The Standard Model (Electroweak Theory)

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Tentative Outline . . .

- ▷ Preliminaries

  Current state of particle physics

  A few words about QCD

  Sources of mass

- ▷ Antecedents of the electroweak theory

  What led to EW theory

  What EW theory needs to explain

- ▷ Some consequences of the Fermi theory

  μ decay

  νe scattering

. . . Outline . . .

▷ $SU(2)_L \otimes U(1)_Y$ theory

Gauge theories

Spontaneous symmetry breaking

Consequences: $W^{\pm}$, Z^0/NC, H, m_f?

Measuring $\sin^2\theta_W$ in νe scattering

GIM / CKM

▷ Phenomena at tree level and beyond

Z^0 pole

W mass and width

Atomic parity violation

Looking for trouble

m_t, M_W, M_Z correlation

Vacuum energy problem

. . . Outline

- ▷ The Higgs boson and the 1-TeV scale

  Why the Higgs boson must exist

  Higgs properties, constraints

  How well can we anticipate M_H?

  Higgs searches

- ▷ The problems of mass

- ▷ The EW scale and beyond

  Hierarchy problem

  Why is the EW scale so small?

  Why is the Planck scale so large?

- ▷ Outlook

General References

- ▷ C. Quigg, "The Electroweak Theory," hep-ph/0204104 (TASI 2000 Lectures)
- ▷ C. Quigg, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions*
- ▷ R. N. Cahn & G. Goldhaber, *Experimental Foundations of Particle Physics*
- ▷ P. B. Renton, "Precision Electroweak Tests of the Standard Model," hep-ph/0206231
- ▷ M. Grünewald, "Electroweak Physics," ICHEP02
- ▷ P. Sphicas, "Physics at the LHC," ICHEP02
- ▷ K. Hagiwara et al. (Particle Data Group), "Review of Particle Physics," *Phys. Rev. D*66 (2002) 010001

Problem sets: http://lutece.fnal.gov/TASI/default.html

Our picture of matter

Pointlike constituents ($r < 10^{-18}$ m)

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s \end{pmatrix}_L \qquad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

Few fundamental forces, derived from gauge symmetries

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

Electroweak symmetry breaking

Higgs mechanism?

Elementarity

▷ Are quarks and leptons structureless?

Symmetry

▷ Electroweak symmetry breaking and the 1-TeV scale

▷ Origin of gauge symmetries

▷ Meaning of discrete symmetries

Unity

▷ Coupling constant unification

▷ Unification of quarks and leptons (new forces!);

of constituents and force particles

▷ Incorporation of gravity

Identity

▷ Fermion masses and mixings; CP violation; ν oscillations

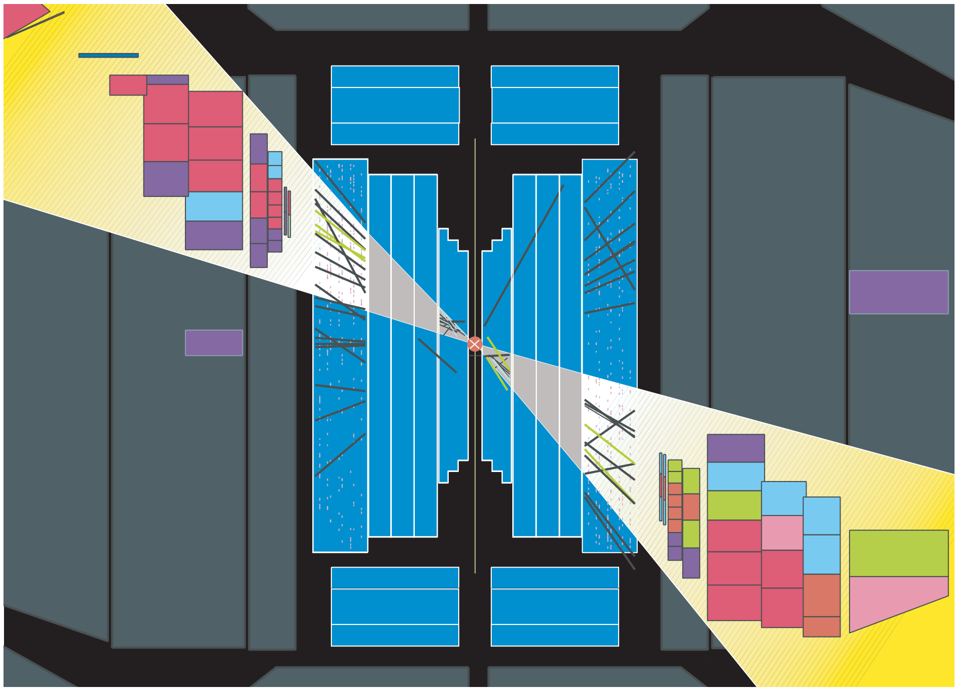
▷ What makes an electron an e and a top quark a t?

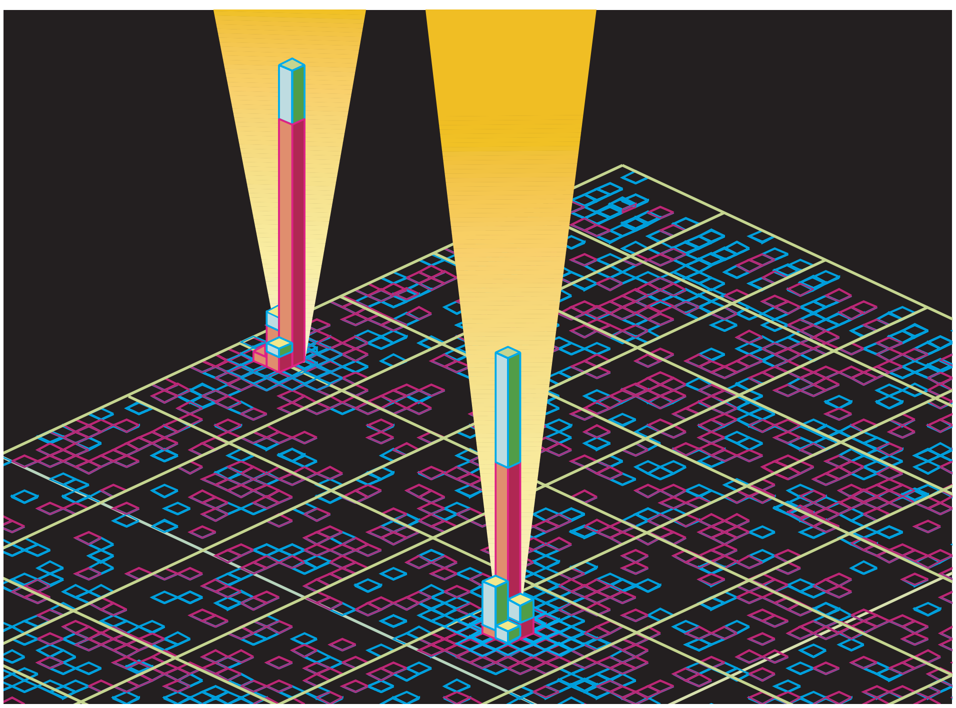
Topography

▷ What is the fabric of space and time?

...the origin of space and time?

Elementarity





Elementarity

Citation: K. Hagiwara *et al.* (Particle Data Group), Phys. Rev. D **66**, 010001 (2002) (URL: http://pdg.lbl.gov)

Quark and Lepton Compositeness, Searches for

Scale Limits Λ for Contact Interactions
(the lowest dimensional interactions with four fermions)

If the Lagrangian has the form

$$\pm \frac{g^2}{2\Lambda^2} \overline{\psi}_L \gamma_\mu \psi_L \overline{\psi}_L \gamma^\mu \psi_L$$

(with $g^2/4\pi$ set equal to 1), then we define $\Lambda \equiv \Lambda^{\pm}_{LL}$. For the full definitions and for other forms, see the Note in the Listings on Searches for Quark and Lepton Compositeness in the full *Review* and the original literature.

$\Lambda^+_{LL}(eeee)$	$>$	8.3 TeV, CL = 95%
$\Lambda^-_{LL}(eeee)$	$>$	10.3 TeV, CL = 95%
$\Lambda^+_{LL}(ee\mu\mu)$	$>$	8.5 TeV, CL = 95%
$\Lambda^-_{LL}(ee\mu\mu)$	$>$	6.3 TeV, CL = 95%
$\Lambda^+_{LL}(ee\tau\tau)$	$>$	5.4 TeV, CL = 95%
$\Lambda^-_{LL}(ee\tau\tau)$	$>$	6.5 TeV, CL = 95%
$\Lambda^+_{LL}(\ell\ell\ell\ell)$	$>$	9.0 TeV, CL = 95%
$\Lambda^-_{LL}(\ell\ell\ell\ell)$	$>$	7.8 TeV, CL = 95%
$\Lambda^+_{LL}(eeuu)$	$>$	23.3 TeV, CL = 95%
$\Lambda^-_{LL}(eeuu)$	$>$	12.5 TeV, CL = 95%
$\Lambda^+_{LL}(eedd)$	$>$	11.1 TeV, CL = 95%
$\Lambda^-_{LL}(eedd)$	$>$	26.4 TeV, CL = 95%
$\Lambda^+_{LL}(eecc)$	$>$	1.0 TeV, CL = 95%
$\Lambda^-_{LL}(eecc)$	$>$	2.1 TeV, CL = 95%
$\Lambda^+_{LL}(eebb)$	$>$	5.6 TeV, CL = 95%
$\Lambda^-_{LL}(eebb)$	$>$	4.9 TeV, CL = 95%
$\Lambda^+_{LL}(\mu\mu qq)$	$>$	2.9 TeV, CL = 95%
$\Lambda^-_{LL}(\mu\mu qq)$	$>$	4.2 TeV, CL = 95%
$\Lambda(\ell\nu\ell\nu)$	$>$	3.10 TeV, CL = 90%
$\Lambda(e\nu qq)$	$>$	2.81 TeV, CL = 95%
$\Lambda^+_{LL}(qqqq)$	$>$	2.7 TeV, CL = 95%
$\Lambda^-_{LL}(qqqq)$	$>$	2.4 TeV, CL = 95%
$\Lambda^+_{LL}(\nu\nu qq)$	$>$	5.0 TeV, CL = 95%
$\Lambda^-_{LL}(\nu\nu qq)$	$>$	5.4 TeV, CL = 95%

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Two views of Symmetry

1. *Indistinguishability*

One object transformed into another

Familiar (and useful!) from

Global Symmetries: isospin, $SU(3)_f$, ...

Spacetime Symmetries

Gauge Symmetries

$\boxed{\text{"EQUIVALENCE"}}$

Idealize more perfect worlds, the better to understand our diverse, changing world

Unbroken unified theory: perfect world of equivalent forces, interchangeable massless particles ... *Perfectly boring?*

$\boxed{\textit{Symmetry} \Leftrightarrow \textit{Disorder}}$

A Perfect World

Two views of Symmetry

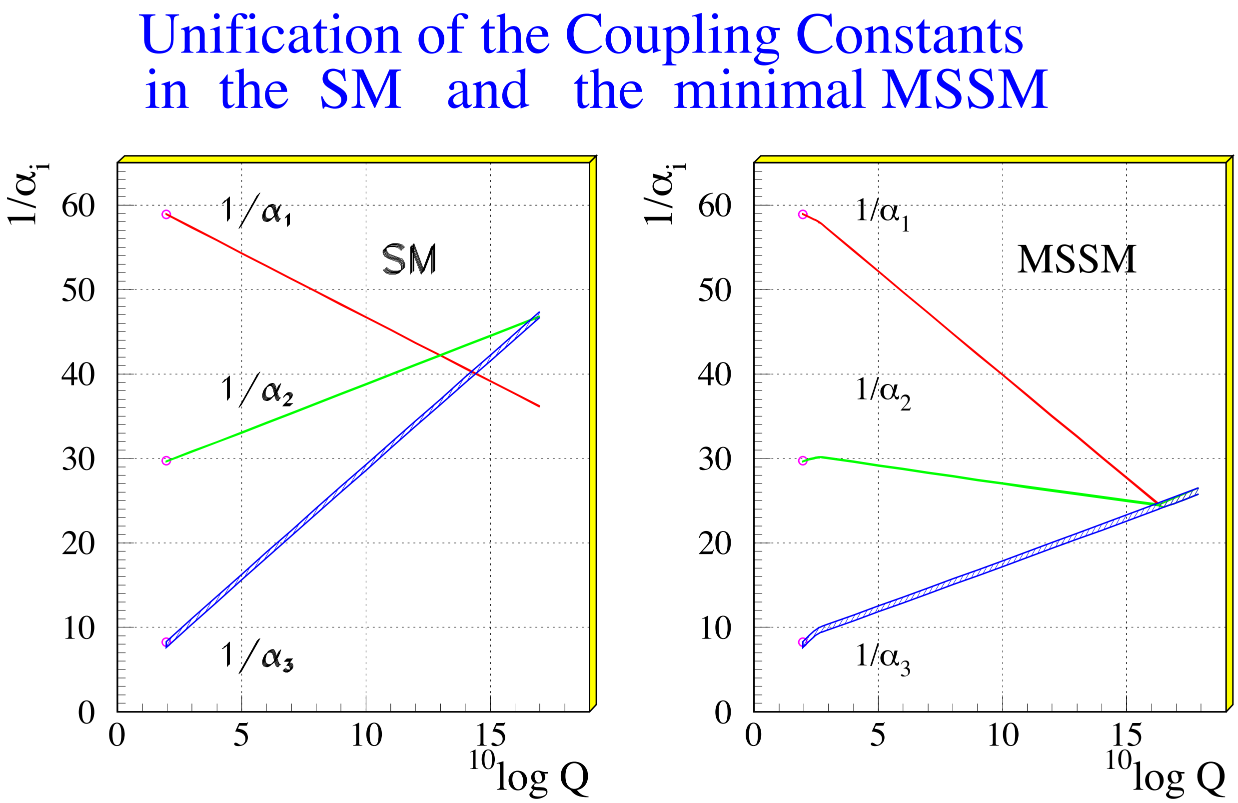
2. *Unobservable*

Goodness of a symmetry means something cannot be measured

e.g., vanishing asymmetry

Un observable	Transformation	Conserved
Absolute position	$\vec{r} \to \vec{r} + \vec{\Delta}$	$\vec{p}$
Absolute time	$t \to t + \delta$	E
Absolute orientation	$\hat{r} \to \hat{r}'$	$\vec{L}$
Absolute velocity	$\vec{v} \to \vec{v} + \vec{w}$	
Absolute right	$\vec{r} \to -\vec{r}$	P
Absolute future	$t \to -t$	T
Absolute charge	$Q \to -Q$	C
Absolute phase		
$\vdots$		

Unity


Unification of the Coupling Constants
in the SM and the minimal MSSM
1/αi
60
50
40
30
20
10
0
0
5
10
15
10log Q
1/α1
1/α2
1/α3
SM
MSSM

QCD is part of the standard model

. . . a remarkably simple, successful, and rich theory

Wilczek, hep-ph/9907340

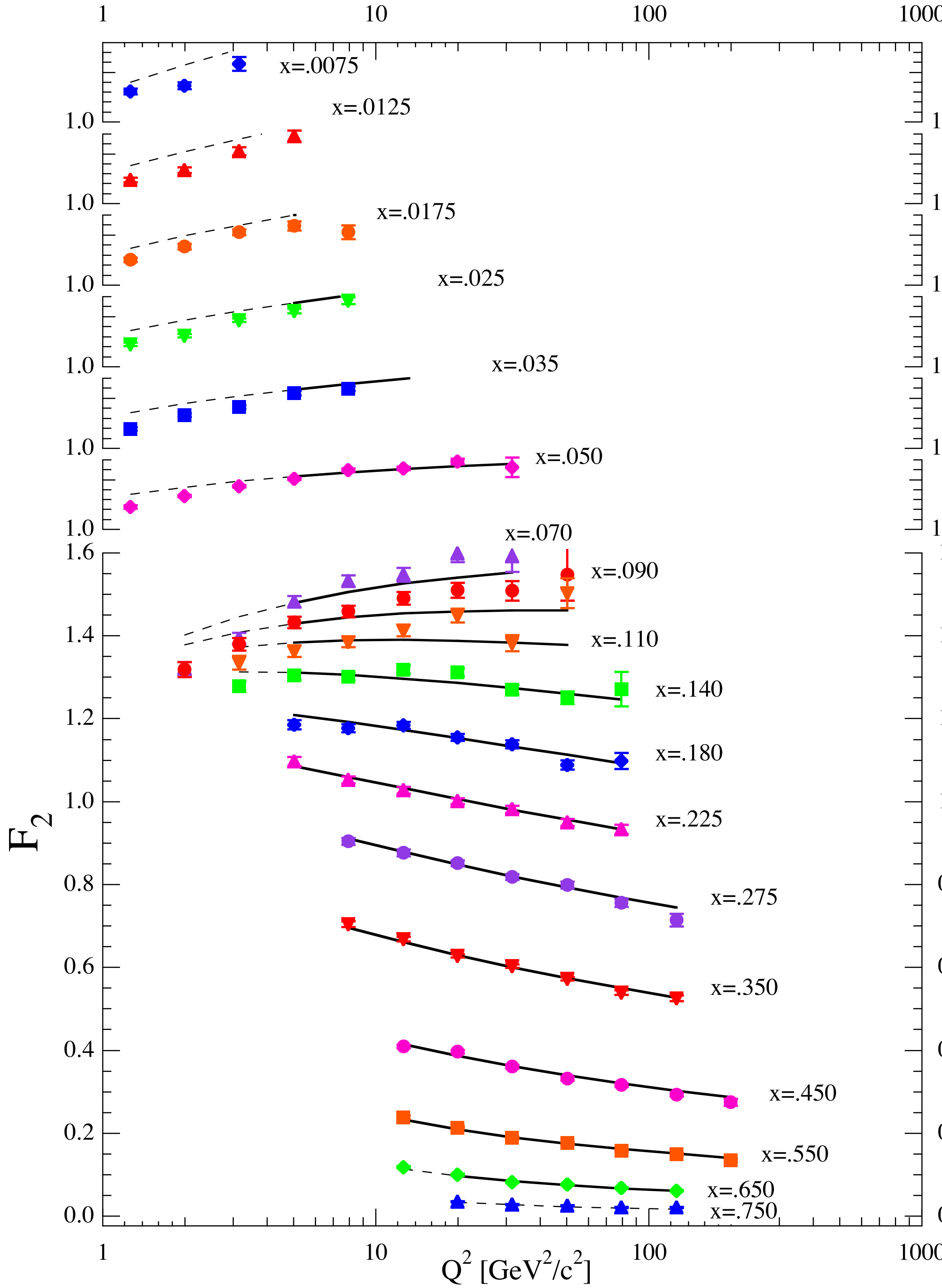
▷ Perturbative QCD

- Exists, thanks to asymptotic freedom
- Describes many phenomena in quantitative detail:
  - ▷ Q^2-evolution of structure functions
  - ▷ Jet production in $\bar{p}p$ collisions
  - ▷ Many decays, event shapes, . . .
- We can measure the running of α_s *(engineering value for unification)*

▷ Nonperturbative (lattice) QCD

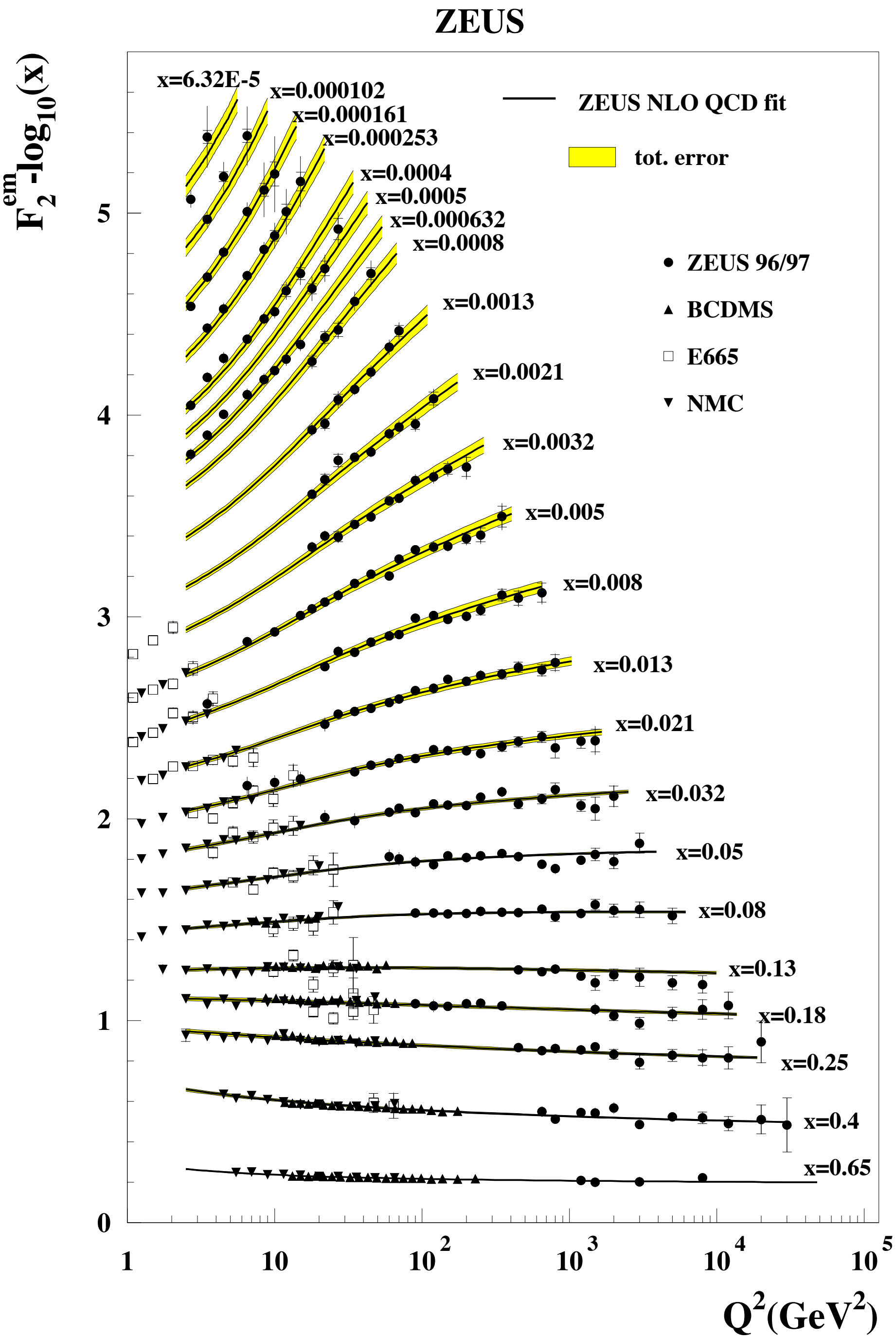
- Predicts the hadron spectrum
- Gives our best information on quark masses, etc. El-Khadra & Luke, hep-ph/0208114

$F_2(x, Q^2)$ in νN interactions (CCFR)



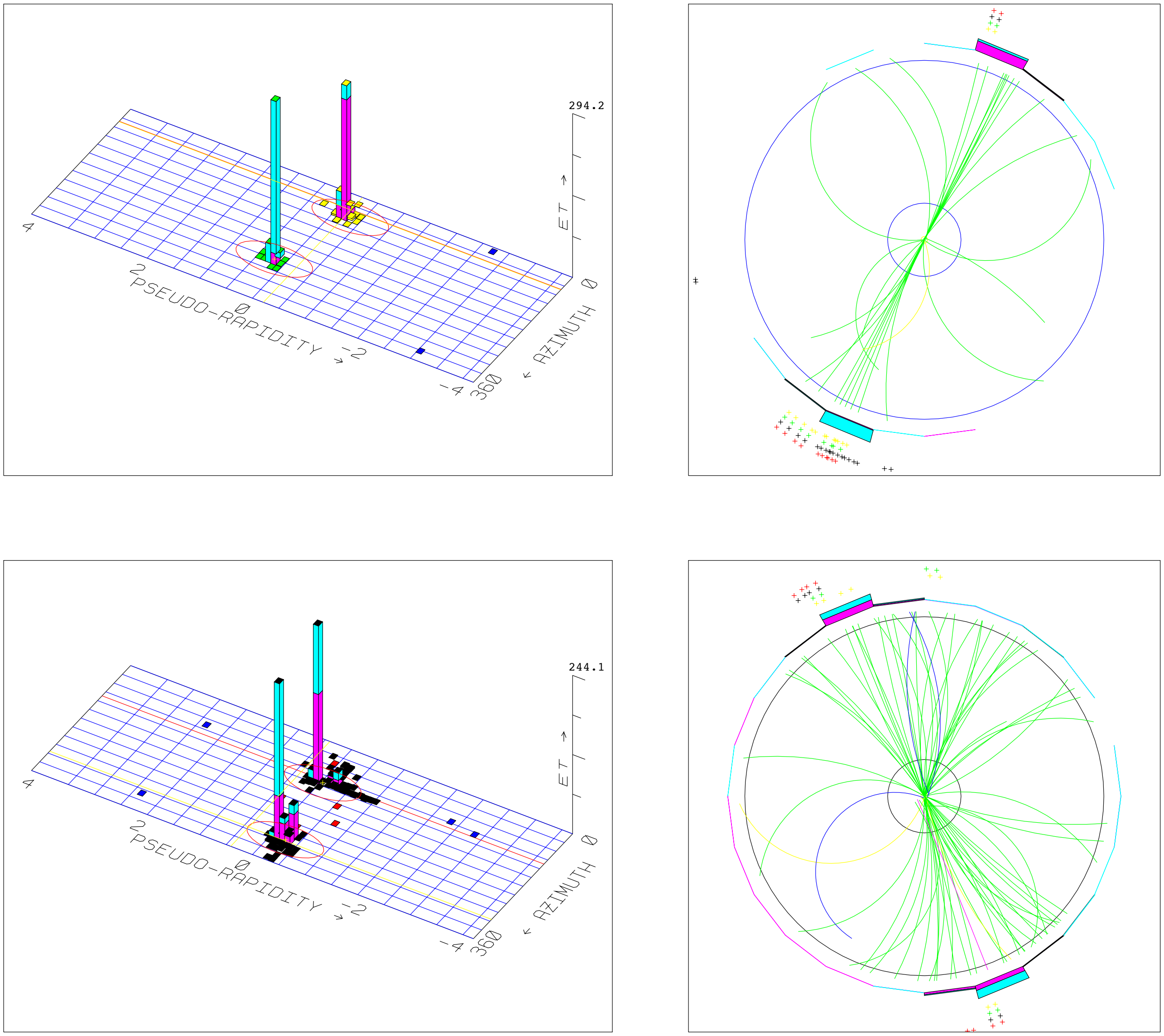
Conrad, Shaevitz, Bolton, *Rev. Mod. Phys.* **70**, 1341 (1998).

$F_2(x, Q^2)$ in ℓN interactions (ZEUS)



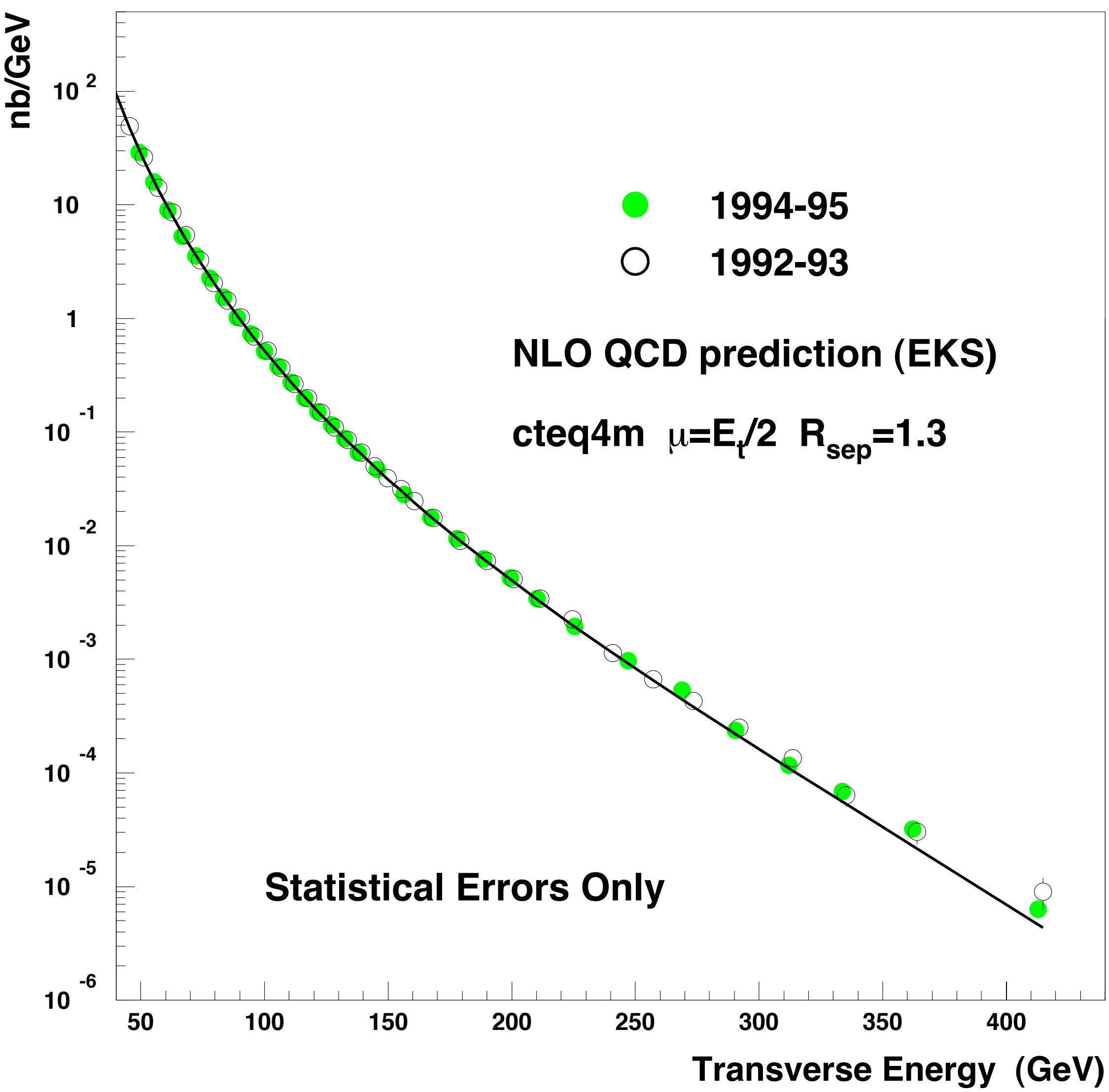
ZEUS, hep-ex/0208023.

$\bar{p}p \rightarrow$ two jets (CDF Run 1)



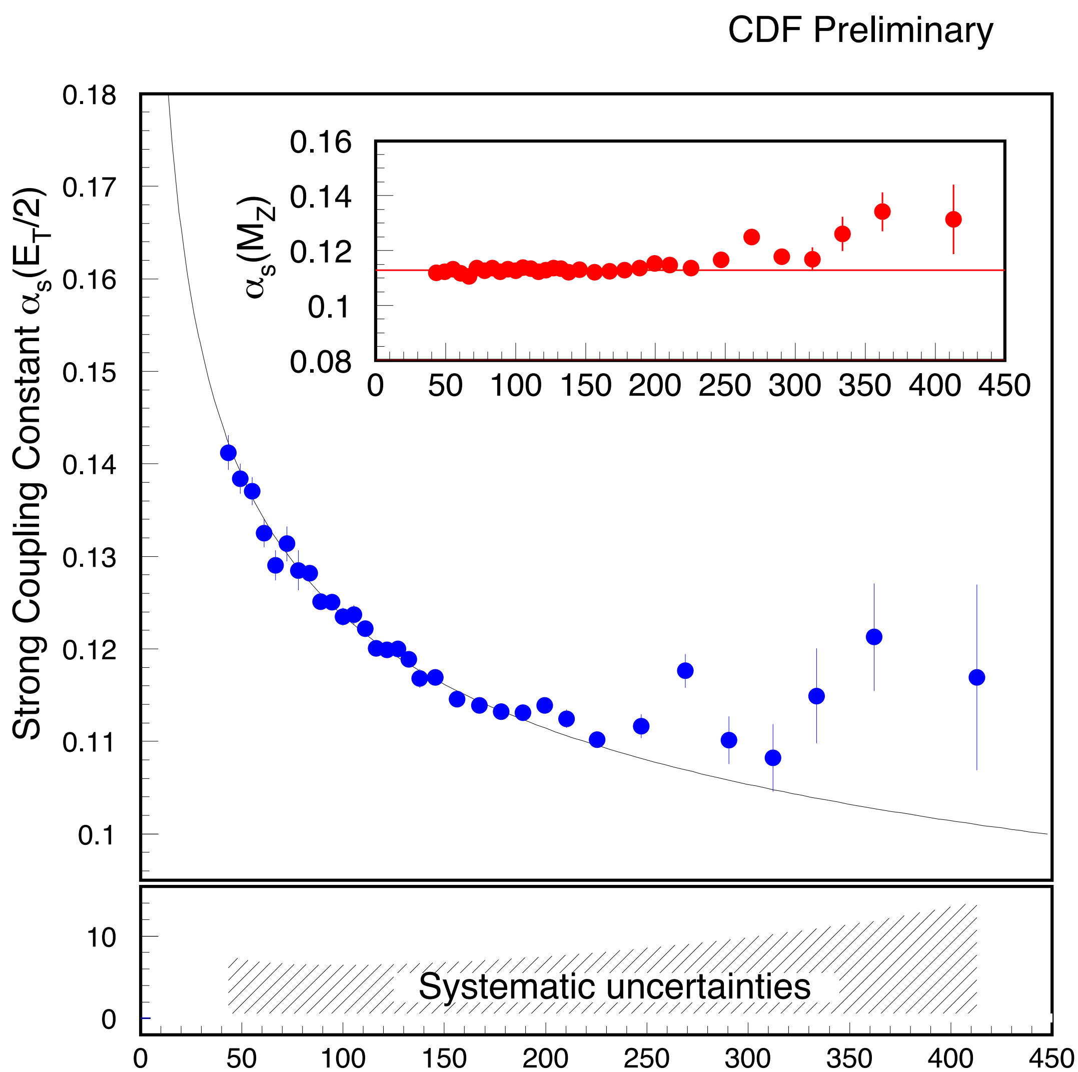
T. Affolder et al. (CDF), *Phys. Rev.* D**64**, 032001 (2001)

Inclusive jet cross section at $\sqrt{s} = 1.8$ TeV (CDF)

