phase factors just the right amount.

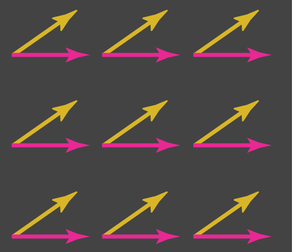
Weyl in 1955: The strongest argument for my theory seemed to be this: the gauge invariance corresponds to the principle of conservation of electric charge as the coordinate invariance corresponds to the conservation law of energy and momentum.

Charge conservation: Borexino, *PRL* (2015)
$\tau(e^- \rightarrow \nu\gamma) \geq 6.6 \times 10^{28}$ yr at 90% CL

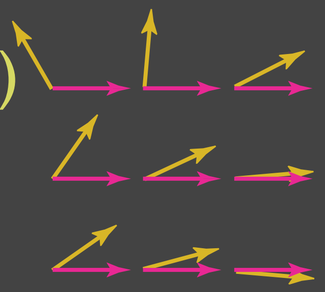
Why is charge conserved?

1) Maxwell's equations. *But they are built to conserve charge (addition of the displacement current to Ampère's law).*

2) *Global* phase invariance (Theorem I) implies a conserved charge, which we *identify* as the electric charge.



3) *Local* phase invariance (Theorem II) gives a theory of electromagnetism, full content of Maxwell's equations.



