The Standard Model
(Electroweak Theory)
and Higgs Physics

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A Decade of Discovery Past . . .

- Electroweak theory → law of nature
- Higgs-boson influence observed in the vacuum
- Neutrino flavor oscillations: $\nu_\mu \rightarrow \nu_\tau$, $\nu_e \rightarrow \nu_\mu/\nu_\tau$
- Understanding QCD
- Discovery of top quark
- Direct $CP$ violation in $K \rightarrow \pi\pi$
- $B$-meson decays violate $CP$
- Flat universe dominated by dark matter, energy
- Detection of $\nu_\tau$ interactions
- Quarks & leptons structureless at TeV scale
A Decade of Discovery Past . . .

- Electroweak theory → law of nature
  \[ Z, e^+e^-, \bar{p}p, \nu N, (g - 2)_\mu, \ldots \]

- Higgs-boson influence observed in the vacuum
  [EW experiments]

- Neutrino flavor oscillations:
  \[ \nu_\mu \rightarrow \nu_\tau, \nu_e \rightarrow \nu_\mu/\nu_\tau \]
  \[ [\nu_\odot, \nu_{atm}, \nu_{reactors}] \]

- Understanding QCD
  [heavy flavor, \( Z^0 \), \( \bar{p}p \), \( \nu N \), \( ep \), ions, lattice]

- Discovery of top quark  \( [\bar{p}p] \)

- Direct \( CP \) violation in \( K \rightarrow \pi\pi \)  \[ [\text{fixed-target}] \]

- \( B \)-meson decays violate \( CP \)
  \[ e^+e^- \rightarrow B\bar{B} \]

- Flat universe dominated by dark matter, energy
  \[ [\text{SN Ia, CMB, LSS}] \]

- Detection of \( \nu_\tau \) interactions \[ [\text{fixed-target}] \]

- Quarks & leptons structureless at TeV scale
  \[ [\text{mainly colliders}] \]
Goal: Understanding the Everyday

- Why are there atoms?
- Why chemistry?
- Why stable structures?
- What makes life possible?
Goal: Understanding the Everyday

- Why are there atoms?
- Why chemistry?
- Why stable structures?
- What makes life possible?

What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?
Searching for the mechanism of electroweak symmetry breaking, we seek to understand

\textit{why the world is the way it is.}

This is one of the deepest questions humans have ever pursued, and

\textit{it is coming within the reach of particle physics.}
Tevatron Collider is running now, breaking new ground in sensitivity
Tevatron Collider in a Nutshell

980-GeV protons, antiprotons

\( (2\pi \text{ km}) \)

*frequency of revolution* \( \approx 45 000 \text{ s}^{-1} \)

392 ns between crossings

\((36 \otimes 36 \text{ bunches})\)

collision rate \( = \mathcal{L} \cdot \sigma_{\text{inelastic}} \approx 10^7 \text{ s}^{-1} \)

c \( \approx 10^9 \text{ km/h} \); \hspace{1em} v_p \approx c - 495 \text{ km/h} \)

Record \( \mathcal{L}_{\text{init}} = 1.27 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1} \)

[ISR: \( pp, 1.4 \)]

Maximum \( \bar{p} \) at Low \( \beta \): \( 1.661 \times 10^{12} \)
The Large Hadron Collider will operate soon, breaking new ground in energy and sensitivity.
LHC in a nutshell

7-TeV protons on protons (27 km);

\[ v_p \approx c - 10 \text{ km/h} \]

Novel two-in-one dipoles (\( \approx 9 \text{ teslas} \))

Startup: 43 \( \otimes \) 43 bunches,

\[ \mathcal{L} \approx 6 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1} \]

Early: 936 bunches,

\[ \mathcal{L} \gtrsim 5 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1} \text{ [75 ns]} \]

First year? 2808 bunches,

\[ \mathcal{L} \rightarrow 2 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1} \]

25 ns bunch spacing

Eventual \( \mathcal{L} \gtrsim 10^{34} \text{ cm}^{-2} \text{ s}^{-1}: \)

100 fb\(^{-1}\)/year
Tentative Outline . . .

▷ Preliminaries
  
  Current state of particle physics
  A few words about QCD
  Sources of mass

▷ Antecedents of the electroweak theory
  
  What led to EW theory
  What EW theory needs to explain

▷ Some consequences of the Fermi theory
  
  $\mu$ decay
  $\nu e$ scattering
. . . Outline . . .

▷ $SU(2)_L \otimes U(1)_Y$ theory

Gauge theories

Spontaneous symmetry breaking

Consequences: $W^\pm$, $Z^0$/NC, $H$, $m_f$?

Measuring $\sin^2 \theta_W$ in $\nu e$ scattering

GIM / CKM

▷ Phenomena at tree level and beyond

$Z^0$ pole

$W$ mass and width

Atomic parity violation

Looking for trouble

$m_t$, $M_W$, $M_Z$ correlation

Vacuum energy problem
Outline

▷ The Higgs boson and the 1-TeV scale
  Why the Higgs boson must exist
  Higgs properties, constraints
  How well can we anticipate $M_H$?
  Higgs searches

▷ The problems of mass

▷ The EW scale and beyond
  Hierarchy problem
  Why is the EW scale so small?
  Why is the Planck scale so large?

▷ Outlook
General References

- C. Quigg, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions*
- R. N. Cahn & G. Goldhaber, *Experimental Foundations of Particle Physics*
- F. Teubert, “Electroweak Physics,” ICHEP04

Problem sets: http://lutece.fnal.gov/TASI/default.html
Our picture of matter

Pointlike constituents \((r < 10^{-18} \text{ m})\)

\[
\begin{pmatrix}
 u \\
 d
\end{pmatrix}_L 
\begin{pmatrix}
 c \\
 s
\end{pmatrix}_L 
\begin{pmatrix}
 t \\
 b
\end{pmatrix}_L 
\begin{pmatrix}
 \nu_e \\
 e^-
\end{pmatrix}_L 
\begin{pmatrix}
 \nu_\mu \\
 \mu^-
\end{pmatrix}_L 
\begin{pmatrix}
 \nu_\tau \\
 \tau^-
\end{pmatrix}_L
\]

Few fundamental forces, derived from gauge symmetries

\[SU(3)_c \otimes SU(2)_L \otimes U(1)_Y\]

Electroweak symmetry breaking

Higgs mechanism?
Elementarity
▷ Are quarks and leptons structureless?

Symmetry
▷ Electroweak symmetry breaking and the 1-TeV scale
▷ Origin of gauge symmetries
▷ Meaning of discrete symmetries

Unity
▷ Coupling constant unification
▷ Unification of quarks and leptons
  (neutrality of atoms ⇒ new forces!);
  of constituents and force particles
▷ Incorporation of gravity

Identity
▷ Fermion masses and mixings; CP violation; $\nu$ oscillations
▷ What makes an electron an $e$ and a top quark a $t$?

Topography
▷ What is the fabric of space and time?
  ...the origin of space and time?
Elementarity

The World’s Most Powerful Microscopes

nanonanophysics

CDF dijet event ($\sqrt{s} = 1.96$ TeV):

$$E_T = 1.364 \text{ TeV}$$

$q\bar{q} \rightarrow \text{jet} + \text{jet}$
Elementarity

If the Lagrangian has the form $\pm \frac{g^2}{2\Lambda^2} \bar{\psi}_L \gamma_{\mu} \psi_L \bar{\psi}_L \gamma_{\mu} \psi_L$ (with $g^2/4\pi$ set equal to 1), then we define $\Lambda \equiv \Lambda_{LL}^{\pm}$. For the full definitions and for other forms, see the Note in the Listings on Searches for Quark and Lepton Compositeness in the full Review and the original literature.

$\Lambda_{LL}^{+}(eeee) > 8.3$ TeV, CL = 95%
$\Lambda_{LL}^{-}(eeee) > 10.3$ TeV, CL = 95%
$\Lambda_{LL}^{+}(ee\mu\mu) > 8.5$ TeV, CL = 95%
$\Lambda_{LL}^{-}(ee\mu\mu) > 6.3$ TeV, CL = 95%
$\Lambda_{LL}^{+}(ee\tau\tau) > 5.4$ TeV, CL = 95%
$\Lambda_{LL}^{-}(ee\tau\tau) > 6.5$ TeV, CL = 95%
$\Lambda_{LL}^{+}(\ell\ell\ell\ell) > 9.0$ TeV, CL = 95%
$\Lambda_{LL}^{-}(\ell\ell\ell\ell) > 7.8$ TeV, CL = 95%
$\Lambda_{LL}^{+}(e\ell\ell\ell) > 23.3$ TeV, CL = 95%
$\Lambda_{LL}^{-}(e\ell\ell\ell) > 12.5$ TeV, CL = 95%
$\Lambda_{LL}^{+}(e\ell\ell\ell) > 11.1$ TeV, CL = 95%
$\Lambda_{LL}^{-}(e\ell\ell\ell) > 26.4$ TeV, CL = 95%
$\Lambda_{LL}^{+}(ee\ell\ell) > 1.0$ TeV, CL = 95%
$\Lambda_{LL}^{-}(ee\ell\ell) > 2.1$ TeV, CL = 95%
$\Lambda_{LL}^{+}(ee\ell\ell) > 5.6$ TeV, CL = 95%
$\Lambda_{LL}^{-}(ee\ell\ell) > 4.9$ TeV, CL = 95%
$\Lambda_{LL}^{+}(\ell\ell\ell\ell) > 2.9$ TeV, CL = 95%
$\Lambda_{LL}^{-}(\ell\ell\ell\ell) > 4.2$ TeV, CL = 95%
$\Lambda(\ell\nu\ell\nu) > 3.10$ TeV, CL = 90%
$\Lambda(e\nu\ell\nu) > 2.81$ TeV, CL = 95%
$\Lambda_{LL}^{+}(\ell\ell\ell\ell) > 2.7$ TeV, CL = 95%
$\Lambda_{LL}^{-}(\ell\ell\ell\ell) > 2.4$ TeV, CL = 95%
$\Lambda_{LL}^{+}(\nu\nu\ell\nu) > 5.0$ TeV, CL = 95%
$\Lambda_{LL}^{-}(\nu\nu\ell\nu) > 5.4$ TeV, CL = 95%
Two views of Symmetry

1. *Indistinguishability*

   One object transformed into another

   Familiar (and useful!) from

   Global Symmetries: isospin, $SU(3)_f$, ...

   Spacetime Symmetries

   Gauge Symmetries

   "EQUIVALENCE"

   Idealize more perfect worlds, the better to understand our diverse, changing world

   Unbroken unified theory: perfect world of equivalent forces, interchangeable massless particles... *Perfectly boring?*

   *Symmetry ⇔ Disorder*
## Two views of Symmetry

### 2. Unobservable

Goodness of a symmetry means something cannot be measured
e.g., vanishing asymmetry

<table>
<thead>
<tr>
<th>Unobservable</th>
<th>Transformation</th>
<th>Conserved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute position</td>
<td>( \vec{r} \rightarrow \vec{r} + \vec{\Delta} )</td>
<td>( \vec{p} )</td>
</tr>
<tr>
<td>Absolute time</td>
<td>( t \rightarrow t + \delta )</td>
<td>( E )</td>
</tr>
<tr>
<td>Absolute orientation</td>
<td>( \hat{r} \rightarrow \hat{r}' )</td>
<td>( \vec{L} )</td>
</tr>
<tr>
<td>Absolute velocity</td>
<td>( \vec{v} \rightarrow \vec{v} + \vec{w} )</td>
<td></td>
</tr>
<tr>
<td>Absolute right</td>
<td>( \vec{r} \rightarrow -\vec{r} )</td>
<td>( P )</td>
</tr>
<tr>
<td>Absolute future</td>
<td>( t \rightarrow -t )</td>
<td>( T )</td>
</tr>
<tr>
<td>Absolute charge</td>
<td>( Q \rightarrow -Q )</td>
<td>( C )</td>
</tr>
<tr>
<td>Absolute phase</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Unity
QCD is part of the standard model

...a remarkably simple, successful, and rich theory

Wilczek, hep-ph/9907340

▷ Perturbative QCD

- Exists, thanks to asymptotic freedom
- Describes many phenomena in quantitative detail:
  ▷ $Q^2$-evolution of structure functions
  ▷ Jet production in $\bar{p}p$ collisions
  ▷ Many decays, event shapes, ...
- We can measure the running of $\alpha_s$
  (engineering value for unification)

▷ Nonperturbative (lattice) QCD

- Predicts the hadron spectrum
- Gives our best information on quark masses, etc.