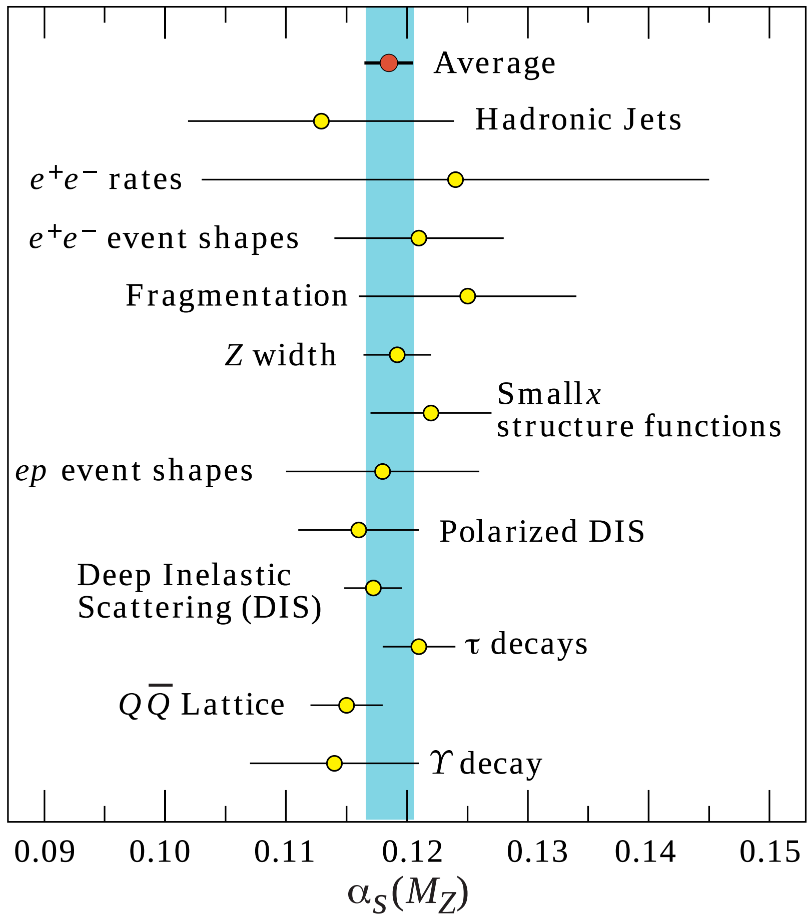


T. Affolder et al. (CDF), *Phys. Rev.* D**64**, 032001 (2001)

$\alpha_s(E_{\mathrm{T}}/2)$ from $\bar{p}p \to$ jets

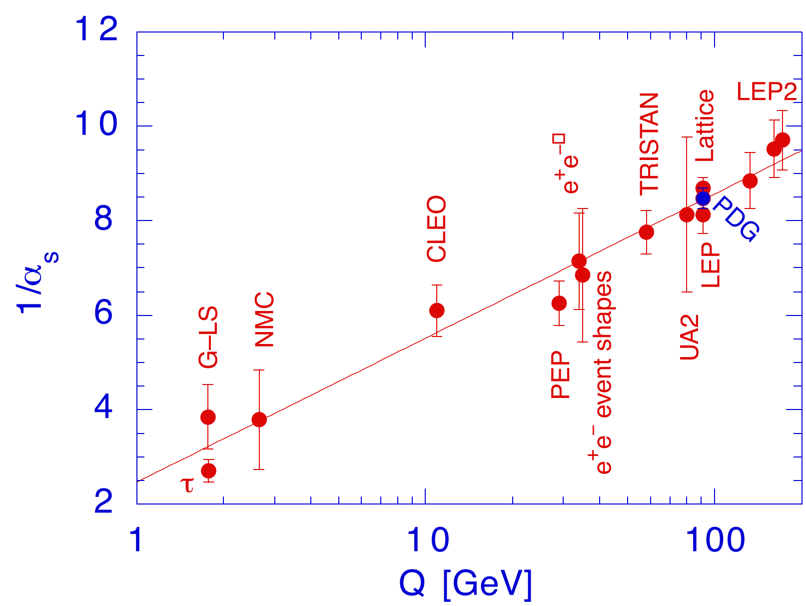


T. Affolder et al. (CDF), *Phys. Rev. Lett.* **88**, 042001 (2002)



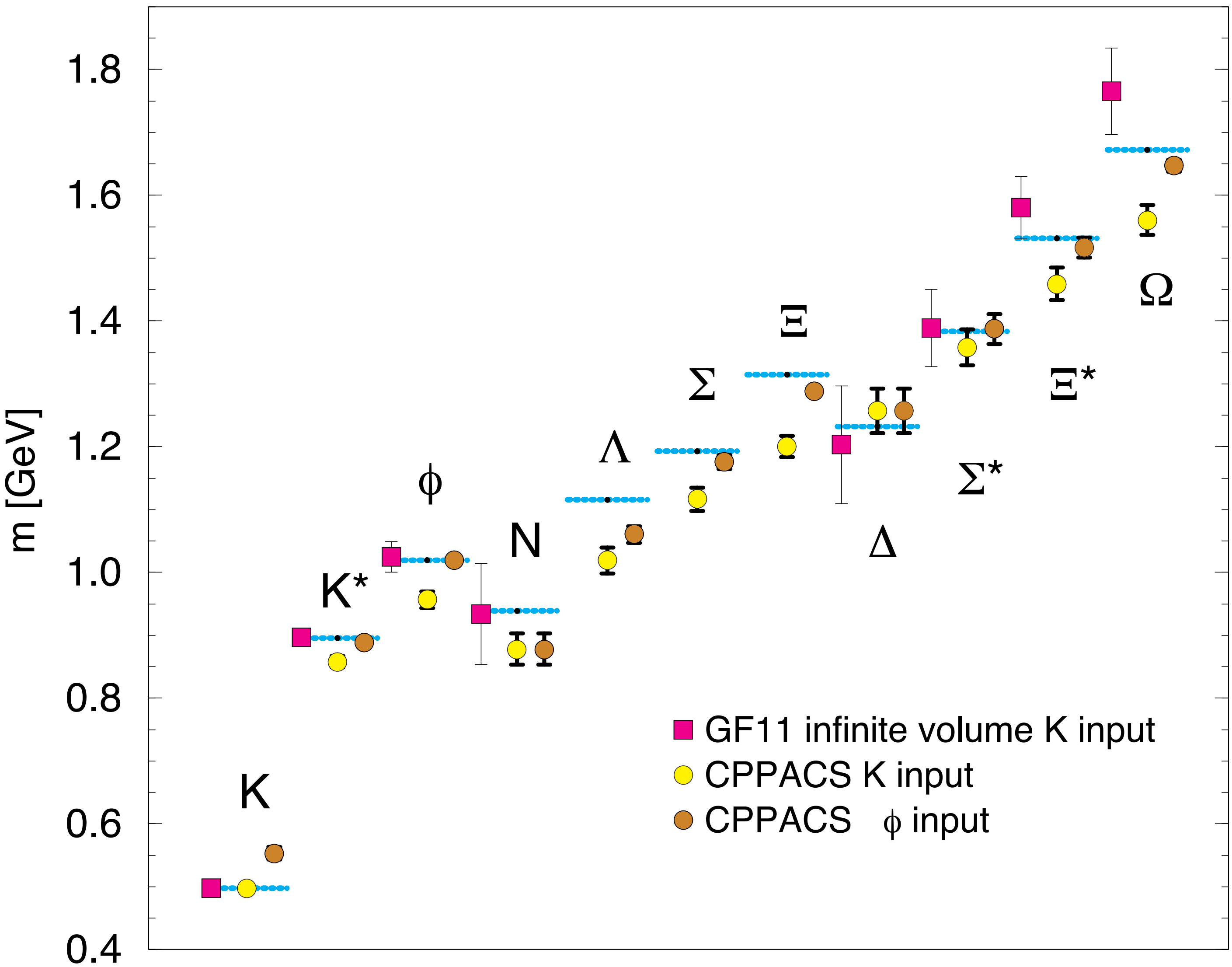
Average
Hadronic Jets
e^+e^- rates
e^+e^- event shapes
Fragmentation
Z width
Small x
structure functions
ep event shapes
Polarized DIS
Deep Inelastic
Scattering (DIS)
τ decays
$Q\overline{Q}$ Lattice
Υ decay
0.09 0.10 0.11 0.12 0.13 0.14 0.15
$\alpha_S(M_Z)$

PDG



$1/\alpha_s$
2 4 6 8 10 12
1 10 100
Q [GeV]
τ
G–LS
NMC
CLEO
PEP
e^+e^- event shapes
TRISTAN
UA2
LEP
Lattice
PDG
LEP2

Quenched hadron spectrum



S. Aoki, *et al.* (CP–PACS), *Phys. Rev. Lett.* **84**, 238 (2000)

(No dynamical fermions)

The Origins of Mass

(masses of nuclei understood)

$p, [\pi], \rho$ understood: QCD

confinement energy is the source

"Mass without mass"

We understand the visible mass of the Universe

. . . without the Higgs mechanism

W, Z electroweak symmetry breaking

$$M_W^2 = \tfrac{1}{2}g^2v^2 = \pi\alpha/G_F\sqrt{2}\sin^2\theta_W$$

$$M_Z^2 = M_W^2/\cos^2\theta_W$$

$q, \ell^\mp$ EWSB + Yukawa couplings

ν_ℓ EWSB + Yukawa couplings; new physics?

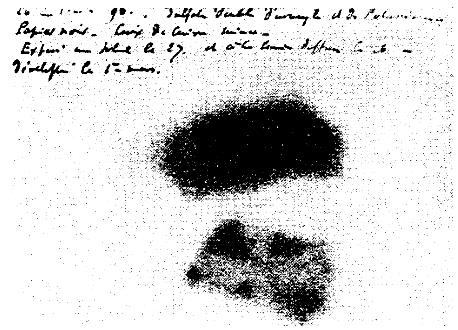
All fermion masses $\Leftrightarrow$ physics beyond standard model

H ?? fifth force ??

Antecedents of EW Theory

Commins & Bucksbaum, *Weak Int. of Lepton & Quarks*

1896: Becquerel radioactivity



Several varieties, including β decay

$$^{A}\mathrm{Z} \to \ ^{A}(\mathrm{Z}+1) + \beta^-$$

Examples:

$$^{3}\mathrm{H}_1 \to \ ^{3}\mathrm{He}_2 + \beta^- \ ,$$

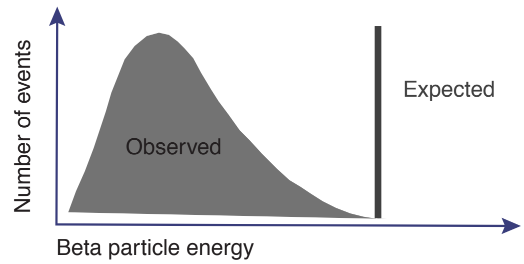
$$n \to p + \beta^- \ ,$$

$$^{214}\mathrm{Pb}_{82} \to \ ^{214}\mathrm{Bi}_{83} + \beta^- \ .$$

β^+-emitters, $^{A}\mathrm{Z} \to \ ^{A}(\mathrm{Z}-1) + \beta^+$, are rare among naturally occurring isotopes. Radio-phosphorus produced 1934 by the Joliot-Curie, *after* positron discovery in cosmic rays.

$^{19}\mathrm{Ne} \to \ ^{19}\mathrm{F} + \beta^+$ studied for right-handed charged currents and time reversal invariance; *positron-emission tomography*

1914: Chadwick β spectrum



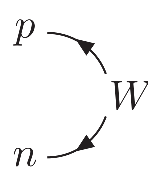
$\boxed{\text{Energy conservation in question}}$

1930: Pauli $\approx$ massless, neutral, penetrating particle

nuclear spin & statistics

↳ neutrino ν

β decay first hint for flavor



$\boxed{\text{charged-current, flavor-changing interactions}}$

1932: Chadwick neutron

$\boxed{\text{↳ isospin symmetry}}$

Neutron & flavor symmetry

$$\begin{aligned} M(n) &= 939.565\,63 \pm 0.000\,28 \text{ MeV}/c^2 \\ M(p) &= 938.272\,31 \pm 0.000\,28 \text{ MeV}/c^2 \\ \Delta M &= 1.293318 \pm 0.000\,009 \text{ MeV}/c^2 \end{aligned}$$

$$\boxed{\Delta M/M \approx 1.4 \times 10^{-3}}$$

Charge-independent nuclear forces?

$$\begin{aligned} {}^3\mathrm{H}(pnn) &= 8.481\,855 \pm 0.000\,013 \text{ MeV} \\ {}^3\mathrm{He}(ppn) &= 7.718\,109 \pm 0.000\,013 \text{ MeV} \\ \Delta(\mathrm{B.E.}) &= 0.763\,46 \text{ MeV} \end{aligned}$$

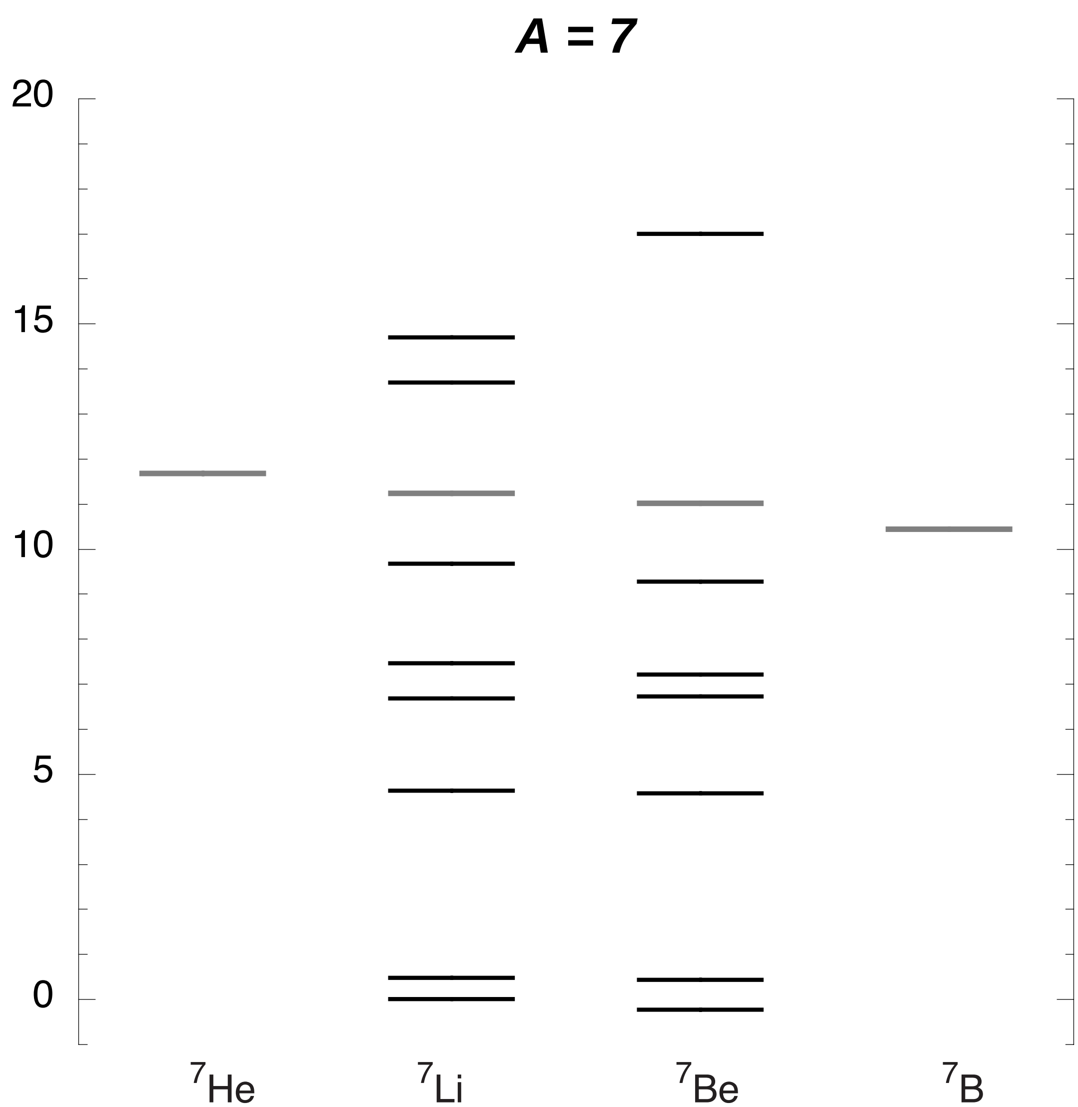
${}^3\mathrm{He}$ charge radius $r = 1.97 \pm 0.015$ fm

$$\boxed{\text{Coulomb energy: } \alpha/r \approx 0.731 \text{ MeV}}$$

Level structures in mirror nuclei. 1

$I_3 = -\frac{1}{2}$: ${}^7\mathrm{Li}(3p + 4n)$ ${}^7\mathrm{Be}(4p + 3n)$: $I_3 = \frac{1}{2}$

$I_3 = -\frac{3}{2}$: ${}^7\mathrm{He}(2p + 3n)$ ${}^7\mathrm{B}(5p + 2n)$: $I_3 = \frac{3}{2}$



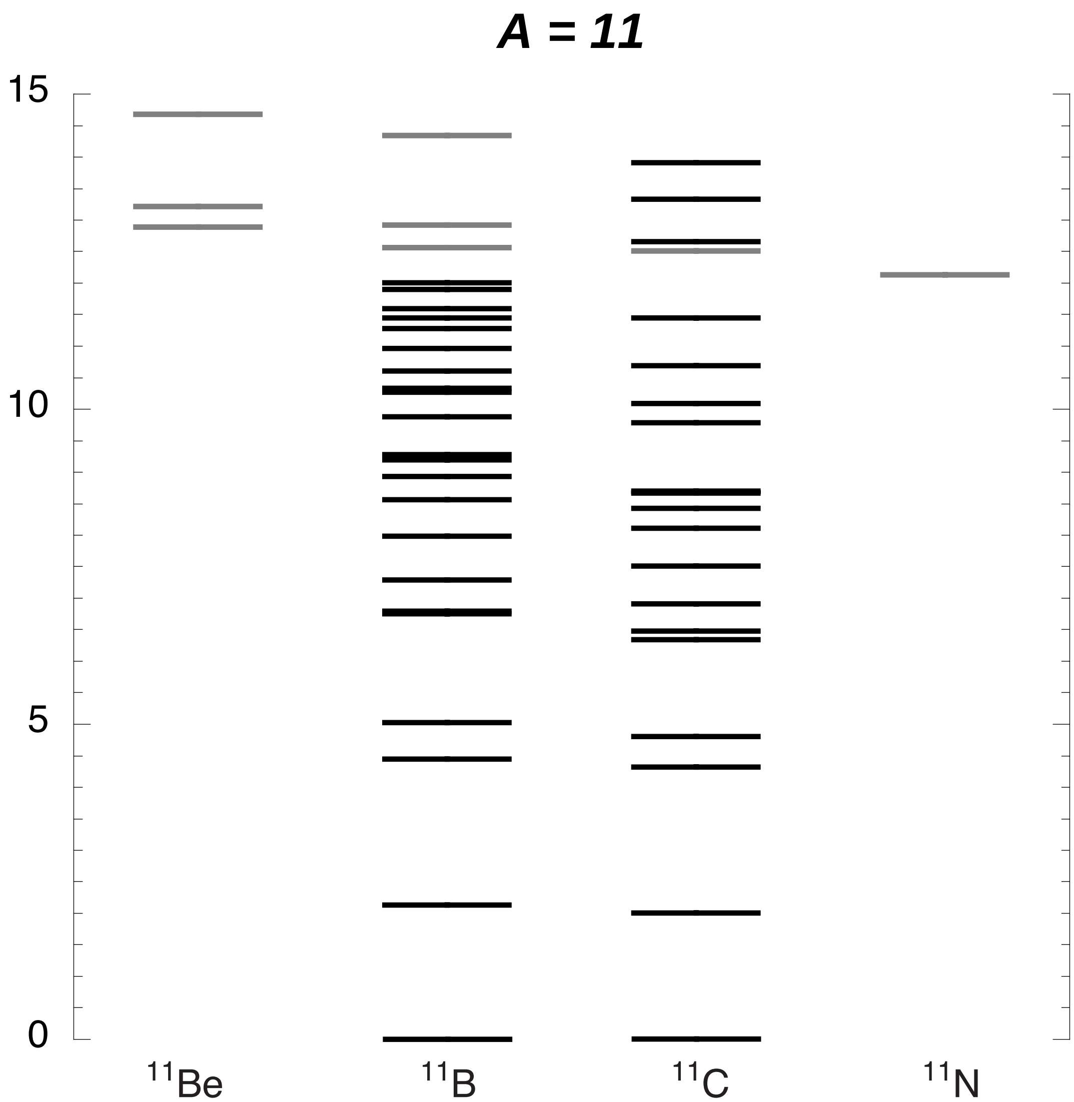
n - p mass difference, Coulomb energy removed

(isobaric analogue states)

Level structures in mirror nuclei. 2

$I_3 = -\frac{1}{2}$: $^{11}\mathrm{B}(5p + 6n)$ $^{11}\mathrm{C}(6p + 5n)$: $I_3 = \frac{1}{2}$

$I_3 = -\frac{3}{2}$: $^{11}\mathrm{Be}(4p + 7n)$ $^{11}\mathrm{N}(7p + 4n)$: $I_3 = \frac{3}{2}$

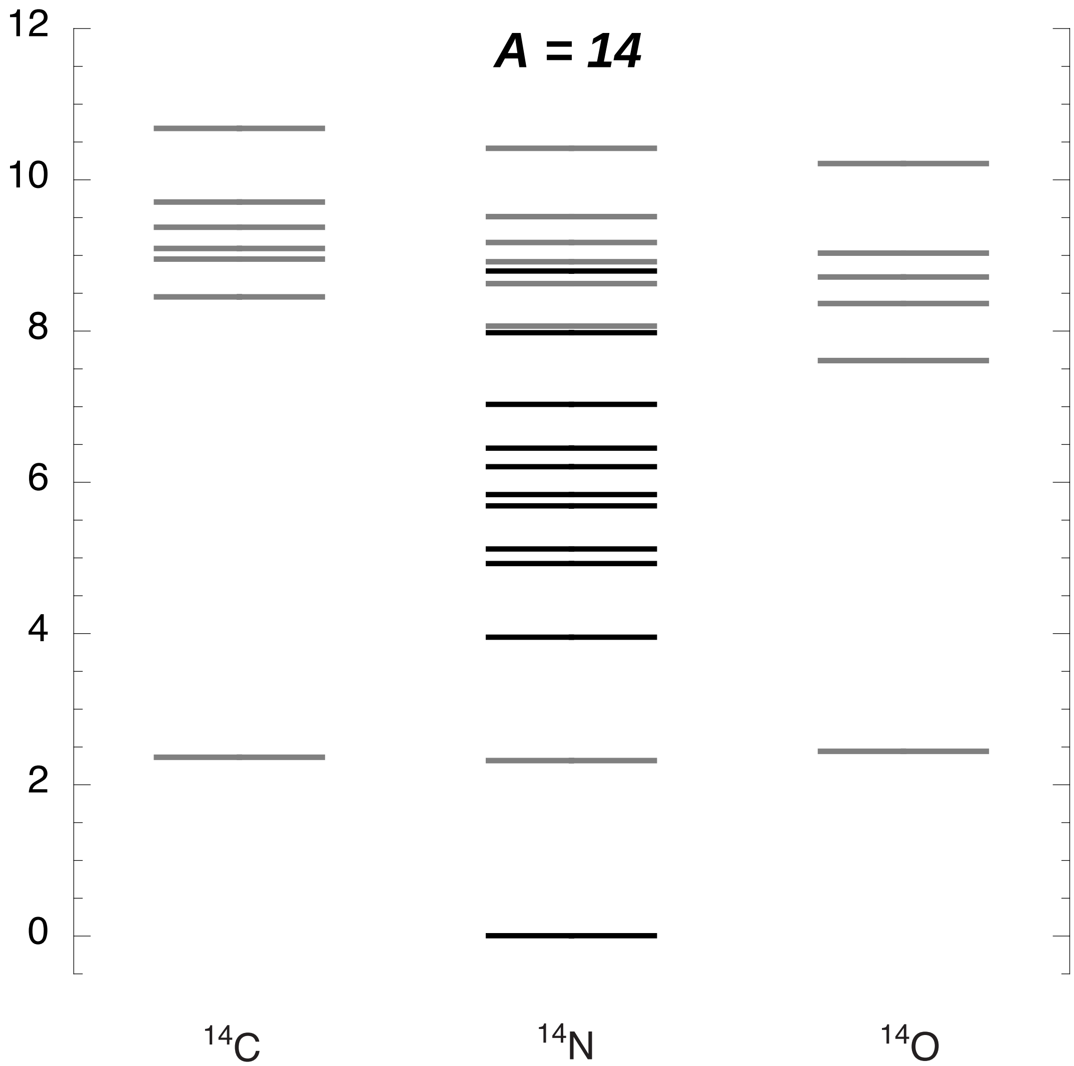


$^{11}\mathrm{Li}(3p + 8n)$ ground state $(34.4$ MeV) $I = \frac{5}{2}$ isobaric analogue

Level structures in mirror nuclei. 3

$A = 14$: NN outside closed core

$$
\begin{array}{llll}
{}^{14}\mathrm{O}: & {}^{12}\mathrm{C} + (pp) & I_3 = +1 \\
{}^{14}\mathrm{N}: & {}^{12}\mathrm{C} + (pn) & I_3 = 0 \\
{}^{14}\mathrm{C}: & {}^{12}\mathrm{C} + (nn) & I_3 = -1
\end{array}
$$



The first flavor symmetry

isospin invariance $\begin{pmatrix} p \\ n \end{pmatrix}$ isospin rotations

In the absence of EM, *convention* determines which (combination) is up

Aside: *Without EM,* how would we know there are two species of nucleons?

Parity violation in weak decays

1956 Wu *et al.*: correlation between
spin vector $\vec{J}$ of polarized $^{60}\mathrm{Co}$ and
direction $\hat{p}_e$ of outgoing β particle

Parity leaves spin (axial vector) unchanged $\boxed{\mathcal{P} : \vec{J} \to \vec{J}}$

Parity reverses electron direction $\boxed{\mathcal{P} : \hat{p}_e \to -\hat{p}_e}$

$\boxed{\text{Correlation } \vec{J} \cdot \hat{p}_e \text{ is } \textit{parity violating}}$

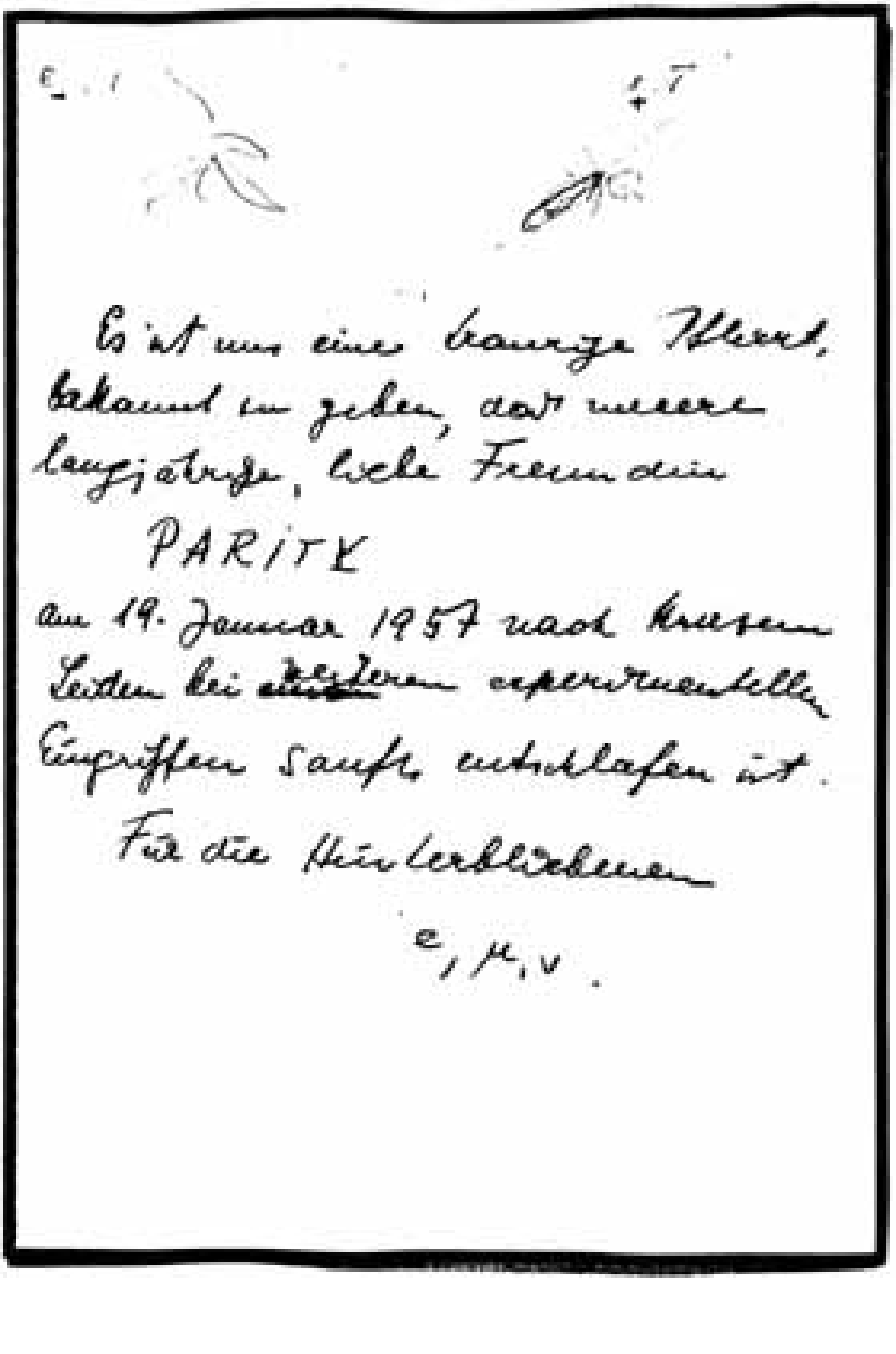
Experiments in late 1950s established that
(charged-current) weak interactions are left-handed

Parity links left-handed, right-handed neutrinos,

$$\nu_L \xrightarrow{\Leftarrow} \mathcal{P} \xleftarrow{\Leftarrow} \cancel{\nu}_R$$

$\Rightarrow$ build a manifestly parity-violating theory with only ν_L.

Pauli's Reaction to the Downfall of Parity



Es ist uns eine traurige Pflicht, bekannt zu geben, daß unsere langjährige, liebe Freundin

PARITY

am 19. Januar 1957 nach kurzem Leiden bei weiteren experimentellen Eingriffen sanft entschlafen ist.

Für die Hinterbliebenen

e, μ, ν.

Pauli's Reaction to the Downfall of Parity

Es ist uns eine traurige Pflicht, bekannt zu geben, daß unsere langjährige ewige Freundin

PARITY

den 19. Januar 1957 nach kurzen Leiden bei weiteren experimentellen Eingriffen sanfte entschlafen ist.

Für die hinterbliebenen

$e \quad \mu \quad \nu$

It is our sad duty to announce that our loyal friend of many years

PARITY

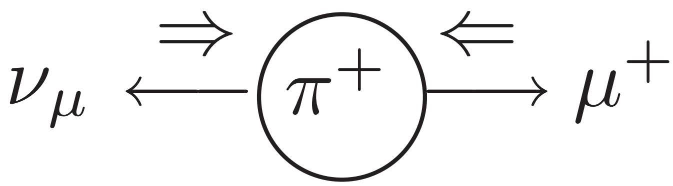
went peacefully to her eternal rest on the nineteenth of January 1957, after a short period of suffering in the face of further experimental interventions.

For those who survive her,

$e \quad \mu \quad \nu$

How do we know ν is LH?

▷ Measure μ^+ helicity in (spin-zero) $\pi^+ \to \mu^+\nu_\mu$



$$\boxed{h(\nu_\mu) = h(\mu^+)}$$

Bardon, *Phys. Rev. Lett.* **7**, 23 (1961)

Possoz, *Phys. Lett.* **70B**, 265 (1977)

μ^+ forced to have "wrong" helicity

...inhibits decay, and inhibits $\pi^+ \to e^+\nu_e$ more

$$\Gamma(\pi^+ \to e^+\nu_e)/\Gamma(\pi^+ \to \mu^+\nu_\mu) = 1.23 \times 10^{-4}$$

▷ Measure longitudinal polarization of recoil nucleus in $\mu^{-\,12}\mathrm{C}(J=0) \to {}^{12}\mathrm{B}(J=1)\nu_\mu$

Infer $h(\nu_\mu)$ by angular momentum conservation

Roesch, *Am. J. Phys.* **50**, 931 (1981)

▷ Measure longitundinal polarization of recoil nucleus in

$$e^- \; {}^{152}\mathrm{Eu}^m(J=0) \quad \to \quad {}^{152}\mathrm{Sm}^*(J=1)\;\nu_e$$
$$\hookrightarrow \gamma \; {}^{152}\mathrm{Sm}$$

$\boxed{\text{Infer } h(\nu_e) \text{ from } \gamma \text{ polarization}}$

Goldhaber, *Phys. Rev.* **109**, 1015 (1958)

Charge conjugation is also violated . . .

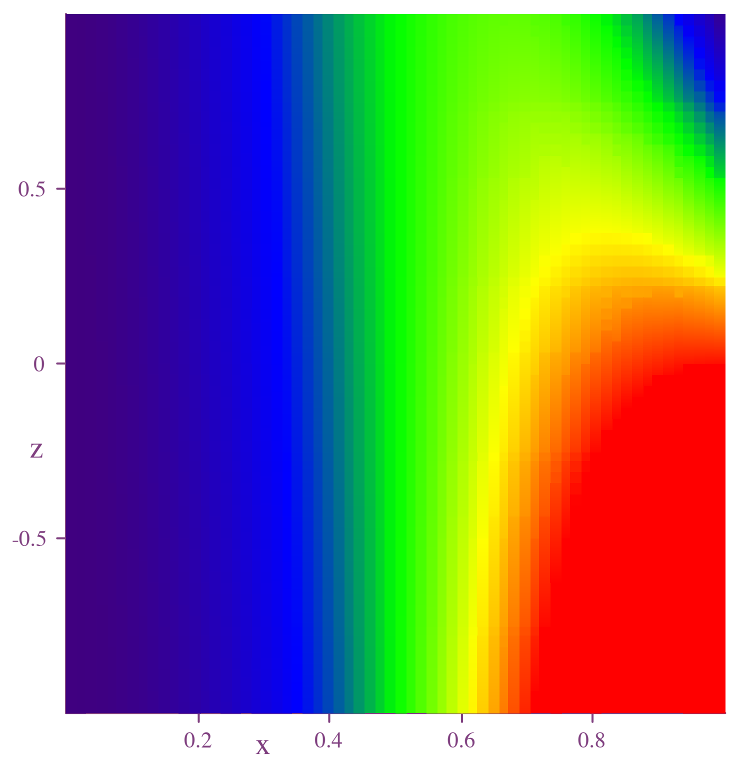
$$\nu_L \xrightarrow{\Leftarrow}\cdot \quad \mathcal{C} \quad \xrightarrow{\Leftarrow} \cancel{\bar{\nu}}_L$$

$\mu^{\pm}$ decay: angular distributions of $e^{\pm}$ reversed

$$\frac{dN(\mu^{\pm} \to e^{\pm} + \ldots)}{dxdz} = x^2(3-2x)\left[1 \pm z\frac{(2x-1)}{(3-2x)}\right]$$

$$x \equiv p_e/p_e^{\max},\ z \equiv \hat{s}_\mu \cdot \hat{p}_e$$

e^+ follows μ^+ spin $\qquad$ e^- avoids μ^- spin



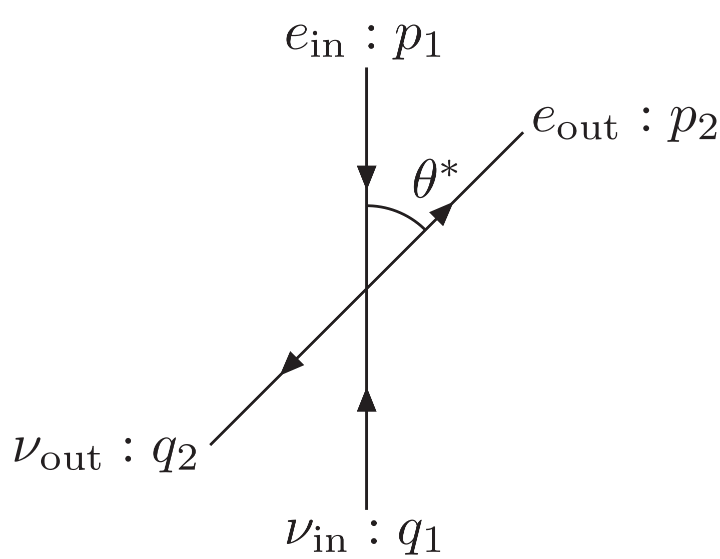
Effective Lagrangian . . .