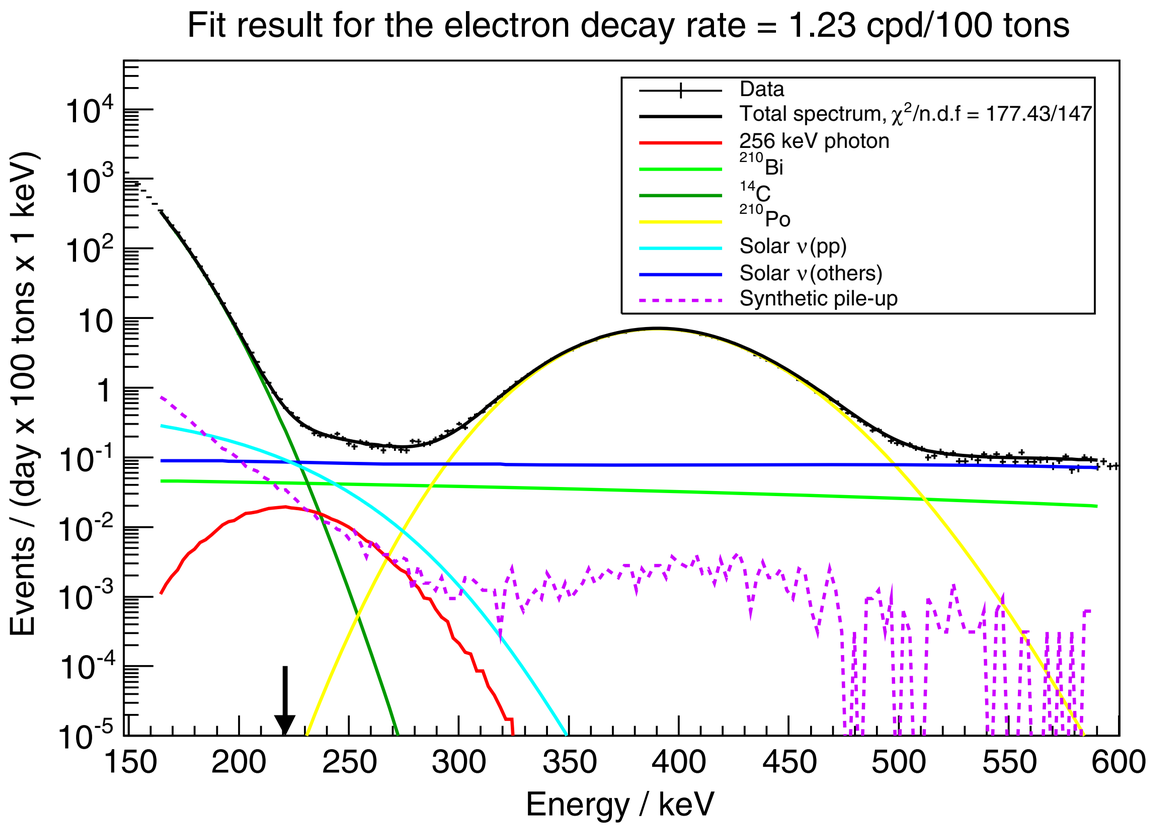


FIG. 1 (color online). Energy spectrum between 150 and 600 keV. The most prominent features are the ^{14}C β spectrum (green line), the peak at about 400 keV from ^{210}Po α decays, and the solar neutrinos, grouped in the blue curve except for the crucial pp neutrinos, which are shown in cyan. The effect of event pileup, mostly overlapping ^{14}C events, is shown in dashed pink. The hypothetical monoenergetic 256 keV γ line is shown in red at its 90% exclusion C.L. with an arrow indicating the mean value of the detected energy, which is lower than 256 keV because of quenching. The fit is done in the range 164–590 keV.

Invariant Variational Problems made waves in GR circles, but was not otherwise an instant sensation: a "Sleeping Beauty"

Heisenberg, who would later say "In the beginning was the symmetry," that is certainly more correct than the Democritean thesis, "in the beginning was the particle." The elementary particles embody the symmetries, they are their simplest representations, but they are only a consequence of the symmetries, probably never read Noether: "[I]t did not penetrate into quantum theory, so I didn't realize the importance to that paper." (cf. YK-S, pp. 85-86)

They were preoccupied with inventing Quantum Mechanics, which unleashes more of the potency of Noether's theorems. Internal symmetries not yet conceived.

W. Heisenberg,
Der Teil und das Ganze, p. 280

Link between spacetime translation invariance and 4-momentum conservation did not dissuade Bohr from asking whether the conservation law might only be satisfied statistically in radiative processes and β-decay

THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[SIXTH SERIES.]

MAY 1924.

γ

LXXVI. *The Quantum Theory of Radiation.*
By N. BOHR, H. A. KRAMERS, *and* J. C. SLATER *.

Introduction.

IN the attempts to give a theoretical interpretation of the mechanism of interaction between radiation and matter, two apparently contradictory aspects of this mechanism have been disclosed. On the one hand, the phenomena of interference, on which the action of all optical instruments essentially depends, claim an aspect of continuity of the same character as that involved in the wave theory of light, especially developed on the basis of the laws of classical electrodynamics. On the other hand, the exchange of energy and momentum between matter and radiation, on which the observation of optical phenomena ultimately depends, claims essentially discontinuous features. These have even led to the introduction of the theory of light-quanta, which in its most extreme form denies the wave constitution of light. At the present state of science it does not seem possible to avoid the formal character of the quantum theory which is shown by the fact that the interpretation of atomic phenomena does not involve a description of the mechanism of the discontinuous processes, which in the quantum theory of spectra are designated as transitions between stationary states of the atom. On the correspondence principle it seems

* Communicated by the Authors.

β

Faraday Lecture.

(DELIVERED BEFORE THE FELLOWS OF THE CHEMICAL SOCIETY AT THE SALTERS' HALL ON MAY 8TH, 1930.)

By NIELS BOHR.

*Chemistry and the Quantum Theory of Atomic Constitution.**

IT is with a feeling of deep reverence that I accept the kind invitation of the Chemical Society to deliver this lecture in commemoration of the great genius to whom we owe so large a part of the common foundation on which chemists and physicists build to-day.

→ 1936

Göttingen: The Mother of Modern Algebra: Rings and Ideals

1922: Außerordentlicher Professor

1928-1929: Moscow

1930: Frankfurt

1932: Alfred Ackermann-Teubner Award with Emil Artin

1932: Plenary lecture at International Congress of Mathematicians, Zürich

edited *Math. Annalen*

Die Noetherknaben / "Der" Noether

„Göttinger Tageblatt" Mittwoch, den 26. April 1933.