$F_2(x, Q^2)$ in $\nu N$ interactions (CCFR)

$F_2(x, Q^2)$ in $\ell N$ interactions (ZEUS)

ZEUS

ZEUS, hep-ex/0208023.
Inclusive jet cross section at $\sqrt{s} = 1.8$ TeV (CDF)

T. Affolder et al. (CDF), *Phys. Rev. D* 64, 032001 (2001)
\( \alpha_s(E_T/2) \) from \( \bar{p}p \rightarrow \text{jets} \)

CDF Preliminary

\begin{figure}
\centering
\includegraphics[width=\textwidth]{CDF_data.png}
\caption{CDF Preliminary Data}
\end{figure}


Comprehensive survey: W. de Boer, hep-ph/0407021

\textit{Electroweak Theory \cdot ICTP 2005}
Hadronic Jets

$\bar{Q}Q$ Lattice

$e^+e^-$ rates

$e^+e^-$ event shapes

Fragmentation

$Z$ width

Small $x$ structure functions

$ep$ event shapes

Polarized DIS

Deep Inelastic Scattering (DIS)

$\tau$ decays

$\gamma$ decay

$1/\alpha_s(Q)$ vs. $Q$ [GeV]

$\alpha_s(M_Z)$ vs. $Q$ [GeV]

PDG

Chris Quigg

Electroweak Theory · ICTP 2005
Quenched hadron spectrum

(No dynamical fermions)
The Origins of Mass

(masses of nuclei “understood”)

\[ p, [\pi], \rho \]

understood: QCD

confinement energy is the source

“Mass without mass”

We understand the visible mass of the Universe

\[ W, Z \]
electroweak symmetry breaking

\[ M_W^2 = \frac{1}{2} g^2 v^2 = \pi \alpha/G_F \sqrt{2} \sin^2 \theta_W \]

\[ M_Z^2 = M_W^2 \cos^2 \theta_W \]

\[ q, \ell^\mp \]
EWSB + Yukawa couplings

\[ \nu_\ell \]
EWSB + Yukawa couplings; new physics?

All fermion masses ⇔ physics beyond standard model

\[ H \]
?? fifth force ??
Antecedents of EW Theory

Commins & Bucksbaum, *Weak Intns of Leptons & Quarks*

1896: Becquerel radioactivity

Several varieties, including $\beta$ decay

$$^A Z \rightarrow ^A (Z + 1) + \beta^-$$

Examples:

$$^3 H_1 \rightarrow ^3 \text{He}_2 + \beta^-,$$
$$n \rightarrow p + \beta^-,$$
$$^{214} \text{Pb}_{82} \rightarrow ^{214} \text{Bi}_{83} + \beta^-.$$  

$\beta^+$-emitters, $^A Z \rightarrow ^A (Z - 1) + \beta^+$, are rare among naturally occurring isotopes. Radio-phosphorus produced 1934 by the Joliot-Curie, *after* positron discovery in cosmic rays. $^{19} \text{Ne} \rightarrow ^{19} \text{F} + \beta^+$ studied for right-handed charged currents and time reversal invariance; *positron-emission tomography*
1914: Chadwick $\beta$ spectrum

Energy conservation in question

1930: Pauli $\approx$ massless, neutral, penetrating particle

nuclear spin & statistics

$\rightarrow$ neutrino $\nu$

$\beta$ decay first hint for flavor

charged-current, flavor-changing interactions

1932: Chadwick neutron

$\rightarrow$ isospin symmetry
Neutron & flavor symmetry

\[ M(n) = 939.56563 \pm 0.00028 \text{ MeV}/c^2 \]
\[ M(p) = 938.27231 \pm 0.00028 \text{ MeV}/c^2 \]
\[ \Delta M = 1.293318 \pm 0.000009 \text{ MeV}/c^2 \]

\[ \Delta M/M \approx 1.4 \times 10^{-3} \]

Charge-independent nuclear forces?

\[ ^3\text{H}(pnn) = 8.481855 \pm 0.000013 \text{ MeV} \]
\[ ^3\text{He}(ppn) = 7.718109 \pm 0.000013 \text{ MeV} \]
\[ \Delta(\text{B.E.}) = 0.76346 \text{ MeV} \]

\[ ^3\text{He} \text{ charge radius } r = 1.97 \pm 0.015 \text{ fm} \]

Coulomb energy: \( \alpha/r \approx 0.731 \text{ MeV} \)
Level structures in mirror nuclei. 1

\[ I_3 = -\frac{1}{2} : \ ^7\text{Li}(3p + 4n) \quad ^7\text{Be}(4p + 3n) : I_3 = \frac{1}{2} \]

\[ I_3 = -\frac{3}{2} : \ ^7\text{He}(2p + 3n) \quad ^7\text{B}(5p + 2n) : I_3 = \frac{3}{2} \]

\[ A = 7 \]

\[ n - p \text{ mass difference, Coulomb energy removed} \]

(isobaric analogue states)
Level structures in mirror nuclei. 2

\[ I_3 = -\frac{1}{2} : \quad ^{11}\text{B}(5p + 6n) \quad ^{11}\text{C}(6p + 5n) : \quad I_3 = \frac{1}{2} \]

\[ I_3 = -\frac{3}{2} : \quad ^{11}\text{Be}(4p + 7n) \quad ^{11}\text{N}(7p + 4n) : \quad I_3 = \frac{3}{2} \]

\[ ^{11}\text{Li}(3p + 8n) \text{ ground state (34.4 MeV)} \quad I = \frac{5}{2} \text{ isobaric analogue} \]
Level structures in mirror nuclei. 3

$A = 14$: $NN$ outside closed core

$^{14}\text{O} : \quad ^{12}\text{C} + (pp) \quad I_3 = +1$

$^{14}\text{N} : \quad ^{12}\text{C} + (pn) \quad I_3 = 0$

$^{14}\text{C} : \quad ^{12}\text{C} + (nn) \quad I_3 = -1$
The first flavor symmetry

isospin invariance \( \begin{pmatrix} p \\ n \end{pmatrix} \) isospin rotations

In the absence of EM, convention determines which (combination) is up

Aside: Without EM, how would we know there are two species of nucleons?
Parity violation in weak decays

1956 Wu et al.: correlation between
spin vector $\vec{J}$ of polarized $^{60}$Co and
direction $\hat{p}_e$ of outgoing $\beta$ particle

Parity leaves spin (axial vector) unchanged

$P: \vec{J} \rightarrow \vec{J}$

Parity reverses electron direction

$P: \hat{p}_e \rightarrow -\hat{p}_e$

Correlation $\vec{J} \cdot \hat{p}_e$ is parity violating

Experiments in late 1950s established that
(charged-current) weak interactions are left-handed

Parity links left-handed, right-handed neutrinos,

$\nu_L \leftrightarrow P \leftrightarrow \nu_R$

$\Rightarrow$ build a manifestly parity-violating theory with only $\nu_L$. 
Pauli’s Reaction to the Downfall of Parity

Es ist nur eine kurze Note, bekannt in gebaren, dass unsere langjährige, liebe Freunde in PARITY
am 19. Januar 1957 nach kurzen
Zeitern bei eigener experimenteller
Eingriffen saft entbliesen ist.
Für die hinterbliebenen
$e, \mu, \nu$. 
Pauli’s Reaction to the Downfall of Parity

It is our sad duty to announce that our loyal friend of many years PARITY went peacefully to her eternal rest on the nineteenth of January 1957, after a short period of suffering in the face of further experimental interventions. For those who survive her, $e \mu \nu$


Für die hinterbliebenen $e \mu \nu$
Pauli’s assertiveness training . . .
How do we know $\nu$ is LH?

- Measure $\mu^+$ helicity in (spin-zero) $\pi^+ \rightarrow \mu^+ \nu_\mu$

$$\nu_\mu \xrightarrow{\Rightarrow} \pi^+ \xleftarrow{\Leftarrow} \mu^+$$

$$h(\nu_\mu) = h(\mu^+)$$


$\mu^+$ forced to have “wrong” helicity

...inhibits decay, and inhibits $\pi^+ \rightarrow e^+ \nu_e$ more

$$\frac{\Gamma(\pi^+ \rightarrow e^+ \nu_e)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)} = 1.23 \times 10^{-4}$$

- Measure longitudinal polarization of recoil nucleus in $\mu^- {^{12}}C(J = 0) \rightarrow {^{12}}B(J = 1)\nu_\mu$

Infer $h(\nu_\mu)$ by angular momentum conservation

Measure longitudinal polarization of recoil nucleus in

\[ e^- \quad ^{152}\text{Eu}^m (J = 0) \rightarrow ^{152}\text{Sm}^* (J = 1) \nu_e \downarrow \to \gamma ^{152}\text{Sm} \]

Infer \( h(\nu_e) \) from \( \gamma \) polarization

Charge conjugation is also violated . . .

\[ \nu_L \leftrightarrow C \leftrightarrow \nu_L \]

\( \mu^\pm \) decay: angular distributions of \( e^\pm \) reversed

\[
\frac{dN(\mu^\pm \to e^\pm + \ldots)}{dx dz} = x^2(3 - 2x) \left[ 1 \pm z \frac{(2x - 1)}{(3 - 2x)} \right]
\]

\( x \equiv p_e/p_{e\text{max}} \), \( z \equiv \hat{s}_\mu \cdot \hat{p}_e \)

\( e^+ \) follows \( \mu^+ \) spin \hspace{1cm} \( e^- \) avoids \( \mu^- \) spin
Effective Lagrangian . . .

Late 1950s: current-current interaction

\[ \mathcal{L}_{V-A} = \frac{-G_F}{\sqrt{2}} \bar{\nu} \gamma_\mu (1 - \gamma_5) e \bar{e} \gamma^\mu (1 - \gamma_5) \nu + \text{h.c.} \]

\[ G_F = 1.16632 \times 10^{-5} \text{ GeV}^{-2} \]

Compute $\bar{\nu} e$ scattering amplitude:

\[ \mathcal{M} = - \frac{iG_F}{\sqrt{2}} \bar{v}(\nu, q_1) \gamma_\mu (1 - \gamma_5) u(e, p_1) \]
\[ \cdot \bar{u}(e, p_2) \gamma^\mu (1 - \gamma_5) v(\nu, q_2) \]
\[
\bar{\nu}e \rightarrow \bar{\nu}e
\]

\[
\frac{d\sigma_{V-A}(\bar{\nu}e \rightarrow \bar{\nu}e)}{d\Omega_{\text{cm}}} = \frac{|M|^2}{64\pi^2 s} = \frac{G_F^2 \cdot 2mE_\nu(1 - z)^2}{16\pi^2}
\]

\[z = \cos \theta^*\]

\[
\sigma_{V-A}(\bar{\nu}e \rightarrow \bar{\nu}e) = \frac{G_F^2 \cdot 2mE_\nu}{3\pi}
\]

\[\approx 0.574 \times 10^{-41} \text{ cm}^2 \left(\frac{E_\nu}{1 \text{ GeV}}\right)\]

Small! \approx 10^{-14} \sigma(pp) at 100 GeV

\[
\nu e \rightarrow \nu e
\]

\[
\frac{d\sigma_{V-A}(\nu e \rightarrow \nu e)}{d\Omega_{\text{cm}}} = \frac{G_F^2 \cdot 2mE_\nu}{4\pi^2}
\]

\[
\sigma_{V-A}(\nu e \rightarrow \nu e) = \frac{G_F^2 \cdot 2mE_\nu}{\pi}
\]

\[\approx 1.72 \times 10^{-41} \text{ cm}^2 \left(\frac{E_\nu}{1 \text{ GeV}}\right)\]
Why $3 \times$ difference?

incoming $e$ \[ J_z = 0 \] outgoing, $z = +1$
\[ e \]
\[ \nu \]
\[ \nu \]

allowed at all angles

incoming $e$ \[ J_z = +1 \] outgoing, $z = +1$
\[ e \]
\[ \bar{\nu} \]
\[ \bar{\nu} \]

forbidden (angular momentum) at $z = +1$
1962: Lederman, Schwartz, Steinberger $\nu_\mu \neq \nu_e$

- Make HE $\pi \rightarrow \mu\nu$ beam
- Observe $\nu N \rightarrow \mu + \text{anything}$
- Don’t observe $\nu N \rightarrow e + \text{anything}$


Suggests family structure

$$\begin{pmatrix}
\nu_e \\
e^{-}
\end{pmatrix}_{L} \approx \begin{pmatrix}
\nu_\mu \\
\mu^{-}
\end{pmatrix}_{L} \approx \text{no interactions known to cross boundaries}$$

Generalize effective (current-current) Lagrangian:

$$\mathcal{L}_{V-A}^{(e\mu)} = \frac{-G_F}{\sqrt{2}} \bar{v}_\mu \gamma_\mu (1 - \gamma_5) \mu \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e + \text{h.c.},$$

Compute muon decay rate

$$\Gamma(\mu \rightarrow e\bar{\nu}_e\nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

accounts for the 2.2-\(\mu\)s muon lifetime
LEPTON FAMILY NUMBER

Lepton family number conservation means separate conservation of each of $L_e$, $L_\mu$, $L_\tau$.