Late 1950s: current-current interaction

$$\mathcal{L}_{V-A} = \frac{-G_F}{\sqrt{2}} \bar{\nu}\gamma_\mu(1-\gamma_5)e \; \bar{e}\gamma^\mu(1-\gamma_5)\nu + \text{h.c.}$$

$$G_F = 1.16632 \times 10^{-5}\ \text{GeV}^{-2}$$

Compute $\bar{\nu}e$ scattering amplitude:



$$\begin{aligned} \mathcal{M} \quad &= \quad -\frac{iG_F}{\sqrt{2}} \bar{v}(\nu, q_1)\gamma_\mu(1-\gamma_5)u(e, p_1) \\ &\qquad \cdot \bar{u}(e, p_2)\gamma^\mu(1-\gamma_5)v(\nu, q_2) \end{aligned}$$

$\bar{\nu}e \longrightarrow \quad \bar{\nu}e$

$$\frac{d\sigma_{V-A}(\bar{\nu}e \to \bar{\nu}e)}{d\Omega_{\rm cm}} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s} = \frac{G_F^2 \cdot 2mE_\nu(1-z)^2}{16\pi^2}$$

$z = \cos\theta^*$

$$\sigma_{V-A}(\bar{\nu}e \to \bar{\nu}e) \quad = \quad \frac{G_F^2 \cdot 2mE_\nu}{3\pi}$$

$$\approx \quad 0.574 \times 10^{-41}\ \mathrm{cm}^2 \left(\frac{E_\nu}{1\ \mathrm{GeV}}\right)$$

Small! $\approx 10^{-14}\ \sigma(pp)$ at 100 GeV

$\nu e \longrightarrow \quad \nu e$

$$\frac{d\sigma_{V-A}(\nu e \to \nu e)}{d\Omega_{\rm cm}} = \frac{G_F^2 \cdot 2mE_\nu}{4\pi^2}$$

$$\sigma_{V-A}(\nu e \to \nu e) \quad = \quad \frac{G_F^2 \cdot 2mE_\nu}{\pi}$$

$$\approx \quad 1.72 \times 10^{-41}\ \mathrm{cm}^2 \left(\frac{E_\nu}{1\ \mathrm{GeV}}\right)$$

Why $3\times$ difference?

incoming $\begin{array}{cc} e & \\ \downarrow & \Uparrow \\ \uparrow & \Downarrow \\ \nu & \end{array}$ $J_z = 0$ outgoing, $z = +1$ $\begin{array}{cc} e & \\ \uparrow & \Downarrow \\ \downarrow & \Uparrow \\ \nu & \end{array}$ $J_z = 0$

$\boxed{\text{allowed at all angles}}$

incoming $\begin{array}{cc} e & \\ \downarrow & \Uparrow \\ \uparrow & \Uparrow \\ \bar{\nu} & \end{array}$ $J_z = +1$ outgoing, $z = +1$ $\begin{array}{cc} e & \\ \uparrow & \Downarrow \\ \downarrow & \Downarrow \\ \bar{\nu} & \end{array}$ $J_z = -1$

$\boxed{\text{forbidden (angular momentum) at } z = +1}$

1962: Lederman, Schwartz, Steinberger $\nu_\mu \neq \nu_e$

▷ Make HE $\pi \rightarrow \mu\nu$ beam

▷ Observe $\nu N \rightarrow \mu +$ anything

▷ Don't observe $\nu N \rightarrow e +$ anything

Danby, *et al.*, *Phys. Rev. Lett.* **9**, 36 (1962)

Suggests family structure

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$$

$\approx$ no interactions known to cross boundaries

Generalize effective (current-current) Lagrangian:

$$\mathcal{L}_{V-A}^{(e\mu)} = \frac{-G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu \; \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e + \text{h.c.} \; ,$$

Compute muon decay rate

$$\Gamma(\mu \rightarrow e \bar{\nu}_e \nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

accounts for the 2.2-μs muon lifetime

TESTS OF NUMBER CONSERVATION LAWS

LEPTON FAMILY NUMBER

Lepton family number conservation means separate conservation of each of L_e, L_μ, L_τ.

$\Gamma(Z \to e^\pm \mu^\mp)/\Gamma_{\text{total}}$	[i]	$<1.7 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to e^\pm \tau^\mp)/\Gamma_{\text{total}}$	[i]	$<9.8 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to \mu^\pm \tau^\mp)/\Gamma_{\text{total}}$	[i]	$<1.2 \times 10^{-5}$, CL = 95%
limit on $\mu^- \to e^-$ conversion		
$\sigma(\mu^- \, {}^{32}\text{S} \to e^- \, {}^{32}\text{S})$ / $\sigma(\mu^- \, {}^{32}\text{S} \to \nu_\mu \, {}^{32}\text{P}^*)$		$<7 \times 10^{-11}$, CL = 90%
$\sigma(\mu^- \, \text{Ti} \to e^- \, \text{Ti})$ / $\sigma(\mu^- \, \text{Ti} \to \text{capture})$		$<4.3 \times 10^{-12}$, CL = 90%
$\sigma(\mu^- \, \text{Pb} \to e^- \, \text{Pb})$ / $\sigma(\mu^- \, \text{Pb} \to \text{capture})$		$<4.6 \times 10^{-11}$, CL = 90%
limit on muonium $\to$ antimuonium conversion $R_g = G_C / G_F$		<0.0030, CL = 90%
$\Gamma(\mu^- \to e^- \nu_e \overline{\nu}_\mu)/\Gamma_{\text{total}}$	[j]	$<1.2 \times 10^{-2}$, CL = 90%
$\Gamma(\mu^- \to e^- \gamma)/\Gamma_{\text{total}}$		$<1.2 \times 10^{-11}$, CL = 90%
$\Gamma(\mu^- \to e^- e^+ e^-)/\Gamma_{\text{total}}$		$<1.0 \times 10^{-12}$, CL = 90%
$\Gamma(\mu^- \to e^- 2\gamma)/\Gamma_{\text{total}}$		$<7.2 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^- \gamma)/\Gamma_{\text{total}}$		$<2.7 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \gamma)/\Gamma_{\text{total}}$		$<1.1 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- \pi^0)/\Gamma_{\text{total}}$		$<3.7 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \pi^0)/\Gamma_{\text{total}}$		$<4.0 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- K^0)/\Gamma_{\text{total}}$		$<1.3 \times 10^{-3}$, CL = 90%
$\Gamma(\tau^- \to \mu^- K^0)/\Gamma_{\text{total}}$		$<1.0 \times 10^{-3}$, CL = 90%
$\Gamma(\tau^- \to e^- \eta)/\Gamma_{\text{total}}$		$<8.2 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \eta)/\Gamma_{\text{total}}$		$<9.6 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- \rho^0)/\Gamma_{\text{total}}$		$<2.0 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \rho^0)/\Gamma_{\text{total}}$		$<6.3 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- K^*(892)^0)/\Gamma_{\text{total}}$		$<5.1 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- K^*(892)^0)/\Gamma_{\text{total}}$		$<7.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- \overline{K}^*(892)^0)/\Gamma_{\text{total}}$		$<7.4 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \overline{K}^*(892)^0)/\Gamma_{\text{total}}$		$<7.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- \phi)/\Gamma_{\text{total}}$		$<6.9 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \phi)/\Gamma_{\text{total}}$		$<7.0 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- e^+ e^-)/\Gamma_{\text{total}}$		$<2.9 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- \mu^+ \mu^-)/\Gamma_{\text{total}}$		$<1.8 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^+ \mu^- \mu^-)/\Gamma_{\text{total}}$		$<1.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- e^+ e^-)/\Gamma_{\text{total}}$		$<1.7 \times 10^{-6}$, CL = 90%

$\Gamma(\tau^- \to \mu^+ e^- e^-)/\Gamma_{\text{total}}$		$<1.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \mu^+ \mu^-)/\Gamma_{\text{total}}$		$<1.9 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- \pi^+ \pi^-)/\Gamma_{\text{total}}$		$<2.2 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \pi^+ \pi^-)/\Gamma_{\text{total}}$		$<8.2 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- \pi^+ K^-)/\Gamma_{\text{total}}$		$<6.4 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- \pi^- K^+)/\Gamma_{\text{total}}$		$<3.8 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- K^+ K^-)/\Gamma_{\text{total}}$		$<6.0 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \pi^+ K^-)/\Gamma_{\text{total}}$		$<7.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \pi^- K^+)/\Gamma_{\text{total}}$		$<7.4 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- K^+ K^-)/\Gamma_{\text{total}}$		$<1.5 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to e^- \pi^0 \pi^0)/\Gamma_{\text{total}}$		$<6.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \pi^0 \pi^0)/\Gamma_{\text{total}}$		$<1.4 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to e^- \eta\eta)/\Gamma_{\text{total}}$		$<3.5 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \eta\eta)/\Gamma_{\text{total}}$		$<6.0 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to e^- \pi^0 \eta)/\Gamma_{\text{total}}$		$<2.4 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \pi^0 \eta)/\Gamma_{\text{total}}$		$<2.2 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to e^-$ light boson$)/\Gamma_{\text{total}}$		$<2.7 \times 10^{-3}$, CL = 95%
$\Gamma(\tau^- \to \mu^-$ light boson$)/\Gamma_{\text{total}}$		$<5 \times 10^{-3}$, CL = 95%

NEUTRINO FLAVOR CHANGE VIA MIXING.

For other lepton mixing, see the Particle Listings. There is now compelling evidence for lepton family number violation in the neutrino sector (also called neutrino flavor change) both from the study of atmospheric neutrino fluxes by SuperKamiokande, and from the combined study of solar neutrino cross sections by SNO (charged and neutral currents) and SuperKamiokande (elastic scattering). For details, see the discussion in the Neutrino Mixing section of the Lepton Summary Table and the notes "Neutrino Physics as Explored by Flavor Change" and " Solar Neutrinos" in the Lepton Particle Listings.

$\Gamma(\pi^+ \to \mu^+ \nu_e)/\Gamma_{\text{total}}$	[k]	$<8.0 \times 10^{-3}$, CL = 90%
$\Gamma(\pi^+ \to \mu^- e^+ e^+ \nu)/\Gamma_{\text{total}}$		$<1.6 \times 10^{-6}$, CL = 90%
$\Gamma(\pi^0 \to \mu^+ e^-)/\Gamma_{\text{total}}$		$<3.8 \times 10^{-10}$, CL = 90%
$\Gamma(\pi^0 \to \mu^- e^+)/\Gamma_{\text{total}}$		$<3.4 \times 10^{-9}$, CL = 90%
$\Gamma(\pi^0 \to \mu^+ e^- + \mu^- e^+)/\Gamma_{\text{total}}$		$<1.72 \times 10^{-8}$, CL = 90%
$\Gamma(\eta \to \mu^+ e^- + \mu^- e^+)/\Gamma_{\text{total}}$		$<6 \times 10^{-6}$, CL = 90%
$\Gamma(\eta'(958) \to e\mu)/\Gamma_{\text{total}}$		$<4.7 \times 10^{-4}$, CL = 90%
$\Gamma(K^+ \to \mu^- \nu e^+ e^+)/\Gamma_{\text{total}}$		$<2.0 \times 10^{-8}$, CL = 90%
$\Gamma(K^+ \to \mu^+ \nu_e)/\Gamma_{\text{total}}$	[k]	$<4 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^+ \mu^+ e^-)/\Gamma_{\text{total}}$		$<2.8 \times 10^{-11}$, CL = 90%
$\Gamma(K^+ \to \pi^+ \mu^- e^+)/\Gamma_{\text{total}}$		$<5.2 \times 10^{-10}$, CL = 90%
$\Gamma(K^0_L \to e^\pm \mu^\mp)/\Gamma_{\text{total}}$	[i]	$<4.7 \times 10^{-12}$, CL = 90%
$\Gamma(K^0_L \to e^\pm e^\pm \mu^\mp \mu^\mp)/\Gamma_{\text{total}}$	[i]	$<1.23 \times 10^{-10}$, CL = 90%
$\Gamma(K^0_L \to \pi^0 \mu^\pm e^\mp)/\Gamma_{\text{total}}$	[i]	$<6.2 \times 10^{-9}$, CL = 90%
$\Gamma(D^+ \to \pi^+ e^\pm \mu^\mp)/\Gamma_{\text{total}}$	[i]	$<3.4 \times 10^{-5}$, CL = 90%
$\Gamma(D^+ \to K^+ e^\pm \mu^\mp)/\Gamma_{\text{total}}$	[i]	$<6.8 \times 10^{-5}$, CL = 90%

$\Gamma(D^0 \to \mu^\pm e^\mp)/\Gamma_{\rm total}$	[i]	$<8.1\times10^{-6}$, CL = 90%
$\Gamma(D^0 \to \pi^0 e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<8.6\times10^{-5}$, CL = 90%
$\Gamma(D^0 \to \eta e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<1.0\times10^{-4}$, CL = 90%
$\Gamma(D^0 \to \pi^+\pi^- e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<1.5\times10^{-5}$, CL = 90%
$\Gamma(D^0 \to \rho^0 e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<4.9\times10^{-5}$, CL = 90%
$\Gamma(D^0 \to \omega e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<1.2\times10^{-4}$, CL = 90%
$\Gamma(D^0 \to K^- K^+ e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<1.8\times10^{-4}$, CL = 90%
$\Gamma(D^0 \to \phi e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<3.4\times10^{-5}$, CL = 90%
$\Gamma(D^0 \to \overline{K}^0 e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<1.0\times10^{-4}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^+ e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<5.53\times10^{-4}$, CL = 90%
$\Gamma(D^0 \to \overline{K}^*(892)^0 e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<8.3\times10^{-5}$, CL = 90%
$\Gamma(D_s^+ \to \pi^+ e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<6.1\times10^{-4}$, CL = 90%
$\Gamma(D_s^+ \to K^+ e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<6.3\times10^{-4}$, CL = 90%
$\Gamma(B^+ \to \pi^+ e^+ \mu^-)/\Gamma_{\rm total}$		$<6.4\times10^{-3}$, CL = 90%
$\Gamma(B^+ \to \pi^+ e^- \mu^+)/\Gamma_{\rm total}$		$<6.4\times10^{-3}$, CL = 90%
$\Gamma(B^+ \to K^+ e^+ \mu^-)/\Gamma_{\rm total}$		$<6.4\times10^{-3}$, CL = 90%
$\Gamma(B^+ \to K^+ e^- \mu^+)/\Gamma_{\rm total}$		$<6.4\times10^{-3}$, CL = 90%
$\Gamma(B^0 \to e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<1.5\times10^{-6}$, CL = 90%
$\Gamma(B^0 \to e^\pm \tau^\mp)/\Gamma_{\rm total}$	[i]	$<5.3\times10^{-4}$, CL = 90%
$\Gamma(B^0 \to \mu^\pm \tau^\mp)/\Gamma_{\rm total}$	[i]	$<8.3\times10^{-4}$, CL = 90%
$\Gamma(B \to e^\pm \mu^\mp s)/\Gamma_{\rm total}$	[i]	$<2.2\times10^{-5}$, CL = 90%
$\Gamma(B_s^0 \to e^\pm \mu^\mp)/\Gamma_{\rm total}$	[i]	$<6.1\times10^{-6}$, CL = 90%

TOTAL LEPTON NUMBER

Violation of total lepton number conservation also implies violation of lepton family number conservation.

$\Gamma(Z \to p e)/\Gamma_{\rm total}$	$<1.8\times10^{-6}$, CL = 95%
$\Gamma(Z \to p \mu)/\Gamma_{\rm total}$	$<1.8\times10^{-6}$, CL = 95%
limit on $\mu^- \to e^+$ conversion	
$\sigma(\mu^- {}^{32}\mathrm{S} \to e^+ {}^{32}\mathrm{Si}^*)$ / $\sigma(\mu^- {}^{32}\mathrm{S} \to \nu_\mu {}^{32}\mathrm{P}^*)$	$<9\times10^{-10}$, CL = 90%
$\sigma(\mu^- {}^{127}\mathrm{I} \to e^+ {}^{127}\mathrm{Sb}^*)$ / $\sigma(\mu^- {}^{127}\mathrm{I} \to \text{anything})$	$<3\times10^{-10}$, CL = 90%
$\sigma(\mu^- \mathrm{Ti} \to e^+ \mathrm{Ca})$ / $\sigma(\mu^- \mathrm{Ti} \to \text{capture})$	$<3.6\times10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^+\pi^-\pi^-)/\Gamma_{\rm total}$	$<1.9\times10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^+\pi^-\pi^-)/\Gamma_{\rm total}$	$<3.4\times10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^+\pi^- K^-)/\Gamma_{\rm total}$	$<2.1\times10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^+ K^- K^-)/\Gamma_{\rm total}$	$<3.8\times10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^+\pi^- K^-)/\Gamma_{\rm total}$	$<7.0\times10^{-6}$, CL = 90%

$\Gamma(\tau^- \to \mu^+ K^- K^-)/\Gamma_{total}$		$<6.0 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\gamma)/\Gamma_{total}$		$<3.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\pi^0)/\Gamma_{total}$		$<1.5 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}2\pi^0)/\Gamma_{total}$		$<3.3 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\eta)/\Gamma_{total}$		$<8.9 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\pi^0\eta)/\Gamma_{total}$		$<2.7 \times 10^{-5}$, CL = 90%
$\Gamma(\pi^+ \to \mu^+\overline{\nu}_e)/\Gamma_{total}$	[k]	$<1.5 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^-\mu^+e^+)/\Gamma_{total}$		$<5.0 \times 10^{-10}$, CL = 90%
$\Gamma(K^+ \to \pi^-e^+e^+)/\Gamma_{total}$		$<6.4 \times 10^{-10}$, CL = 90%
$\Gamma(K^+ \to \pi^-\mu^+\mu^+)/\Gamma_{total}$	[k]	$<3.0 \times 10^{-9}$, CL = 90%
$\Gamma(K^+ \to \mu^+\overline{\nu}_e)/\Gamma_{total}$	[k]	$<3.3 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^0e^+\overline{\nu}_e)/\Gamma_{total}$		$<3 \times 10^{-3}$, CL = 90%
$\Gamma(D^+ \to \pi^-e^+e^+)/\Gamma_{total}$		$<9.6 \times 10^{-5}$, CL = 90%
$\Gamma(D^+ \to \pi^-\mu^+\mu^+)/\Gamma_{total}$		$<1.7 \times 10^{-5}$, CL = 90%
$\Gamma(D^+ \to \pi^-e^+\mu^+)/\Gamma_{total}$		$<5.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^+ \to \rho^-\mu^+\mu^+)/\Gamma_{total}$		$<5.6 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^-e^+e^+)/\Gamma_{total}$		$<1.2 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^-\mu^+\mu^+)/\Gamma_{total}$		$<1.2 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^-e^+\mu^+)/\Gamma_{total}$		$<1.3 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^*(892)^-\mu^+\mu^+)/\Gamma_{total}$		$<8.5 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to \pi^-\pi^-e^+e^+ + \text{c.c.})/\Gamma_{total}$		$<1.12 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to \pi^-\pi^-\mu^+\mu^+ + \text{c.c.})/\Gamma_{total}$		$<2.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-e^+e^+ + \text{c.c.})/\Gamma_{total}$		$<2.06 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-\mu^+\mu^+ + \text{c.c.})/\Gamma_{total}$		$<3.9 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to K^-K^-e^+e^+ + \text{c.c.})/\Gamma_{total}$		$<1.52 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to K^-K^-\mu^+\mu^+ + \text{c.c.})/\Gamma_{total}$		$<9.4 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \pi^-\pi^-e^+\mu^+ + \text{c.c.})/\Gamma_{total}$		$<7.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-e^+\mu^+ + \text{c.c.})/\Gamma_{total}$		$<2.18 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to K^-K^-e^+\mu^+ + \text{c.c.})/\Gamma_{total}$		$<5.7 \times 10^{-5}$, CL = 90%
$\Gamma(D_s^+ \to \pi^-e^+e^+)/\Gamma_{total}$		$<6.9 \times 10^{-4}$, CL = 90%
$\Gamma(D_s^+ \to \pi^-\mu^+\mu^+)/\Gamma_{total}$		$<8.2 \times 10^{-5}$, CL = 90%
$\Gamma(D_s^+ \to \pi^-e^+\mu^+)/\Gamma_{total}$		$<7.3 \times 10^{-4}$, CL = 90%
$\Gamma(D_s^+ \to K^-e^+e^+)/\Gamma_{total}$		$<6.3 \times 10^{-4}$, CL = 90%
$\Gamma(D_s^+ \to K^-\mu^+\mu^+)/\Gamma_{total}$		$<1.8 \times 10^{-4}$, CL = 90%
$\Gamma(D_s^+ \to K^-e^+\mu^+)/\Gamma_{total}$		$<6.8 \times 10^{-4}$, CL = 90%
$\Gamma(D_s^+ \to K^*(892)^-\mu^+\mu^+)/\Gamma_{total}$		$<1.4 \times 10^{-3}$, CL = 90%
$\Gamma(B^+ \to \pi^-e^+e^+)/\Gamma_{total}$		$<3.9 \times 10^{-3}$, CL = 90%
$\Gamma(B^+ \to \pi^-\mu^+\mu^+)/\Gamma_{total}$		$<9.1 \times 10^{-3}$, CL = 90%
$\Gamma(B^+ \to \pi^-e^+\mu^+)/\Gamma_{total}$		$<6.4 \times 10^{-3}$, CL = 90%
$\Gamma(B^+ \to K^-e^+e^+)/\Gamma_{total}$		$<3.9 \times 10^{-3}$, CL = 90%
$\Gamma(B^+ \to K^-\mu^+\mu^+)/\Gamma_{total}$		$<9.1 \times 10^{-3}$, CL = 90%

Cross section for inverse muon decay

$$\sigma(\nu_\mu e \to \mu\nu_e) = \sigma_{V-A}(\nu_e e \to \nu_e e)\left[1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu}\right]^2$$

agrees with CHARM II, CCFR data ($E_\nu \lesssim 600$ GeV)

$$\boxed{\text{PW unitarity: } |\mathcal{M}_J| < 1}$$

$V - A$ theory:

$$\mathcal{M}_0 = \frac{G_F \cdot 2m_e E_\nu}{\pi\sqrt{2}}\left[1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu}\right]$$

satisfies pw unitarity for

$$E_\nu < \pi/G_F m_e\sqrt{2} \approx 3.7 \times 10^8 \text{ GeV}$$

$$\boxed{\Rightarrow V - A \text{ theory cannot be complete}}$$

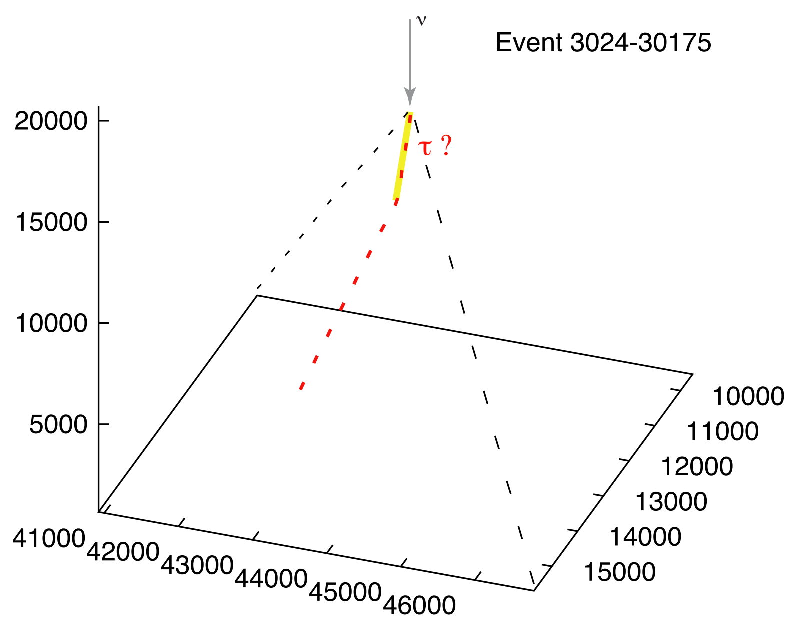
physics must change before $\sqrt{s} \approx 600$ GeV

2000: DONuT Three-Neutrino Experiment

▷ Prompt (beam-dump) ν_τ beam produced in

$$D_s^+ \rightarrow \tau^+ \nu_\tau \\ \hookrightarrow X^+ \bar{\nu}_\tau$$

▷ Try to observe $\nu_\tau N \rightarrow \tau +$ anything in emulsion; τ lifetime is 0.3 ps



Candidate event in ECC1. The three tracks with full emulsion data are shown. The red track shows a 100 mrad kink 4.5mm from the interaction vertex. The scale units are microns.

Kodama, *et al.*, *Phys. Lett.* **B504**, 218 (2001)

Leptons are seen as free particles

Table 1: Some properties of the leptons.

Lepton	Mass	Lifetime
e^-	$0.510\,998\,902(21)$ MeV/c^2	$> 4.6 \times 10^{26}$ y $(90\%$ CL$)$
ν_e	< 3 eV/c^2	
μ^-	$105.658\,357(5)$ MeV/c^2	$2.197\,03(4) \times 10^{-6}$ s
ν_μ	< 0.19 MeV/c^2 $(90\%$ CL$)$	
τ^-	$1776.99^{+0.29}_{-0.26}$ MeV/c^2	$290.6 \pm 1.1 \times 10^{-15}$ s
ν_τ	< 18.2 MeV/c^2 $(95\%$ CL$)$	

All spin-$\frac{1}{2}$, pointlike ($\lesssim$ few $\times\, 10^{-17}$ cm)

kinematically determined ν masses consistent with 0

(ν oscillations $\Rightarrow$ nonzero, unequal masses)

Universal weak couplings

Rough and ready test

Fermi constant from muon decay

$$G_\mu = \left[\frac{192\pi^3\hbar}{\tau_\mu m_\mu^5}\right]^{\frac{1}{2}} = 1.1638 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Meticulous analysis yields $G_\mu = 1.16637(1) \times 10^{-5}$ GeV^{-2}

Fermi constant from tau decay

$$G_\tau = \left[\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)}{\Gamma(\tau \to \mathrm{all})}\frac{192\pi^3\hbar}{\tau_\tau m_\tau^5}\right]^{\frac{1}{2}} = 1.1642\times10^{-5}\ \mathrm{GeV}^{-2}$$

Excellent agreement with $G_\beta = 1.16639(2) \times 10^{-5}$ GeV^{-2}

Charged currents acting in leptonic and semileptonic interactions are of universal strength; $\Rightarrow$ *universality of current-current form, or whatever lies behind it*

Nonleptonic enhancement

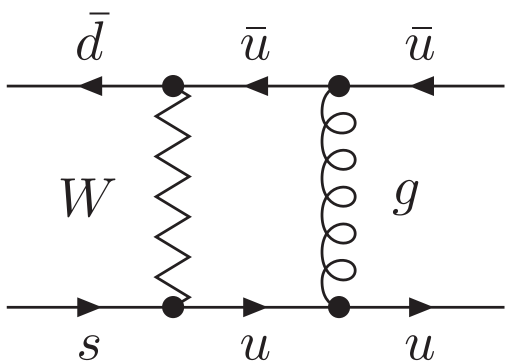
Certain NL transitions are more rapid than universality suggests

$$\underbrace{\Gamma(K_S \to \pi^+\pi^-)}_{I=0,2} \approx 450 \times \underbrace{\Gamma(K^+ \to \pi^+\pi^0)}_{I=2}$$

$$\boxed{A_0 \approx 22 \times A_2}$$

$|\Delta I| = \frac{1}{2}$ rule; "octet dominance" (over **27**)

Origin of this phenomenological rule is only partly understood. Short-distance *(perturbative)* QCD corrections arise from



... explain $\approx \sqrt{\text{enhancement}}$

SYMMETRIES $\Longrightarrow$ INTERACTIONS
Phase Invariance (Symmetry) in Quantum Mechanics

QM STATE: COMPLEX SCHRÖDINGER WAVE FUNCTION $\psi(x)$

OBSERVABLES

$$\langle O \rangle = \int d^n x \psi^* O \psi$$

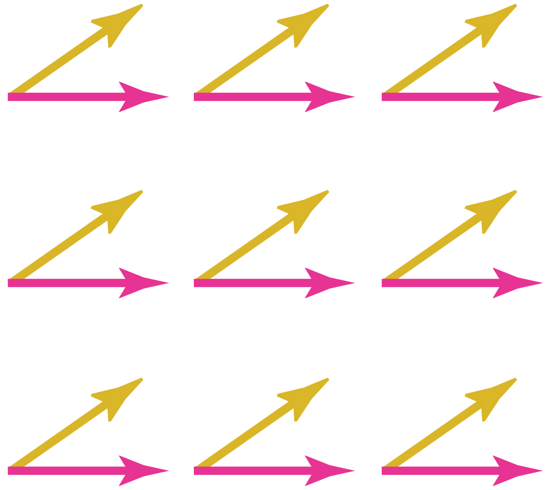
ARE UNCHANGED

UNDER A GLOBAL PHASE ROTATION

$$\psi(x) \to e^{i\theta}\psi(x)$$
$$\psi^*(x) \to e^{-i\theta}\psi^*(x)$$

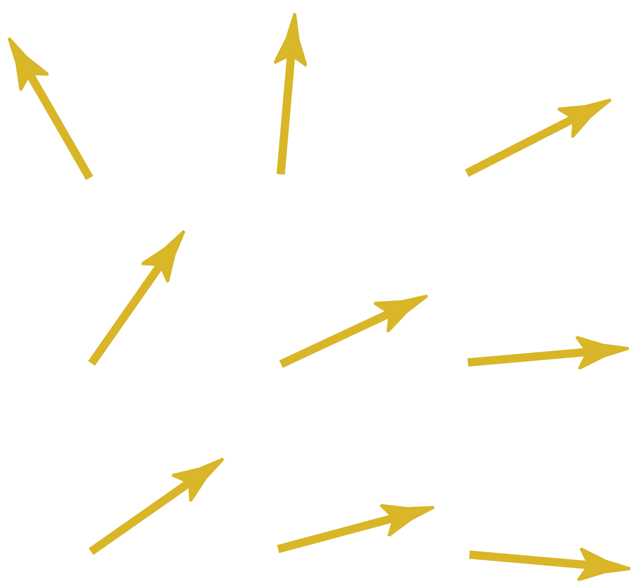
- Absolute phase of the wave function cannot be measured (is a matter of convention).
- Relative phases (interference experiments) are unaffected by a global phase rotation.

GLOBAL ROTATION — SAME EVERYWHERE



MIGHT WE CHOOSE ONE PHASE CONVENTION IN $\Pi\acute{\upsilon}\lambda o\varsigma$ AND ANOTHER IN BATAVIA?

A DIFFERENT CONVENTION AT EACH POINT?



$$\boxed{\psi(x) \to e^{iq\alpha(x)}\psi(x)}$$

THERE IS A PRICE.

Some variables (e.g., momentum) and the Schrödinger equation itself contain derivatives. Under the transformation

$$\psi(x) \to e^{iq\alpha(x)}\psi(x)$$

the gradient of the wave function transforms as

$$\nabla\psi(x) \to e^{iq\alpha(x)}[\nabla\psi(x)+iq(\nabla\alpha(x))\psi(x)]$$

The $\nabla\alpha(x)$ term spoils local phase invariance.

TO RESTORE LOCAL PHASE INVARIANCE . . .

Modify the equations of motion and observables.

$$\boxed{\text{Replace } \nabla \text{ by } \nabla + iq\vec{A}}$$

"Gauge-covariant derivative"

If the vector potential $\vec{A}$ transforms under local phase rotations as

$$\vec{A}(x) \to \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x),$$

then $(\nabla + iq\vec{A})\psi \to e^{iq\alpha(x)}(\nabla + iq\vec{A})\psi$ and $\psi^*(\nabla + iq\vec{A})\psi$ is invariant under local rotations.

NOTE . . .

- $\vec{A}(x) \to \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x)$ has the form of a gauge transformation in electrodynamics.
- The replacement $\nabla \to (\nabla + iq\vec{A})$ corresponds to $\vec{p} \to \vec{p} - q\vec{A}$

FORM OF INTERACTION IS DEDUCED
FROM LOCAL PHASE INVARIANCE

$\Longrightarrow$ MAXWELL'S EQUATIONS

DERIVED

FROM A SYMMETRY PRINCIPLE

QED is the gauge theory based on
$U(1)$ phase symmetry

GENERAL PROCEDURE

- Recognize a symmetry of Nature.
- Build it into the laws of physics.
  (Connection with conservation laws)
- Impose symmetry in stricter (local) form.

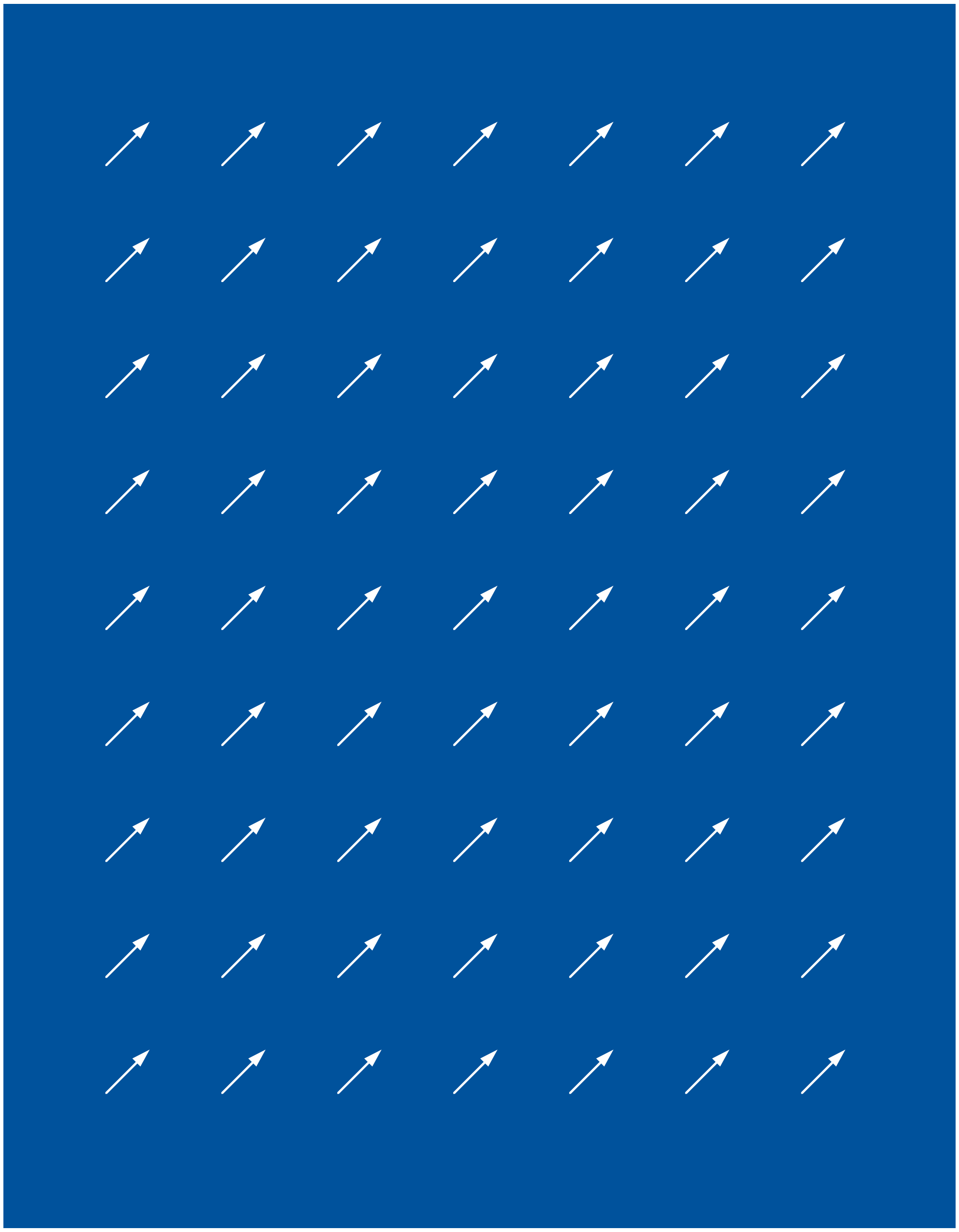
$\Longrightarrow$ INTERACTIONS

- Massless vector fields (gauge fields)
- Minimal coupling to the conserved current
- Interactions among the gauge fields, if symmetry is non-Abelian

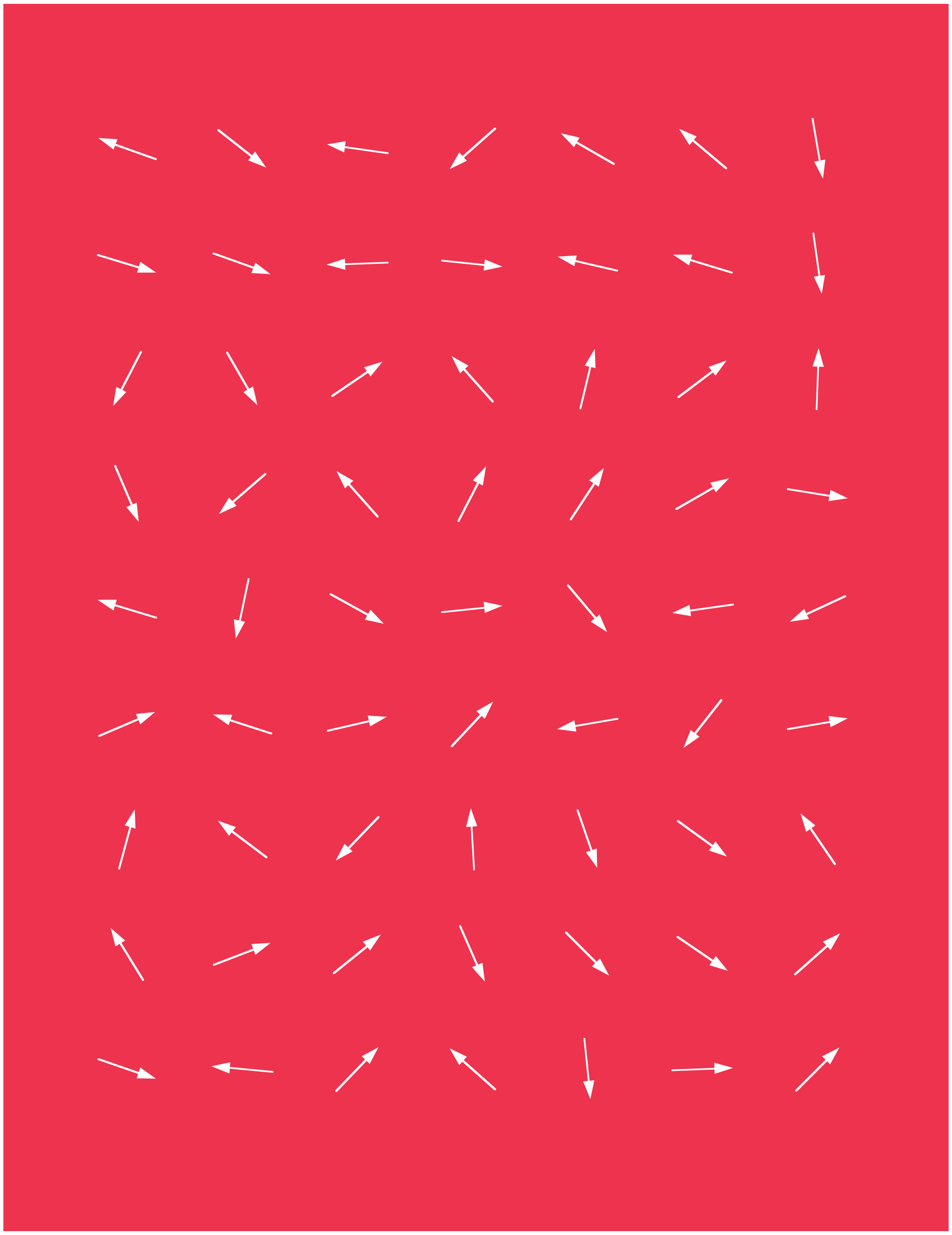
Posed as a problem in mathematics, construction of a gauge theory is always possible (at the level of a classical $\mathcal{L}$; consistent quantum theory may require additional vigilance).