6 Göttinger Professoren beurlaubt

Weitere werden folgen

Kultusminister Rust hat, wie uns aus Berlin gedrahtet wird, bis zur endgültigen Entscheidung auf Grund des Beamtengesetzes an der Universität Göttingen nachstehende Dozenten beurlaubt: die Professoren Honig, Courant, Born, Emmy Noether, Bernstein und Bondi. Weitere Beurlaubungen werden folgen.

Außer den Beurlaubungen behält sich das Kultusministerium vor, in der nächsten Zeit zur Umgestaltung der Hochschulen und zur Wiederherstellung ihres bodenständigen Charakters auf Grund des § 5 des Gesetzes zur Wiederherstellung des Berufsbeamtentums vom 7. April 1933 eine Reihe von Versetzungen vorzunehmen.

*

Richard Honig wurde 1890 in Gnesen geboren. Nach seinem Studium in München und Breslau habilitierte er sich 1919 in Göttingen und wurde hier am 1. Juli 1931 zum ordentlichen Professor des Strafrechts ernannt.

Richard Courant, der 1888 zu Lublinitz geboren wurde, studierte an den Universitäten Breslau und Göttingen und am Züricher Polytechnikum und habilitierte sich später in Göttingen für Mathematik.

Auch Max Born ist gebürtiger Schlesier. Nach dem Besuch der Universitäten Breslau, Heidelberg, Zürich und Göttingen und einem Studienaufenthalt in England übernahm er 1909 in Göttingen nach dem Tode Minkowskis die Fortführung seiner Arbeiten auf dem Gebiete der Elektrodynamik.

Fräulein Emmy Noether, die in Erlangen geboren ist, studierte in ihrer Heimatstadt und in Göttingen und habilitierte sich 1919 in Göttingen, wo sie vor etwa einem Jahr zum Professor der Matthematik ernannt wurde.

Felix Bernstein wurde 1878 zu Halle geboren. Nach dem Studium in München, Halle, Berlin und Göttingen habilitierte er sich 1903 in Halle für Mathematik und kam im Herbst 1909 nach Göttingen.

Curt Werner Bondy ist Hamburger und studierte in Göttingen und Kiel, sowie nach Kriegsende in Hamburg. Hier habilitierte er sich 1925 für Sozialpsychologie und Sozialpädagogik und erhielt 1930 in Göttingen einen Lehrauftrag für Sozialpädagogik.

*

Weitere Beurlaubungen erfolgten:

an der Universität Frankfurt a. M.: Salomon, Mennicke, M. Wertheimer, Strupp, Weil, Pribram, Richard Koch, Glatzer, Pleßner, Sommerfeld, Walter Fränkel, Fritz Mayer, Ernst Kahn, Neumark, Ernst Cohn, Braun, Ludwig Wertheimer und Altschul;

an der Universität Marburg (Lahn): Röpke, Jacobsohn;

an der Universität Königsberg i. Pr.: Professor Hensel; eine Wiederverwendung von Professor Hensel ist in Aussicht genommen,

an der Handelshochschule Königsberg i. Pr.: die Professoren Rogowsky, Hänsler und Kürbs;

an der Universität Kiel: die Professoren Colm, Neisser, Adolf Fraenkel, Hussert, Stenzel, Liepe, Rauch, Schücking, Opet. Ueber Professor Harms (Kiel) und Professor v. Hentig (Kiel) bleibt Verfügung vorbehalten.

Weitere Beurlaubungen werden folgen. Außer den Beurlaubungen behält sich das Kultusministerium vor, in der nächsten Zeit zur Umgestaltung der Hochschulen und zur Wiederherstellung ihres bodenständigen Charakters vor allem an den Grenzuniversitäten Breslau, Kiel und Königsberg auf Grund des § 5 des Gesetzes zur Wiederherstellung des Berufsbeamtentums vom 7. April 1933 eine Reihe von Versetzungen vorzunehmen.

Professor Kahrstedt über die Judenfrage

in der „Morningpost".