$\Gamma(Z \rightarrow e^+ \mu^+)/\Gamma_{\text{total}}$  
$\Gamma(Z \rightarrow e^+ \tau^+)/\Gamma_{\text{total}}$  
$\Gamma(Z \rightarrow \mu^+ \tau^+)/\Gamma_{\text{total}}$

limit on $\mu^- \rightarrow e^-$ conversion

$\sigma(\mu^- 32S \rightarrow e^- 32S) / \sigma(\mu^- 32S \rightarrow \nu_\mu 32P^*)$  
$\sigma(\mu^- Ti \rightarrow e^- Ti) / \sigma(\mu^- Ti \rightarrow capture)$  
$\sigma(\mu^- Pb \rightarrow e^- Pb) / \sigma(\mu^- Pb \rightarrow capture)$

limit on muonium $\rightarrow$ antimuonium conversion $R_g = G_C / G_F$  
$\Gamma(\mu^- \rightarrow e^- e^- e^- e^-) / \Gamma_{\text{total}}$  
$\Gamma(\mu^- \rightarrow e^- \gamma) / \Gamma_{\text{total}}$  
$\Gamma(\mu^- \rightarrow e^- e^+ e^-) / \Gamma_{\text{total}}$  
$\Gamma(\mu^- \rightarrow e^- 2\gamma) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow e^- \gamma) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow \mu^- \gamma) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow e^- 0) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow \mu^- 0) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow e^- K^0) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow \mu^- K^0) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow e^- \eta) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow \mu^- \eta) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow e^- \rho^0) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow \mu^- \rho^0) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow e^- K^*(892)^0) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow \mu^- K^*(892)^0) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow e^- \phi) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow \mu^- \phi) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow e^- e^+ e^-) / \Gamma_{\text{total}}$  
$\Gamma(\tau^- \rightarrow e^- \mu^+ \mu^-) / \Gamma_{\text{total}}$

[i] $< 1.7 \times 10^{-6}$, CL = 95%  
[i] $< 9.8 \times 10^{-5}$, CL = 95%  
[i] $< 1.2 \times 10^{-5}$, CL = 95%  
$< 7 \times 10^{-11}$, CL = 90%  
$< 4.3 \times 10^{-12}$, CL = 90%  
$< 4.6 \times 10^{-11}$, CL = 90%  
$< 0.0030$, CL = 90%  
[j] $< 1.2 \times 10^{-2}$, CL = 90%  
$< 1.2 \times 10^{-11}$, CL = 90%  
$< 1.0 \times 10^{-12}$, CL = 90%  
$< 7.2 \times 10^{-11}$, CL = 90%  
$< 2.7 \times 10^{-6}$, CL = 90%  
$< 1.1 \times 10^{-6}$, CL = 90%  
$< 3.7 \times 10^{-6}$, CL = 90%  
$< 4.0 \times 10^{-6}$, CL = 90%  
$< 9.1 \times 10^{-7}$, CL = 90%  
$< 9.5 \times 10^{-7}$, CL = 90%  
$< 8.2 \times 10^{-6}$, CL = 90%  
$< 9.6 \times 10^{-6}$, CL = 90%  
$< 2.0 \times 10^{-6}$, CL = 90%  
$< 6.3 \times 10^{-6}$, CL = 90%  
$< 5.1 \times 10^{-6}$, CL = 90%  
$< 7.5 \times 10^{-6}$, CL = 90%  
$< 7.4 \times 10^{-6}$, CL = 90%  
$< 7.5 \times 10^{-6}$, CL = 90%  
$< 6.9 \times 10^{-6}$, CL = 90%  
$< 7.0 \times 10^{-6}$, CL = 90%  
$< 2.9 \times 10^{-6}$, CL = 90%  
$< 1.8 \times 10^{-6}$, CL = 90%
TOTAL LEPTON NUMBER

Violation of total lepton number conservation also implies violation of lepton family number conservation.

\[ \Gamma(Z \rightarrow p e)/\Gamma_{\text{total}} < 1.8 \times 10^{-6}, \text{ CL = 95%} \]
\[ \Gamma(Z \rightarrow p \mu)/\Gamma_{\text{total}} < 1.8 \times 10^{-6}, \text{ CL = 95%} \]

limit on \( \mu^- \rightarrow e^+ \) conversion

\[ \frac{\sigma(\mu^- 32S \rightarrow e^+ 32S^*)}{\sigma(\mu^- 32S \rightarrow \nu^\mu 32p^*)} < 9 \times 10^{-10}, \text{ CL = 90%} \]
\[ \frac{\sigma(\mu^- 127I \rightarrow e^+ 127Sb^*)}{\sigma(\mu^- 127I \rightarrow \text{ anything})} < 3 \times 10^{-10}, \text{ CL = 90%} \]
\[ \frac{\sigma(\mu^- Ti \rightarrow e^+ Ca)}{\sigma(\mu^- Ti \rightarrow \text{ capture})} < 3.6 \times 10^{-11}, \text{ CL = 90%} \]

\[ \Gamma(\tau^- \rightarrow e^+ \pi^- \pi^-)/\Gamma_{\text{total}} < 1.9 \times 10^{-6}, \text{ CL = 90%} \]
\[ \Gamma(\tau^- \rightarrow \mu^+ \pi^- \pi^-)/\Gamma_{\text{total}} < 3.4 \times 10^{-6}, \text{ CL = 90%} \]
\[ \Gamma(\tau^- \rightarrow e^+ \pi^- K^-)/\Gamma_{\text{total}} < 2.1 \times 10^{-6}, \text{ CL = 90%} \]
\[ \Gamma(\tau^- \rightarrow e^+ K^- K^-)/\Gamma_{\text{total}} < 3.8 \times 10^{-6}, \text{ CL = 90%} \]
\[ \Gamma(\tau^- \rightarrow \mu^+ \pi^- K^-)/\Gamma_{\text{total}} < 7.0 \times 10^{-6}, \text{ CL = 90%} \]
\[ \Gamma(\tau^- \rightarrow \mu^+ e^- K^-)/\Gamma_{\text{total}} < 6.0 \times 10^{-6}, \text{ CL = 90%} \]
\[ \Gamma(\tau^- \rightarrow \overline{\nu}_\gamma)/\Gamma_{\text{total}} < 3.5 \times 10^{-6}, \text{ CL = 90%} \]
\[ \Gamma(\tau^- \rightarrow \overline{\nu}_\pi^0)/\Gamma_{\text{total}} < 1.5 \times 10^{-5}, \text{ CL = 90%} \]
\[ \Gamma(\tau^- \rightarrow \overline{\nu}_2\pi^0)/\Gamma_{\text{total}} < 3.3 \times 10^{-5}, \text{ CL = 90%} \]
\[ \Gamma(\tau^- \rightarrow \overline{\nu}_\eta)/\Gamma_{\text{total}} < 8.9 \times 10^{-6}, \text{ CL = 90%} \]
\[ \Gamma(\tau^- \rightarrow \overline{\nu}_\pi^0\eta)/\Gamma_{\text{total}} < 2.7 \times 10^{-5}, \text{ CL = 90%} \]

\[ t_{1/2}(\text{ }^{76}\text{Ge} \rightarrow \text{ }^{76}\text{Ge} + 2 \ e^- ) > 1.9 \times 10^{25} \text{ yr, CL = 90%} \]

\[ \Gamma(\pi^+ \rightarrow \mu^+ \overline{\nu}_e)/\Gamma_{\text{total}} < 1.5 \times 10^{-3}, \text{ CL = 90%} \]
\[ \Gamma(K^+ \rightarrow \pi^- \mu^+ e^+)/\Gamma_{\text{total}} < 5.0 \times 10^{-10}, \text{ CL = 90%} \]
\[ \Gamma(K^+ \rightarrow \pi^- e^+ e^+)/\Gamma_{\text{total}} < 6.4 \times 10^{-10}, \text{ CL = 90%} \]
\[ \Gamma(K^+ \rightarrow \pi^- \mu^+ \mu^+)/\Gamma_{\text{total}} < 3.0 \times 10^{-9}, \text{ CL = 90%} \]
\[ \Gamma(K^+ \rightarrow \mu^+ \overline{\nu}_e)/\Gamma_{\text{total}} < 3.3 \times 10^{-3}, \text{ CL = 90%} \]
\[ \Gamma(K^+ \rightarrow 0^+ e^+ \overline{\nu}_e)/\Gamma_{\text{total}} < 3 \times 10^{-3}, \text{ CL = 90%} \]
\[ \Gamma(D^+ \rightarrow \pi^- e^+ e^+)/\Gamma_{\text{total}} < 9.6 \times 10^{-5}, \text{ CL = 90%} \]
\[ \Gamma(D^+ \rightarrow \pi^- \mu^+ \mu^+)/\Gamma_{\text{total}} < 4.8 \times 10^{-6}, \text{ CL = 90%} \]
\[ \Gamma(D^+ \rightarrow \pi^- e^+ \mu^+)/\Gamma_{\text{total}} < 5.0 \times 10^{-5}, \text{ CL = 90%} \]
\[ \Gamma(D^+ \rightarrow \rho^- \mu^+ \mu^+)/\Gamma_{\text{total}} < 5.6 \times 10^{-4}, \text{ CL = 90%} \]
\[ \Gamma(D^+ \rightarrow K^- e^+ e^+)/\Gamma_{\text{total}} < 1.2 \times 10^{-4}, \text{ CL = 90%} \]
\[ \Gamma(D^+ \rightarrow K^- \mu^+ \mu^+)/\Gamma_{\text{total}} < 1.3 \times 10^{-5}, \text{ CL = 90%} \]
\[ \Gamma(D^+ \rightarrow K^- e^+ \mu^+)/\Gamma_{\text{total}} < 1.3 \times 10^{-4}, \text{ CL = 90%} \]
\[ \Gamma(D^+ \rightarrow K^- (892) e^+ \mu^+)/\Gamma_{\text{total}} < 8.5 \times 10^{-4}, \text{ CL = 90%} \]
\[ \Gamma(D^0 \rightarrow \pi^- e^+ e^+ + c.c.)/\Gamma_{\text{total}} < 1.12 \times 10^{-4}, \text{ CL = 90%} \]
\[ \Gamma(D^0 \rightarrow \pi^- \mu^+ \mu^+ + c.c.)/\Gamma_{\text{total}} < 2.9 \times 10^{-5}, \text{ CL = 90%} \]
\[ \Gamma(D^0 \rightarrow K^- \pi^- e^+ e^+ + c.c.)/\Gamma_{\text{total}} < 2.06 \times 10^{-4}, \text{ CL = 90%} \]
Cross section for inverse muon decay

\[ \sigma(\nu_\mu e \to \mu \nu_e) = \sigma_{V-A}(\nu_e e \to \nu_e e) \left[ 1 - \left( \frac{m_\mu^2 - m_e^2}{2m_e E_\nu} \right) \right]^2 \]

agrees with CHARM II, CCFR data (\( E_\nu \lesssim 600 \text{ GeV} \))

**PW unitarity:** \(|\mathcal{M}_J| < 1\)

\( V - A \) theory:

\[ \mathcal{M}_0 = \frac{G_F \cdot 2m_e E_\nu}{\pi \sqrt{2}} \left[ 1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu} \right] \]

satisfies pw unitarity for

\[ E_\nu < \frac{\pi}{G_F m_e \sqrt{2}} \approx 3.7 \times 10^8 \text{ GeV} \]

\[ \Rightarrow V - A \text{ theory cannot be complete} \]

"physics must change" before \( \sqrt{s} \approx 600 \text{ GeV} \)
2000: DONuT Three-Neutrino Experiment

▷ Prompt (beam-dump) $\nu_\tau$ beam produced in

$$D_s^+ \rightarrow \tau^+ \nu_\tau \quad \text{\Downarrow} \quad X^+ \bar{\nu}_\tau$$

▷ Try to observe $\nu_\tau N \rightarrow \tau + \text{anything in emulsion;}$ $\tau$ lifetime is 0.3 ps

Candidate event in ECC1. The three tracks with full emulsion data are shown. The red track shows a 100 mrad kink 4.5mm from the interaction vertex. The scale units are microns.