Formalism is no guarantee that the gauge symmetry was chosen wisely.

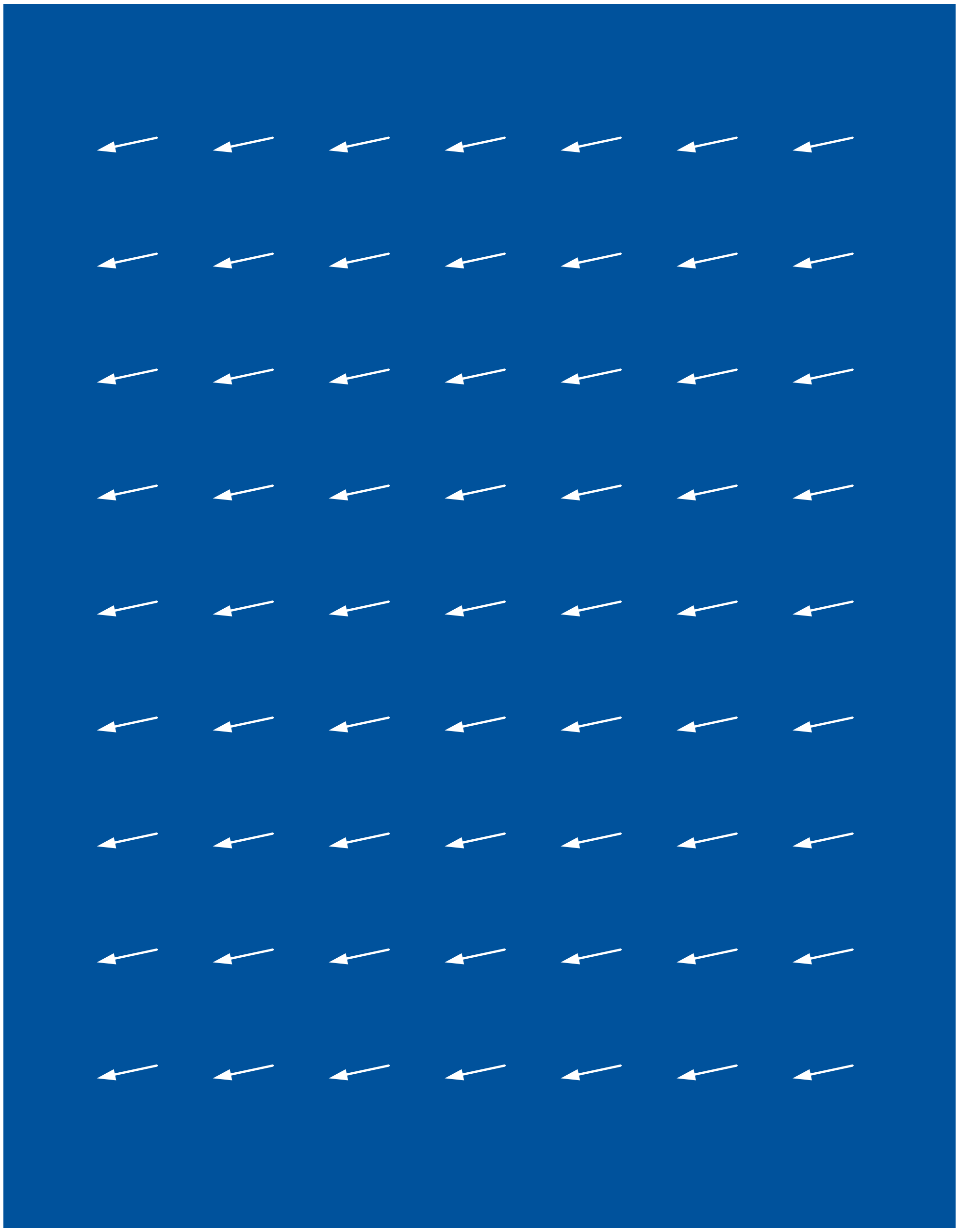
The Crystal World



The Crystal World



The Crystal World



The Perfect World

Massive Photon? *Hiding Symmetry*

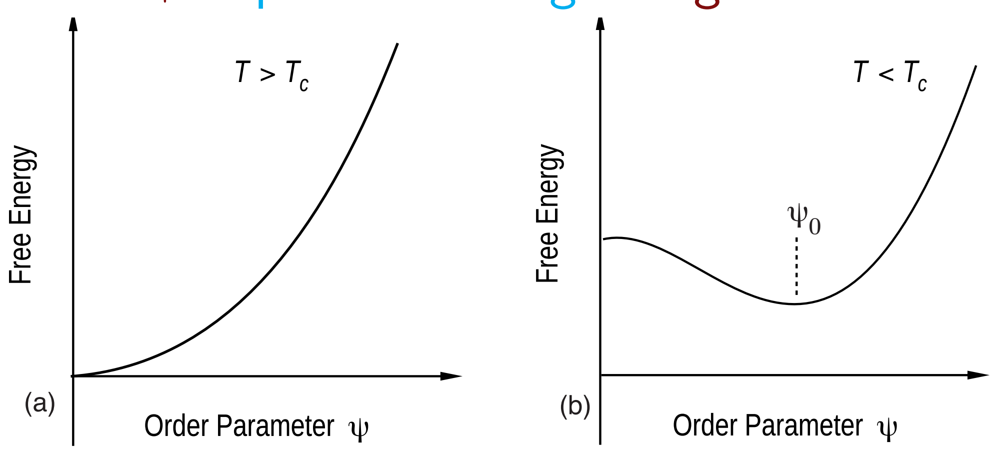
Recall $\boxed{2}$ miracles of superconductivity:

- ▷ No resistance
- ▷ Meissner effect (exclusion of $\mathbf{B}$)

Ginzburg–Landau Phenomenology
(not a theory from first principles)

normal, resistive charge carriers . . .

. . . + superconducting charge carriers



$\mathbf{B} = 0$:

$$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha\,|\psi|^2 + \beta\,|\psi|^4$$

$$T > T_c : \quad \alpha > 0 \quad \langle|\psi|^2\rangle_0 = 0$$

$$T < T_c : \quad \alpha < 0 \quad \langle|\psi|^2\rangle_0 \neq 0$$

NONZERO MAGNETIC FIELD

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{8\pi} + \frac{1}{2m^*}\left|-i\hbar\nabla\psi - \frac{e^*}{c}\mathbf{A}\psi\right|^2$$

$$\left.\begin{array}{c} e^* = -2 \\ m^* \end{array}\right\} \text{of superconducting carriers}$$

Weak, slowly varying field

$$\psi \approx \psi_0 \neq 0, \; \nabla\psi \approx 0$$

Variational analysis $\Longrightarrow$

$$\boxed{\nabla^2\mathbf{A} - \frac{4\pi e^*}{m^* c^2}\left|\psi_0\right|^2\mathbf{A} = 0}$$

wave equation of a *massive photon*

Photon— *gauge boson* — acquires mass
within superconductor

origin of Meissner effect

Formulate electroweak theory

three crucial clues from experiment:

▷ Left-handed weak-isospin doublets,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

and

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad \begin{pmatrix} t \\ b' \end{pmatrix}_L ;$$

▷ Universal strength of the (charged-current) weak interactions;

▷ Idealization that neutrinos are massless.

First two clues suggest $SU(2)_L$ gauge symmetry

A theory of leptons

$$\mathsf{L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \mathsf{R} \equiv e_R$$

weak hypercharges $Y_L = -1$, $Y_R = -2$

Gell-Mann–Nishijima connection, $Q = I_3 + \frac{1}{2}Y$

$SU(2)_L \otimes U(1)_Y$ gauge group $\Rightarrow$ gauge fields:

⋆ weak isovector $\vec{b}_\mu$, coupling g

⋆ weak isoscalar $\mathcal{A}_\mu$, coupling $g'/2$

Field-strength tensors

$$F^{\ell}_{\mu\nu} = \partial_\nu b^{\ell}_\mu - \partial_\mu b^{\ell}_\nu + g\varepsilon_{jk\ell} b^j_\mu b^k_\nu \;, SU(2)_L$$

and

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu \;, U(1)_Y$$

Interaction Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}} \ ,$$

with

$$\mathcal{L}_{\text{gauge}} = -\tfrac{1}{4}F^{\ell}_{\mu\nu}F^{\ell\mu\nu} - \tfrac{1}{4}f_{\mu\nu}f^{\mu\nu},$$

and

$$\begin{aligned}\mathcal{L}_{\text{leptons}} &= \overline{\mathsf{R}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y\right)\mathsf{R} \\ &+ \overline{\mathsf{L}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_{\mu}\right)\mathsf{L}.\end{aligned}$$

Electron mass term

$$\mathcal{L}_e = -m_e(\bar{e}_{\mathsf{R}}e_{\mathsf{L}} + \bar{e}_{\mathsf{L}}e_{\mathsf{R}}) = -m_e\bar{e}e$$

would violate local gauge invariance Theory has four massless gauge bosons

$$\mathcal{A}_{\mu} \quad b^1_{\mu} \quad b^2_{\mu} \quad b^3_{\mu}$$

Nature has but one (γ)

Hiding EW Symmetry

Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition

▷ Introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1$$

▷ Add to $\mathcal{L}$ (gauge-invariant) terms for interaction and propagation of the scalars,

$$\mathcal{L}_{\mathrm{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi),$$

where $\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_\mu$ and

$$\boxed{V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + |\lambda|\,(\phi^\dagger \phi)^2}$$

▷ Add a Yukawa interaction

$$\mathcal{L}_{\mathrm{Yukawa}} = -\zeta_e \left[\overline{\mathsf{R}}(\phi^\dagger \mathsf{L}) + (\overline{\mathsf{L}}\phi)\mathsf{R}\right]$$

▷ Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^2 < 0$
Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks) $SU(2)_L$ and $U(1)_Y$

but preserves $U(1)_{\mathrm{em}}$ invariance

Invariance under $\mathcal{G}$ means $e^{i\alpha\mathcal{G}}\langle\phi\rangle_0 = \langle\phi\rangle_0$, so $\mathcal{G}\langle\phi\rangle_0 = 0$

$$\tau_1\langle\phi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_2\langle\phi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_3\langle\phi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$Y\langle\phi\rangle_0 = Y_\phi\langle\phi\rangle_0 = +1\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

Examine electric charge operator Q on the (electrically neutral) vacuum state

$$
\begin{aligned}
Q\langle\phi\rangle_0 &= \tfrac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 \\
&= \tfrac{1}{2}\begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix}\langle\phi\rangle_0 \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \textit{unbroken!}
\end{aligned}
$$

Four original generators are broken

electric charge is not

▷ $SU(2)_L \otimes U(1)_Y \to U(1)_{\text{em}}$ (will verify)

▷ Expect massless photon

▷ Expect gauge bosons corresponding to

$$\tau_1,\ \tau_2,\ \tfrac{1}{2}(\tau_3 - Y) \equiv K$$

to acquire masses

Expand about the vacuum state

Let $\phi = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}$; in *unitary gauge*

$$\begin{aligned} \mathcal{L}_{\text{scalar}} &= \tfrac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2\eta^2 \\ &\quad + \frac{v^2}{8}[g^2 \left| b_1 - ib_2 \right|^2 + (g'\mathcal{A}_\mu - gb_\mu^3)^2] \\ &\quad + \text{interaction terms} \end{aligned}$$

Higgs boson η has acquired $(\text{mass})^2$ $M_H^2 = -2\mu^2 > 0$

$$\frac{g^2v^2}{8}(\left|W_\mu^+\right|^2 + \left|W_\mu^-\right|^2) \Longleftrightarrow M_{W^\pm} = gv/2$$

Now define othogonal combinations

$$Z_\mu = \frac{-g'\mathcal{A}_\mu + gb_\mu^3}{\sqrt{g^2+g'^2}} \qquad A_\mu = \frac{g\mathcal{A}_\mu + g'b_\mu^3}{\sqrt{g^2+g'^2}}$$

$$M_{Z^0} = \sqrt{g^2+g'^2}\, v/2 = M_W\sqrt{1+g'^2/g^2}$$

$\boxed{A_\mu \text{ remains massless}}$

$$
\begin{aligned}
\mathcal{L}_{\mathsf{Yukawa}} &= -\zeta_e \frac{(v+\eta)}{\sqrt{2}}(\bar{e}_{\mathsf{R}} e_{\mathsf{L}} + \bar{e}_{\mathsf{L}} e_{\mathsf{R}}) \\
&= -\frac{\zeta_e v}{\sqrt{2}} \bar{e}e - \frac{\zeta_e \eta}{\sqrt{2}} \bar{e}e
\end{aligned}
$$

electron acquires $m_e = \zeta_e v/\sqrt{2}$

$\boxed{\text{Higgs coupling to electrons: } m_e/v \quad (\propto \text{ mass})}$

Desired particle content . . . + Higgs scalar

Values of couplings, electroweak scale v?

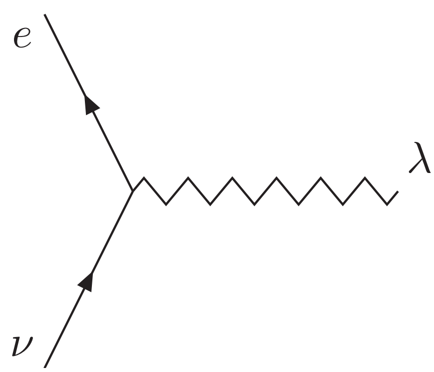
What about interactions?

Interactions . . .

$$\mathcal{L}_{W\text{-}\ell} = -\frac{g}{2\sqrt{2}}[\bar{\nu}\gamma^{\mu}(1-\gamma_5)eW_{\mu}^{+} + \bar{e}\gamma^{\mu}(1-\gamma_5)\nu W_{\mu}^{-}]$$

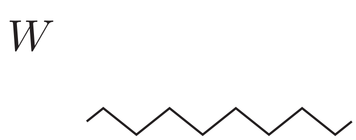
$+$ similar terms for μ and τ

Feynman rule:



$$\frac{-ig}{2\sqrt{2}}\gamma_{\lambda}(1-\gamma_5)$$

gauge-boson propagator:



$$= \frac{-i(g_{\mu\nu} - k_{\mu}k_{\nu}/M_W^2)}{k^2 - M_W^2}\,.$$

Compute $\nu_\mu e \to \mu\nu_e$

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1-(m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1+2m_e E_\nu/M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\begin{aligned} \frac{g^4}{16M_W^4} &= 2G_F^2 \\ \Rightarrow g^4 &= 32(G_F M_W^2)^2 = 64\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^2 \\ \Rightarrow \frac{g}{2\sqrt{2}} &= \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}} \end{aligned}$$

Using $M_W = gv/2$, determine

$$\boxed{v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV}}$$

the electroweak scale

$$\Rightarrow \langle\phi^0\rangle_0 = (G_F\sqrt{8})^{-\frac{1}{2}} \approx 174 \text{ GeV}$$

W-propagator modifies HE behavior

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

$$\lim_{E_\nu \to \infty} \sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\sqrt{2}}$$

independent of energy!

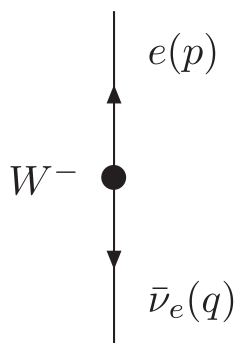
partial-wave unitarity respected for

$$s < M_W^2[\exp(\pi\sqrt{2}/G_F M_W^2) - 1]$$

W-boson properties

No prediction yet for M_W (haven't determined g)

Leptonic decay $W^- \to e^-\nu_e$



$$p \approx \left(\frac{M_W}{2}; \frac{M_W \sin\theta}{2}, 0, \frac{M_W \cos\theta}{2}\right)$$

$$q \approx \left(\frac{M_W}{2}; -\frac{M_W \sin\theta}{2}, 0, -\frac{M_W \cos\theta}{2}\right)$$

$$\mathcal{M} = -i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}} \bar{u}(e,p)\gamma_\mu(1-\gamma_5)v(\nu,q)\,\varepsilon^\mu$$

$\varepsilon^\mu = (0;\hat{\varepsilon})$: W polarization vector in its rest frame

$$\begin{aligned} |\mathcal{M}|^2 &= \frac{G_F M_W^2}{\sqrt{2}} \mathrm{tr}\left[\not{\varepsilon}(1-\gamma_5)\not{q}(1+\gamma_5)\not{\varepsilon}^*\not{p}\right] \; ; \\ \mathrm{tr}[\cdots] &= \left[\varepsilon\cdot q\,\varepsilon^*\cdot p - \varepsilon\cdot\varepsilon^*\,q\cdot p + \varepsilon\cdot p\,\varepsilon^*\cdot q + i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma\right] \end{aligned}$$

decay rate is independent of W polarization; look first at longitudinal pol. $\varepsilon^\mu = (0;0,0,1) = \varepsilon^{*\mu}$, eliminate $\epsilon_{\mu\nu\rho\sigma}$

$$|\mathcal{M}|^2 = \frac{4G_F M_W^4}{\sqrt{2}}\sin^2\theta$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2}\,\frac{\mathcal{S}_{12}}{M_W^3}$$

$$\mathcal{S}_{12} = \sqrt{[M_W^2 - (m_e + m_\nu)^2][M_W^2 - (m_e - m_\nu)^2]} = M_W^2$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2\sqrt{2}} \sin^2\theta$$

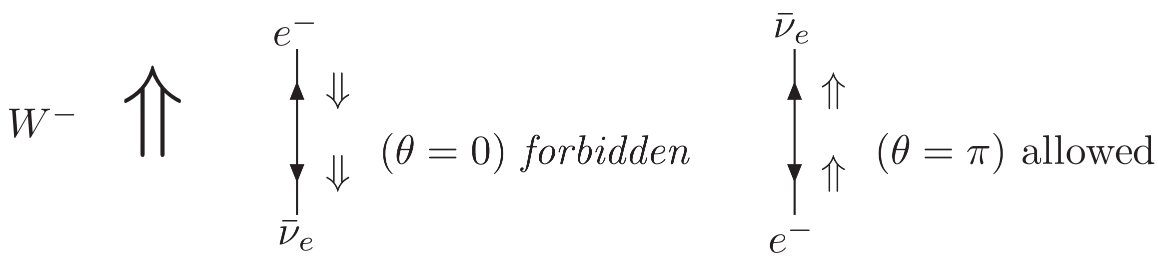
and

$$\boxed{\Gamma(W \to e\nu) = \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

Other helicities: $\varepsilon^\mu_{\pm 1} = (0; -1, \mp i, 0)/\sqrt{2}$

$$\frac{d\Gamma_{\pm 1}}{d\Omega} = \frac{G_F M_W^3}{32\pi^2\sqrt{2}}(1 \mp \cos\theta)^2$$

Extinctions at $\cos\theta = \pm 1$ are consequences of angular momentum conservation:



(situation reversed for $W^+ \to e^+\nu_e$)

e^+ follows polarization direction of W^+

e^- avoids polarization direction of W^-

important for discovery of $W^\pm$ in $\bar{p}p$ ($\bar{q}q$)   C violation

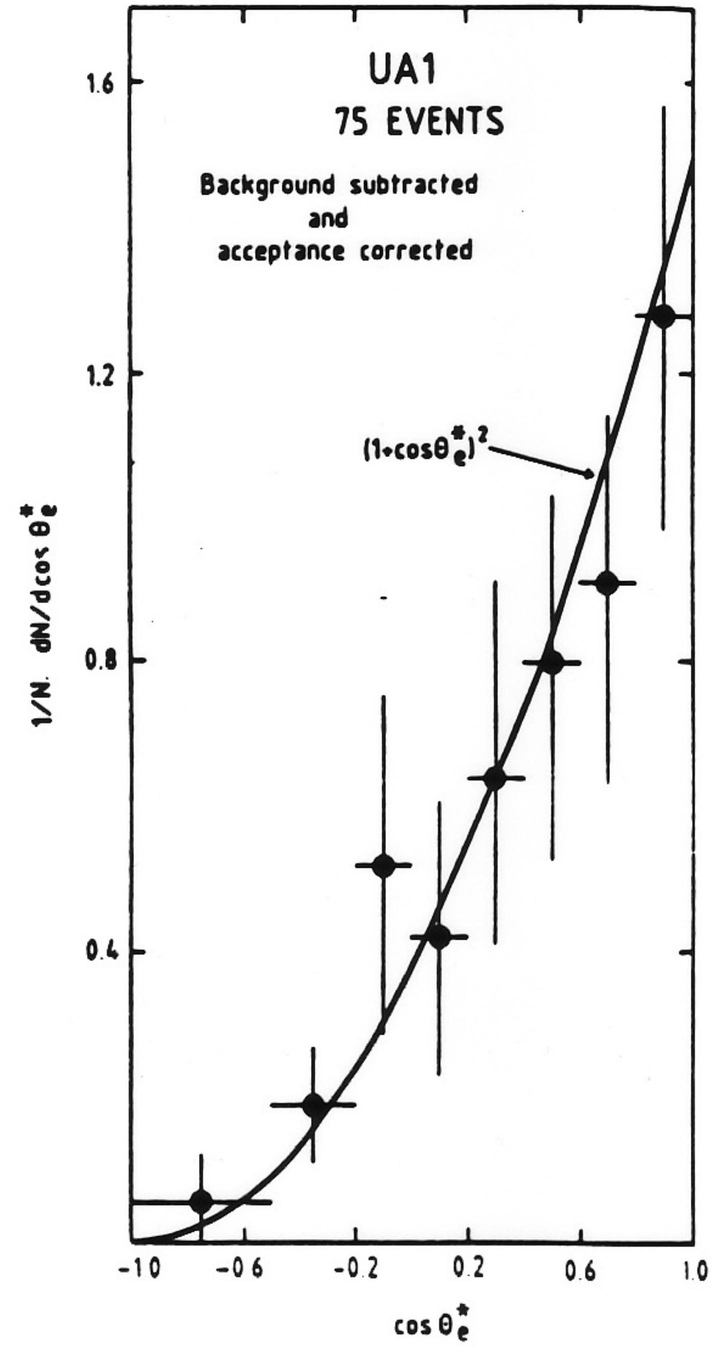


Fig. 2. The W decay angular distribution of the emission angle θ^* of the electron (positron) with respect to the proton (antiproton) direction in the rest frame of the W. Only those events for which the lepton charge and the decay kinematics are well determined have been used. The curve shows the $(V - A)$ expectation of $(1 + \cos\theta^*)^2$.

Interactions . . .

$$\mathcal{L}_{A\text{-}\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e}\gamma^\mu e A_\mu$$

. . . vector interaction; $\Rightarrow A_\mu$ as γ, provided

$$\boxed{gg'/\sqrt{g^2 + g'^2} \equiv e}$$

Define $g' = g \tan\theta_W$ θ_W: weak mixing angle

$$\begin{aligned} g &= e/\sin\theta_W \geq e \\ g' &= e/\cos\theta_W \geq e \end{aligned}$$

$Z_\mu = b_\mu^3 \cos\theta_W - \mathcal{A}_\mu \sin\theta_W$ $A_\mu = \mathcal{A}_\mu \cos\theta_W + b_\mu^3 \sin\theta_W$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4\cos\theta_W} \bar{\nu}\gamma^\mu (1 - \gamma_5)\nu \, Z_\mu$$

$$\mathcal{L}_{Z-e} = \frac{-g}{4\cos\theta_W} \bar{e} \left[L_e \gamma^\mu (1 - \gamma_5) + R_e \gamma^\mu (1 + \gamma_5) \right] e Z_\mu$$

$$\begin{aligned} L_e &= 2\sin^2\theta_W - 1 = 2x_W + \tau_3 \\ R_e &= 2\sin^2\theta_W = 2x_W \end{aligned}$$

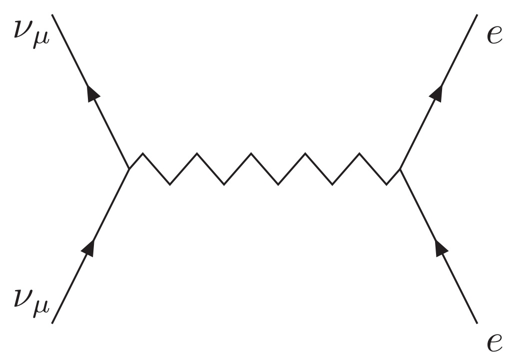
Z-boson properties

Decay calculation analogous to $W^{\pm}$

$$\begin{aligned}\Gamma(Z \to \nu\bar{\nu}) &= \frac{G_F M_Z^3}{12\pi\sqrt{2}} \\ \Gamma(Z \to e^+e^-) &= \Gamma(Z \to \nu\bar{\nu}) \left[L_e^2 + R_e^2\right]\end{aligned}$$

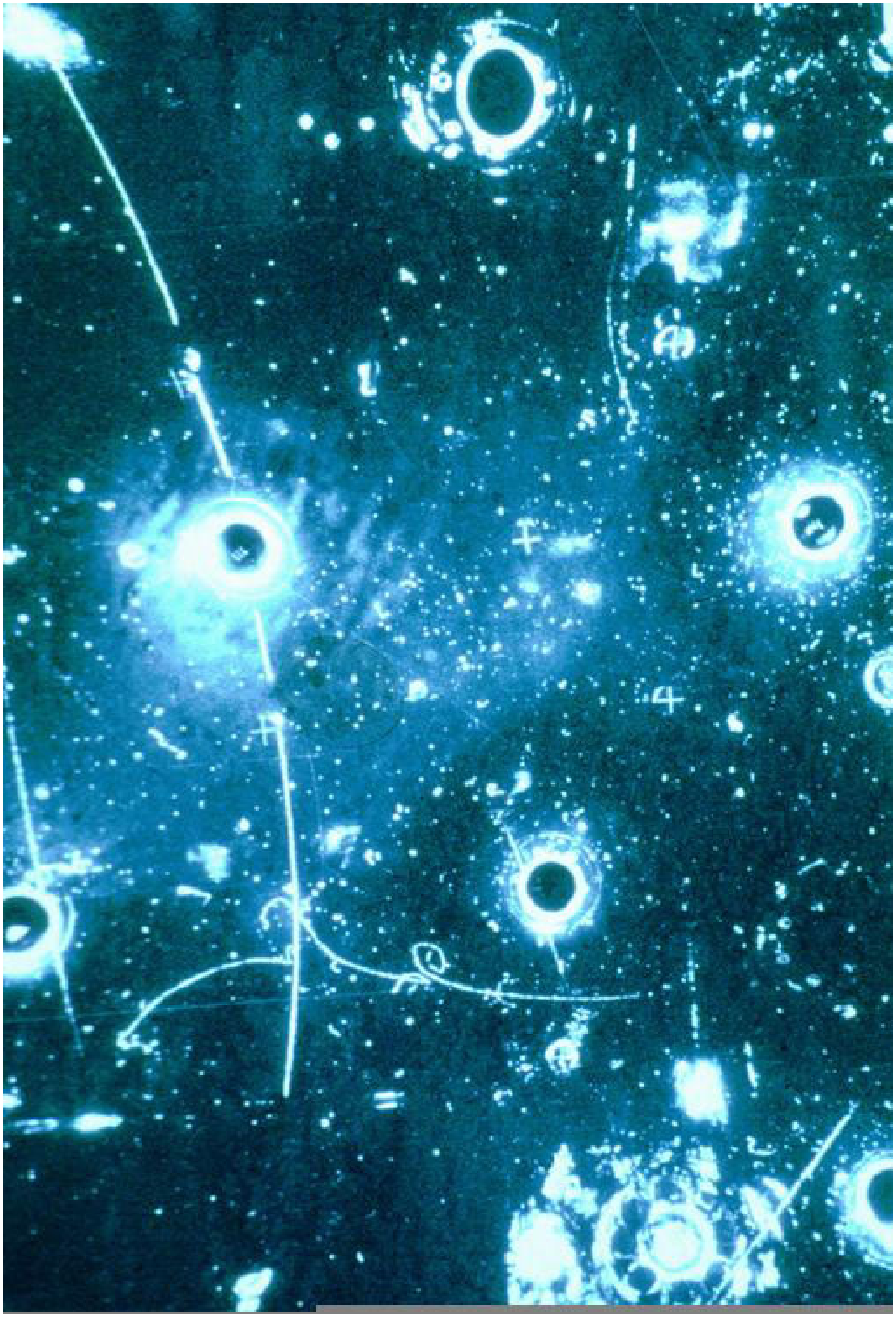
Neutral-current interactions

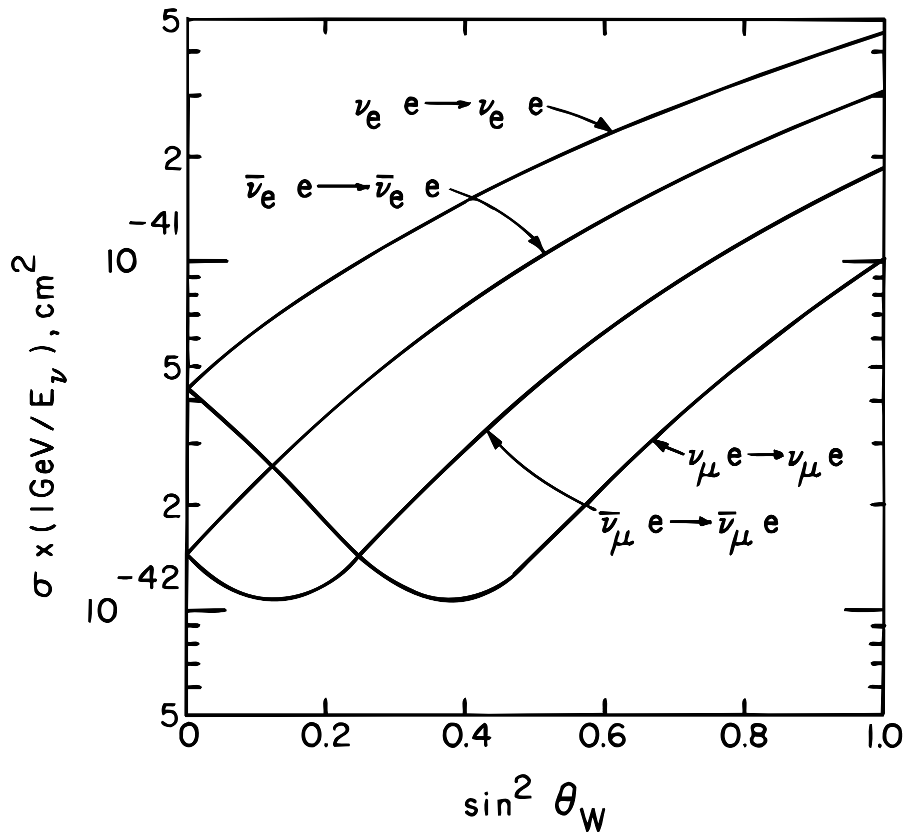
New νe reaction, not present in $V - A$



$$\begin{aligned}\sigma(\nu_\mu e \to \nu_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2 + R_e^2/3\right] \\ \sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2/3 + R_e^2\right] \\ \sigma(\nu_e e \to \nu_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2 + R_e^2/3\right] \\ \sigma(\bar{\nu}_e e \to \bar{\nu}_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2/3 + R_e^2\right]\end{aligned}$$

Gargamelle $\nu_\mu e$ Event





"Model-independent" analysis

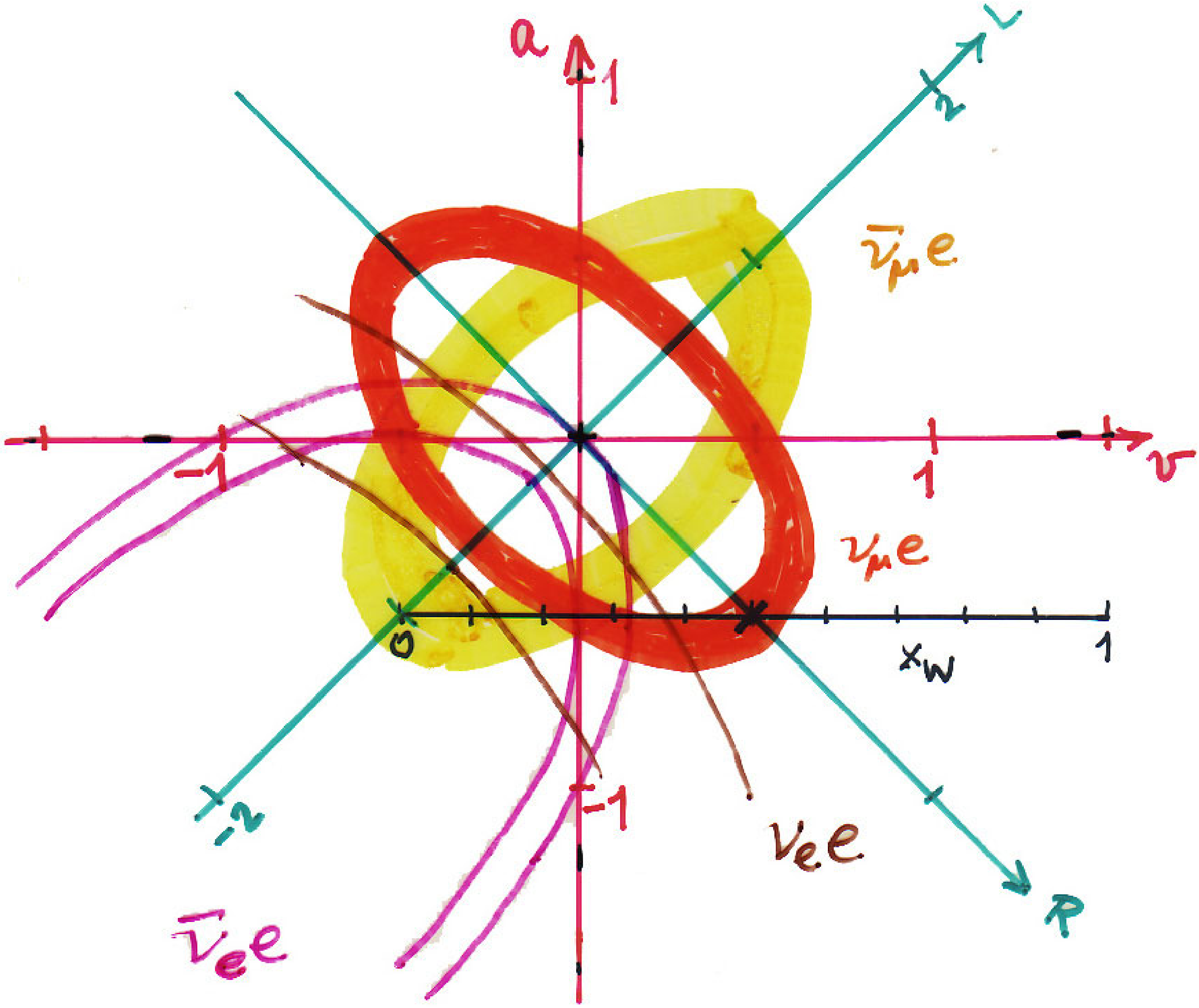
Measure all cross sections to determine chiral couplings L_e and R_e or traditional vector and axial couplings v and a

$$a = \tfrac{1}{2}(L_e - R_e) \qquad v = \tfrac{1}{2}(L_e - R_e)$$

$$L_e = v + a \qquad R_e = v - a$$

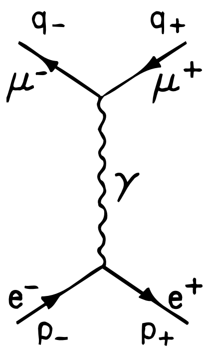
model-independent in V, A framework

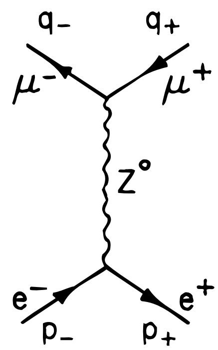
Neutrino–electron scattering



Twofold ambiguity remains even after measuring all four cross sections: same cross sections result if we interchange $R_e \leftrightarrow -R_e$ $(v \leftrightarrow a)$