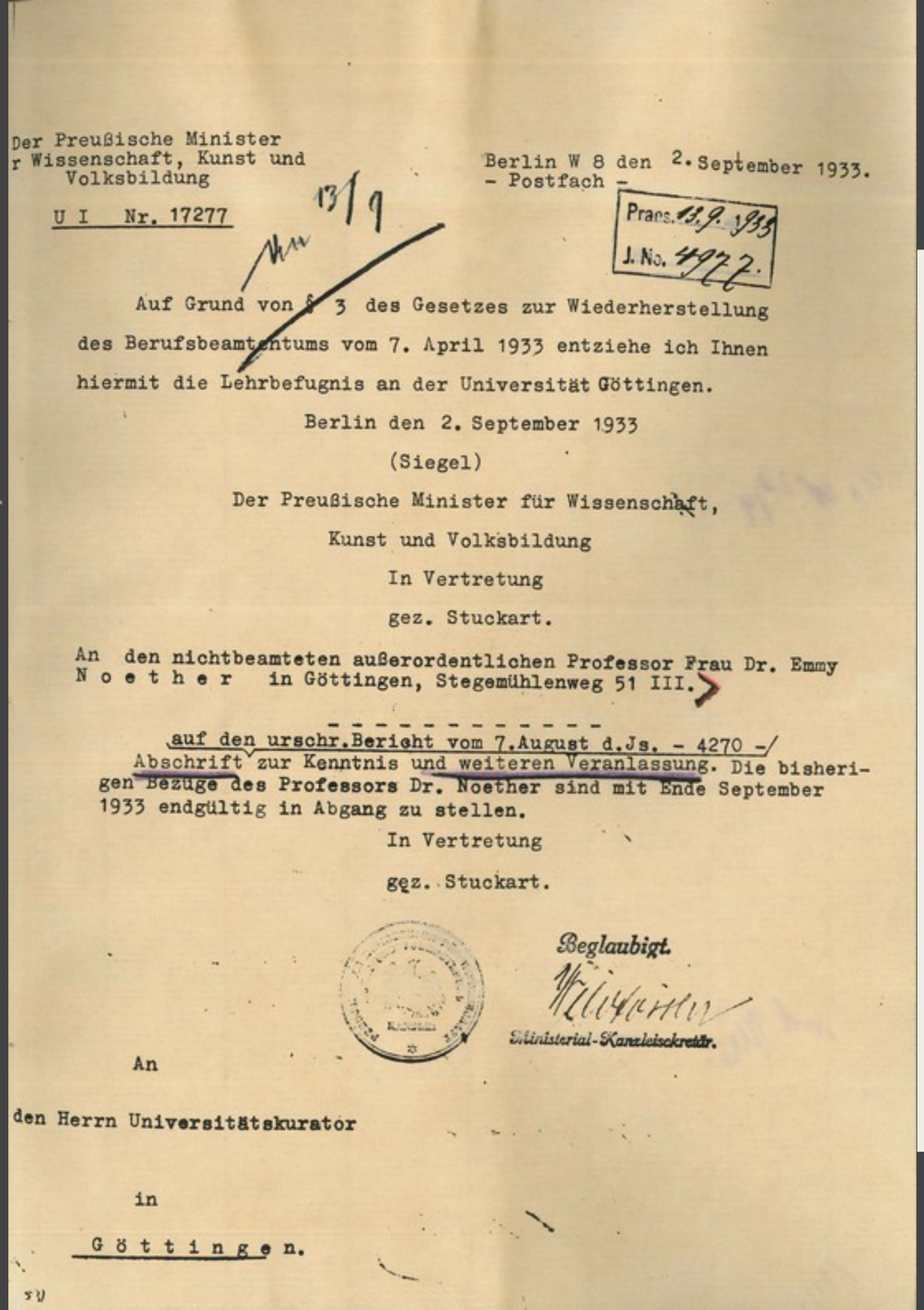
Die konservative „Morningpost" veröffentlicht einen ausführlichen Bericht des Göttinger Geschichtsprofessors Ulrich Kahrstedt, in dem die antisemitische Politik der Nationalsozialisten historisch begründet und verteidigt wird. Kahrstedt weist besonders auf die kommunistische Seite der Judenfrage hin.