Leptons are seen as free particles

Table 1: Some properties of the leptons.

<table>
<thead>
<tr>
<th>Lepton</th>
<th>Mass</th>
<th>Lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^{-}$</td>
<td>$0.51099892(4) \text{ MeV/c}^2$</td>
<td>$&gt; 4.6 \times 10^{26} \text{ y (90% CL)}$</td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>$&lt; 3 \text{ eV/c}^2$</td>
<td>$\tau/m &gt; 7 \times 10^9 \text{ s/eV}$</td>
</tr>
<tr>
<td>$\mu^{-}$</td>
<td>$105.658369(9) \text{ MeV/c}^2$</td>
<td>$2.19703(4) \times 10^{-6} \text{ s}$</td>
</tr>
<tr>
<td>$\nu_\mu$</td>
<td>$&lt; 0.19 \text{ MeV/c}^2 \text{ (90% CL)}$</td>
<td>$\tau/m &gt; 15.4 \text{ s/eV}$</td>
</tr>
<tr>
<td>$\tau^{-}$</td>
<td>$1776.99^{+0.29}_{-0.26} \text{ MeV/c}^2$</td>
<td>$290.6 \pm 1.1 \times 10^{-15} \text{ s}$</td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td>$&lt; 18.2 \text{ MeV/c}^2 \text{ (95% CL)}$</td>
<td></td>
</tr>
</tbody>
</table>

All spin-$\frac{1}{2}$, pointlike ($\ll \text{ few } \times \text{ 10}^{-17} \text{ cm}$)

kinematically determined $\nu$ masses consistent with 0
($\nu$ oscillations $\Rightarrow$ nonzero, unequal masses)
Universal weak couplings

*Rough and ready test*

Fermi constant from muon decay

\[ G_\mu = \left[ \frac{192\pi^3 \hbar}{\tau_\mu m^5_\mu} \right]^{\frac{1}{2}} = 1.1638 \times 10^{-5} \text{ GeV}^{-2} \]

Meticulous analysis yields \( G_\mu = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2} \)

Fermi constant from tau decay

\[ G_\tau = \left[ \frac{\Gamma(\tau \rightarrow e\bar{\nu}_e\nu_\tau)}{\Gamma(\tau \rightarrow \text{all})} \frac{192\pi^3 \hbar}{\tau_\tau m^5_\tau} \right]^{\frac{1}{2}} = 1.1642 \times 10^{-5} \text{ GeV}^{-2} \]

Excellent agreement with \( G_\beta = 1.16639(2) \times 10^{-5} \text{ GeV}^{-2} \)

Charged currents acting in leptonic and semileptonic interactions are of universal strength; \( \Rightarrow \text{ universality of current-current form, or whatever lies behind it} \)
**Nonleptonic enhancement**

Certain NL transitions are more rapid than universality suggests

\[
\Gamma(K_S \rightarrow \pi^+\pi^-) \approx 450 \times \Gamma(K^+ \rightarrow \pi^+\pi^0)
\]

\[I=0, 2\]

\[I=2\]

\[A_0 \approx 22 \times A_2\]

\[|\Delta I| = \frac{1}{2}\] rule; “octet dominance” (over 27)

Origin of this phenomenological rule is only partly understood. Short-distance (perturbative) QCD corrections arise from

\[W\]

\[g\]

\[s\]

\[u\]

\[\ldots\text{explain } \approx \sqrt{\text{enhancement}}\]
SYMMETRIES $\rightarrow$ INTERACTIONS

Phase Invariance (Symmetry) in Quantum Mechanics

QM STATE: COMPLEX SCHRÖDINGER WAVE FUNCTION $\psi(x)$

OBSERVABLES

$$\langle O \rangle = \int dx \psi^* O \psi$$

ARE UNCHANGED

UNDER A GLOBAL PHASE ROTATION

$$\psi(x) \rightarrow e^{i\theta} \psi(x)$$
$$\psi^*(x) \rightarrow e^{-i\theta} \psi^*(x)$$

- Absolute phase of the wave function cannot be measured (is a matter of convention).

- Relative phases (interference experiments) are unaffected by a global phase rotation.

NEW

ORIGINAL

Chris Quigg

Electroweak Theory · ICTP 2005
GLOBAL ROTATION — SAME EVERYWHERE

MIGHT WE CHOOSE ONE PHASE CONVENTION IN MIRAMARE AND ANOTHER IN BATAVIA?

A DIFFERENT CONVENTION AT EACH POINT?

\[ \psi(x) \to e^{i\alpha(x)} \psi(x) \]
THERE IS A PRICE.

Some variables (e.g., momentum) and the Schrödinger equation itself contain derivatives. Under the transformation

$$\psi(x) \rightarrow e^{iq\alpha(x)}\psi(x)$$

the gradient of the wave function transforms as

$$\nabla\psi(x) \rightarrow e^{iq\alpha(x)}[\nabla\psi(x) + iq(\nabla\alpha(x))\psi(x)]$$

The $\nabla\alpha(x)$ term spoils local phase invariance.

TO RESTORE LOCAL PHASE INVARIANCE . . .

Modify the equations of motion and observables.

Replace $\nabla$ by $\nabla + iq\vec{A}$

“Gauge-covariant derivative”

If the vector potential $\vec{A}$ transforms under local phase rotations as

$$\vec{A}(x) \rightarrow \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x),$$

then $(\nabla + iq\vec{A})\psi \rightarrow e^{iq\alpha(x)}(\nabla + iq\vec{A})\psi$ and $\psi^*(\nabla + iq\vec{A})\psi$ is invariant under local rotations.
NOTE . . .

• $\vec{A}(x) \rightarrow \vec{A}'(x) \equiv \vec{A}(x) - \nabla \alpha(x)$ has the form of a gauge transformation in electrodynamics.

• The replacement $\nabla \rightarrow (\nabla + iq\vec{A})$ corresponds to $\vec{p} \rightarrow \vec{p} - q\vec{A}$

FORM OF INTERACTION IS DEDUCED FROM LOCAL PHASE INVARiance

$\implies$ MAXWELL'S EQUATIONS DERIVED FROM A SYMMETRY PRINCIPLE

QED is the gauge theory based on $U(1)$ phase symmetry
GENERAL PROCEDURE

- Recognize a symmetry of Nature.
- Build it into the laws of physics.
  (Connection with conservation laws)
- Impose symmetry in stricter (local) form.

$\implies$ INTERACTIONS

- Massless vector fields (gauge fields)
- Minimal coupling to the conserved current
- Interactions among the gauge fields, if symmetry is non-Abelian

Posed as a problem in mathematics, construction of a gauge theory is always possible (at the level of a classical $\mathcal{L}$; consistent quantum theory may require additional vigilance).

Formalism is no guarantee that the gauge symmetry was chosen wisely.
The Crystal World
The Crystal World
The Perfect World
Massive Photon?  

Hiding Symmetry

Recall the two miracles of superconductivity:

- No resistance
- Meissner effect (exclusion of $\mathbf{B}$)

Ginzburg–Landau Phenomenology
(not a theory from first principles)

normal, resistive charge carriers . . .

. . . + superconducting charge carriers

\[ \begin{align*}
\text{B} = 0: \\
G_{\text{super}}(0) &= G_{\text{normal}}(0) + \alpha |\psi|^2 + \beta |\psi|^4 \\
T > T_c &: \quad \alpha > 0 \quad \langle |\psi|^2 \rangle_0 = 0 \\
T < T_c &: \quad \alpha < 0 \quad \langle |\psi|^2 \rangle_0 \neq 0
\end{align*} \]
NONZERO MAGNETIC FIELD

\[ G_{\text{super}}(B) = G_{\text{super}}(0) + \frac{B^2}{8\pi} + \frac{1}{2m^*} \left| -i\hbar \nabla \psi - \frac{e^*}{c} A \psi \right|^2 \]

\[ e^* = -2 \]

\[ m^* \]

\{ of superconducting carriers \}

Weak, slowly varying field

\[ \psi \approx \psi_0 \neq 0, \ \nabla \psi \approx 0 \]

Variational analysis \Rightarrow

\[ \nabla^2 A - \frac{4\pi e^*}{m^* c^2} |\psi_0|^2 A = 0 \]

wave equation of a \textit{massive photon}

Photon—\textit{gauge boson}—acquires mass within superconductor

origin of Meissner effect
Formulate electroweak theory

three crucial clues from experiment:

▷ Left-handed weak-isospin doublets,

\[
\begin{pmatrix}
\nu_e \\ e
\end{pmatrix}_L,
\begin{pmatrix}
\nu_\mu \\ \mu
\end{pmatrix}_L,
\begin{pmatrix}
\nu_\tau \\ \tau
\end{pmatrix}_L
\]

and

\[
\begin{pmatrix}
u_u \\ d'
\end{pmatrix}_L,
\begin{pmatrix}
u_c \\ s'
\end{pmatrix}_L,
\begin{pmatrix}
u_t \\ b'
\end{pmatrix}_L;
\]

▷ Universal strength of the (charged-current) weak interactions;

▷ Idealization that neutrinos are massless.

First two clues suggest $SU(2)_L$ gauge symmetry
A theory of leptons

\[ L = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \]

weak hypercharges \( Y_L = -1, Y_R = -2 \)

Gell-Mann–Nishijima connection, \( Q = I_3 + \frac{1}{2} Y \)

\( SU(2)_L \otimes U(1)_Y \) gauge group \( \Rightarrow \) gauge fields:

\* weak isovector \( \vec{b}_\mu \), coupling \( g \)

\* weak isoscalar \( A_\mu \), coupling \( g'/2 \)

Field-strength tensors

\[ F_{\mu\nu}^\ell = \partial_\nu b_\mu^\ell - \partial_\mu b_\nu^\ell + g \varepsilon_{j k \ell} b_j^\mu b_k^\nu , SU(2)_L \]

and

\[ f_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu , U(1)_Y \]
**Interaction Lagrangian**

\[ \mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}}, \]

with

\[ \mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu \nu}^{\ell} F^{\ell \mu \nu} - \frac{1}{4} f_{\mu \nu} f^{\mu \nu}, \]

and

\[ \mathcal{L}_{\text{leptons}} = \bar{R} i \gamma^\mu \left( \partial_\mu + i \frac{g'}{2} A_\mu Y \right) R \]
\[ + \bar{L} i \gamma^\mu \left( \partial_\mu + i \frac{g'}{2} A_\mu Y + i \frac{g}{2} \vec{\tau} \cdot \vec{b}_\mu \right) L. \]

Electron mass term

\[ \mathcal{L}_e = -m_e (\bar{e}_R e_L + \bar{e}_L e_R) = -m_e \bar{e} e \]

would violate local gauge invariance. Theory has four massless gauge bosons

\[ A_\mu, b_\mu^1, b_\mu^2, b_\mu^3 \]

Nature has but one (\( \gamma \))
Hiding EW Symmetry

Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition

▷ Introduce a complex doublet of scalar fields

\[ \phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1 \]

▷ Add to \( \mathcal{L} \) (gauge-invariant) terms for interaction and propagation of the scalars,

\[ \mathcal{L}_{\text{scalar}} = (\mathcal{D}_\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi), \]

where \( \mathcal{D}_\mu = \partial_\mu + ig'2 A_\mu Y + ig 2 \tau \cdot \vec{b}_\mu \) and

\[ V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2 \]

▷ Add a Yukawa interaction

\[ \mathcal{L}_{\text{Yukawa}} = -\zeta_e \left[ \bar{R}(\phi^\dagger L) + (\bar{L}\phi)R \right] \]
Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^2 < 0$

Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks) $SU(2)_L$ and $U(1)_Y$

but preserves $U(1)_{em}$ invariance

Invariance under $G$ means $e^{i\alpha G} \langle \phi \rangle_0 = \langle \phi \rangle_0$, so $G \langle \phi \rangle_0 = 0$

$$
\tau_1 \langle \phi \rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}
$$

$$
\tau_2 \langle \phi \rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}
$$

$$
\tau_3 \langle \phi \rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}
$$

$$
Y \langle \phi \rangle_0 = Y_\phi \langle \phi \rangle_0 = +1 \langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}
$$
Examine electric charge operator $Q$ on the (electrically neutral) vacuum state

$$Q\langle \phi \rangle_0 = \frac{1}{2} (\tau_3 + Y) \langle \phi \rangle_0$$

$$= \frac{1}{2} \begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix} \langle \phi \rangle_0$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ unbroken!}$$