Consider $e^+e^- \to \mu^+\mu^-$





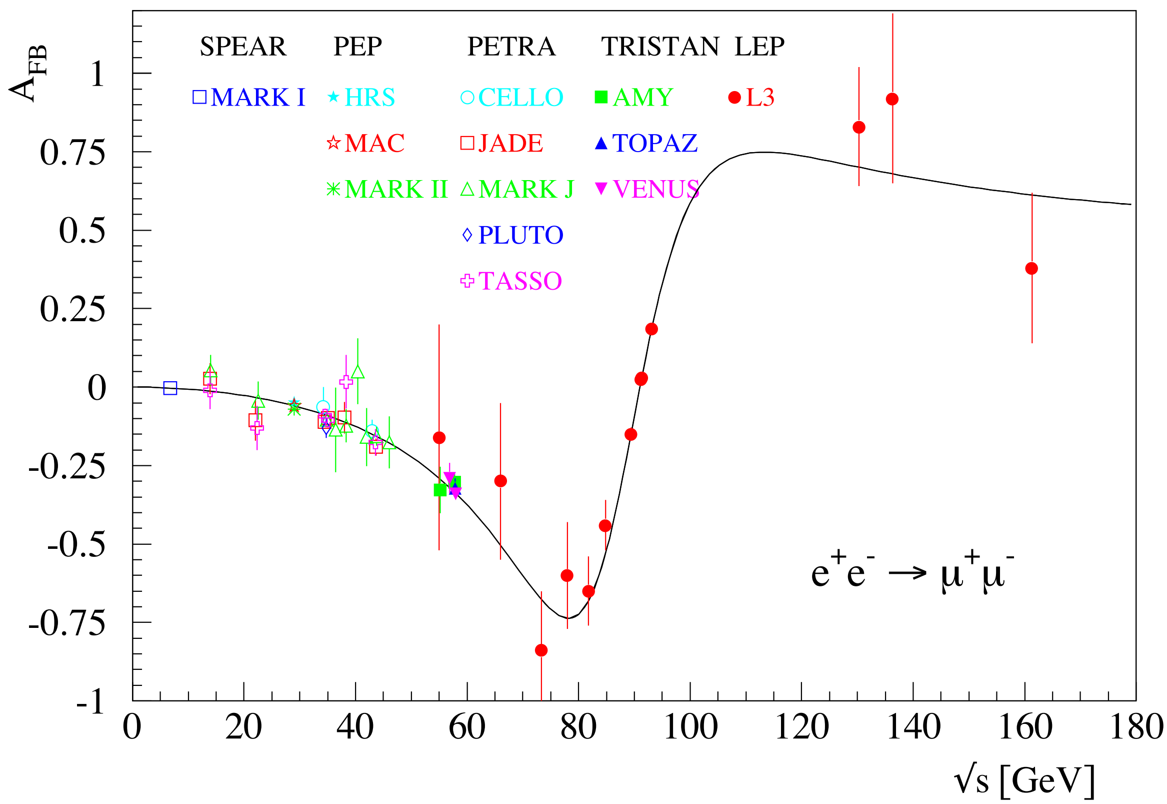
$$
\begin{aligned}
\mathcal{M} \quad &= \quad -ie^2\bar{u}(\mu,q_-)\gamma_\lambda Q_\mu v(\mu,q_+)\frac{g^{\lambda\nu}}{s}\bar{v}(e,p_+)\gamma_\nu u(e.p_-) \\
&+\frac{i}{2}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)\bar{u}(\mu,q_-)\gamma_\lambda[R_\mu(1+\gamma_5)+L_\mu(1-\gamma_5)]v(\mu,q_+) \\
&\times\frac{g^{\lambda\nu}}{s-M_Z^2}\bar{v}(e,p_+)\gamma_\nu[R_e(1+\gamma_5)+L_e(1-\gamma_5)]u(e,p_-)
\end{aligned}
$$

muon charge $Q_\mu = -1$

$$
\begin{aligned}
\frac{d\sigma}{dz} \quad &= \quad \frac{\pi\alpha^2 Q_\mu^2}{2s}(1+z^2) \\
&-\frac{\alpha Q_\mu G_F M_Z^2(s-M_Z^2)}{8\sqrt{2}[(s-M_Z^2)^2+M_Z^2\Gamma^2]} \\
&\times[(R_e+L_e)(R_\mu+L_\mu)(1+z^2)+2(R_e-L_e)(R_\mu-L_\mu)z] \\
&+\frac{G_F^2 M_Z^4 s}{64\pi[(s-M_Z^2)^2+M_Z^2\Gamma^2]} \\
&\times[(R_e^2+L_e^2)(R_\mu^2+L_\mu^2)(1+z^2)+2(R_e^2-L_e^2)(R_\mu^2-L_\mu^2)z]
\end{aligned}
$$

F-B asymmetry $A \equiv \dfrac{\int_0^1 \; dz\, d\sigma/dz - \int_{-1}^0 \; dz\, d\sigma/dz}{\int_{-1}^1 \; dz\, d\sigma/dz}$

$$\begin{aligned} \lim_{s/M_Z^2 \ll 1} A &= \frac{3G_F s}{16\pi\alpha Q_\mu \sqrt{2}}(R_e - L_e)(R_\mu - L_\mu) \\ &\approx -6.7\times 10^{-5}\left(\frac{s}{1\ \mathrm{GeV}^2}\right)(R_e - L_e)(R_\mu - L_\mu) \\ &= -3G_F s a^2/4\pi\alpha\sqrt{2} \end{aligned}$$

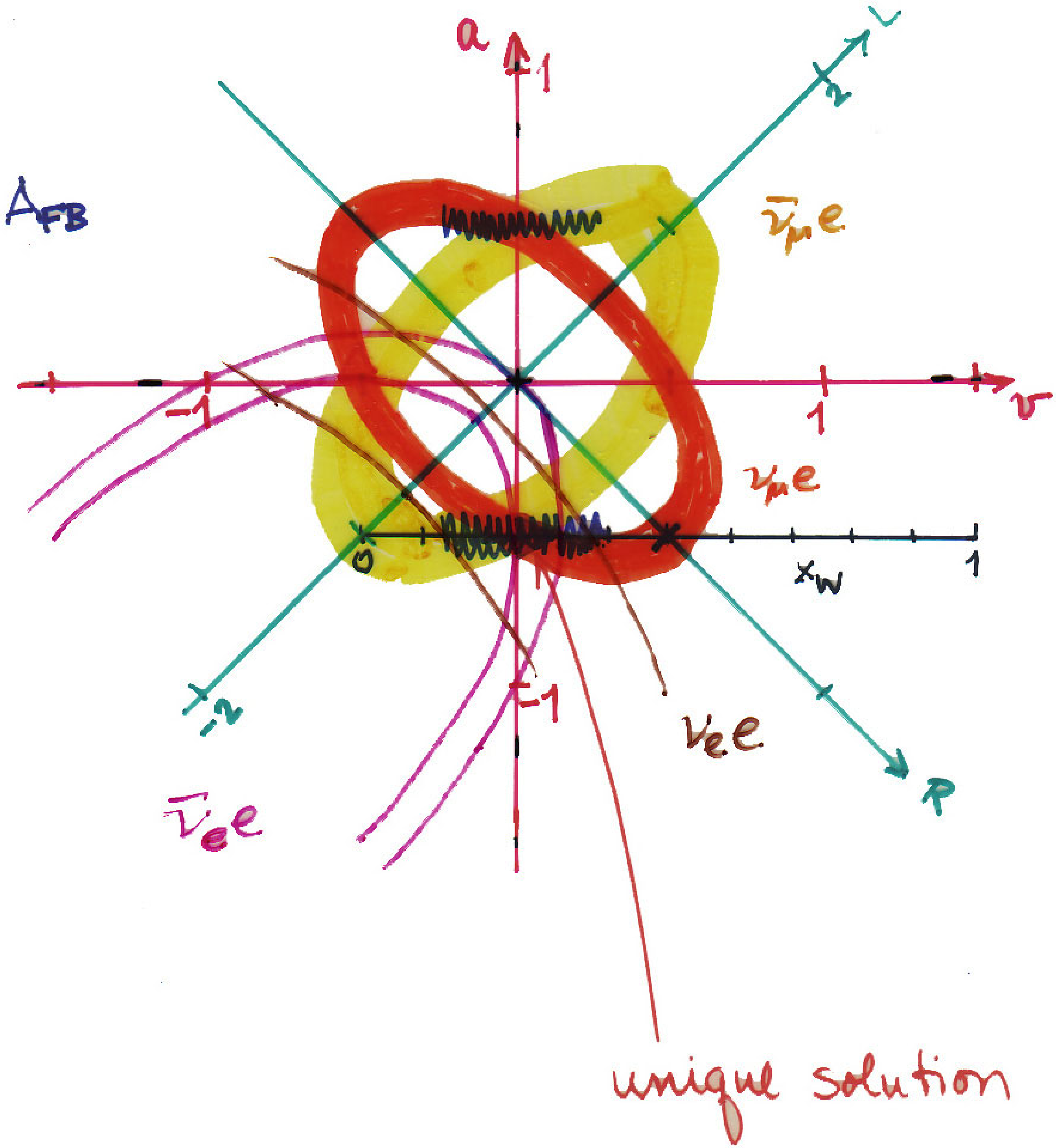


J. Mnich *Phys. Rep.* **271**, 181-266 (1996)

Measuring A resolves ambiguity

Validate EW theory, measure $\sin^2\theta_W$

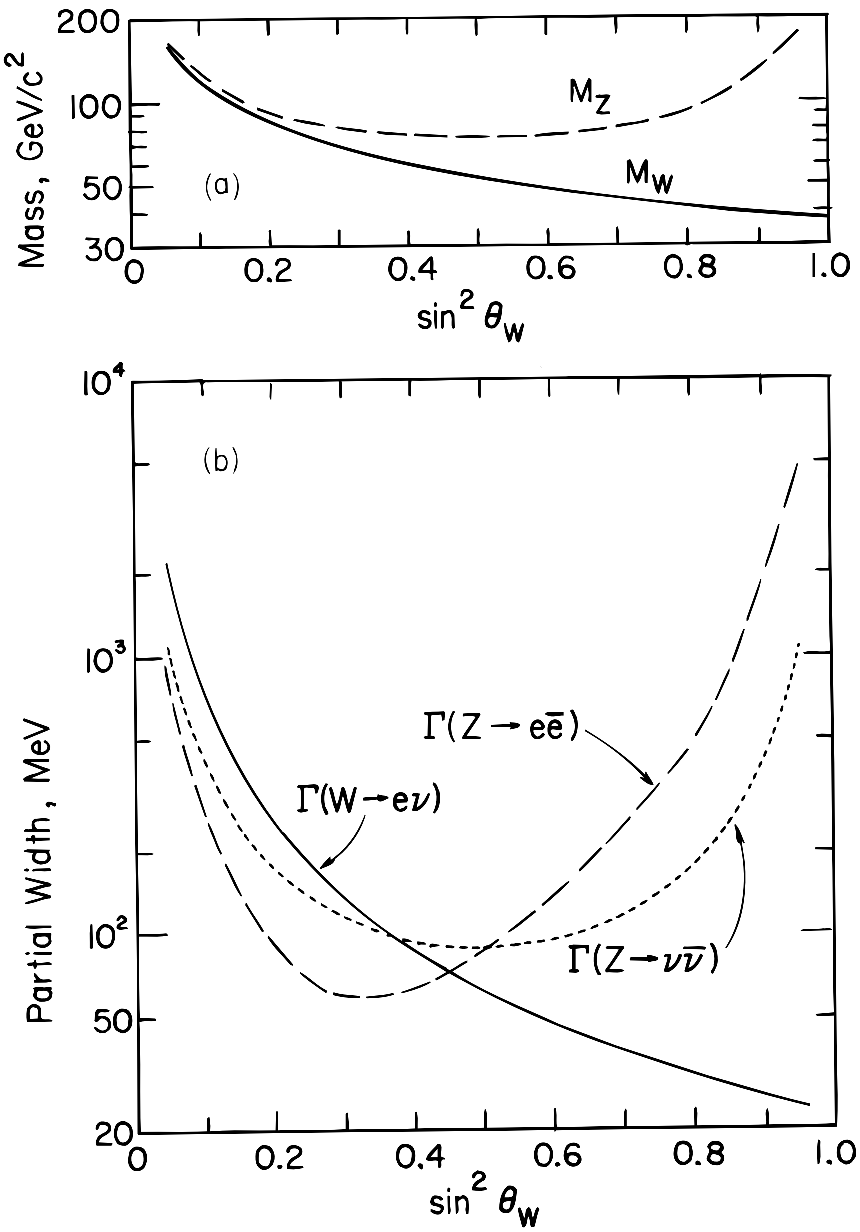
Neutrino–electron scattering



With a measurement of $\sin^2\theta_W$, predict

$$M_W^2 = g^2v^2/4 = e^2/4G_F\sqrt{2}\sin^2\theta_W \approx (37.3 \text{ GeV}/c^2)^2/\sin^2\theta_W$$

$$M_Z^2 = M_W^2/\cos^2\theta_W$$



568 Intermediate Vector Bosons W^+, W^-, and Z^0

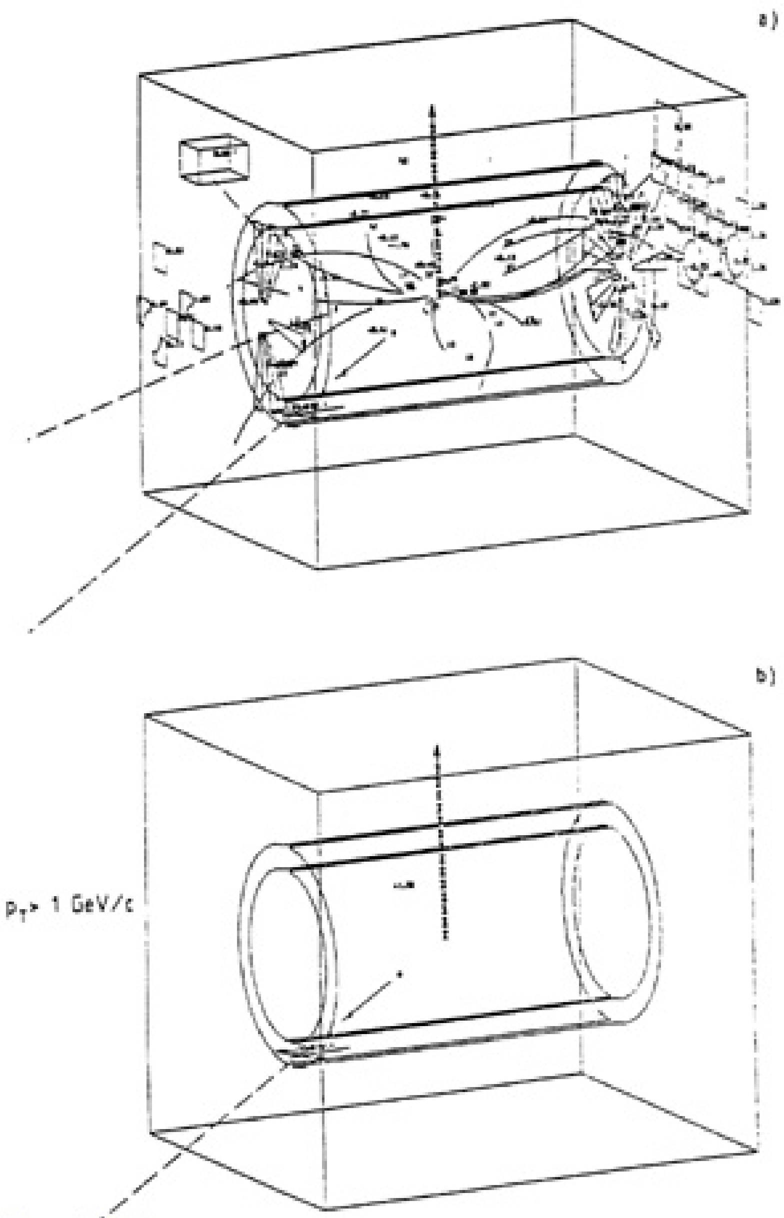
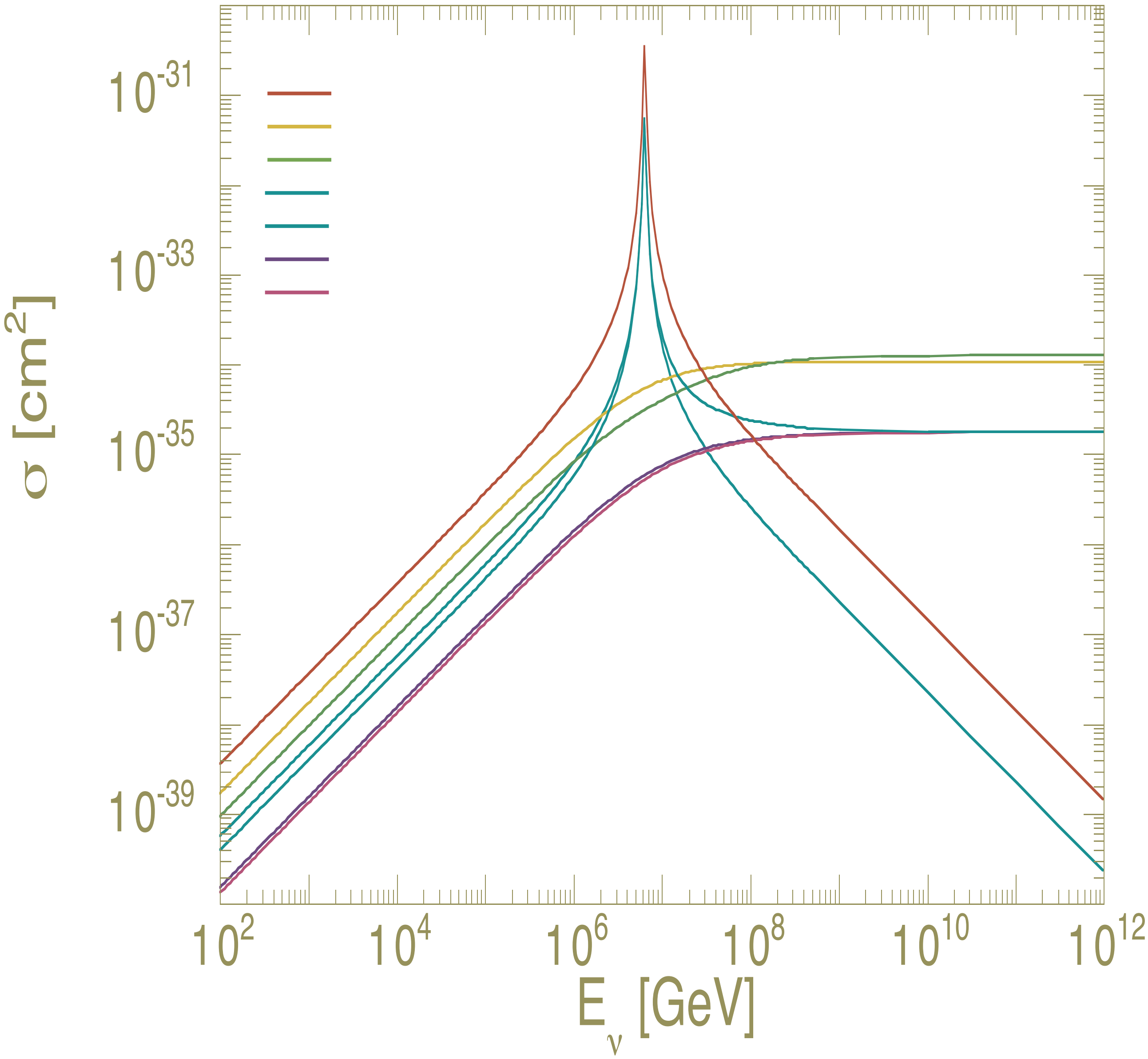


Fig. 16a,b

UA1



At low energies: $\sigma(\bar{\nu}_e e \to$ hadrons$) > \sigma(\nu_\mu e \to \mu\nu_e) >$
$\sigma(\nu_e e \to \nu_e e) > \sigma(\bar{(}\nu_e e \to \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \to \bar{\nu}_e e) >$
$\sigma(\nu_\mu e \to \nu_\mu e) > \sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$

EW interactions of quarks

▷ Left-handed doublet

$$
\mathsf{L}_q = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad
\begin{array}{ccc}
I_3 & Q & Y = 2(Q - I_3) \\
\frac{1}{2} & +\frac{2}{3} & \\
 & & \frac{1}{3} \\
-\frac{1}{2} & -\frac{1}{3} &
\end{array}
$$

▷ two right-handed singlets

	I_3	Q	$Y = 2(Q - I_3)$
$\mathsf{R}_u = u_R$	0	$+\frac{2}{3}$	$+\frac{4}{3}$
$\mathsf{R}_d = d_R$	0	$-\frac{1}{3}$	$-\frac{2}{3}$

▷ CC interaction

$$
\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{u}_e \gamma^\mu (1 - \gamma_5) d \, W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u \, W_\mu^- \right]
$$

identical in form to $\mathcal{L}_{W\text{-}\ell}$: universality ⇔ weak isospin

▷ NC interaction

$$
\mathcal{L}_{Z\text{-}q} = \frac{-g}{4 \cos\theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu \left[L_i (1 - \gamma_5) + R_i (1 + \gamma_5) \right] q_i Z_\mu
$$

$$
L_i = \tau_3 - 2 Q_i \sin^2\theta_W \qquad R_i = -2 Q_i \sin^2\theta_W
$$

equivalent in form (not numbers) to $\mathcal{L}_{Z\text{-}\ell}$

Trouble in Paradise

Universal $u \leftrightarrow d$, $\nu_e \leftrightarrow e$ *not quite right*

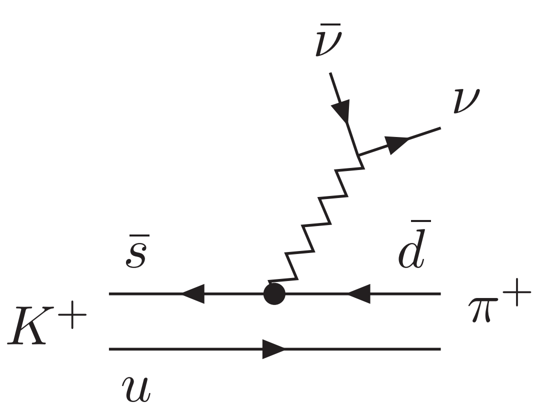
$$\text{Good:} \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow \text{Better:} \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

$$d_\theta \equiv d\,\cos\theta_C + s\,\sin\theta_C \qquad \cos\theta_C = 0.9736 \pm 0.0010$$

"Cabibbo-rotated" doublet perfects CC interaction (up to small third-generation effects) but $\Rightarrow$ serious trouble for NC

$$\begin{aligned}
\mathcal{L}_{Z\text{-}q} \quad = \quad & \frac{-g}{4\cos\theta_W} Z_\mu \left\{ \bar{u}\gamma^\mu \left[L_u(1-\gamma_5) + R_u(1+\gamma_5) \right] u \right. \\
& + \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d\,\cos^2\theta_C \\
& + \bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s\,\sin^2\theta_C \\
& + \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s\,\sin\theta_C\cos\theta_C \\
& \left. + \; \bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d\,\sin\theta_C\cos\theta_C \right\}
\end{aligned}$$

Strangeness-changing NC interactions highly suppressed!



BNL E-787 detected two $K^+ \rightarrow \pi^+\nu\bar{\nu}$ candidates, with $\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) = 1.57^{+1.75}_{-0.82} \times 10^{-10}$

(SM: 0.8 ± 0.3)

Phys. Rev. Lett. **88**, 041803 (2002)

Glashow–Iliopoulos–Maiani

two left-handed doublets

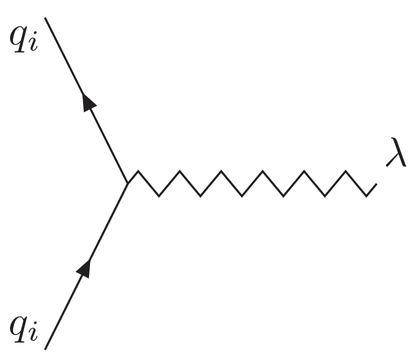
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$$

$$(s_\theta = s \cos\theta_C - d \sin\theta_C)$$

$+$ right-handed singlets, e_R, μ_R, u_R, d_R, c_R, s_R

Required new charmed quark, c

Cross terms vanish in $\mathcal{L}_{Z\text{-}q}$,



$$\frac{-ig}{4\cos\theta_W}\gamma_\lambda[(1-\gamma_5)L_i + (1+\gamma_5)R_i] \quad ,$$

$$L_i = \tau_3 - 2Q_i \sin^2\theta_W \quad R_i = -2Q_i \sin^2\theta_W$$

flavor-diagonal interaction!

Straightforward generalization to n quark doublets

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{\Psi}\gamma^\mu(1-\gamma_5)\mathcal{O}\Psi\, W_\mu^+ + \mathrm{h.c.}\right]$$

composite $\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ \\ d \\ s \\ \vdots \end{pmatrix}$ flavor structure $\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$

U: unitary quark mixing matrix

Weak-isospin part: $\mathcal{L}_{Z\text{-}q}^{\mathrm{iso}} = \dfrac{-g}{4\cos\theta_W}\bar{\Psi}\gamma^\mu(1-\gamma_5)\left[\mathcal{O},\mathcal{O}^\dagger\right]\Psi$

Since $\left[\mathcal{O},\mathcal{O}^\dagger\right] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

$\boxed{\Rightarrow \text{NC interaction is flavor-diagonal}}$

General $n \times n$ quark-mixing matrix U:

$n(n-1)/2$ real $\angle$, $(n-1)(n-2)/2$ complex phases

3×3 (Cabibbo–Kobayashi-Maskawa): 3 $\angle$ + 1 phase

$\Rightarrow$ CP violation

Qualitative successes of $SU(2)_L \otimes U(1)_Y$ theory:

- ▷ neutral-current interactions
- ▷ necessity of charm
- ▷ existence and properties of $W^{\pm}$ and Z^0

Decade of precision tests EW (one-per-mille)

M_Z	$91\,187.6 \pm 2.1\ \mathrm{MeV}/c^2$
Γ_Z	$2495.2 \pm 2.3\ \mathrm{MeV}$
$\sigma^0_{\text{hadronic}}$	$41.541 \pm 0.037\ \mathrm{nb}$
Γ_{hadronic}	$1744.4 \pm 2.0\ \mathrm{MeV}$
Γ_{leptonic}	$83.984 \pm 0.086\ \mathrm{MeV}$
$\Gamma_{\text{invisible}}$	$499.0 \pm 1.5\ \mathrm{MeV}$

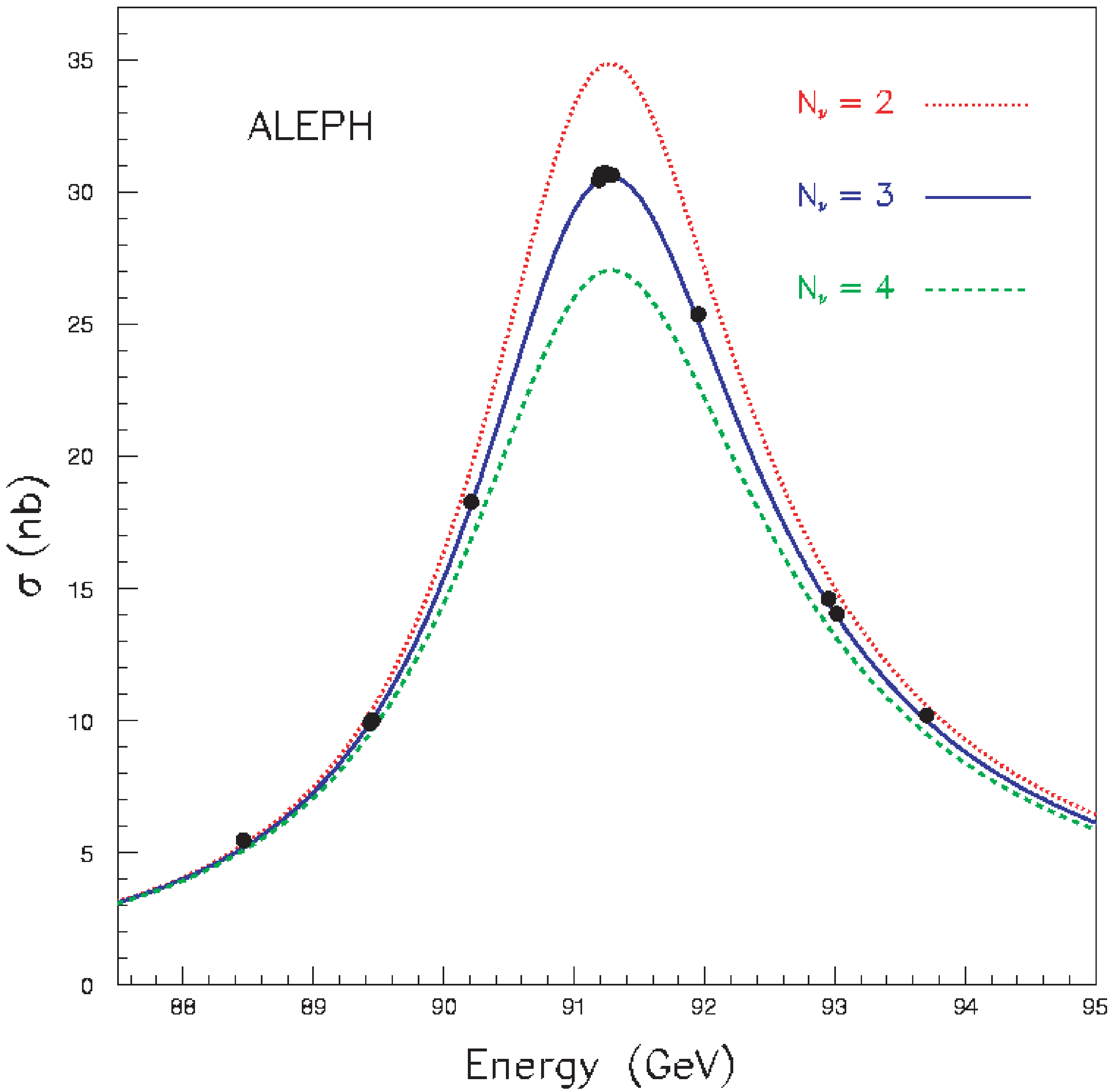
where $\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$

light neutrinos $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \to \nu_i\bar{\nu}_i)$

Current value: $\boxed{N_\nu = 2.994 \pm 0.012}$

. . . excellent agreement with ν_e, ν_μ, and ν_τ

Three light neutrinos



The top quark must exist

▷ Two families

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L$$

don't account for CP violation. Need a third family . . . or another answer.

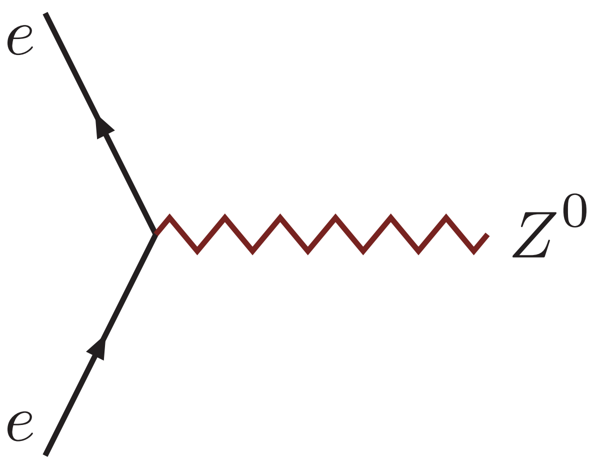
Given the existence of b, (τ)

▷ top is needed for an anomaly-free EW theory

▷ absence of FCNC in b decay ($b \not\to s\ell^+\ell^-$, etc.)