Expelled from Math/Physics Faculty, 1933

Felix Bernstein
Max Born
Richard Courant
Emmy Noether

S. Mac Lane, "Mathematics at Göttingen under the Nazis"

Edelstein Collection, National Library of Israel



Der Preußische Minister
r Wissenschaft, Kunst und
Volksbildung

U I Nr. 17277

Berlin W 8 den 2. September 1933.
- Postfach -

Praes. 13.9. 1933
J. No. 4972.

Auf Grund von § 3 des Gesetzes zur Wiederherstellung des Berufsbeamtentums vom 7. April 1933 entziehe ich Ihnen hiermit die Lehrbefugnis an der Universität Göttingen.

Berlin den 2. September 1933

(Siegel)

Der Preußische Minister für Wissenschaft,
Kunst und Volksbildung
In Vertretung
gez. Stuckart.

An den nichtbeamteten außerordentlichen Professor Frau Dr. Emmy Noether in Göttingen, Stegemühlenweg 51 III.

- - - - - - - - - - - -

auf den urschr. Bericht vom 7. August d. Js. - 4270 -/
Abschrift zur Kenntnis und weiteren Veranlassung. Die bisherigen Bezüge des Professors Dr. Noether sind mit Ende September 1933 endgültig in Abgang zu stellen.

In Vertretung
gez. Stuckart.

Beglaubigt.
Ministerial-Kanzleisekretär.

An
den Herrn Universitätskurator
in
Göttingen.

Letter of the Prussian Minister for Science, Art and Adult Education, Berlin, September 2, 1933

The Prussian minister responsible for science, art and adult education,,

On the basis of article 3 of "The Law for the Restoration of the Professional Civil Service" from April 1933, I nullify your teaching permit in Göttingen University.

Signed, Stuckhart, on behalf of the minister.

To Professor, the external lecturer Professor Ms. Dr. Emmy Noether in Göttingen.

True copy of the report from August 7, 1933.

For your information, please note and carry out.

The wages of Professor Emmy Noether must be ceased as of end of September 1933.

TO JOIN BRYN MAWR.

Dr. Emmy Noether, Ousted by Nazis, Will Be on Faculty.

Special to THE NEW YORK TIMES.

BRYN MAWR, Pa., Oct. 3.—President Marion Edwards Park at the opening of Bryn Mawr College today announced that Bryn Mawr was to have in its faculty for two years Dr. Emmy Noether, formerly of the University of Göttingen, She was asked, with other members of the Göttingen faculty, to resign last Spring, under the Nazi regime.

The appointment of Dr. Noether was made possible by a gift from the Institute of International Education and the Rockefeller Foundation.

The New York Times

Published: October 4, 1933

Copyright © The New York Times

1934

Cf. Qinna Shen, "A Refugee Scholar from Nazi Germany: Emmy Noether and Bryn Mawr College"

Bryn Mawr, 1933–1935: graduate students weekly trips to IAS, Princeton for seminars and lecture courses.

Spring vacation, 1935: surgery, sepsis, death.

Famous Mathematician, Dr. Emmy Noether, Dies

The College was shocked and grieved to hear that Dr. Emmy Noether, one of the most eminent modern mathematicians, and visiting lecturer in mathematics at Bryn Mawr, died on April 14 after undergoing a serious operation.

Dr. Noether was born at Erlangen in 1882, the university at which her father, Dr. Max Noether, was a professor of mathematics of great note. Dr. Emmy Noether received the degree of Doctor of Philosophy from Erlangen in 1907. She was Privatdozent and Professor of Mathematics at the University of Göttingen. Some of the most distinguished German students of mathematics have been her pupils. The Rockefeller Foundation and the Emergency Committee in Aid of Displaced German Scholars made it possible for the Department of Mathematics to invite her to Bryn Mawr. Her special field was modern algebra, in which she solved difficult problems of calculus, and about which she wrote in many German mathematical periodicals.