Four original generators are broken

electric charge is not

\[ \triangleright \quad SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em} \text{ (will verify)} \]

\[ \triangleright \quad \text{Expect massless photon} \]

\[ \triangleright \quad \text{Expect gauge bosons corresponding to} \]

$$\tau_1, \tau_2, \frac{1}{2}(\tau_3 - Y) \equiv K$$

to acquire masses
Expand about the vacuum state

Let \( \phi = \begin{pmatrix} 0 \\ (v + \eta)/\sqrt{2} \end{pmatrix} \); in unitary gauge

\[
\mathcal{L}_{\text{scalar}} = \frac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2 \eta^2 + \frac{v^2}{8} [g^2 |b_1 - ib_2|^2 + (g' A_\mu - gb_3^\mu)^2] + \text{interaction terms}
\]

Higgs boson \( \eta \) has acquired (mass)\(^2 \) \( M_H^2 = -2\mu^2 > 0 \)

\[
\frac{g^2 v^2}{8} (|W_\mu^+|^2 + |W_\mu^-|^2) \iff M_{W^\pm} = gv/2
\]

Now define orthogonal combinations

\[
Z_\mu = \frac{-g' A_\mu + gb^3_\mu}{\sqrt{g^2 + g'^2}} \quad A_\mu = \frac{g A_\mu + g'b^3_\mu}{\sqrt{g^2 + g'^2}}
\]

\[
M_{Z^0} = \sqrt{g^2 + g'^2} \ v/2 = M_W \sqrt{1 + g'^2/g^2}
\]

\( A_\mu \) remains massless
$\mathcal{L}_{\text{Yukawa}} = -\zeta_e \frac{(v + \eta)}{\sqrt{2}} (\bar{e}_R e_L + \bar{e}_L e_R)$

$= -\frac{\zeta_e v}{\sqrt{2}} \bar{e} e - \frac{\zeta_e \eta}{\sqrt{2}} \bar{e} e$

Electron acquires $m_e = \zeta_e v / \sqrt{2}$

Higgs coupling to electrons: $m_e / v \propto \text{mass}$

Desired particle content $\ldots +$ Higgs scalar

Values of couplings, electroweak scale $v$?

What about interactions?
**Interactions . . .**

\[ \mathcal{L}_{W-\ell} = -\frac{g}{2\sqrt{2}} [\bar{\nu}\gamma^\mu (1-\gamma_5) e W^\mu_\mu + \bar{e}\gamma^\mu (1-\gamma_5) \nu W^-_\mu] \]

+ similar terms for \( \mu \) and \( \tau \)

**Feynman rule:**

\[ \gamma \]

\[ e \]

\[ \nu \]

\[ \lambda \]

\[ \frac{-ig}{2\sqrt{2}} \gamma^\lambda (1-\gamma_5) \]

**gauge-boson propagator:**

\[ \frac{-i(g_{\mu\nu} - k_{\mu} k_{\nu} / M_W^2)}{k^2 - M_W^2} \].
Compute $\nu_\mu e \rightarrow \mu \nu_e$

$$\sigma(\nu_\mu e \rightarrow \mu \nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \left[1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu}\right]^2 \frac{1}{(1 + 2m_e E_\nu/M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\frac{g^4}{16M_W^4} = 2G_F^2$$

$$\Rightarrow g^4 = 32(G_F M_W^2)^2 = 64 \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^2$$

$$\Rightarrow \frac{g}{2\sqrt{2}} = \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}}$$

Using $M_W = g\nu/2$, determine

$$v = (G_F \sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV}$$

the electroweak scale

$$\Rightarrow \langle \phi^0 \rangle_0 = (G_F \sqrt{8})^{-\frac{1}{2}} \approx 174 \text{ GeV}$$
$W$-propagator modifies HE behavior

$$
\sigma(\nu_\mu e \rightarrow \mu \nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}
$$

$$
\lim_{E_\nu \rightarrow \infty} \sigma(\nu_\mu e \rightarrow \mu \nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\sqrt{2}}
$$

independent of energy!

partial-wave unitarity respected for

$$
s < M_W^2 [\exp (\pi \sqrt{2}/G_F M_W^2) - 1]
$$
**$W$-boson properties**

No prediction yet for $M_W$ (haven’t determined $g$)

Leptonic decay $W^- \rightarrow e^- \nu_e$

\[
\begin{align*}
\begin{array}{c}
e(p) \\
W^- \\
\bar{\nu}_e(q)
\end{array}
\end{align*}
\]

\[
\begin{align*}
p & \approx \left( \frac{M_W}{2} ; \frac{M_W \sin \theta}{2}, 0, \frac{M_W \cos \theta}{2} \right) \\
q & \approx \left( \frac{M_W}{2} ; -\frac{M_W \sin \theta}{2}, 0, -\frac{M_W \cos \theta}{2} \right)
\end{align*}
\]

\[
\mathcal{M} = -i \left( \frac{G_F M_W^2}{\sqrt{2}} \right)^{\frac{1}{2}} \bar{u}(e, p) \gamma_\mu (1 - \gamma_5) v(\nu, q) \varepsilon^\mu
\]

$\varepsilon^\mu = (0; \hat{\varepsilon})$: $W$ polarization vector in its rest frame

\[
\begin{align*}
|\mathcal{M}|^2 &= \frac{G_F M_W^2}{\sqrt{2}} \text{tr} \left[ \gamma(1 - \gamma_5) \gamma(1 + \gamma_5) \gamma^* \right] ; \\
\text{tr}[\cdots] &= [\varepsilon \cdot q \varepsilon^* \cdot p - \varepsilon \cdot \varepsilon^* q \cdot p + \varepsilon \cdot p \varepsilon^* \cdot q + i\epsilon_{\mu\nu\rho\sigma} \varepsilon^\mu q^\nu \varepsilon^* p^\rho]
\end{align*}
\]

*decay rate* is independent of $W$ polarization; look first at longitudinal pol. $\varepsilon^\mu = (0; 0, 0, 1) = \varepsilon^* \mu$, eliminate $\epsilon_{\mu\nu\rho\sigma}$

\[
|\mathcal{M}|^2 = \frac{4G_F M_W^4}{\sqrt{2}} \sin^2 \theta
\]
\[ \frac{d\Gamma_0}{d\Omega} = \frac{|M|^2}{64\pi^2} \frac{S_{12}}{M_W^3} \]

\[ S_{12} = \sqrt{[M_W^2 - (m_e + m_\nu)^2][M_W^2 - (m_e - m_\nu)^2]} = M_W^2 \]

\[ \frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2 \sqrt{2}} \sin^2 \theta \]

and

\[ \Gamma(W \rightarrow e\nu) = \frac{G_F M_W^3}{6\pi \sqrt{2}} \]

Other helicities: \( \varepsilon_{\pm 1}^\mu = (0; -1, \mp i, 0)/\sqrt{2} \)

\[ \frac{d\Gamma_{\pm 1}}{d\Omega} = \frac{G_F M_W^3}{32\pi^2 \sqrt{2}} (1 - \cos \theta)^2 \]

Extinctions at \( \cos \theta = \pm 1 \) are consequences of angular momentum conservation:

\[ W^- \uparrow \quad e^- \quad \downarrow \quad (\theta = 0) \text{ forbidden} \quad \bar{\nu}_e \uparrow \quad (\theta = \pi) \text{ allowed} \]

(situation reversed for \( W^+ \rightarrow e^+\nu_e \))

\( e^+ \) follows polarization direction of \( W^+ \)

\( e^- \) avoids polarization direction of \( W^- \)

important for discovery of \( W^\pm \) in \( \bar{p}p \ (\bar{q}q) \quad C \text{ violation} \)
Fig. 2. The $W$ decay angular distribution of the emission angle $\theta^*$ of the electron (positron) with respect to the proton (antiproton) direction in the rest frame of the $W$. Only those events for which the lepton charge and the decay kinematics are well determined have been used. The curve shows the $\langle V^-A \rangle$ expectation of $(1 + \cos \theta^*)^2$. 

UA1
75 EVENTS

Background subtracted and acceptance corrected
Interactions . . .

\[ \mathcal{L}_{A-\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e}\gamma^\mu eA_\mu \]

. . . vector interaction; \( A_\mu \) as \( \gamma \), provided

\[ gg' / \sqrt{g^2 + g'^2} \equiv e \]

Define \( g' = g \tan \theta_W \)

\( \theta_W \): weak mixing angle

\[ g = e / \sin \theta_W \geq e \]
\[ g' = e / \cos \theta_W \geq e \]

\[ Z_\mu = b_\mu^3 \cos \theta_W - A_\mu \sin \theta_W \quad A_\mu = A_\mu \cos \theta_W + b_\mu^3 \sin \theta_W \]

\[ \mathcal{L}_{Z-\nu} = \frac{-g}{4 \cos \theta_W} \bar{\nu}\gamma^\mu (1 - \gamma_5) \nu Z_\mu \]

\[ \mathcal{L}_{Z-e} = \frac{-g}{4 \cos \theta_W} \bar{e} [L_e \gamma^\mu (1 - \gamma_5) + R_e \gamma^\mu (1 + \gamma_5)] eZ_\mu \]

\[ L_e = 2 \sin^2 \theta_W - 1 = 2x_W + \tau_3 \]
\[ R_e = 2 \sin^2 \theta_W = 2x_W \]
**Z-boson properties**

Decay calculation analogous to $W^\pm$

\[
\Gamma(Z \rightarrow \nu\bar{\nu}) = \frac{G_F M_Z^3}{12\pi\sqrt{2}}
\]

\[
\Gamma(Z \rightarrow e^+ e^-) = \Gamma(Z \rightarrow \nu\bar{\nu}) \left[ L_e^2 + R_e^2 \right]
\]

**Neutral-current interactions**

New $\nu e$ reaction, not present in $V - A$

\[
\sigma(\nu_\mu e \rightarrow \nu_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[ L_e^2 + R_e^2 / 3 \right]
\]

\[
\sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[ L_e^2 / 3 + R_e^2 \right]
\]

\[
\sigma(\nu_e e \rightarrow \nu_e e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[ (L_e + 2)^2 + R_e^2 / 3 \right]
\]

\[
\sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[ (L_e + 2)^2 / 3 + R_e^2 \right]
\]
Gargamelle $\nu_\mu e$ Event
“Model-independent” analysis

Measure all cross sections to determine chiral couplings $L_e$ and $R_e$ or traditional vector and axial couplings $v$ and $a$

\[
a = \frac{1}{2}(L_e - R_e) \quad \quad v = \frac{1}{2}(L_e - R_e)
\]

$L_e = v + a \quad \quad R_e = v - a$

model-independent in $V, A$ framework
Neutrino–electron scattering
Twofold ambiguity remains even after measuring all four cross sections: same cross sections result if we interchange $R_e \leftrightarrow -R_e$ ($v \leftrightarrow a$)