▷ b has weak isospin $I_{3L} = -\frac{1}{2}$; needs partner

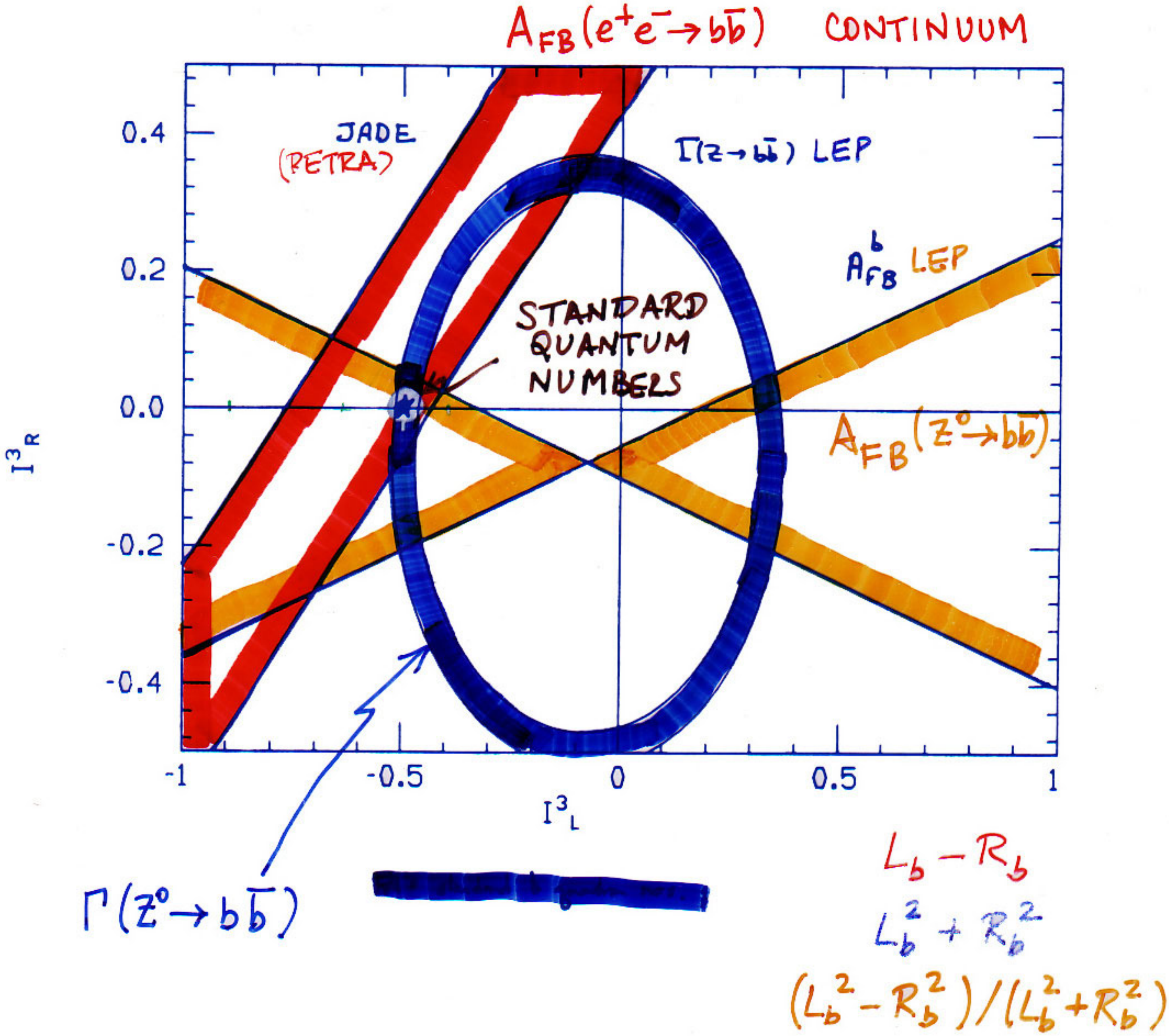
$$\begin{pmatrix} t \\ b \end{pmatrix}_L$$



$$L_b = I_{3L} - Q_b \sin^2\theta_W$$

$$R_b = I_{3R} - Q_b \sin^2\theta_W$$

Measure $I_{3L}^{(b)} = -0.490^{+0.015}_{-0.012}$ $I_{3R}^{(b)} = -0.028 \pm 0.056$



Needed: top with $I_{3L} = +\frac{1}{2}$

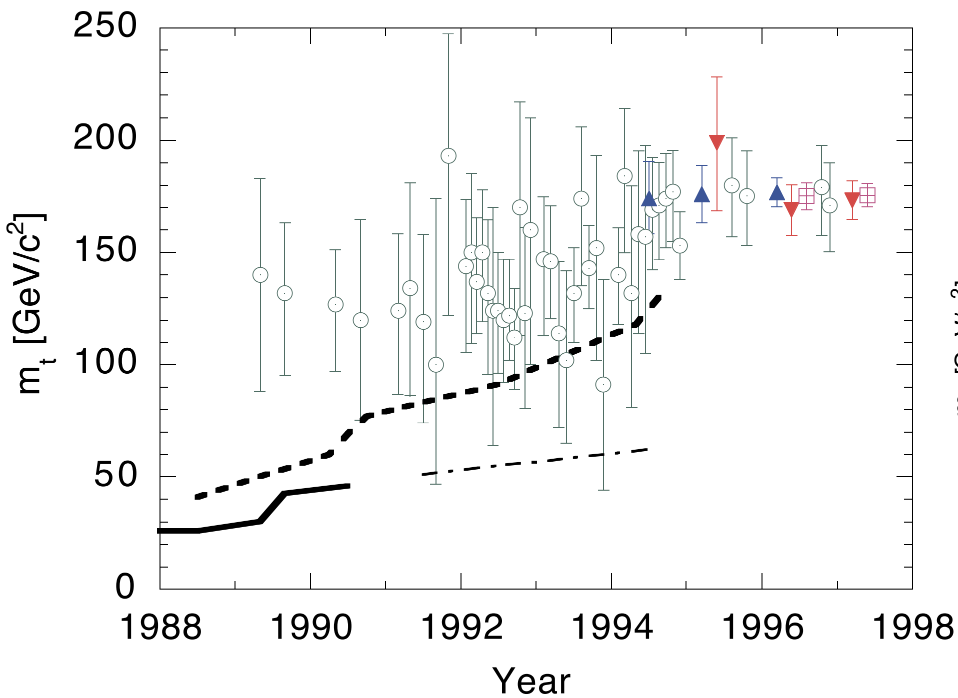
D. Schaile & P. Zerwas, *Phys. Rev.* D**45**, 3262 (1992)

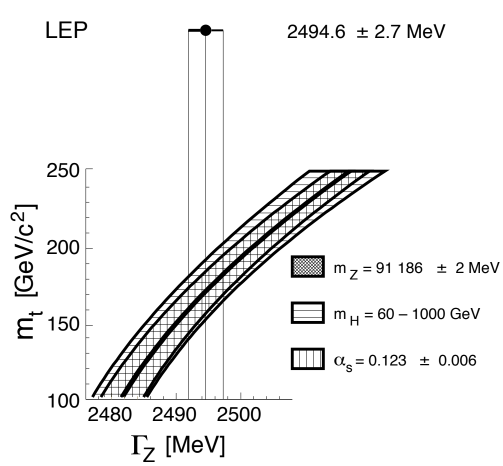
Global fits . . .

to precision EW measurements:

▷ precision improves with time

▷ calculations improve with time



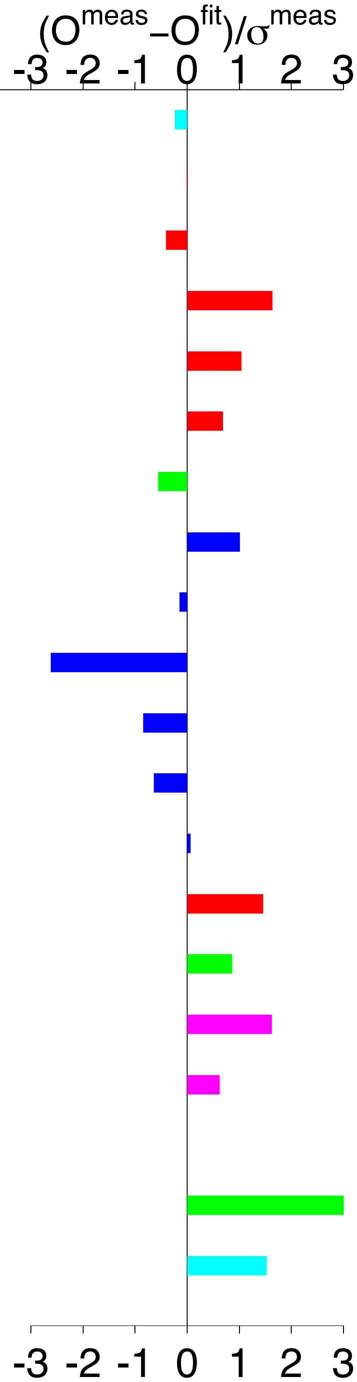


11.94, LEPEWWG: $m_t = 178 \pm 11^{+18}_{-19}$ GeV/c^2

Direct measurements: $m_t = 174.3 \pm 5.1$ GeV/c^2

Summer 2002

	Measurement	Pull
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	-0.24
m_Z [GeV]	91.1875 ± 0.0021	0.00
Γ_Z [GeV]	2.4952 ± 0.0023	-0.41
σ^0_{had} [nb]	41.540 ± 0.037	1.63
R_l	20.767 ± 0.025	1.04
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.68
$A_l(P_\tau)$	0.1465 ± 0.0032	-0.55
R_b	0.21644 ± 0.00065	1.01
R_c	0.1718 ± 0.0031	-0.15
$A^{0,b}_{fb}$	0.0995 ± 0.0017	-2.62
$A^{0,c}_{fb}$	0.0713 ± 0.0036	-0.84
A_b	0.922 ± 0.020	-0.64
A_c	0.670 ± 0.026	0.06
A_l(SLD)	0.1513 ± 0.0021	1.46
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.87
m_W [GeV]	80.449 ± 0.034	1.62
Γ_W [GeV]	2.136 ± 0.069	0.62
m_t [GeV]	174.3 ± 5.1	0.00
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	3.00
Q_W(Cs)	-72.18 ± 0.46	1.52



Parity violation in atoms

Nucleon appears elementary at very low Q^2; effective Lagrangian for nucleon β-decay

$$\mathcal{L}_\beta = -\frac{G_F}{\sqrt{2}}\bar{e}\gamma_\lambda(1-\gamma_5)\nu\,\bar{p}\gamma^\lambda(1-g_A\gamma_5)n$$

$g_A \approx 1.26$: axial charge

NC interactions ($x_W \equiv \sin^2\theta_W$):

$$\begin{aligned}\mathcal{L}ep &= \frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-4x_W-\gamma_5)e\,\bar{p}\gamma^\lambda(1-4x_W-\gamma_5)p\ ,\\ \mathcal{L}en &= \frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-4x_W-\gamma_5)e\,\bar{n}\gamma^\lambda(1-\gamma_5)n\end{aligned}$$

$\triangleright$ Regard nucleus as a noninteracting collection of Z protons and N neutrons $\triangleright$ Perform NR reduction; nucleons contribute coherently to A_eV_N coupling, so dominant P-violating contribution to eN amplitude is

$$\mathcal{M}_{\text{pv}} = \frac{-iG_F}{2\sqrt{2}}Q^W\,\bar{e}\rho_N(\mathbf{r})\gamma_5 e$$

$\rho_N(\mathbf{r})$: nucleon density at e^- coordinate $\mathbf{r}$

$Q^W \equiv Z(1-4x_W) - N$: weak charge

Bennett & Wieman (Boulder) determined weak charge of Cesium by measuring 6S-7S transition polarizability

$$Q_W(\text{Cs}) = -72.06 \pm 0.28\ (\text{expt}) \pm 0.34\ (\text{theory})$$

about 2.5σ above SM prediction

The vacuum energy problem

Higgs potential $V(\varphi^\dagger\varphi) = \mu^2(\varphi^\dagger\varphi) + |\lambda|\,(\varphi^\dagger\varphi)^2$

At the minimum,

$$V(\langle\varphi^\dagger\varphi\rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda|\,v^4}{4} < 0.$$

Identify $M_H^2 = -2\mu^2$

contributes field-independent vacuum energy density

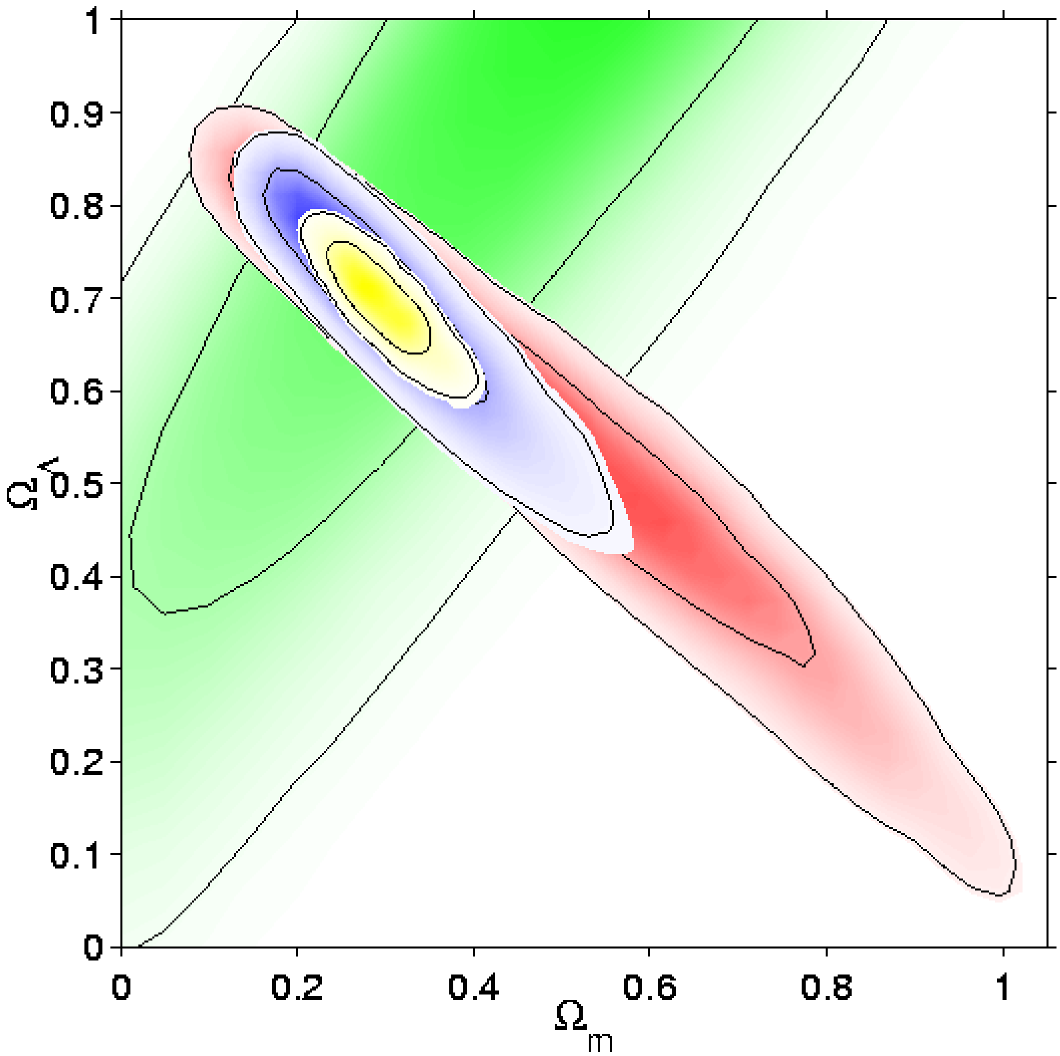
$$\boxed{\varrho_H \equiv \frac{M_H^2 v^2}{8}}$$

Adding vacuum energy density $\varrho_{\mathsf{vac}} \Leftrightarrow$ adding cosmological constant Λ to Einstein's equation

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = \frac{8\pi G_N}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$$\Lambda = \frac{8\pi G_N}{c^4}\varrho_{\mathsf{vac}}$$

observed vacuum energy density $\varrho_{\mathsf{vac}} \lesssim 10^{-46}\ \mathrm{GeV}^4$



Lewis & Bridle, astro-ph/0205436

But $M_H \gtrsim 114\ \mathrm{GeV}/c^2 \Rightarrow$

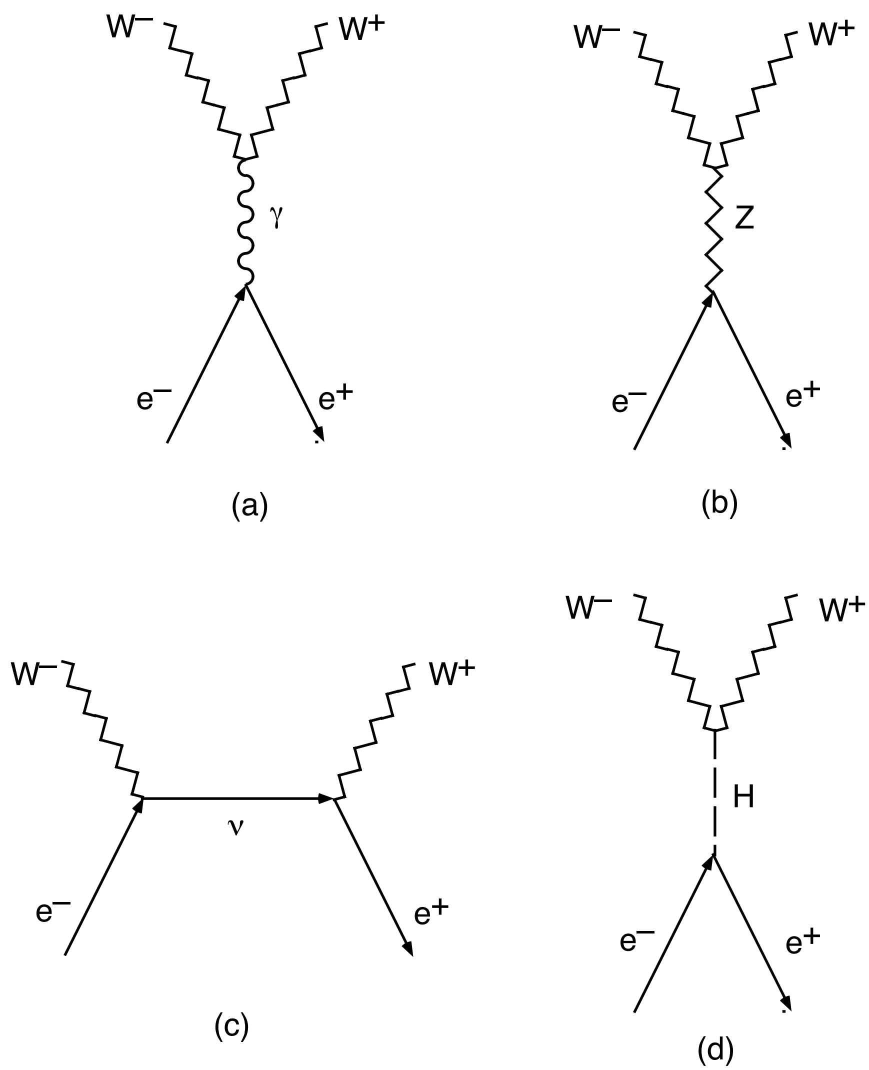
$$\varrho_H \gtrsim 10^8\ \mathrm{GeV}^4$$

MISMATCH BY 54 ORDERS OR MAGNITUDE

Why a Higgs Boson Must Exist

▷ Role in canceling high-energy divergences

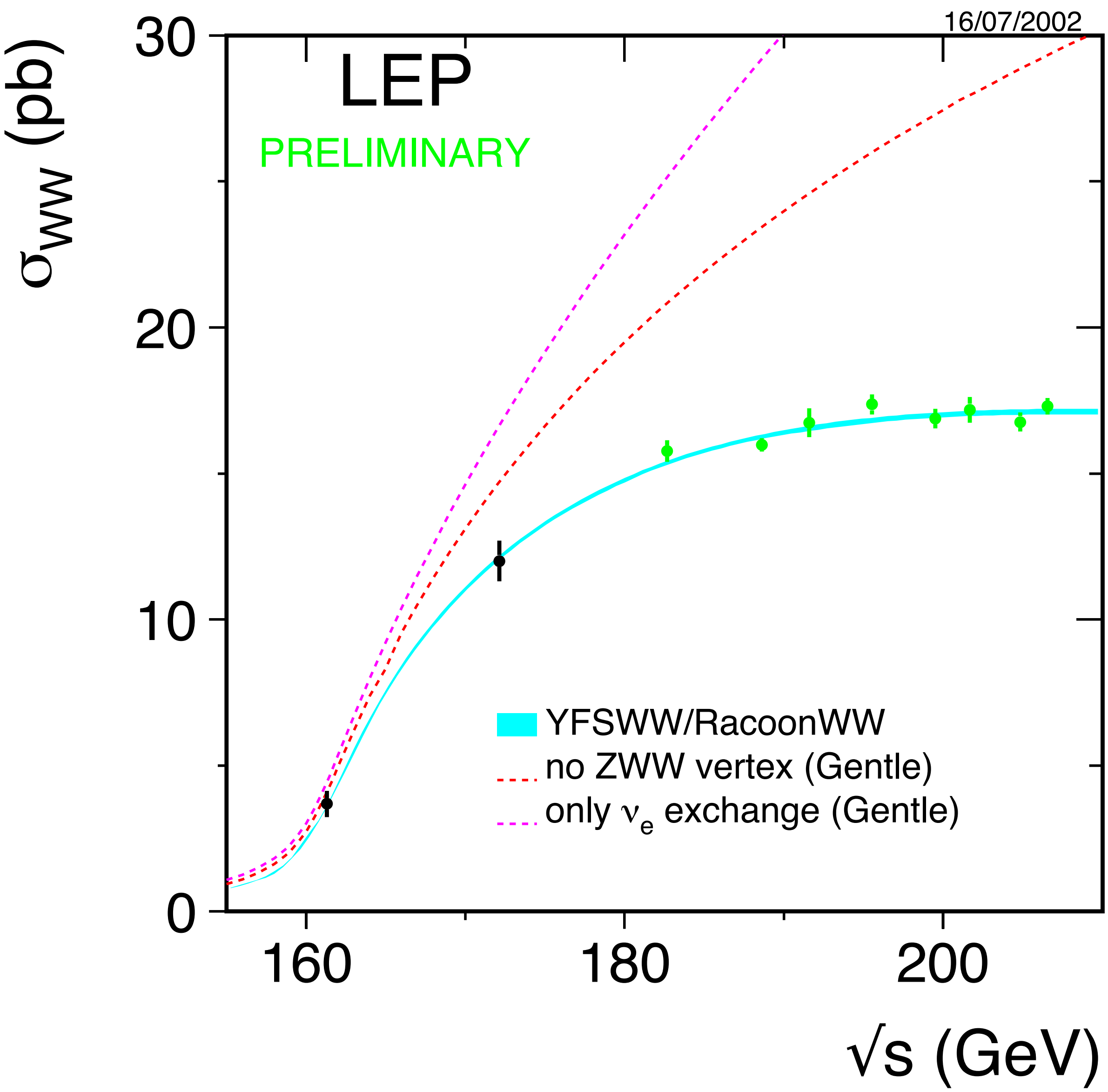
S-matrix analysis of $e^+e^- \to W^+W^-$



$J = 1$ partial-wave amplitudes $\mathcal{M}_\gamma^{(1)}$, $\mathcal{M}_Z^{(1)}$, $\mathcal{M}_\nu^{(1)}$ have—individually—unacceptable high-energy behavior $(\propto s)$

. . . But sum is well-behaved

"Gauge cancellation" observed at LEP2, Tevatron



$J = 0$ amplitude exists because electrons have mass, and can be found in "wrong" helicity state

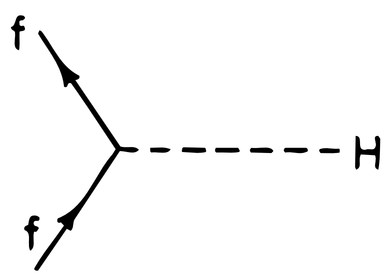
$$\mathcal{M}_{\nu}^{(0)} \propto s^{\frac{1}{2}}$$: unacceptable HE behavior

(no contributions from γ and Z)

This divergence is canceled by the Higgs-boson contribution

$$\boxed{\Rightarrow H e\bar{e} \text{ coupling must be } \propto m_e,}$$

because "wrong-helicity" amplitudes $\propto m_e$



$$\frac{-im_f}{v} = -im_f(G_F\sqrt{2})^{1/2}$$

If the Higgs boson did not exist, *something else* would have to cure divergent behavior

IF gauge symmetry were unbroken . . .

▷ no Higgs boson

▷ no longitudinal gauge bosons

▷ no extreme divergences

▷ no wrong-helicity amplitudes

. . . and no viable low-energy phenomenology

In spontaneously broken theory . . .

▷ gauge structure of couplings eliminates the most severe divergences

▷ lesser—but potentially fatal—divergence arises because the electron has mass

. . . due to the Higgs mechanism

▷ SSB provides its own cure—the Higgs boson

A similar interplay and compensation *must exist* in any acceptable theory

Bounds on M_H

EW theory does not predict Higgs-boson mass

Self-consistency $\Rightarrow$ plausible lower and upper bounds

▷ Conditional *upper bound* from Unitarity

Compute amplitudes $\mathcal{M}$ for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta)$$

Most channels decouple—pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies)—for any M_H.

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0/\sqrt{2} \quad HH/\sqrt{2} \quad HZ_L^0$$

L: longitudinal, $1/\sqrt{2}$ for identical particles

In HE limit,[a] s-wave amplitudes $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2}(a_0) \to \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect the partial-wave unitarity condition $|a_0| \leq 1$

$$\Longrightarrow \quad M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F}\right)^{1/2} = 1 \text{ TeV}/c^2$$

condition for perturbative unitarity

[a] Convenient to calculate using *Goldstone-boson equivalence theorem*, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by $\mathcal{L}_{\text{int}} = -\lambda v h(2w^+w^- + z^2 + h^2) - (\lambda/4)(2w^+w^- + z^2 + h^2)^2$, with $1/v^2 = G_F\sqrt{2}$ and $\lambda = G_F M_H^2/\sqrt{2}$.

▷ If the bound is respected

- ⋆ weak interactions remain weak at all energies
- ⋆ perturbation theory is everywhere reliable

▷ If the bound is violated

- ⋆ perturbation theory breaks down
- ⋆ weak interactions among $W^{\pm}$, Z, and H become strong on the 1-TeV scale

$\Rightarrow$ features of *strong* interactions at GeV energies will characterize *electroweak* gauge boson interactions at TeV energies

Threshold behavior of the pw amplitudes a_{IJ} follows from chiral symmetry

$a_{00} \approx$	$G_F s/8\pi\sqrt{2}$	attractive
$a_{11} \approx$	$G_F s/48\pi\sqrt{2}$	attractive
$a_{20} \approx$	$-G_F s/16\pi\sqrt{2}$	repulsive

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

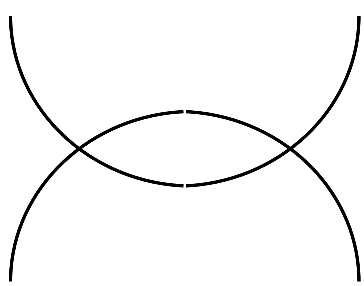
$\triangleright$ Triviality of scalar field theory

Only *noninteracting* scalar field theories make sense on all energy scales

Quantum field theory vacuum is a dielectric medium that screens charge $\Rightarrow$ *effective charge* is a function of the distance or, equivalently, of the energy scale

$$\boxed{\text{running coupling constant}}$$

In $\lambda\phi^4$ theory, it is easy to calculate the variation of the coupling constant λ in perturbation theory by summing bubble graphs



$\lambda(\mu)$ is related to a higher scale Λ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2}\log\left(\Lambda/\mu\right)$$

(Perturbation theory reliable only when λ is small, lattice field theory treats strong-coupling regime)

For stable Higgs potential (*i.e.*, for vacuum energy not to race off to $-\infty$), *require* $\lambda(\Lambda) \geq 0$

Rewrite RGE as an inequality

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log\left(\Lambda/\mu\right) \ .$$

implies an *upper bound*

$$\boxed{\lambda(\mu) \leq 2\pi^2/3\log\left(\Lambda/\mu\right)}$$

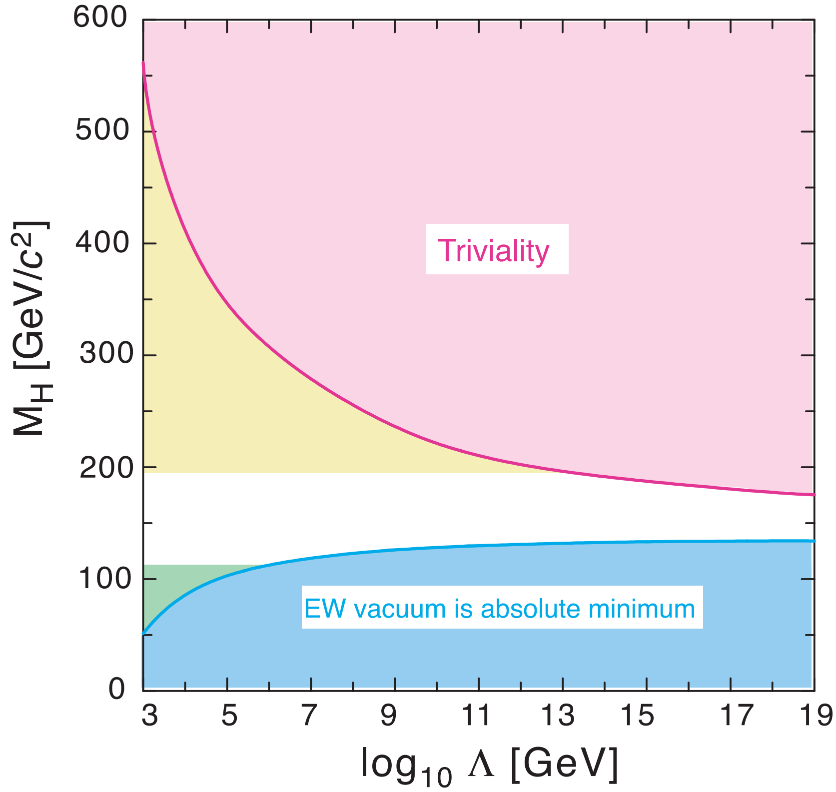
If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit $\Lambda \to \infty$ while holding μ fixed at some reasonable physical scale. In this limit, the bound forces $\lambda(\mu)$ to zero. $\longrightarrow$ free field theory "trivial"

Rewrite as bound on M_H:

$$\Lambda \leq \mu \exp\left(\frac{2\pi^2}{3\lambda(\mu)}\right)$$

Choose $\mu = M_H$, and recall $M_H^2 = 2\lambda(M_H)v^2$

$$\boxed{\Lambda \leq M_H \exp\left(4\pi^2 v^2/3M_H^2\right)}$$



Moral: For any M_H, there is a *maximum energy scale* $\Lambda^\star$ at which the theory ceases to make sense.

The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies

Perturbative analysis breaks down when $M_H \to 1$ TeV/c^2 and interactions become strong

Lattice analyses $\Longrightarrow M_H \lesssim 710 \pm 60$ GeV/c^2 if theory describes physics to a few percent up to a few TeV

If $M_H \to 1$ TeV EW theory lives on brink of instability

▷ *Lower bound* by requiring EWSB vacuum $V(v) < V(0)$

Requiring that $\langle\phi\rangle_0 \neq 0$ be an absolute minimum of the one-loop potential up to a scale Λ yields the vacuum-stability condition

$$M_H^2 > \frac{3G_F\sqrt{2}}{8\pi^2}(2M_W^4 + M_Z^4 - 4m_t^4)\log(\Lambda^2/v^2)$$

... for $m_t \lesssim M_W$

(No illuminating analytic form for heavy m_t)

If the Higgs boson is relatively light—which would itself require explanation—then the theory can be self-consistent up to very high energies

If EW theory is to make sense all the way up to a unification scale $\Lambda^\star = 10^{16}$ GeV, then

$$134\ \mathrm{GeV}/c^2 \lesssim M_H \lesssim 177\ \mathrm{GeV}/c^2$$

Higgs-Boson Properties

$$\Gamma(H \to f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

$\propto M_H$ in the limit of large Higgs mass

$$\Gamma(H \to W^+W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}}(1-x)^{1/2}(4-4x+3x^2)$$

$$x \equiv 4M_W^2/M_H^2$$

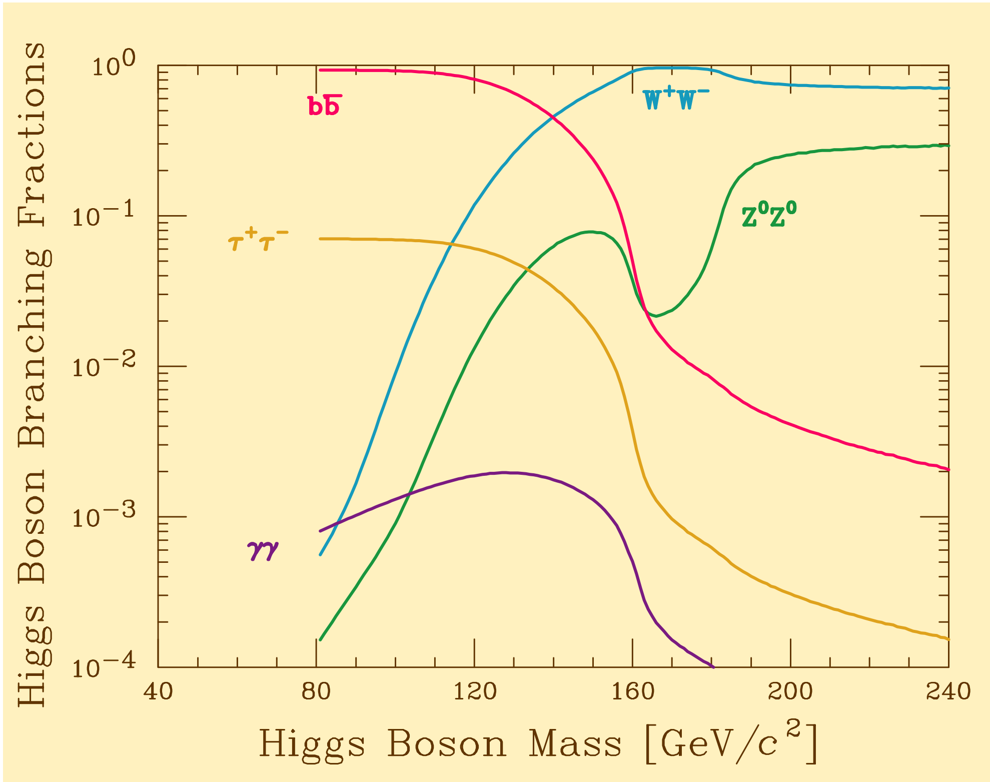
$$\Gamma(H \to Z^0Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}}(1-x')^{1/2}(4-4x'+3x'^2)$$

$$x' \equiv 4M_Z^2/M_H^2$$

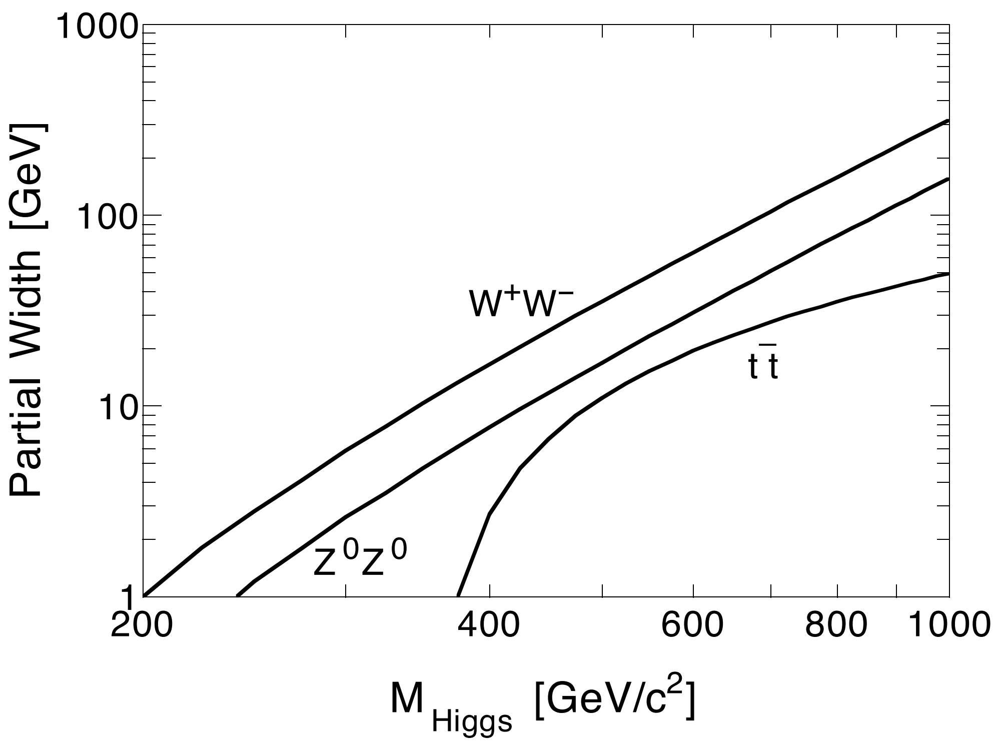
asymptotically $\propto M_H^3$ and $\frac{1}{2}M_H^3$, respectively
($\frac{1}{2}$ from weak isospin)

$2x^2$ and $2x'^2$ terms $\Leftrightarrow$ decays into transversely polarized gauge bosons

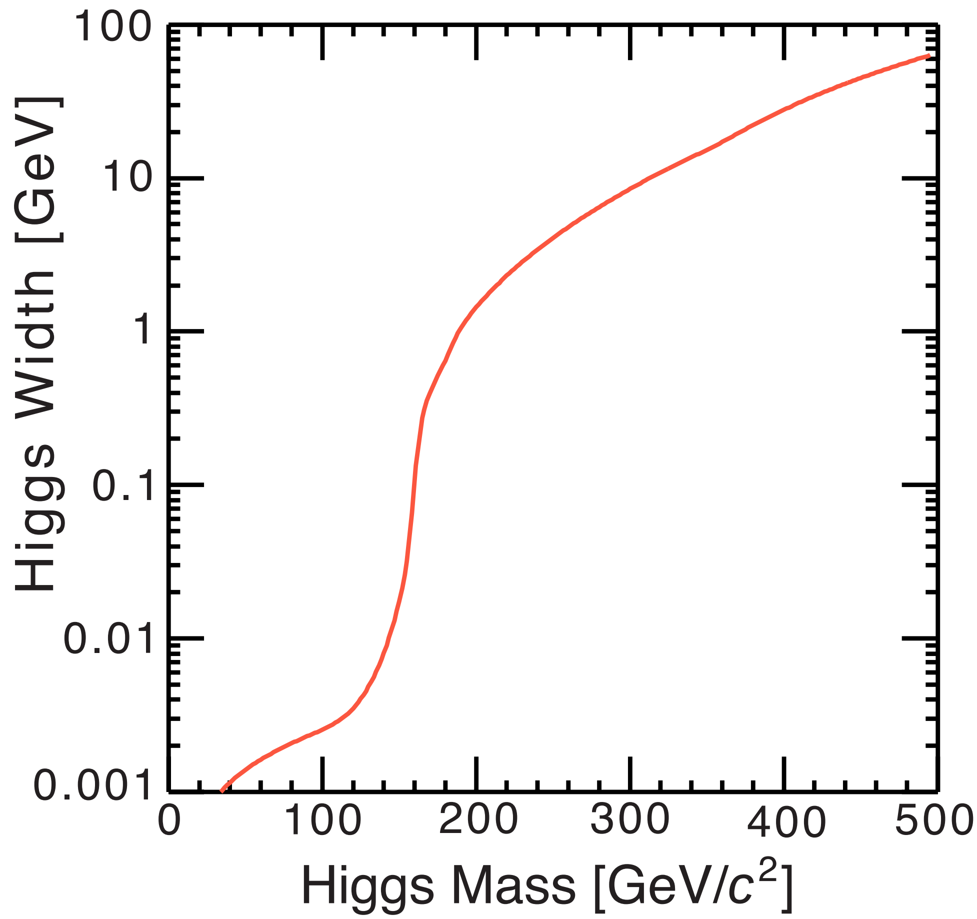
Dominant decays for large M_H into pairs of longitudinally polarized weak bosons



Higgs Boson Branching Fractions
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$b\bar{b}$
W^+W^-
Z^0Z^0
$\tau^+\tau^-$
$\gamma\gamma$
40
80
120
160
200
240
Higgs Boson Mass [GeV/c^2]



Partial Width [GeV]
1000
100
10
1
W^+W^-
Z^0Z^0
$t\bar{t}$
200
400
600
800
1000
M_{Higgs} [GeV/c^2]



Below W^+W^- threshold, $\Gamma_H \lesssim 1$ GeV

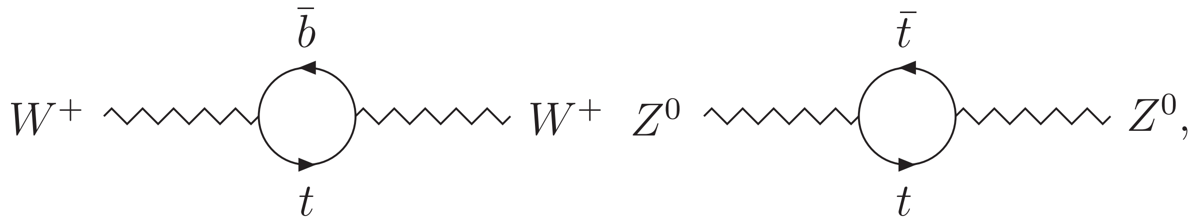
Far above W^+W^- threshold, $\Gamma_H \propto M_H^3$

For $M_H \to 1$ TeV/c^2, Higgs boson is an *ephemeron*, with a perturbative width approaching its mass.