Dr. Noether came to Bryn Mawr in the fall of 1933. Last year she gave a course in Modern Algebra to four graduate students, and this year three research fellows with unusual previous records were especially invited to work with her. Miss Stauffer has just completed the thesis on which she was working with Dr. Noether. Miss Monroe is the only undergraduate who has studied with her.

THE LATE EMMY NOETHER.

Professor Einstein Writes in Appreciation of a Fellow-Mathematician.

To the Editor of The New York Times:

The efforts of most human beings are consumed in the struggle for their daily bread, but most of those who are, either through fortune or some special gift, relieved of this struggle are largely absorbed in further improving their worldly lot. Beneath the effort directed toward the accumulation of worldly goods lies all too frequently the illusion that this is the most substantial and desirable end to be achieved; but there is, fortunately, a minority composed of those who recognize early in their lives that the most beautiful and satisfying experiences open to humankind are not derived from the outside, but are bound up with the development of the individual's own feeling, thinking and acting. The genuine artists, investigators and thinkers have always been persons of this kind. However inconspicuously the life of these individuals runs its course, none the less the fruits of their endeavors are the most valuable contributions which one generation can make to its successors.

Within the past few days a distinguished mathematician, Professor Emmy Noether, formerly connected with the University of Goettingen and for the past two years at Bryn Mawr College, died in her fifty-third year. In the judgment of the most competent living mathematicians, Fraeulein Noether was the most significant creative mathematical genius thus far produced since the higher education of women began. In the realm of algebra, in which the most gifted mathematicians have been busy for centuries, she discovered methods which have proved of enormous importance in the development of the present-day younger generation of mathematicians. Pure mathematics is, in its way, the poetry of logical ideas. One seeks the most general ideas of operation which will bring together in simple, logical and unified form the largest possible circle of formal relationships. In this effort toward logical beauty spiritual formulae are discovered necessary for the deeper penetration into the laws of nature.

Born in a Jewish family distinguished for the love of learning, Emmy Noether, who, in spite of the efforts of the great Goettingen mathematician, Hilbert, never reached the academic standing due her in her own country, none the less surrounded herself with a group of students and investigators at Goettingen, who have already become distinguished as teachers and investigators. Her unselfish, significant work over a period of many years was rewarded by the new rulers of Germany with a dismissal, which cost her the means of maintaining her simple life and the opportunity to carry on her mathematical studies. Farsighted friends of science in this country were fortunately able to make such arrangements at Bryn Mawr College and at Princeton that she found in America up to the day of her death not only colleagues who esteemed her friendship but grateful pupils whose enthusiasm made her last years the happiest and perhaps the most fruitful of her entire career.

ALBERT EINSTEIN.

Princeton University, May 1, 1935.

Emmy Noether marker, Bryn Mawr College Cloisters

Hermann Weyl:
I have a vivid recollection of her when I was in Göttingen as visiting professor in the winter semester of 1926-1927, and lectured on representations of continuous groups. She was in the audience; for just at that time the hypercomplex number systems and their representations had caught her interest and I remember many discussions when I walked home after the lectures, with her and von Neumann, who was in Göttingen as a Rockefeller Fellow, through the cold, dirty, rain-wet streets of Göttingen. When I was called permanently to Göttingen in 1930, I earnestly tried to obtain from the Ministerium a better position for her, because I was ashamed to occupy such a preferred position beside her whom I knew to be my superior as a mathematician in many respects.