Consider $e^+e^- \rightarrow \mu^+\mu^-$

\[
\mathcal{M} = -ie^2 \bar{u}(\mu, q_-) \gamma_\lambda Q_\mu v(\mu, q_+)^g_{\lambda\nu} \frac{g_{\lambda\nu}}{s} \bar{v}(e, p_+)\gamma_\nu u(e, p_-) + \frac{i}{2} \left( \frac{G_F M_Z^2}{\sqrt{2}} \right) \bar{u}(\mu, q_-) \gamma_\lambda [R_\mu (1 + \gamma_5) + L_\mu (1 - \gamma_5)] v(\mu, q_+) \\
\times \frac{g_{\lambda\nu}}{s - M_Z^2} \bar{v}(e, p_+)\gamma_\nu [R_e (1 + \gamma_5) + L_e (1 - \gamma_5)] u(e, p_-)
\]

muon charge $Q_\mu = -1$

\[
\frac{d\sigma}{dz} = \frac{\pi \alpha^2 Q_\mu^2}{2s} (1 + z^2) \\
- \frac{\alpha Q_\mu G_F M_Z^2 (s - M_Z^2)}{8\sqrt{2}[(s - M_Z^2)^2 + M_Z^2 \Gamma^2]} \times [(R_e + L_e)(R_\mu + L_\mu)(1 + z^2) + 2(R_e - L_e)(R_\mu - L_\mu)z] \\
+ \frac{G_F^2 M_Z^4 s}{64\pi[(s - M_Z^2)^2 + M_Z^2 \Gamma^2]} \times [(R_e^2 + L_e^2)(R_\mu^2 + L_\mu^2)(1 + z^2) + 2(R_e^2 - L_e^2)(R_\mu^2 - L_\mu^2)z]
\]
F-B asymmetry \( A \equiv \frac{\int_0^1 dz d\sigma/dz - \int_{-1}^0 dz d\sigma/dz}{\int_{-1}^1 dz d\sigma/dz} \)

\[
\lim_{s/M_Z^2 \ll 1} A = \frac{3G_F s}{16\pi\alpha Q\mu}\sqrt{2} (R_e - L_e)(R_\mu - L_\mu) \\
\approx -6.7 \times 10^{-5} \left( \frac{s}{1 \text{ GeV}^2} \right) (R_e - L_e)(R_\mu - L_\mu) \\
= -3G_F sa^2/4\pi\alpha\sqrt{2}
\]
Neutrino–electron scattering
With a measurement of $\sin^2 \theta_W$, predict

$$M_W^2 = g^2 v^2 / 4 = e^2 / 4 G_F \sqrt{2} \sin^2 \theta_W \approx (37.3 \text{ GeV/c}^2)^2 / \sin^2 \theta_W$$

$$M_Z^2 = M_W^2 / \cos^2 \theta_W$$
At low energies: \(\sigma(\bar{\nu}_e e \rightarrow \text{hadrons}) > \sigma(\nu_\mu e \rightarrow \mu \nu_e) > \sigma(\nu_e e \rightarrow \nu_e e) > \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) > \sigma(\nu_\mu e \rightarrow \nu_\mu e) > \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e)\)
EW interactions of quarks

▷ Left-handed doublet

\[ L_q = \begin{pmatrix} u \\ d \end{pmatrix} _L \]

\[ \begin{array}{ccc}
I_3 & Q & Y = 2(Q - I_3) \\
\frac{1}{2} & +\frac{2}{3} & \frac{1}{3} \\
-\frac{1}{2} & -\frac{1}{3} & \end{array} \]

▷ two right-handed singlets

\[ \begin{array}{ccc}
I_3 & Q & Y = 2(Q - I_3) \\
R_u = u_R & 0 & +\frac{2}{3} & +\frac{4}{3} \\
R_d = d_R & 0 & -\frac{1}{3} & -\frac{2}{3} \end{array} \]

▷ CC interaction

\[ \mathcal{L}_{W-q} = \frac{-g}{2\sqrt{2}} [\bar{u}_e \gamma^\mu (1 - \gamma_5) d \ W^+ _\mu + \bar{d} \gamma^\mu (1 - \gamma_5) u \ W^- _\mu] \]

identical in form to \( \mathcal{L}_{W-\ell} \): universality \( \Leftrightarrow \) weak isospin

▷ NC interaction

\[ \mathcal{L}_{Z-q} = \frac{-g}{4 \cos \theta_W} \sum _{i=u,d} \bar{q}_i \gamma^\mu [L_i (1 - \gamma_5) + R_i (1 + \gamma_5)] q_i Z^\mu \]

\[ L_i = \tau_3 - 2Q_i \sin^2 \theta_W \quad R_i = -2Q_i \sin^2 \theta_W \]

equivalent in form (not numbers) to \( \mathcal{L}_{Z-\ell} \)
Trouble in Paradise

Universal $u \leftrightarrow d$, $\nu_e \leftrightarrow e$ not quite right

Good: \[
\begin{pmatrix}
u \\ d
\end{pmatrix}_L \rightarrow \text{Better:} \begin{pmatrix}
u \\ d_{\theta}
\end{pmatrix}_L
\]

\[d_{\theta} \equiv d \cos \theta_C + s \sin \theta_C \quad \cos \theta_C = 0.9736 \pm 0.0010\]

“Cabibbo-rotated” doublet perfects CC interaction (up to small third-generation effects) but $\Rightarrow$ serious trouble for NC

\[
\mathcal{L}_{Z-q} = \frac{-g}{4 \cos \theta_W} Z_{\mu} \left\{ \bar{u} \gamma^\mu \left[ L_u (1 - \gamma_5) + R_u (1 + \gamma_5) \right] u \\
+ \bar{d} \gamma^\mu \left[ L_d (1 - \gamma_5) + R_d (1 + \gamma_5) \right] d \cos^2 \theta_C \\
+ \bar{s} \gamma^\mu \left[ L_d (1 - \gamma_5) + R_d (1 + \gamma_5) \right] s \sin^2 \theta_C \\
+ \bar{d} \gamma^\mu \left[ L_d (1 - \gamma_5) + R_d (1 + \gamma_5) \right] s \sin \theta_C \cos \theta_C \\
+ \bar{s} \gamma^\mu \left[ L_d (1 - \gamma_5) + R_d (1 + \gamma_5) \right] d \sin \theta_C \cos \theta_C \right\}
\]

Strangeness-changing NC interactions highly suppressed!

BNL E-787/E-949 has three $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ candidates, with $B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1.47^{+1.30}_{-0.89} \times 10^{-10}$

(Phys. Rev. Lett. 93, 031801 (2004))
two left-handed doublets
\[
\begin{pmatrix}
\nu_e \\
e^-
\end{pmatrix}_L 
\begin{pmatrix}
\nu_\mu \\
\mu^-
\end{pmatrix}_L 
\begin{pmatrix}
u \\
d_\theta
\end{pmatrix}_L 
\begin{pmatrix}
u c \\
s_\theta
\end{pmatrix}_L
\]

\( (s_\theta = s \cos \theta_C - d \sin \theta_C) \)

+ right-handed singlets, \( e_R, \mu_R, u_R, d_R, c_R, s_R \)

Required new charmed quark, \( c \)

Cross terms vanish in \( \mathcal{L}_{Z-q} \),

\[ -ig \frac{\gamma \lambda [(1 - \gamma_5)L_i + (1 + \gamma_5)R_i]}{4 \cos \theta_W}, \]

\[ L_i = \tau_3 - 2Q_i \sin^2 \theta_W \]

\[ R_i = -2Q_i \sin^2 \theta_W \]

flavor-diagonal interaction!
Straightforward generalization to \( n \) quark doublets

\[
\mathcal{L}_{W-q} = \frac{-g}{2\sqrt{2}} \left[ \bar{\Psi} \gamma^\mu (1 - \gamma_5) \mathcal{O} \Psi \right] W_\mu^+ + \text{h.c.}
\]

composite \( \Psi = \begin{pmatrix} u \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix} \)

\( \mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix} \)

\( U: \) unitary quark mixing matrix

Weak-isospin part:

\[
\mathcal{L}^{\text{iso}}_{Z-q} = \frac{-g}{4 \cos \theta_W} \bar{\Psi} \gamma^\mu (1 - \gamma_5) \left[ \mathcal{O}, \mathcal{O}^\dagger \right] \Psi
\]

Since \( \left[ \mathcal{O}, \mathcal{O}^\dagger \right] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3 \)

\( \Rightarrow \) NC interaction is flavor-diagonal

General \( n \times n \) quark-mixing matrix \( U \):

\( n(n - 1)/2 \) real \( \angle \), \( (n - 1)(n - 2)/2 \) complex phases

3 × 3 (Cabibbo–Kobayashi–Maskawa): \( 3 \angle + 1 \) phase

\( \Rightarrow \) CP violation
Qualitative successes of $SU(2)_L \otimes U(1)_Y$ theory:

- neutral-current interactions
- necessity of charm
- existence and properties of $W^\pm$ and $Z^0$

Decade of precision tests EW (one-per-mille)

\begin{align*}
M_Z & \quad 91187.6 \pm 2.1 \text{ MeV}/c^2 \\
\Gamma_Z & \quad 2495.2 \pm 2.3 \text{ MeV} \\
\sigma^0_{\text{hadronic}} & \quad 41.541 \pm 0.037 \text{ nb} \\
\Gamma_{\text{hadronic}} & \quad 1744.4 \pm 2.0 \text{ MeV} \\
\Gamma_{\text{leptonic}} & \quad 83.984 \pm 0.086 \text{ MeV} \\
\Gamma_{\text{invisible}} & \quad 499.0 \pm 1.5 \text{ MeV}
\end{align*}

where $\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$

light neutrinos $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{SM}(Z \to \nu_i \bar{\nu}_i)$

Current value: $N_\nu = 2.994 \pm 0.012$

... excellent agreement with $\nu_e$, $\nu_\mu$, and $\nu_\tau$
Three light neutrinos

![Graph showing the relationship between energy (GeV) and cross-section (nb) for different neutrino numbers (Nv = 2, 3, 4)]
The top quark must exist

- Two families

\[
\begin{pmatrix}
u \\ d
\end{pmatrix}_L \begin{pmatrix}c \\ s
\end{pmatrix}_L
\]

don’t account for CP violation. Need a third family . . . or another answer.

Given the existence of \( b, (\tau) \)

- top is needed for an anomaly-free EW theory
- absence of FCNC in \( b \) decay (\( b \nrightarrow s\ell^+\ell^- \), etc.)
- \( b \) has weak isospin \( I_{3L} = -\frac{1}{2} \); needs partner

\[
\begin{pmatrix}t \\ b
\end{pmatrix}_L
\]

\[
L_b = I_{3L} - Q_b \sin^2 \theta_W
\]

\[
R_b = I_{3R} - Q_b \sin^2 \theta_W
\]
Measure $I^{(b)}_{3L} = -0.490^{+0.015}_{-0.012} \quad I^{(b)}_{3R} = -0.028 \pm 0.056$

Needed: top with $I_{3L} = +\frac{1}{2}$

Global fits . . .

to precision EW measurements:

▷ precision improves with time

▷ calculations improve with time

11.94, LEPEWWG: $m_t = 178 \pm 11^{+18}_{-19} \text{ GeV}/c^2$

Direct measurements: $m_t = 174.3 \pm 5.1 \text{ GeV}/c^2$
<table>
<thead>
<tr>
<th>Measurement</th>
<th>Fit</th>
<th>( \Delta \alpha_{\text{meas}}^{\text{fit}} / \sigma_{\text{meas}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \alpha_{\text{had}}^{(5)}(m_Z) )</td>
<td>0.02761 ± 0.00036</td>
<td>0.02770</td>
</tr>
<tr>
<td>( m_Z ) [GeV]</td>
<td>91.1875 ± 0.0021</td>
<td>91.1874</td>
</tr>
<tr>
<td>( \Gamma_Z ) [GeV]</td>
<td>2.4952 ± 0.0023</td>
<td>2.4965</td>
</tr>
<tr>
<td>( \sigma_{\text{had}} ) [nb]</td>
<td>41.540 ± 0.037</td>
<td>41.481</td>
</tr>
<tr>
<td>( R_l )</td>
<td>20.767 ± 0.025</td>
<td>20.739</td>
</tr>
<tr>
<td>( A_{fb}^{0,l} )</td>
<td>0.01714 ± 0.00095</td>
<td>0.01642</td>
</tr>
<tr>
<td>( A_l(P_{\tau}) )</td>
<td>0.1465 ± 0.0032</td>
<td>0.1480</td>
</tr>
<tr>
<td>( R_b )</td>
<td>0.21630 ± 0.00066</td>
<td>0.21562</td>
</tr>
<tr>
<td>( R_c )</td>
<td>0.1723 ± 0.0031</td>
<td>0.1723</td>
</tr>
<tr>
<td>( A_{fb}^{0,b} )</td>
<td>0.0992 ± 0.0016</td>
<td>0.1037</td>
</tr>
<tr>
<td>( A_{fb}^{0,c} )</td>
<td>0.0707 ± 0.0035</td>
<td>0.0742</td>
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<tr>
<td>( A_b )</td>
<td>0.923 ± 0.020</td>
<td>0.935</td>
</tr>
<tr>
<td>( A_c )</td>
<td>0.670 ± 0.027</td>
<td>0.668</td>
</tr>
<tr>
<td>( A_l(\text{SLD}) )</td>
<td>0.1513 ± 0.0021</td>
<td>0.1480</td>
</tr>
<tr>
<td>( \sin^2 \theta_{\text{lep}}(Q_{\text{fb}}) )</td>
<td>0.2324 ± 0.0012</td>
<td>0.2314</td>
</tr>
<tr>
<td>( m_W ) [GeV]</td>
<td>80.425 ± 0.034</td>
<td>80.390</td>
</tr>
<tr>
<td>( \Gamma_W ) [GeV]</td>
<td>2.133 ± 0.069</td>
<td>2.093</td>
</tr>
<tr>
<td>( m_t ) [GeV]</td>
<td>178.0 ± 4.3</td>
<td>178.4</td>
</tr>
</tbody>
</table>