Clues to the Higgs-boson mass

Sensitivity of EW observables to m_t gave early indications for massive top

quantum corrections to SM predictions for M_W and M_Z arise from different quark loops



...alter link between the M_W and M_Z:

$$M_W^2 = M_Z^2 \left(1 - \sin^2\theta_W\right)(1 + \Delta\rho)$$

where $\Delta\rho \approx \Delta\rho^{(\text{quarks})} = 3G_F m_t^2 / 8\pi^2\sqrt{2}$

strong dependence on m_t^2 accounts for precision of m_t estimates derived from EW observables

$\boxed{m_t \text{ known to } \pm 3\% \text{ from Tevatron } \ldots}$

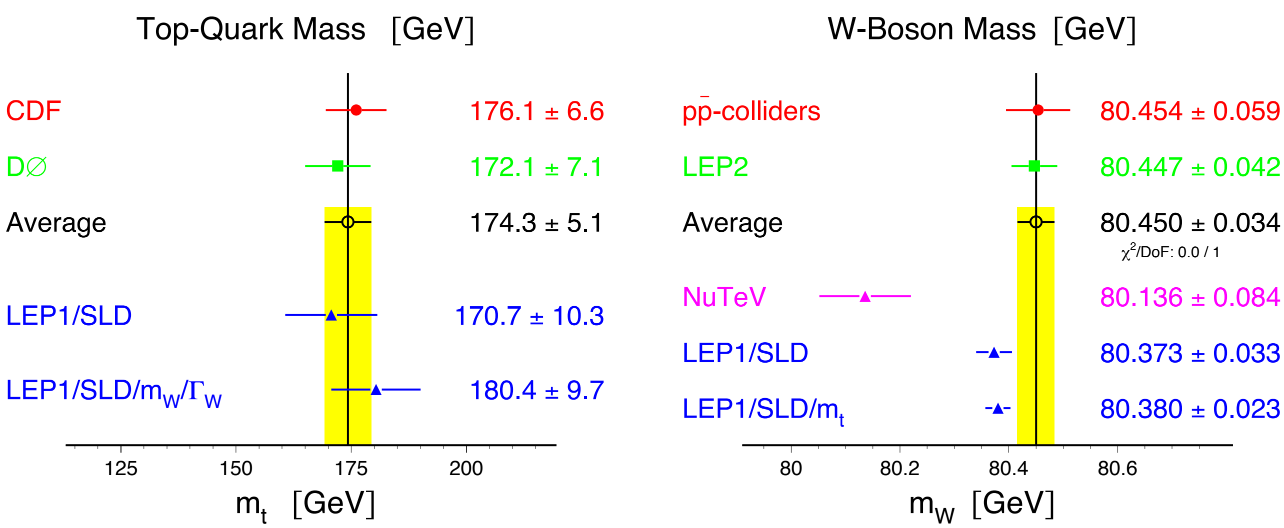
$\Longrightarrow$ look beyond the quark loops to next most important quantum corrections:

$\boxed{\text{Higgs-boson effects}}$

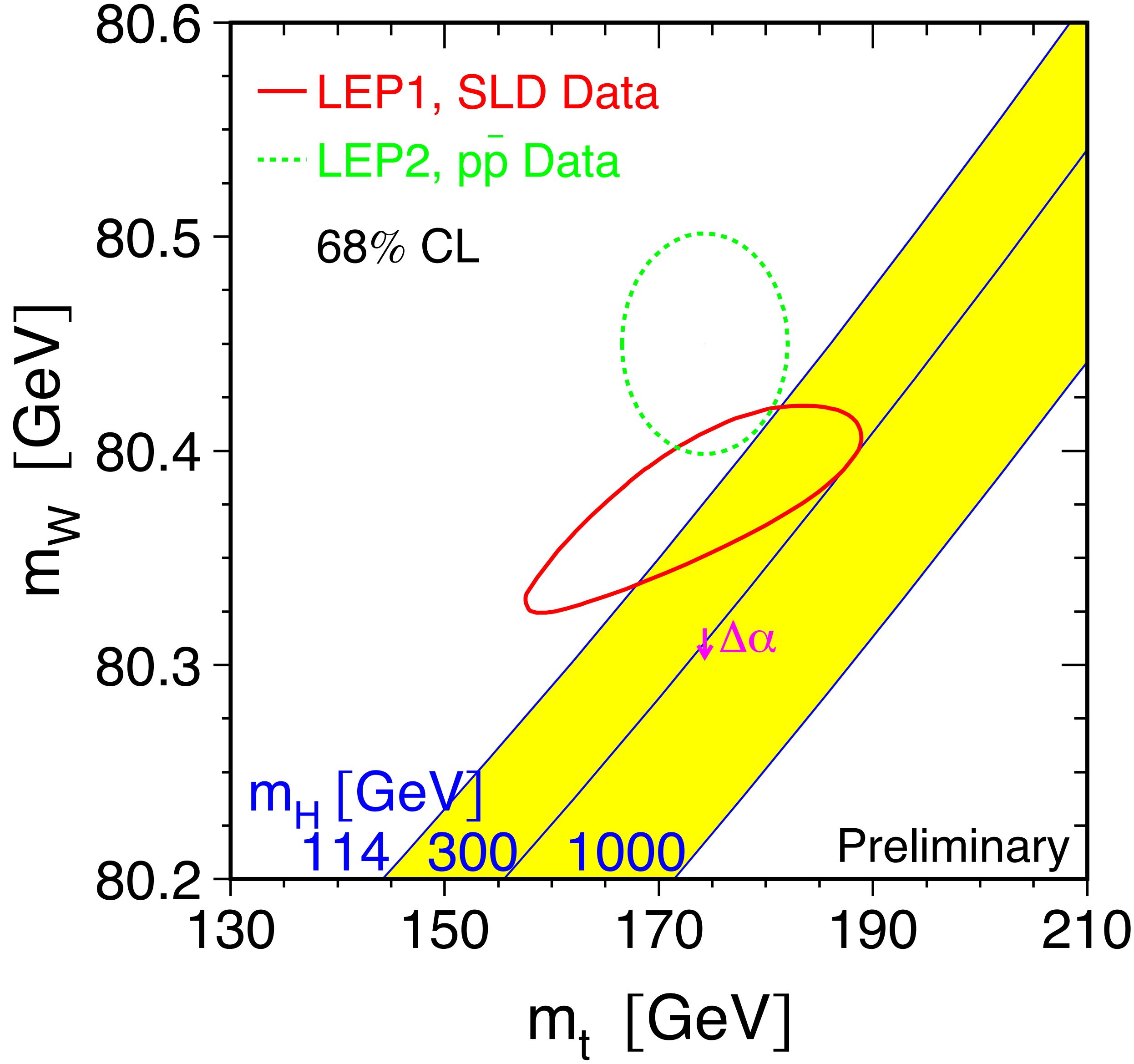
H quantum corrections smaller than t corrections, exhibit more subtle dependence on M_H than the m_t^2 dependence of the top-quark corrections

$$\Delta\rho^{(\mathrm{Higgs})} = \mathcal{C} \cdot \ln\left(\frac{M_H}{v}\right)$$

M_Z known to 23 ppm, m_t and M_W well measured



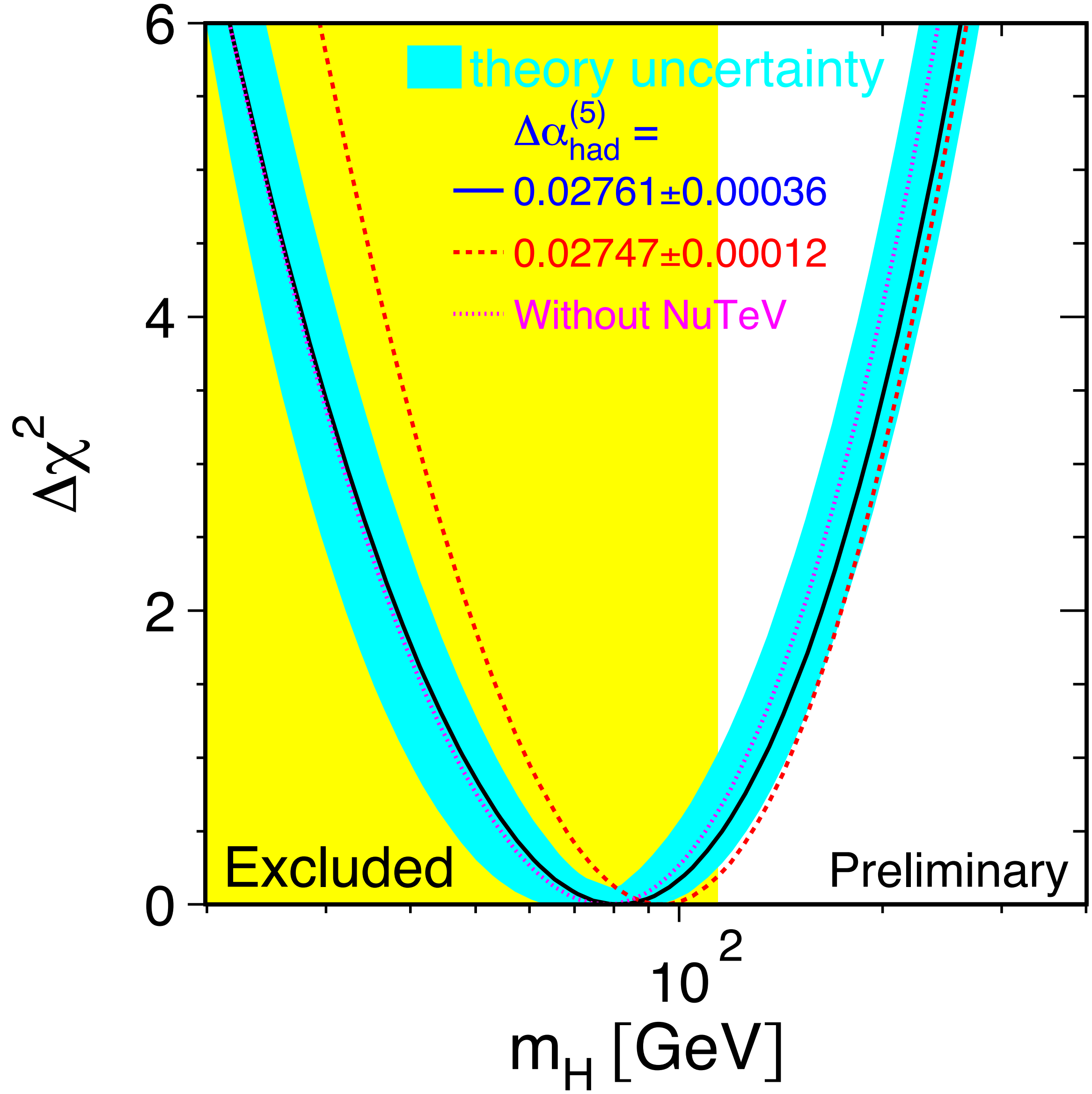
so examine dependence of M_W upon m_t and M_H



Direct, indirect determinations agree reasonably
Both favor a light Higgs boson,

within framework of SM analysis.

Fit to a universe of data



Within SM, LEPEWWG deduce a 95% CL upper limit, $M_H \lesssim 193$ GeV/c^2.

Direct searches at LEP $\Rightarrow M_H > 114.4$ GeV/c^2, excluding much of the favored region

either the Higgs boson is just around the corner, or SM analysis is misleading

Things will soon be popping!

$\boxed{\text{Expect progress from } M_W\text{-}m_t\text{-}M_H \text{ correlation}}$

▷ Tevatron and LHC measurements will determine m_t within 1 or 2 GeV/c^2

▷ . . . and improve δM_W to about 15 MeV/c^2

▷ As the Tevatron's integrated luminosity grows past 10 fb^{-1}, CDF and DØ will begin to explore the region of M_H not excluded by LEP

▷ ATLAS and CMS will carry on the exploration of the Higgs sector at the LHC

Assessment

25 YEARS OF CONFIRMATIONS OF
$SU(2)_L \otimes U(1)_Y$

- $\star$ neutral currents
- $\star$ $W^{\pm}$, Z^0
- $\star$ charm

($+$ experimental guidance)

- $\star$ τ, ν_τ
- $\star$ b, t

$+$ experimental surprises

- $\star$ narrowness of ψ, ψ'
- $\star$ long B lifetime
- $\star$ large B^0–$\bar{B}^0$ mixing
- $\star$ heavy top
- $\star$ neutrino oscillations

10 YEARS OF PRECISION MEASUREMENTS...
...FIND NO SIGNIFICANT DEVIATIONS

QUANTUM CORRECTIONS TESTED AT $\pm 10^{-3}$

NO "NEW" PHYSICS ...YET!

Theory tested at distances
from 10^{-17} cm
to $\sim 10^{22}$ cm

origin Coulomb's law (tabletop experiments)

smaller $\begin{cases} \text{Atomic physics} \longrightarrow \text{QED} \\ \text{high-energy experiments} \longrightarrow \text{EW theory} \end{cases}$

larger $M_\gamma \approx 0$ in planetary ... measurements

IS EW THEORY TRUE ?
COMPLETE ??

Higgs search in e^+e^- collisions

▷ $\sigma(e^+e^- \to H \to \text{all})$ is *minute*, $\propto m_e^2$

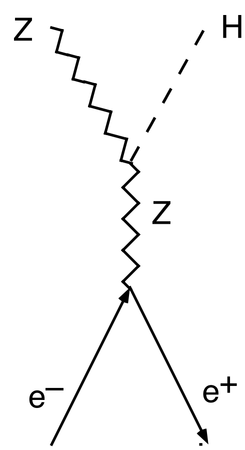
Even narrowness of low-mass H is not enough to make it visible . . . Sets aside a traditional strength of e^+e^- machines—*pole physics*

▷ *Most promising:*

associated production

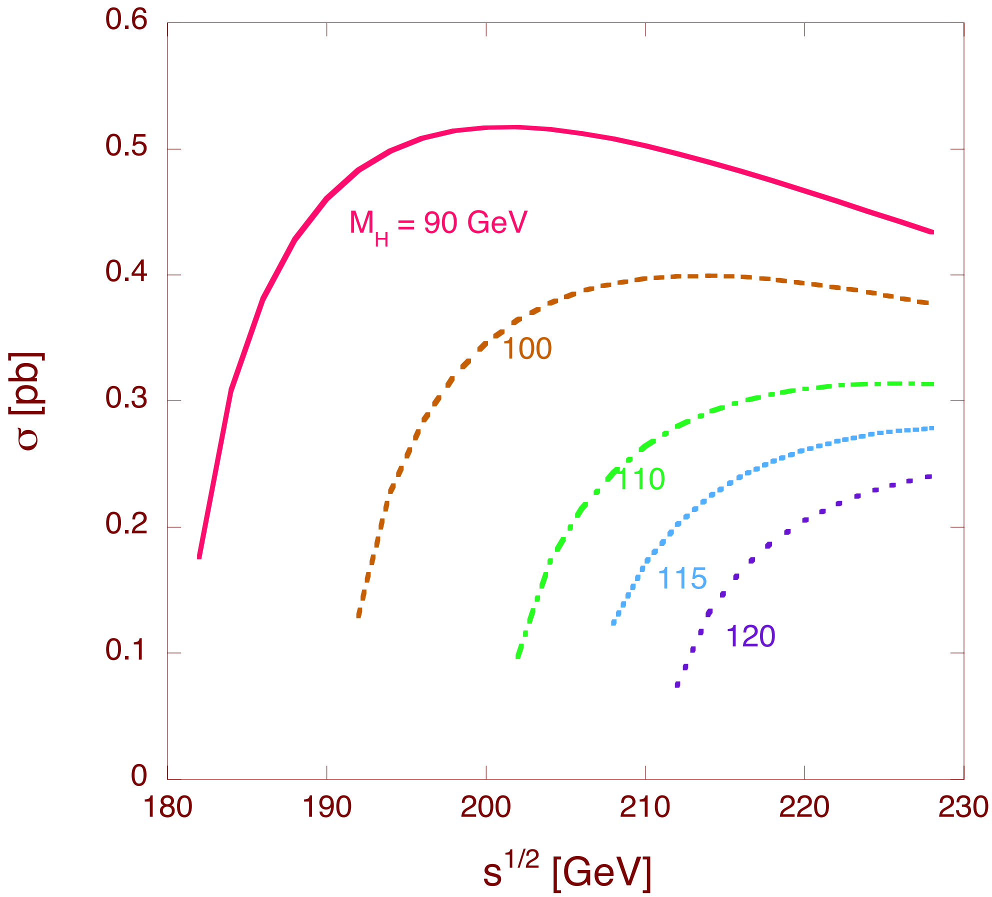
$$e^+e^- \to HZ$$

has no small couplings



$$\boxed{\sigma = \frac{\pi\alpha^2}{24\sqrt{s}} \frac{K(K^2 + 3M_Z^2)[1 + (1 - 4x_W)^2]}{(s - M_Z^2)^2 \quad x_W^2(1 - x_W)^2}}$$

K: c.m. momentum of H $\qquad x_W \equiv \sin^2\theta_W$



$+$ important effect of ISR

LEP 2: sensitive nearly to kinematical limit

$$\boxed{M_H^{\max} = \sqrt{s} - M_Z}$$

LC: sensitive for $M_H \lesssim 0.7\sqrt{s}$

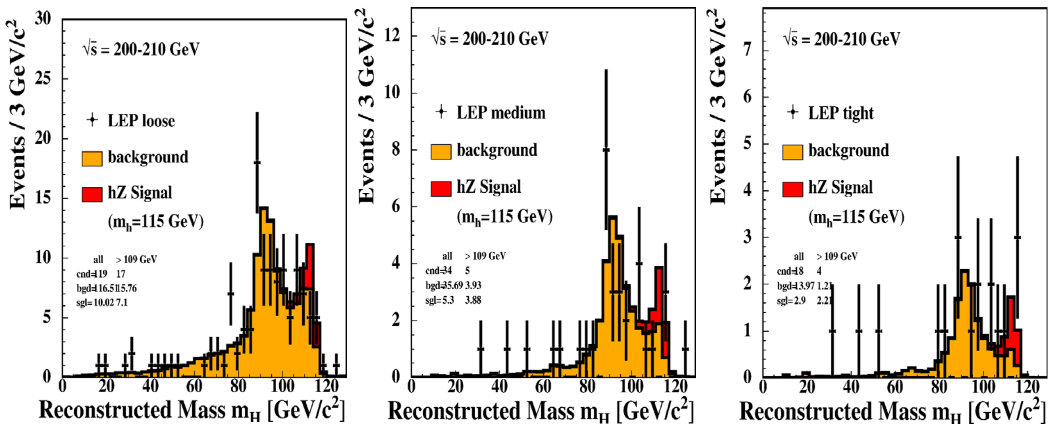
& measure excitation curve to determine

$$\delta M_H \approx 60 \text{ MeV} \sqrt{100 \text{ fb}^{-1}/\mathcal{L}} \text{ for } M_H = 100 \text{ GeV}$$

Direct Search for the SM Higgs Boson

LEP-2: mainly $e^+e^- \to Z\,H \to f\,f\,b\,b$

Selection (mass independent) and mass reconstruction



Full statistical analysis for search based on:

Global discriminating variable and reconstructed mass

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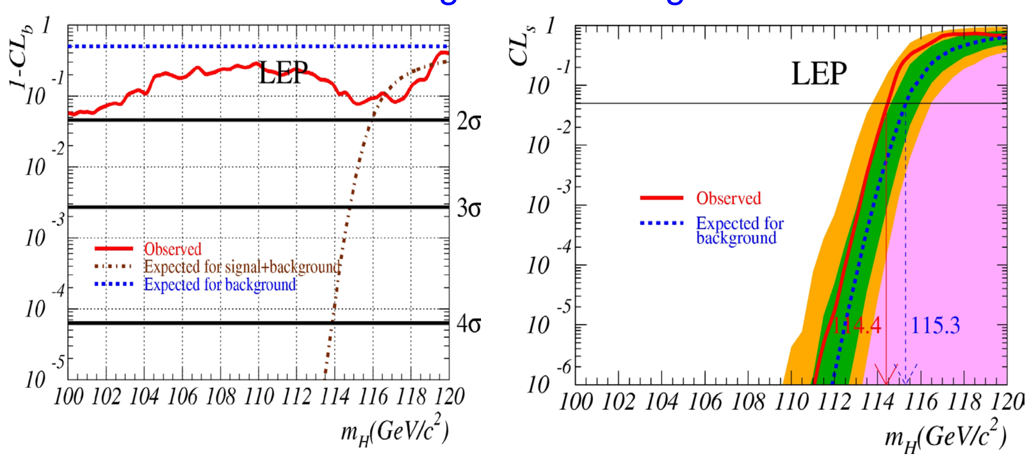
Direct Search for the SM Higgs Boson

Confidence level for background and signal:



1.7 σ excess (P=8%) over expected SM background

One experiment (ALEPH, 2.8-3.0 σ), one channel (qqbb)

Final LEP-2 SM Higgs-boson mass limit (95% C.L.):

$M_{Higgs} > 114.4$ GeV (expected limit: 115.3 GeV)

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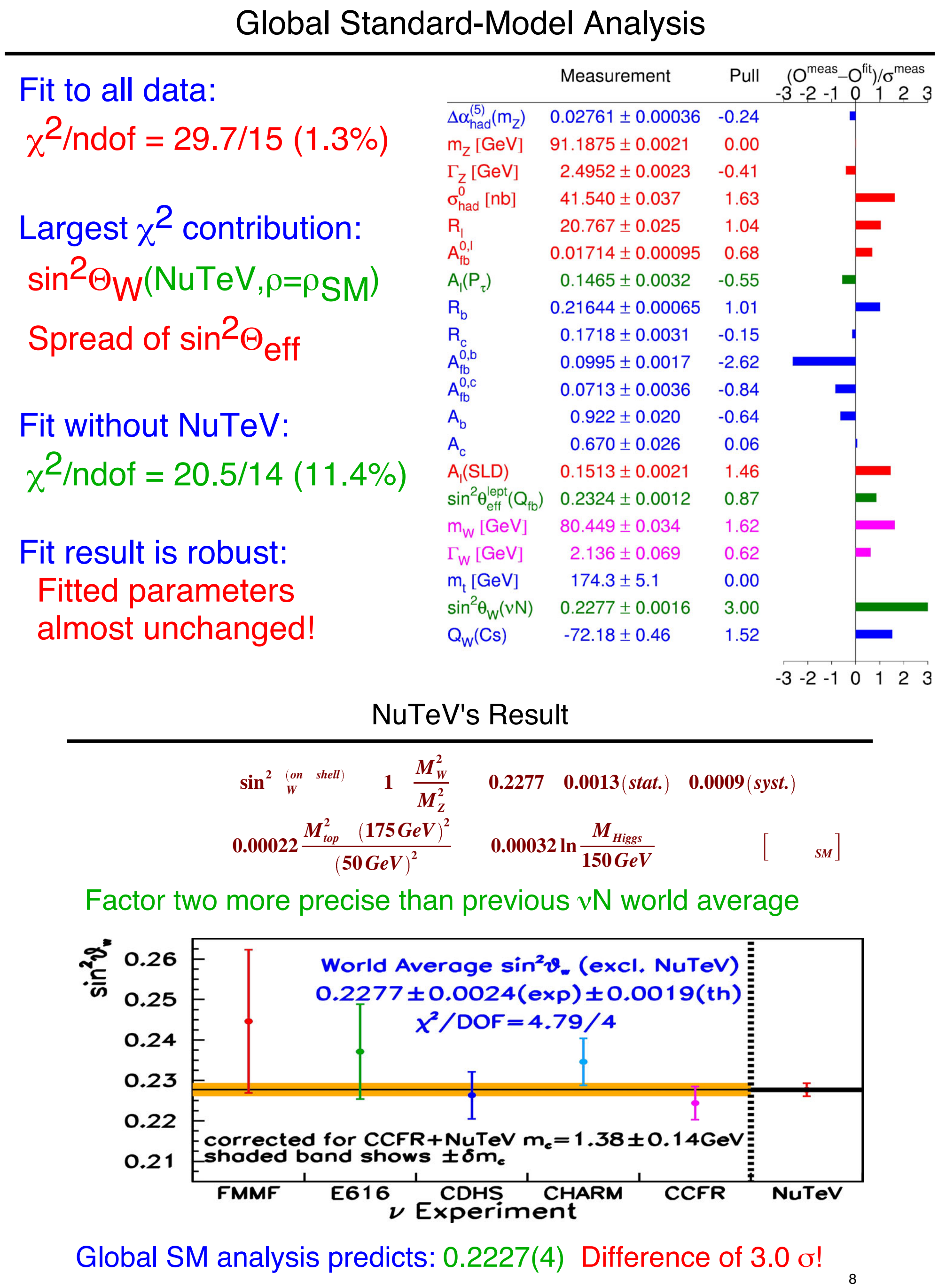
Martin Grunewald, ICHEP02

Can we believe clues for M_H?

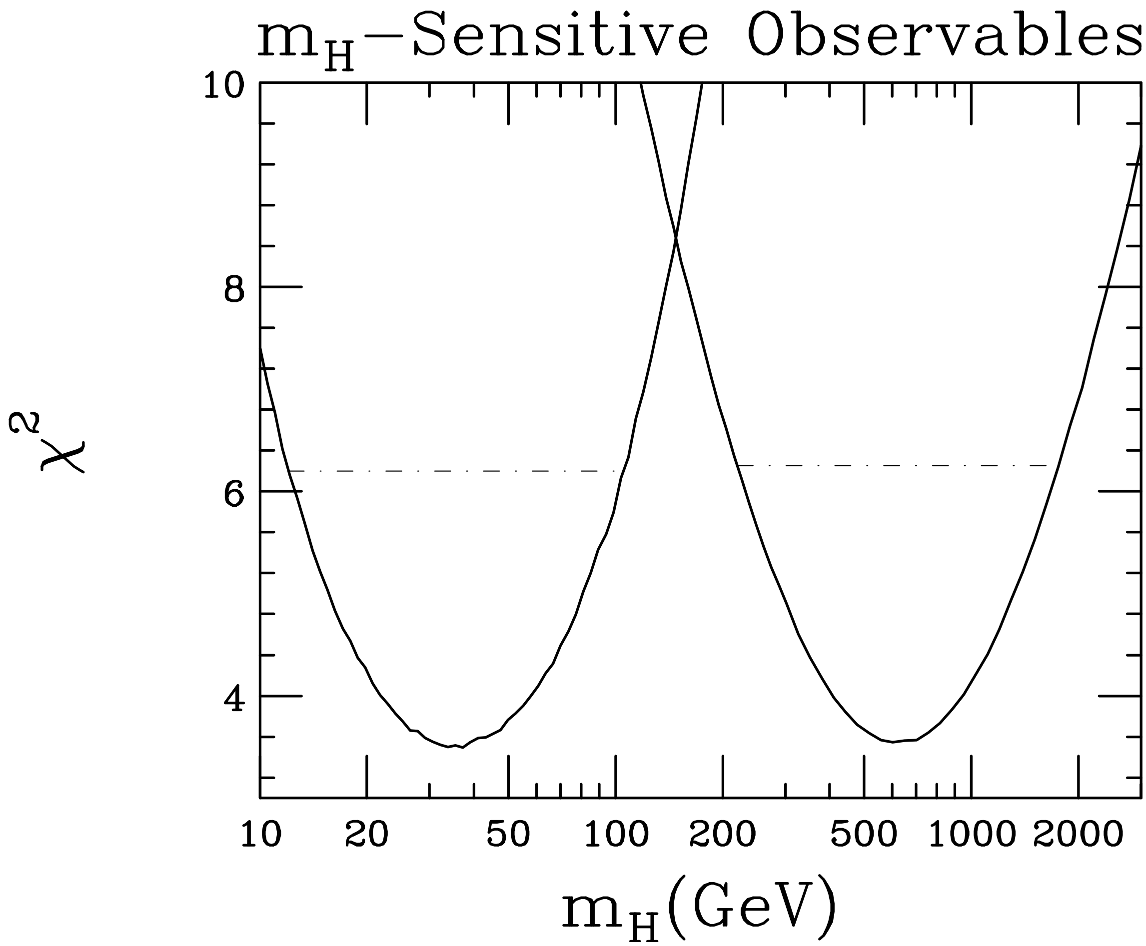
Establishment view



Martin Grunewald, ICHEP02

Can we believe clues for M_H?

Do precision measurements imply new physics?