Pavel Alexandrov:
With the death of Emmy Noether I lost the acquaintance of one of the most captivating human beings I have ever known. Her extraordinary kindness of heart, alien to any affectation or insincerity; her cheerfulness and simplicity; her ability to ignore everything that was unimportant in life-created around her an atmosphere of warmth, peace and good will which could never be forgotten by those who associated with her. … Though mild and forgiving, her nature was also passionate, temperamental, and strong-willed; she always stated her opinions forthrightly, and did not fear objections. It was moving to see her love for her students, who comprised the basic milieu in which she lived and replaced the family she did not have. Her concern for her students' needs, both scientific and worldly, her sensitivity and responsiveness, were rare qualities. Her great sense of humor, which made both her public appearances and informal association with her especially pleasant, enabled her to deal lightly and without ill will with all of the injustices and absurdities which befell her in her academic career. Instead of taking offense in these situations, she laughed.

van der Waerden & Noether, 1929

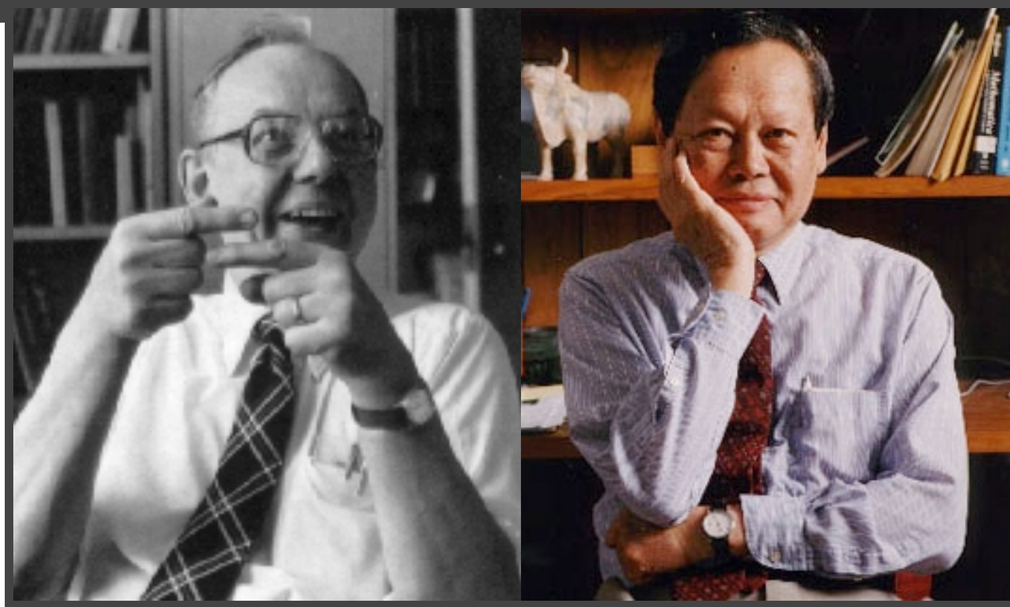
This entirely non-visual and noncalculative mind of hers was probably one of the main reasons why her lectures were difficult to follow. She was without didactic talent, and the touching efforts she made to clarify her statements, even before she had finished pronouncing them, by rapidly adding explanations, tended to produce the opposite effect. And yet, how profound the impact of her lecturing was. Her small, loyal audience, usually consisting of a few advanced students and often of an equal number of professors and guests, had to strain enormously in order to follow her. Yet those who succeeded gained far more than they would have from the most polished lecture. She almost never presented completed theories; usually they were in the process of being developed. Each of her lectures was a program. And no one was happier than she herself when this program was carried out by her students. Entirely free of egotism and vanity she never asked anything for herself but first of all fostered the work of her students. She always wrote the introductions to our papers …

Isotopic Spin Conservation and a Generalized Gauge Invariance*

by C. N. Yang and R. Mills in Upton, N.Y.

Phys. Rev. 95 (1954) 631

The conservation of isotopic spin points to the existence of a fundamental invariance law similar to the conservation of electric charge. In the latter case, the electric charge serves as a source of electromagnetic field; an important concept in this case is that a gauge invariance which is closely connected with (1) the equation of motion of the electromagnetic field, (2) the existence of a current density, and (3) the possible interactions between a charged field and the electromagnetic field. We have tried to generalize this concept of gauge invariance to apply to isotopic spin conservation. It turns out that a very natural generalization is possible. The field that plays the role of the electromagnetic field is here a vector field that satisfies a nonlinear equation even in the absence of other fields. (This is because unlike the electromagnetic field this field has an isotopic spin and consequently acts as a source of itself.) The existence of a current density is automatic, and the interaction of this field with any fields of arbitrary isotopic spin is of a definite form (except for possible terms similar to the anomalous magnetic moment interaction terms in electrodynamics.)