LEP Electroweak Working Group, Winter 2005
Parity violation in atoms

Nucleon appears elementary at very low $Q^2$; effective Lagrangian for nucleon $\beta$-decay

$$\mathcal{L}_\beta = -\frac{G_F}{\sqrt{2}} \bar{e}\gamma_\lambda (1 - \gamma_5)\nu \bar{p}\gamma^\lambda (1 - g_A \gamma_5)n$$

$g_A \approx 1.26$: axial charge

NC interactions ($x_W \equiv \sin^2 \theta_W$):

$$\mathcal{L}_{ep} = \frac{G_F}{2\sqrt{2}} \bar{e}\gamma_\lambda (1 - 4x_W - \gamma_5)e \bar{p}\gamma^\lambda (1 - 4x_W - \gamma_5)p,$$

$$\mathcal{L}_{en} = \frac{G_F}{2\sqrt{2}} \bar{e}\gamma_\lambda (1 - 4x_W - \gamma_5)e \bar{n}\gamma^\lambda (1 - \gamma_5)n$$

▷ Regard nucleus as a noninteracting collection of $Z$ protons and $N$ neutrons ▷ Perform NR reduction; nucleons contribute coherently to $A_e V_N$ coupling, so dominant $P$-violating contribution to $eN$ amplitude is

$$\mathcal{M}_{pv} = \frac{-iG_F}{2\sqrt{2}} Q^W \bar{e}\rho_N(r)\gamma_5 e$$

$\rho_N(r)$: nucleon density at $e^-$ coordinate $r$

$$Q^W \equiv Z(1 - 4x_W) - N: \text{weak charge}$$

Bennett & Wieman (Boulder) determined weak charge of Cesium by measuring 6S-7S transition polarizability

$$Q_W(\text{Cs}) = -72.06 \pm 0.28 \text{ (expt)} \pm 0.34 \text{ (theory)}$$

about $2.5\sigma$ above SM prediction
The vacuum energy problem

Higgs potential \( V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2 \)

At the minimum,

\[
V(\langle \phi^\dagger \phi \rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda| v^4}{4} < 0.
\]

Identify \( M_H^2 = -2\mu^2 \)

contributes field-independent vacuum energy density

\[
\rho_H = \frac{M_H^2 v^2}{8}
\]

Adding vacuum energy density \( \rho_{\text{vac}} \Leftrightarrow \) adding cosmological constant \( \Lambda \) to Einstein’s equation

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu}
\]

\[
\Lambda = \frac{8\pi G_N}{c^4} \rho_{\text{vac}}
\]
observed vacuum energy density $\varrho_{\text{vac}} \lesssim 10^{-46}$ GeV$^4$

But $M_H \gtrsim 114 \text{ GeV}/c^2$ $\Rightarrow$

$\varrho_H \gtrsim 10^8$ GeV$^4$

MISMATCH BY 54 ORDERS OR MAGNITUDE
Why a Higgs Boson Must Exist

- Role in canceling high-energy divergences

\( S \)-matrix analysis of \( e^+e^- \rightarrow W^+W^- \)

\[ J = 1 \] partial-wave amplitudes \( \mathcal{M}_{\gamma}^{(1)} \), \( \mathcal{M}_Z^{(1)} \), \( \mathcal{M}_\nu^{(1)} \)

have—individually—unacceptable high-energy behavior \( (\propto s) \)
...But sum is well-behaved

“Gauge cancellation” observed at LEP2, Tevatron

![Graph showing σ_{WW} (pb) versus √s (GeV)]

- YFSWW/RacoonWW
- no ZWW vertex (Gentle)
- only ν_e exchange (Gentle)
$J = 0$ amplitude exists because electrons have mass, and can be found in “wrong” helicity state

\[ M_{\nu}^{(0)} \propto s^{\frac{1}{2}} : \text{unacceptable HE behavior} \]

(no contributions from $\gamma$ and $Z$)

This divergence is canceled by the Higgs-boson contribution

\[ \Rightarrow H e \bar{e} \text{ coupling must be } \propto m_e, \]

because “wrong-helicity” amplitudes $\propto m_e$

If the Higgs boson did not exist, something else would have to cure divergent behavior
IF gauge symmetry were unbroken . . .

- no Higgs boson
- no longitudinal gauge bosons
- no extreme divergences
- no wrong-helicity amplitudes

... and no viable low-energy phenomenology

In spontaneously broken theory . . .

- gauge structure of couplings eliminates the most severe divergences
- lesser—but potentially fatal—divergence arises because the electron has mass
  ... due to the Higgs mechanism
- SSB provides its own cure—the Higgs boson

A similar interplay and compensation *must exist* in any acceptable theory
**Bounds on** \( M_H \)

EW theory does not predict Higgs-boson mass

Self-consistency \( \Rightarrow \) plausible lower and upper bounds

▷ **Conditional upper bound** from Unitarity

Compute amplitudes \( \mathcal{M} \) for gauge boson scattering at high energies, make a partial-wave decomposition

\[
\mathcal{M}(s, t) = 16\pi \sum_J (2J + 1)a_J(s)P_J(\cos \theta)
\]

Most channels decouple—pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies)—for any \( M_H \).

Four interesting channels:

\[
W_L^+ W_L^- \quad Z_L^0 Z_L^0/\sqrt{2} \quad HH/\sqrt{2} \quad HZ_L^0
\]

\( L \): longitudinal, \( 1/\sqrt{2} \) for identical particles
In HE limit,\(^a\) \(s\)-wave amplitudes \(\propto G_F M_H^2\)

\[
\lim_{s \gg M_H^2} (a_0) \rightarrow \frac{-G_F M_H^2}{4\pi \sqrt{2}} \cdot \begin{bmatrix}
1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\
1/\sqrt{8} & 3/4 & 1/4 & 0 \\
1/\sqrt{8} & 1/4 & 3/4 & 0 \\
0 & 0 & 0 & 1/2
\end{bmatrix}
\]

Require that largest eigenvalue respect the partial-wave unitarity condition \(|a_0| \leq 1\)

\[
\implies M_H \leq \left(\frac{8\pi \sqrt{2}}{3G_F}\right)^{1/2} = 1 \text{ TeV/c}^2
\]

condition for perturbative unitarity

\(^a\)Convenient to calculate using Goldstone-boson equivalence theorem, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by \(\mathcal{L}_{\text{int}} = -\lambda \pi h(2w^+w^- + z^2 + h^2) - (\lambda/4)(2w^+w^- + z^2 + h^2)^2\), with \(1/v^2 = G_F \sqrt{2}\) and \(\lambda = G_F M_H^2/\sqrt{2}\).
If the bound is respected

- weak interactions remain weak at all energies
- perturbation theory is everywhere reliable

If the bound is violated

- perturbation theory breaks down
- weak interactions among $W^\pm$, $Z$, and $H$ become strong on the 1-TeV scale

⇒ features of strong interactions at GeV energies will characterize electroweak gauge boson interactions at TeV energies

Threshold behavior of the pw amplitudes $a_{IJ}$ follows from chiral symmetry

$$
\begin{align*}
    a_{00} &\approx G_F s/8\pi\sqrt{2} \quad \text{attractive} \\
    a_{11} &\approx G_F s/48\pi\sqrt{2} \quad \text{attractive} \\
    a_{20} &\approx -G_F s/16\pi\sqrt{2} \quad \text{repulsive}
\end{align*}
$$

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV
Triviality of scalar field theory

Only *noninteracting* scalar field theories make sense on all energy scales.

Quantum field theory vacuum is a dielectric medium that screens charge ⇒ *effective charge* is a function of the distance or, equivalently, of the energy scale.

In $\lambda \phi^4$ theory, it is easy to calculate the variation of the coupling constant $\lambda$ in perturbation theory by summing bubble graphs.

$\lambda(\mu)$ is related to a higher scale $\Lambda$ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log \left( \frac{\Lambda}{\mu} \right)$$

(Perturbation theory reliable only when $\lambda$ is small, lattice field theory treats strong-coupling regime.)
For stable Higgs potential (i.e., for vacuum energy not to race off to \(-\infty\)), require \(\lambda(\Lambda) \geq 0\)

Rewrite RGE as an inequality

\[
\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log (\Lambda/\mu) .
\]

implies an upper bound

\[
\lambda(\mu) \leq \frac{2\pi^2}{3} \log (\Lambda/\mu)
\]

If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit \(\Lambda \to \infty\) while holding \(\mu\) fixed at some reasonable physical scale. In this limit, the bound forces \(\lambda(\mu)\) to zero. \(\rightarrow\) free field theory “trivial”

Rewrite as bound on \(M_H\):

\[
\Lambda \leq \mu \exp \left( \frac{2\pi^2}{3\lambda(\mu)} \right)
\]

Choose \(\mu = M_H\), and recall \(M_H^2 = 2\lambda(M_H)v^2\)

\[
\Lambda \leq M_H \exp \left( \frac{4\pi^2 v^2}{3M_H^2} \right)
\]
Moral: For any $M_H$, there is a maximum energy scale $\Lambda^*$ at which the theory ceases to make sense. The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies.

Perturbative analysis breaks down when $M_H \rightarrow 1$ TeV/c$^2$ and interactions become strong.

Lattice analyses $\Rightarrow M_H \lesssim 710 \pm 60$ GeV/c$^2$ if theory describes physics to a few percent up to a few TeV.

If $M_H \rightarrow 1$ TeV EW theory lives on brink of instability.
**Lower bound** by requiring EWSB vacuum \( V(v) < V(0) \)

Requiring that \( \langle \phi \rangle_0 \neq 0 \) be an absolute minimum of the one-loop potential up to a scale \( \Lambda \) yields the vacuum-stability condition

\[
M_H^2 > \frac{3G_F \sqrt{2}}{8\pi^2} (2M_W^4 + M_Z^4 - 4m_t^4) \log(\Lambda^2/v^2)
\]

\[
\ldots \text{for } m_t \lesssim M_W
\]

(No illuminating analytic form for heavy \( m_t \))

If the Higgs boson is relatively light—which would itself require explanation—then the theory can be self-consistent up to very high energies

If EW theory is to make sense all the way up to a unification scale \( \Lambda^* = 10^{16} \text{ GeV} \), then

\[
134 \text{ GeV}/c^2 \lesssim M_H \lesssim 177 \text{ GeV}/c^2
\]
Higgs-Boson Properties

$$\Gamma(H \to f \bar{f}) = \frac{G_F m_f^2 M_H}{4\pi \sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

$$\propto M_H$$ in the limit of large Higgs mass

$$\Gamma(H \to W^+ W^-) = \frac{G_F M_H^3}{32\pi \sqrt{2}} (1-x)^{1/2} (4 - 4x + 3x^2)$$

$$x \equiv 4M_W^2/M_H^2$$

$$\Gamma(H \to Z^0 Z^0) = \frac{G_F M_H^3}{64\pi \sqrt{2}} (1-x')^{1/2} (4 - 4x' + 3x'^2)$$

$$x' \equiv 4M_Z^2/M_H^2$$

asymptotically $$\propto M_H^3$$ and $$\frac{1}{2} M_H^3$$, respectively

$$\left(\frac{1}{2} \text{ from weak isospin}\right)$$

$$2x^2$$ and $$2x'^2$$ terms $$\Leftrightarrow$$ decays into transversely polarized gauge bosons

Dominant decays for large $$M_H$$ into pairs of longitudinally polarized weak bosons
Below $W^+W^-$ threshold, $\Gamma_H \ll 1$ GeV

Far above $W^+W^-$ threshold, $\Gamma_H \propto M_H^3$

For $M_H \to 1$ TeV/$c^2$, Higgs boson is an ephemeron, with a perturbative width approaching its mass.
Clues to the Higgs-boson mass

Sensitivity of EW observables to $m_t$ gave early indications for massive top quantum corrections to SM predictions for $M_W$ and $M_Z$ arise from different quark loops

\[
\begin{align*}
\bar{b} & \quad \bar{t} \\
W^+ & \sim W^+ \quad Z^0 & \sim Z^0,
\end{align*}
\]

... alter link between the $M_W$ and $M_Z$:

\[M_W^2 = M_Z^2 \left(1 - \sin^2 \theta_W \right) (1 + \Delta \rho)\]

where $\Delta \rho \approx \Delta \rho^{(\text{quarks})} = 3G_F m_t^2 / 8\pi^2 \sqrt{2}$

strong dependence on $m_t^2$ accounts for precision of $m_t$ estimates derived from EW observables

$m_t$ known to $\pm 3\%$ from Tevatron ...