Robert L. Mills & C. N. Yang, 1954:

To learn more …

Auguste Dick (transl. H. I. Blocher), *Emmy Noether 1882–1935* (1981)

M. K. Smith and J. W. Brewer, *Emmy Noether: A Tribute to Her Life and Work* (1981)

B. Srinivasan and J. Sally, *Emmy Noether in Bryn Mawr* (1983)

H. A. Kastrup, "The contributions of Emmy Noether, Felix Klein and Sophus Lie to the modern concept of symmetries in physical systems" (1983)

L. M. Lederman and C. T. Hill, *Symmetry and the Beautiful Universe* (2007)

Y. Kosmann-Schwarzbach, *The Noether Theorems* (2011)

Celebrating Emmy Noether, a symposium at the Institute for Advanced Study (2016)

Peter J. Olver, "Emmy Noether's Enduring Legacy in Symmetry" (2018)

Cordula Tollmien, "Emmy Noether (1882–1935)," emmy-noether.net (in German)

Ferdinand Ihringer, "Emmy Noether's Habilitation" (in English)

Chronology

1882 March 23. Emmy Noether born in Erlangen

1900 April. Bavarian examinations for female teachers of French and English, in Ansbach

1903 July 14. *Reifeprüfung**, Königliches Realgymnasium, Nuremberg

1907 December 13. Doctor's degree (Dr. phil.) in Erlangen

1915 April. Moved to Göttingen

1915 May 9. Mother dies in Erlangen

1919 June 4. *Habilitation** in Göttingen

1921 December 13. Father dies in Erlangen

1922 April 6. Named *"ausserordentlicher Professor"**

1923 Summer. Teaching assignment in algebra

1925 August. Completion of manuscript, "Abstrakter Aufbau der Idealtheorie in Zahl- und Funktionenkörpern"

1928 September 5. Communication to the International Congress of Mathematics in Bologna, "Hyperkomplexe Grössen und Darstellungstheorie in arithmetischer Auffassung," Internationaler Mathematiker—Kongress, Bologna

1928–1929 Visiting professor in Moscow

1930 Visiting professor in Frankfurt/Main

1932 June 1. Completion of the manuscript, "Nichtkommutative Algebren"

1932 Ackermann-Teubner Gedächtnispreis (memorial award)

1932 September 7. Lecture, "Hyperkomplexe Grössen und ihre Beziehungen zur kommutativen Algebra und zur Zahlentheorie," Internationaler Mathematiker-Kongress, Zurich

1933 April. Withdrawal of permission to teach, with reference to § 3 of the law "zur Wiederherstellung des Berufsbeamtentums"

1933 October. Aboard the "Bremen," to become visiting professor in the United States

1934 Last publication, "Zerfallende verschränkte Produkte und ihre Maximalordnungen. Von Emmy Noether in Göttingen z. Zt. Bryn Mawr, Penna"

1934 Trip to Germany; final emigration to the USA

1935 April 7. Last letter to Helmut Hasse

1935 April 14. Unexpected death following an operation in Bryn Mawr, PA, USA

Auguste Dick, *Emmy Noether 1882–1935*

Stephanie Magdziak sculpture

Role of women in US institutions (examples)

Yale: undergraduate women admitted in 1969,
graduate students, 1892
first science Ph.D.s, 1894 (astronomy & chemistry)
first physics Ph.D., 1932 ?
women on faculty since 1920, first tenured 1950s
first tenured in physics, 2001; in math, 2013

Princeton: undergraduate women admitted in 1969,
first full-time graduate student, 1961
first physics Ph.D., 1971
first tenured professor, 1968; in physics, 1998

Berkeley: undergraduate women from 1870
first physics Ph.D., 1926
first on physics faculty, 1981 (tenured)