$\implies$ look beyond the quark loops to next most important quantum corrections:

Higgs-boson effects
$H$ quantum corrections smaller than $t$ corrections, exhibit more subtle dependence on $M_H$ than the $m_t^2$ dependence of the top-quark corrections

$$\Delta \rho^{(\text{Higgs})} = C \cdot \ln \left( \frac{M_H}{v} \right)$$

$M_Z$ known to 23 ppm, $m_t$ and $M_W$ well measured

so examine dependence of $M_W$ upon $m_t$ and $M_H$
Direct, indirect determinations agree reasonably.
Both favor a light Higgs boson,

within framework of SM analysis.
Direct, indirect determinations agree reasonably
Both favor a light Higgs boson,

*within framework of SM analysis.*
Direct, indirect determinations agree reasonably
Both favor a light Higgs boson,

\textit{within framework of SM analysis.}
CDF Run 2 Preliminary (June 2 2005)

- Run 1 World Average (Run 1 only): \(178.0 \pm 2.7 \pm 3.3\)
- Run 1 D0 Lepton+Jets (Run 1 only): \(180.1 \pm 3.6 \pm 3.9\)
- Run 1 CDF Lepton+Jets (Run 1 only): \(176.1 \pm 5.1 \pm 5.3\)
- Lepton+Jets: \(M_{\text{reco}}+W \rightarrow jj\) (L= 318 pb\(^{-1}\)): \(173.5 \pm 2.7 \pm 3.0\)
- Lepton+Jets: DLM (L= 318 pb\(^{-1}\)): \(173.8 \pm 2.7 \pm 3.3\)
- Dilepton: \(\nu\) weighting (L= 200 pb\(^{-1}\)): \(168.1 \pm 11.0 \pm 8.6\)
- Dilepton: \(\phi\) of \(\nu\) (L= 193 pb\(^{-1}\)): \(170.0 \pm 16.6 \pm 7.4\)
- Dilepton: \(P_{Z}(t\bar{t})\) (L= 193 pb\(^{-1}\)): \(176.5 \pm 17.2 \pm 6.9\)
Fit to a universe of data

\[ \Delta \chi^2 \]

\[
\Delta \alpha_{\text{had}}^{(5)} = \begin{cases} 
0.02761 \pm 0.00036 \\
0.02749 \pm 0.00012 
\end{cases}
\]

incl. low \( Q^2 \) data

Excluded

\( m_H [\text{GeV}] \)
Fit to a universe of data
Within SM, LEPEWWG deduce a 95% CL upper limit, $M_H \lesssim 280 \text{ GeV}/c^2$.

Direct searches at LEP $\Rightarrow M_H > 114.4 \text{ GeV}/c^2$, excluding much of the favored region.

either the Higgs boson is just around the corner, or SM analysis is misleading

Things will soon be popping!

Expect progress from $M_W$-$m_t$-$M_H$ correlation

▷ Tevatron and LHC measurements will determine $m_t$ within 1 or 2 GeV/c$^2$

▷ ...and improve $\delta M_W$ to about 15 MeV/c$^2$

▷ As the Tevatron’s integrated luminosity approaches 10 fb$^{-1}$, CDF and DØ will begin to explore the region of $M_H$ not excluded by LEP

▷ ATLAS and CMS will carry on the exploration of the Higgs sector at the LHC
Penumbra
Synthetic Spring
Neptune
Big Technology
I'm With You
Cooled
Faith (Yourself)
Travel
Perpetual Symmetry

Produced By
Gareth Young
and Higgs Boson

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Assessment

25 YEARS OF CONFIRMATIONS OF
$SU(2)_L \otimes U(1)_Y$

★ neutral currents

★ $W^\pm, Z^0$

★ charm

(+ experimental guidance)

★ $\tau, \nu_\tau$

★ $b, t$

+ experimental surprises

★ narrowness of $\psi, \psi'$

★ long $B$ lifetime

★ large $B^0-\bar{B}^0$ mixing

★ heavy top

★ neutrino oscillations
10 YEARS OF PRECISION MEASUREMENTS... 
... FIND NO SIGNIFICANT DEVIATIONS

QUANTUM CORRECTIONS TESTED AT $\pm 10^{-3}$

NO “NEW” PHYSICS ... YET!

Theory tested at distances
from $10^{-17}$ cm
to $\sim 10^{22}$ cm

origin Coulomb’s law (tabletop experiments)

smaller \[
\left\{
\begin{align*}
\text{Atomic physics} & \rightarrow \text{QED} \\
\text{high-energy experiments} & \rightarrow \text{EW theory}
\end{align*}
\right.
\]

larger $M_\gamma \approx 0$ in planetary ... measurements

IS EW THEORY TRUE?
COMPLETE ??
EWSB: another path?

Modeled EWSB on Ginzburg–Landau description of SC phase transition

had to introduce new, elementary scalars

GL is not the last word on superconductivity: dynamical Bardeen–Cooper–Schrieffer theory

The elementary fermions—electrons—and gauge interactions—QED—needed to generate the scalar bound states are already present in the case of superconductivity. Could a scheme of similar economy account for EWSB?

\[ SU(3)_c \otimes SU(2)_L \otimes U(1)_Y + \text{massless } u \text{ and } d \]

Treat \( SU(2)_L \otimes U(1)_Y \) as perturbation

\( m_u = m_d = 0 \): QCD has exact \( SU(2)_L \otimes SU(2)_R \)

chiral symmetry. At an energy scale \( \sim \Lambda_{QCD} \), strong interactions become strong, fermion condensates appear, and \( SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V \)

\[ \Rightarrow \] 3 Goldstone bosons, one for each broken generator: 3 massless pions (Nambu)
Broken generators: 3 axial currents; couplings to $\pi$ measured by pion decay constant $f_\pi$

**Turn on $SU(2)_L \otimes U(1)_Y$:** EW gauge bosons couple to axial currents, acquire masses of order $\sim g f_\pi$

$$\mathcal{M}^2 = \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & gg' \\ 0 & 0 & gg' & g'^2 \end{pmatrix} \frac{f_\pi^2}{4},$$

where $(W^+, W^-, W_3, A)$ same structure as standard EW theory. Diagonalize:

- $M^2_W = g^2 f_\pi^2 / 4$, $M^2_Z = (g^2 + g'^2)f_\pi^2 / 4$, $M^2_A = 0$, so

$$\frac{M^2_Z}{M^2_W} = \frac{(g^2 + g'^2)}{g^2} = \frac{1}{\cos^2 \theta_W}$$

Massless pions disappear from physical spectrum, to become longitudinal components of weak bosons

$$M_W \approx 30 \text{ MeV}/c^2$$
With no Higgs mechanism . . .

- Quarks and leptons would remain massless
- QCD would confine them in color-singlet hadrons
- *Nucleon mass would be little changed*, but proton outweighs neutron
- QCD breaks EW symmetry, gives \((1/2500 \times \text{observed})\) masses to \(W, Z\), so weak-isospin force doesn’t confine
- Rapid! \(\beta\)-decay \(\Rightarrow\) lightest nucleus is one neutron; no hydrogen atom
- Probably some light elements in BBN, but \(\infty\) Bohr radius
- No atoms (as we know them) means no chemistry, no stable composite structures like the solids and liquids we know

. . . the character of the physical world would be profoundly changed
In a decade or two, we can hope to...

Understand electroweak symmetry breaking
*Observe the Higgs boson*
Measure neutrino masses and mixings
*Establish Majorana neutrinos ($\beta\beta_0\nu$)*
Thoroughly explore CP violation in $B$ decays
*Exploit rare decays ($K, D, \ldots$)*
Observe neutron EDM, pursue electron EDM
*Use top as a tool*
Observe new phases of matter
*Understand hadron structure quantitatively*
Uncover QCD’s full implications
*Observe proton decay*
Understand the baryon excess
*Catalogue matter and energy of the universe*
Measure dark energy equation of state
*Search for new macroscopic forces*
Determine GUT symmetry

Detect neutrinos from the universe
Learn how to quantize gravity
*Learn why empty space is nearly weightless*
Test the inflation hypothesis
*Understand discrete symmetry violation*
Resolve the hierarchy problem
*Discover new gauge forces*
Directly detect dark-matter particles
*Explore extra spatial dimensions*
Understand the origin of large-scale structure
*Observe gravitational radiation*
Solve the strong CP problem
*Learn whether supersymmetry is TeV-scale*
Seek TeV-scale dynamical symmetry breaking
*Search for new strong dynamics*
Explain the highest-energy cosmic rays
*Formulate problem of identity*

... and learn to ask the right questions
Appendix: The EW scale and beyond

EWSB scale, \( v = (G_F \sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV} \), sets

\[
M_W^2 = \frac{g^2 v^2}{2} \quad M_Z^2 = \frac{M_W^2}{\cos^2 \theta_W}
\]

But it is not the only scale of physical interest

quasi-certain: \( M_{\text{Planck}} = 1.22 \times 10^{19} \text{ GeV} \)

probable: \( SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \) unification scale
\(~ 10^{15-16} \text{ GeV} \)

somewhere: flavor scale

How to keep the distant scales from mixing in the face of quantum corrections?

\text{OR}

How to stabilize the mass of the Higgs boson on the electroweak scale?

\text{OR}

Why is the electroweak scale small?
Higgs potential \( V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2 \)

\( \mu^2 < 0: SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}}, \) as

\[
\langle \phi \rangle_0 = \begin{pmatrix}
0 \\
\sqrt{-\mu^2/2|\lambda|}
\end{pmatrix} = \begin{pmatrix}
0 \\
(G_F \sqrt{8})^{-1/2} \\
175 \, \text{GeV}
\end{pmatrix}
\]

Beyond classical approximation, quantum corrections to scalar mass parameters:

\[
m^2(p^2) = m_0^2 + \text{J=1} + \text{J=1/2} + \text{J=0}
\]

Loop integrals are potentially divergent.

\[
m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \cdots
\]

\( \Lambda: \) reference scale at which \( m^2 \) is known

\( g: \) coupling constant of the theory

\( C: \) coefficient calculable in specific theory
\[ m^2(p^2) = m^2(\Lambda^2) + C g^2 \int_{p^2}^{\Lambda^2} dk^2 + \cdots \]

For the mass shifts induced by radiative corrections to remain under control (not greatly exceed the value measured on the laboratory scale), \textit{either}

\( \Lambda \) must be small, \textit{or}

\( \Lambda \) must be small, \textit{or}

\( \text{new physics must intervene to cut off the integral} \)

\[ \Lambda \sim M_{\text{Planck}} = \left( \frac{\hbar c}{G_{\text{Newton}}} \right)^{1/2} \approx 1.22 \times 10^{19} \text{ GeV} \]

\( \text{for } SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \)

\( \text{OR} \)

\[ \Lambda \sim M_U \approx 10^{15} - 10^{16} \text{ GeV} \]

\( \text{for unified theory} \)

Both \( \gg v/\sqrt{2} \approx 175 \text{ GeV} \implies \text{New Physics at } E \lesssim 1 \text{ TeV} \)
Fine tuning the Higgs
\( \Lambda = 10 \text{ TeV} \)

- tree
- gauge loops
- higgs loop
- top loop

actual Higgs mass \(^2\)

\( (200 \text{ GeV})^2 \)

Yuck!!!

Martin Schmaltz, IHEP02
Only a few distinct scenarios . . .

▷ Supersymmetry: balance contributions of fermion loops \((-1)\) and boson loops \((+1)\)

*Exact supersymmetry,*

\[
\sum_{i=\text{fermions}+\text{bosons}} C_i \int dk^2 = 0
\]

*Broken supersymmetry,* shifts acceptably small if superpartner mass splittings are not too large

\[g^2 \Delta M^2 \text{ “small enough” } \Rightarrow \tilde{M} \lesssim 1 \text{ TeV}/c^2\]

▷ Composite scalars (technicolor): New physics arises on scale of composite Higgs-boson binding,

\[\Lambda_{TC} \simeq O(1 \text{ TeV})\]

“Form factor” cuts effective range of integration

▷ Strongly interacting gauge sector: \(WW\) resonances, multiple \(W\) production, probably scalar bound state “quasiHiggs” with \(M < 1 \text{ TeV}\)

▷ Extra spacetime dimensions: pseudo-Nambu–Goldstone bosons, extra particles to cancel integrand, . . .