M. S. Chanowitz, hep-ph/0207123

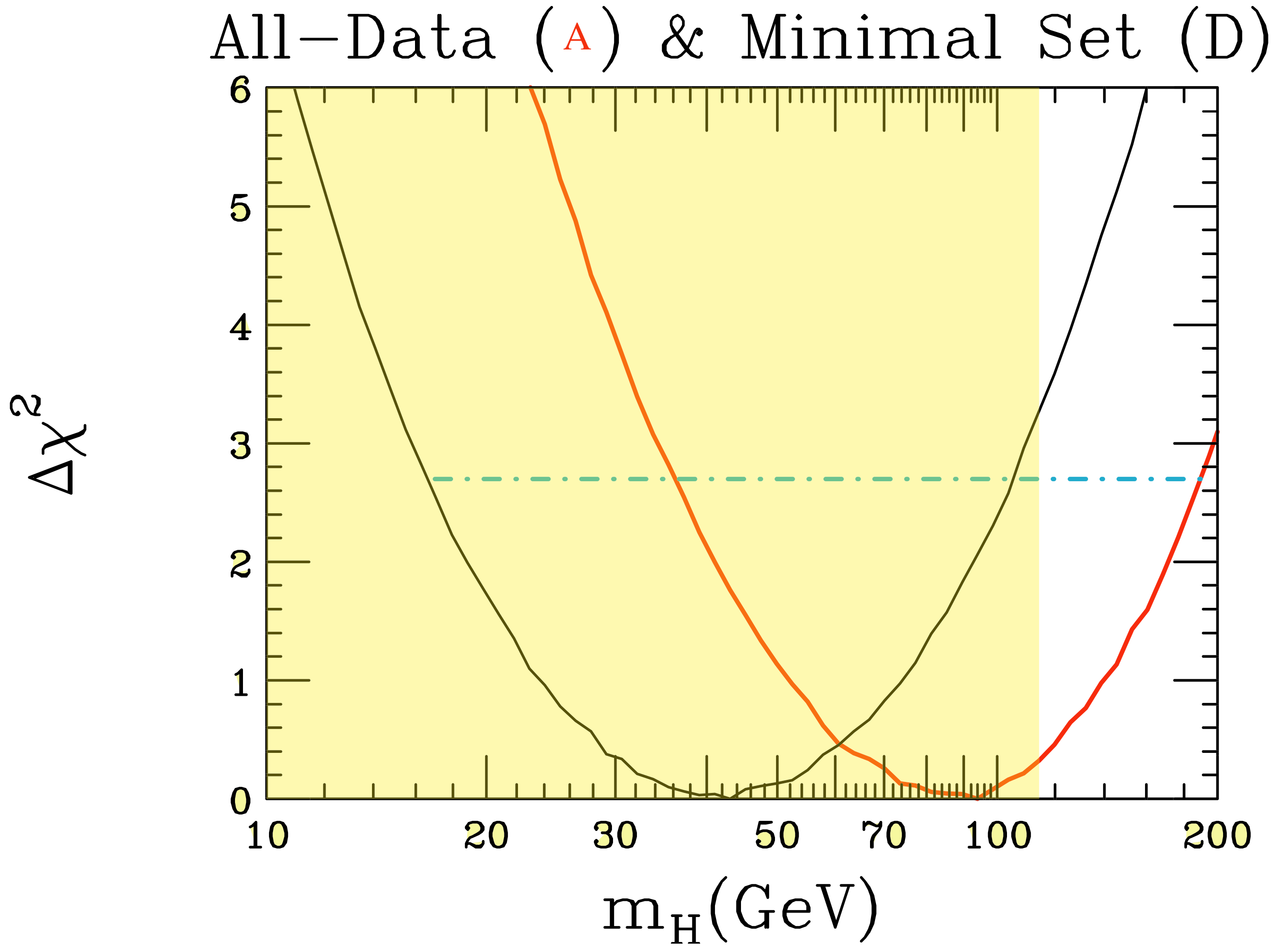
Left curve: M_H-sensitive observables

$$A_{\mathrm{LR}},\ A_{\mathrm{FB}}^{\ell},\ A_{e,\tau},\ M_W,\ \Gamma_Z,\ R_\ell$$

Right curve: remaining M_H-sensitive observables

$$A_{\mathrm{FB}}^{b},\ A_{\mathrm{FB}}^{c},\ Q_{\mathrm{FB}},\ \text{and}\ \sin^2\theta_W^{\mathrm{OS}}(\overset{(-)}{\nu}N)$$

More food for thought:



Chanowitz

Right curve: all data

Left curve: omit $\sin^2\theta_W(\nu N)$, hadronic asymmetries

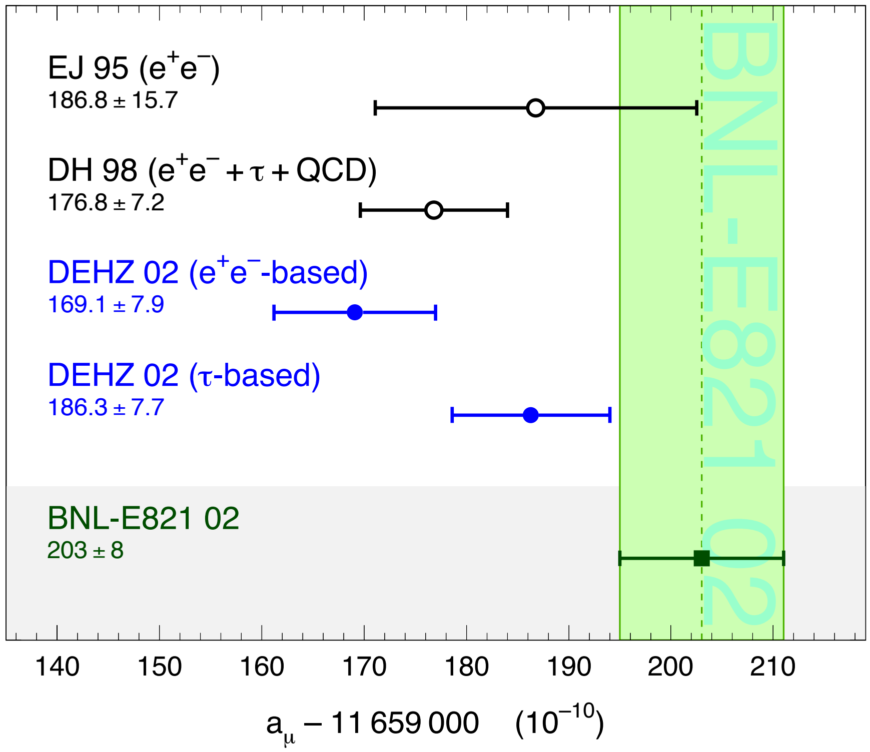
$\Longrightarrow$ *outliers make inferred M_H consistent with searches*

Excellent introduction to SM analyses: J. L. Rosner, "The SM in 2001" (Scottish Summer School, hep-ph/0108195)

+ guide to literature on SM and beyond (hep-ph/0206176)

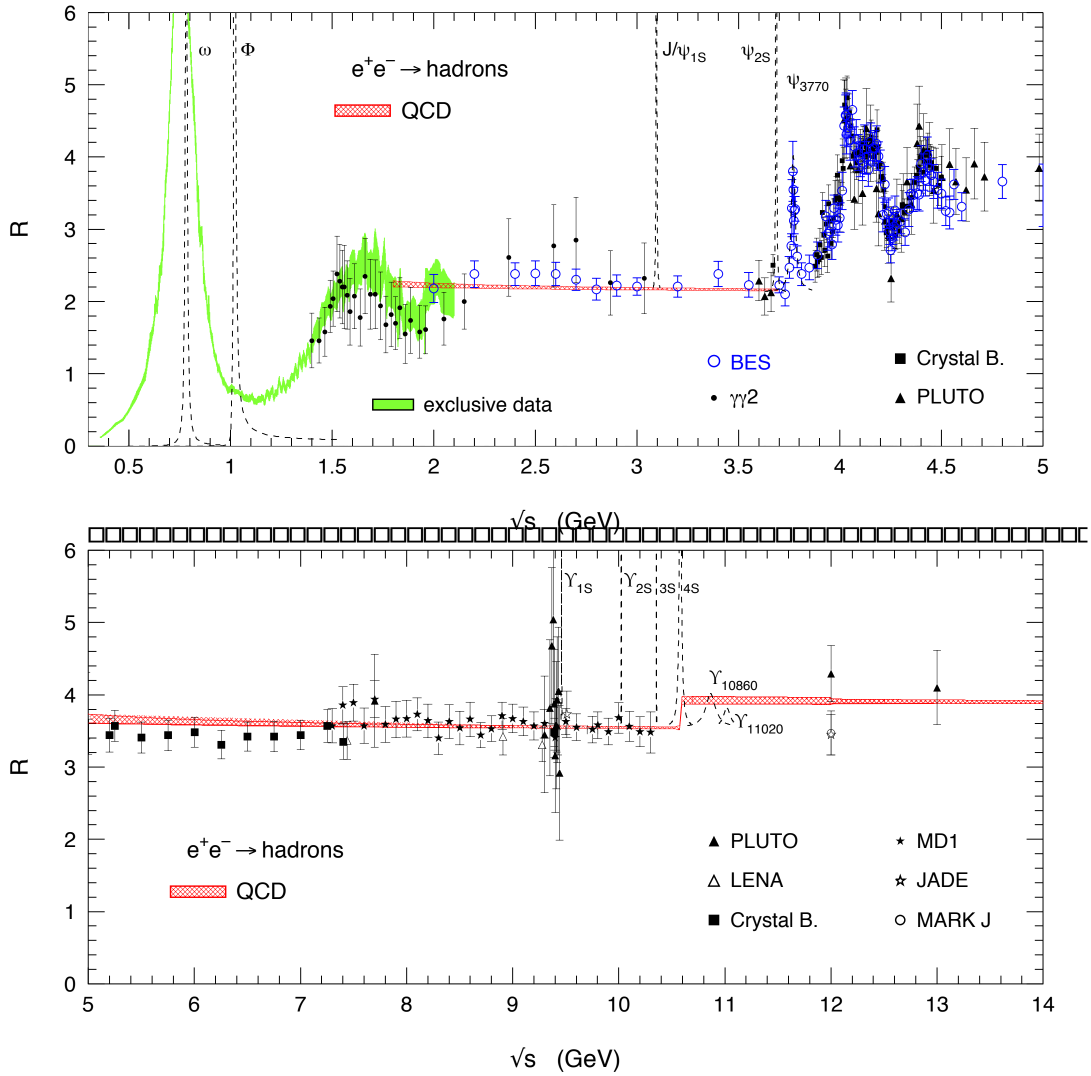
Chinks in the SM armor?

Examine $(g-2)_\mu$



- ▷ $(g-2)_\mu$ Measurement: Bennett, *et al.*, hep-ex/0208001
- ▷ $(g-2)_\mu$ theory: K. Melnikov, talk at 2002 SLAC Summer Institute
- ▷ Vacuum polarization contributions: Davier, *et al.*, hep-ph/0208177
- ▷ Have they measured $(g-2)_\mu$?: Feng–Matchev–Shadmi, hep-ph/0208106
- ▷ Classic $(g-2)_\mu$ experiments at CERN: Combley–Farley–Picasso, *Phys. Rep.* **68**, 93 (1981)

Raw materials for vacuum polarization



Davier, *et al.* (hep-ph/0208177)

EWSB: another path?

Modeled EWSB on Ginzburg–Landau description of SC phase transition

had to introduce new, elementary scalars

GL is not the last word on superconductivity: *dynamical* Bardeen–Cooper–Schrieffer theory

The elementary fermions—electrons—and gauge interactions—QED—needed to generate the scalar bound states are already present in the case of superconductivity. Could a scheme of similar economy account for EWSB?

$\boxed{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y + \text{massless } u \text{ and } d}$

Treat $SU(2)_L \otimes U(1)_Y$ as perturbation

$m_u = m_d = 0$: QCD has exact $SU(2)_L \otimes SU(2)_R$ chiral symmetry. At an energy scale $\sim \Lambda_{\text{QCD}}$, strong interactions become strong, fermion condensates appear, and $SU(2)_L \otimes SU(2)_R \to SU(2)_V$

$\Longrightarrow$ 3 Goldstone bosons, one for each broken generator: 3 massless pions (Nambu)

Broken generators: 3 axial currents; couplings to π measured by pion decay constant f_π

Turn on $SU(2)_L \otimes U(1)_Y$: EW gauge bosons couple to axial currents, acquire masses of order $\sim g f_\pi$

$$\mathcal{M}^2 = \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & gg' \\ 0 & 0 & gg' & g'^2 \end{pmatrix} \frac{f_\pi^2}{4},$$

$(W^+, W^-, W_3, \mathcal{A})$

same structure as standard EW theory. Diagonalize: $M_W^2 = g^2 f_\pi^2/4$, $M_Z^2 = (g^2 + g'^2) f_\pi^2/4$, $M_A^2 = 0$, so

$$\frac{M_Z^2}{M_W^2} = \frac{(g^2 + g'^2)}{g^2} = \frac{1}{\cos^2\theta_W}$$

Massless pions disappear from physical spectrum, to become longitudinal components of weak bosons

$$\boxed{M_W \approx 30\ \mathrm{MeV}/c^2}$$

The EW scale and beyond

EWSB scale, $v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246$ GeV, sets

$$M_W^2 = g^2v^2/2 \qquad M_Z^2 = M_W^2/\cos^2\theta_W$$

But it is not the only scale of physical interest

quasi-certain: $M_{\text{Planck}} = 1.22 \times 10^{19}$ GeV

probable: $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ unification scale $\sim 10^{15-16}$ GeV

somewhere: flavor scale

How to keep the distant scales from mixing in the face of quantum corrections?

OR

How to stabilize the mass of the Higgs boson on the electroweak scale?

OR

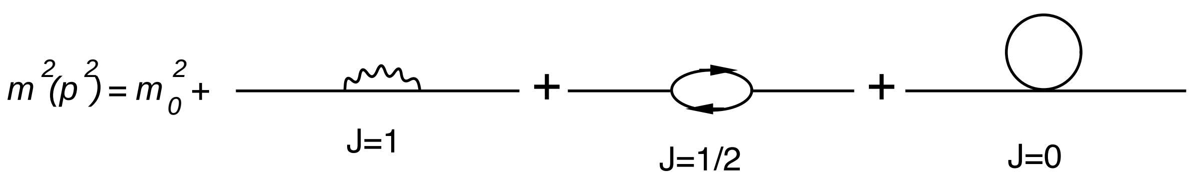
Why is the electroweak scale small?

Higgs potential $V(\phi^\dagger\phi) = \mu^2(\phi^\dagger\phi) + |\lambda|\,(\phi^\dagger\phi)^2$

$\mu^2 < 0$: $SU(2)_L \otimes U(1)_Y \to U(1)_{\text{em}}$, as

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ \sqrt{-\mu^2/2|\lambda|} \end{pmatrix} \equiv \begin{pmatrix} 0 \\ \underbrace{(G_F\sqrt{8})^{-1/2}}_{175\ \text{GeV}} \end{pmatrix}$$

Beyond classical approximation, quantum corrections to scalar mass parameters:



Loop integrals are potentially divergent.

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \cdots$$

Λ: reference scale at which m^2 is known

g: coupling constant of the theory

C: coefficient calculable in specific theory

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \cdots$$

For the mass shifts induced by radiative corrections to remain under control (not greatly exceed the value measured on the laboratory scale), *either*

▷ Λ must be small, *or*

▷ new physics must intervene to cut off the integral

BUT natural reference scale for Λ is

$$\Lambda \sim M_{\text{Planck}} = \left(\frac{\hbar c}{G_{\text{Newton}}}\right)^{1/2} \approx 1.22 \times 10^{19} \text{ GeV}$$

for $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

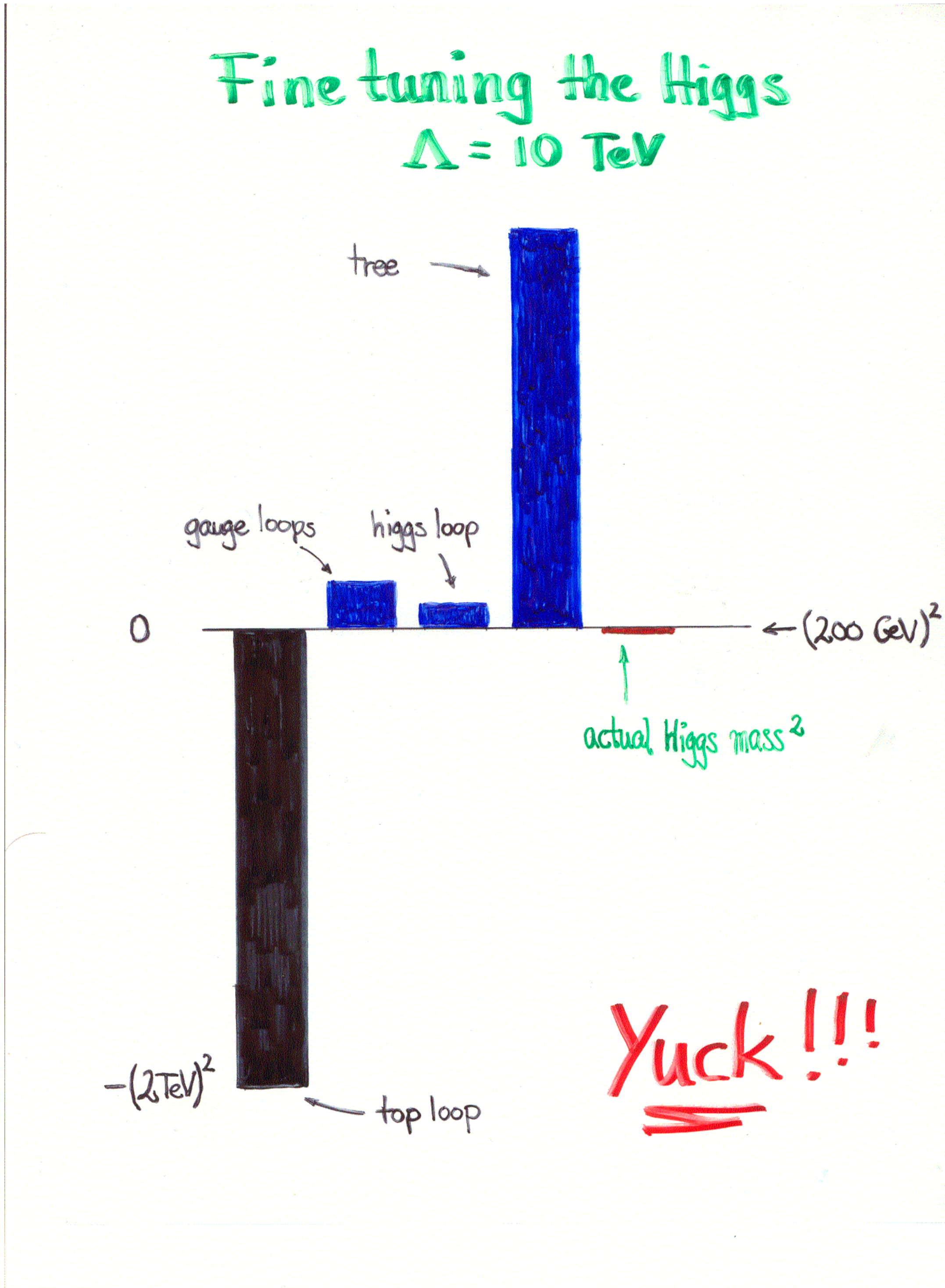
OR

$$\Lambda \sim M_U \approx 10^{15}\text{-}10^{16} \text{ GeV}$$

for unified theory

Both $\gg v/\sqrt{2} \approx 175$ GeV $\Longrightarrow$

$\boxed{\text{New Physics at } E \lesssim 1 \text{ TeV}}$



Martin Schmaltz, ICHEP02

Only a few distinct scenarios . . .

- Supersymmetry: balance contributions of fermion loops (-1) and boson loops $(+1)$

  Exact supersymmetry,

$$\sum_{i=\substack{\text{fermions}\\+\text{bosons}}} C_i \int dk^2 = 0$$

  Broken supersymmetry, shifts acceptably small if superpartner mass splittings are not too large
  $g^2 \Delta M^2$ "small enough" $\Rightarrow \widetilde{M} \lesssim 1$ TeV/c^2

- Composite scalars (technicolor): New physics arises on scale of composite Higgs-boson binding,

$$\Lambda_{\mathsf{TC}} \simeq O(1 \text{ TeV})$$

  "Form factor" cuts effective range of integration

- Strongly interacting gauge sector: WW resonances, multiple W production, probably scalar bound state "quasiHiggs" with $M < 1$ TeV

- Extra spacetime dimensions: pseudo-Nambu–Goldstone bosons, extra particles to cancel integrand, . . .

Why Supersymmetry?

▷ Closely approximates the standard model

▷ Maximal (unique) extension of Poincaré invariance

▷ A path to the incorporation of gravity: local supersymmetry $\longrightarrow$ supergravity

▷ Solution to the naturalness problem: allows a fundamental scalar at low energies

($+$ unification):

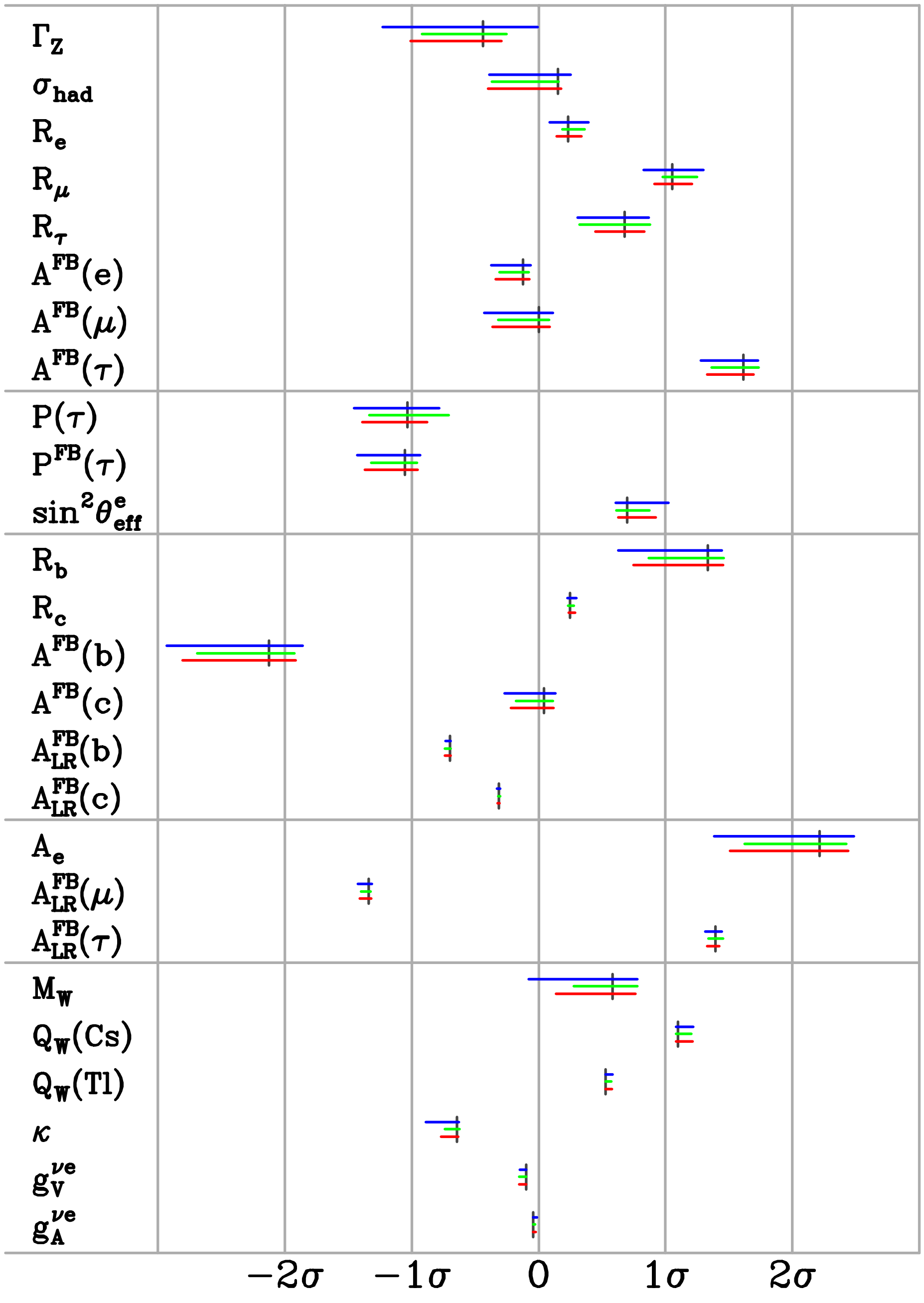
▷ $\sin^2\theta_W$, coupling constant unification

▷ Can generate SSB potential ($+$ universality)

($+R$-parity):

▷ LSP as dark matter candidate

MSSM closely resembles the standard EW Theory

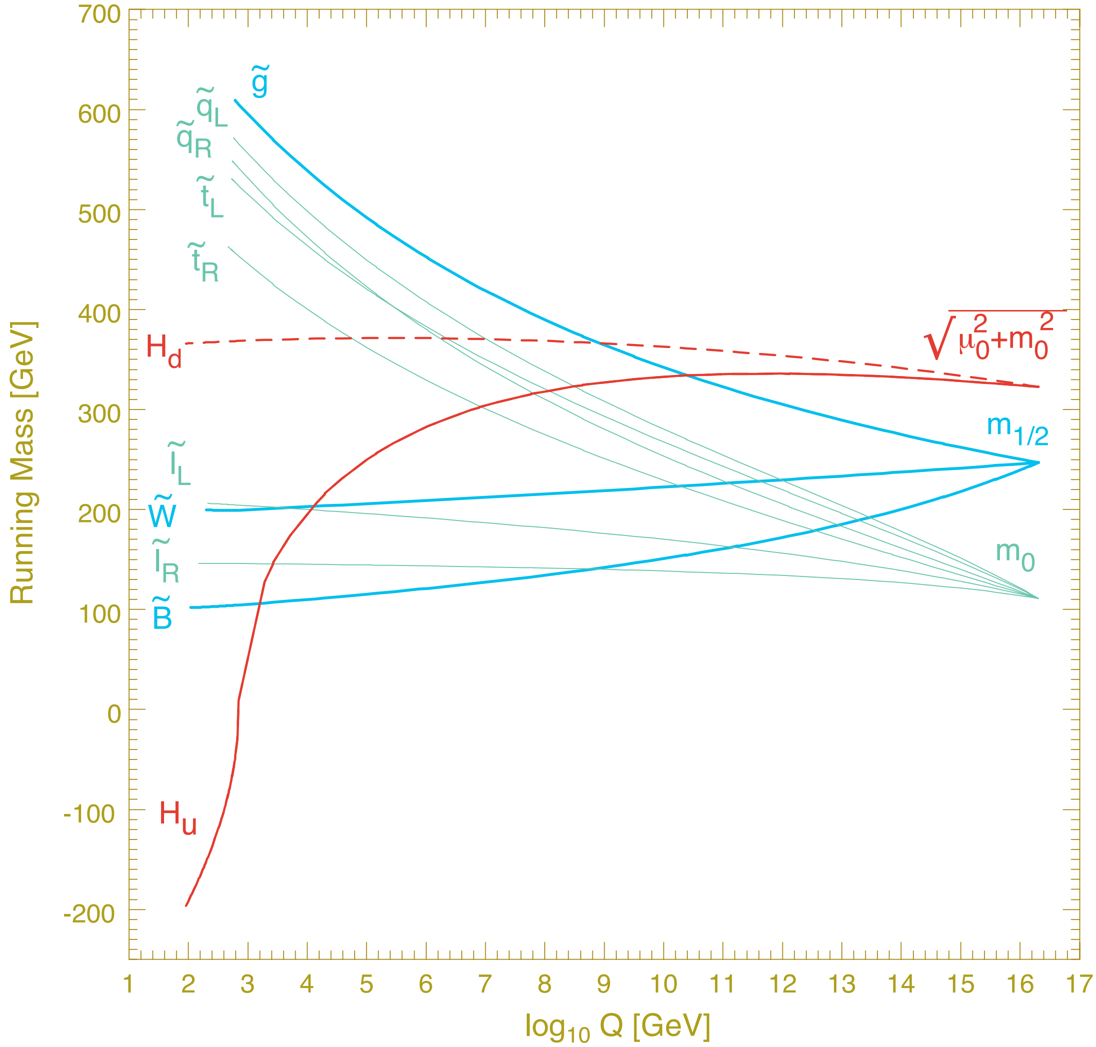


Erler & Pierce: SUSY *vs.* SM, hep-ph/9801238

Cho & Hagiwara, hep-ph/9912260

| SM — SUGRA — $5 \oplus 5^\star$ GMSB — $10 \oplus 10^\star$ GMSB

For heavy top, SSB may follow naturally in SUSY



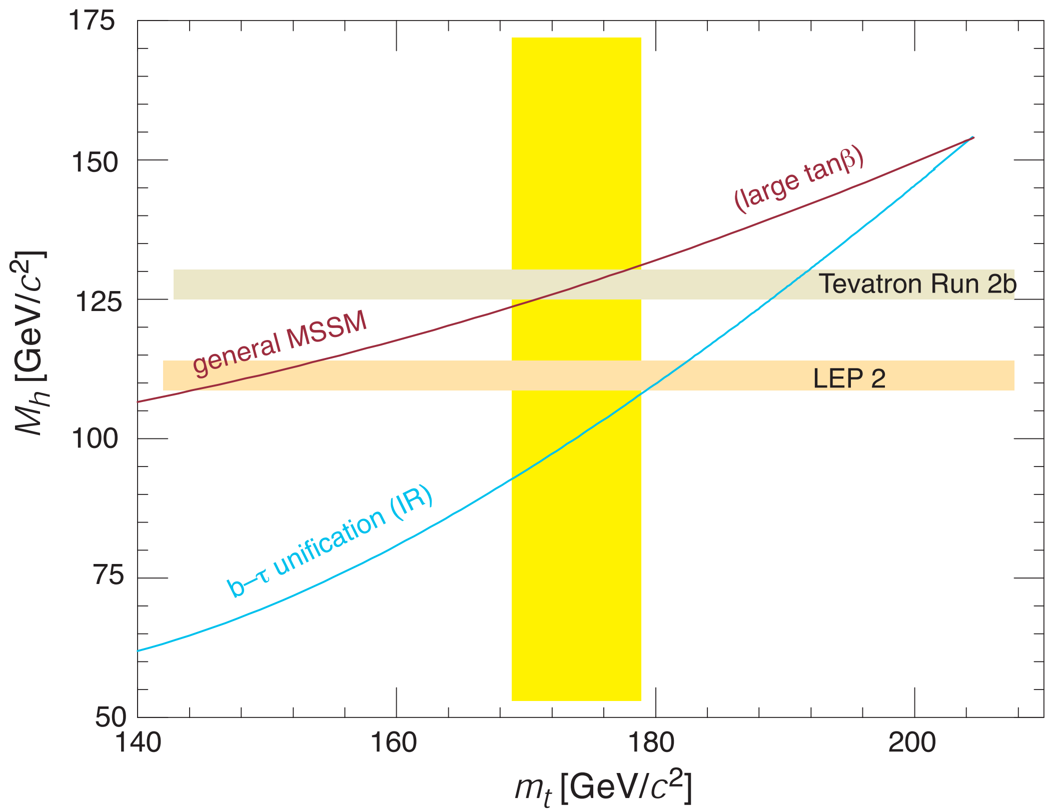
...(sign of M^2 indicated)

Kane, *et al.* (hep-ph/9312272, *Phys. Rev.* D**49**, 6173 (1994))

Upper bounds on M_h in the MSSM

$$M_h^2 = M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 M_W^2}\left[\log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \cdots\right] \lesssim \left(130\ \text{GeV}/c^2\right)^2$$

Upper bound on $M_h \Leftrightarrow$ large M_A limit, $(M_s = 1\ \text{TeV})$



Carena, *et al.*, *Phys. Lett.* **B355**, 209 (1995)

If nonminimal SUSY Higgs couplings are perturbative up to M_U,

$$\boxed{M_h \lesssim 150\ \text{GeV}}$$

SUSY doubles the spectrum

If $m_{\tilde{e}} < m_e$ ...

... no Pauli principle to dictate the integrity of molecules

Dyson & Lieb: If basic constituents of matter were bosons, individual molecules would join into a

shrinking

insatiable

undifferentiated

$\boxed{\text{B L O B !}}$

SUPERSYMMETRY MENACES US

WITH AN AMORPHOUS DEATH

Full understanding of SUSY would show us why we live in a world ruled by the *EXCLUSION PRINCIPLE*

SUSY Challenges

- ▷ Extra dynamics needed to break SUSY

  "Soft" SUSY breaking $\Longrightarrow$

  MSSM with $\boxed{124}$ parameters

- ▷ Weak-scale SUSY protects M_H, but does not explain the weak scale ("μ problem")

- ▷ Global SUSY must deal with the threat of flavor-changing neutral currents

- ▷ (Like SM) Clear predictions for gauge-boson masses, not so clear for squarks and sleptons

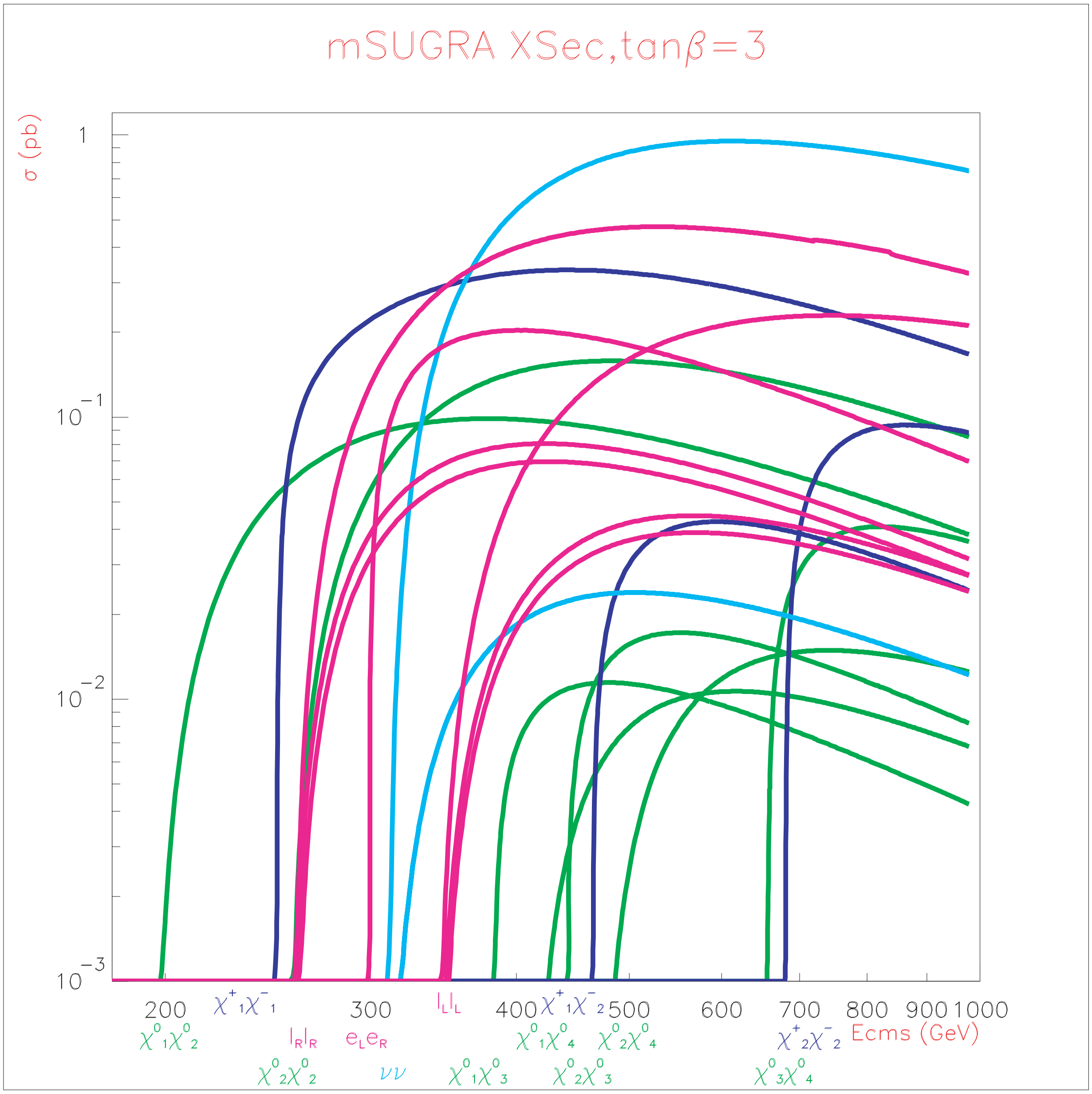
- ▷ So far, SUSY is well hidden

If weak-scale SUSY is present, we should see it soon

... in the Higgs sector and beyond

$\boxed{\text{We will live in "interesting times"}}$

Example SUSY thresholds in e^+e^-



Grahame Blair

My view

Supersymmetry is (almost) certain to be true . . .

. . . as a path to the incorporation of gravity

Whether SUSY resolves the problems of the 1-TeV scale is a logically separate question

. . . answer less obvious

$$\boxed{\boxed{\boxed{\text{EXPERIMENT WILL DECIDE}}}}$$

A look at technicolor

Follow the "other path" to EWSB, but with a new interaction, new constituents

massless u, d quarks	$\longrightarrow$	new fermions "technifermions"
color QCD	$\longrightarrow$	new interaction "technicolor"

Choose scale of interaction so that

$$f_\pi \qquad \longrightarrow \qquad F_\pi = v = (G_F\sqrt{2})^{-\frac{1}{2}}$$

Generates correct M_W, M_Z, *but* produces no Yukawa couplings, so no fermion masses

Shows possibility that

gauge-boson masses

&

fermion masses

... have *different* origins

To generate fermion mass, embed technicolor in a larger extended technicolor gauge group

$$G_{\mathrm{ETC}} \supset G_{\mathrm{TC}}$$

that couples quarks and leptons to technifermions

If

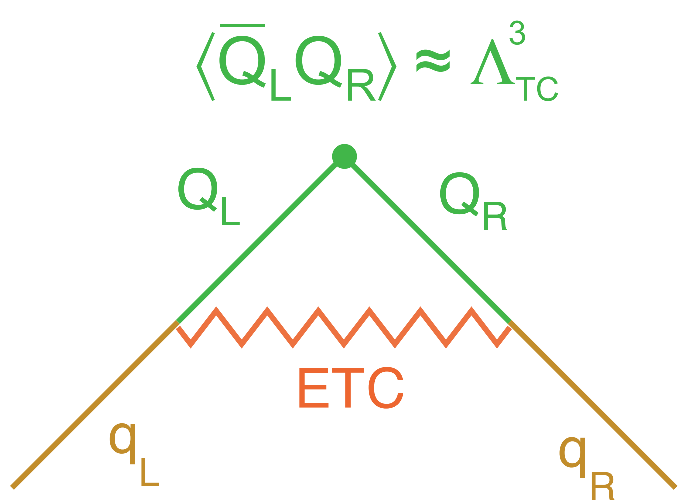
$$G_{\mathrm{ETC}} \rightarrow G_{\mathrm{TC}}$$

at scale Λ_{ETC},

then quarks and leptons may acquire masses

$$\boxed{m \sim \Lambda_{\mathrm{TC}}^3 / \Lambda_{\mathrm{ETC}}^2}$$

“radiative” mechanism



Standard ETC is challenged by problems of reproducing wide range of quark masses while avoiding FCNC traps

Consider $|\Delta S| = 2$ interactions

$$\mathcal{L}_{|\Delta S|=2} = \frac{g_{\mathrm{ETC}}^2 \theta_{sd}^2}{M_{\mathrm{ETC}}^2} (\bar{s}\Gamma^\mu d)(\bar{s}\Gamma'_\mu d) + \cdots$$

$\Delta M_K < 3.5 \times 10^{-12}$ MeV $\Longrightarrow$

$$\boxed{\frac{M_{\mathrm{ETC}}^2}{g_{\mathrm{ETC}}^2 \left|\theta_{sd}\right|^2} \text{ very large}}$$

$\Longrightarrow$ hard to generate heavy enough c, s, t, b

Multiscale TC (Eichten & Lane)

Many fermions (in different TC reps)
$\Longrightarrow$ many technipions
light ρ_{T}, ω_{T}, π_{T}

Generation of fermion mass is where all the *experimental threats* to Technicolor arise:

Flavor-changing neutral currents
Matter content (S parameter)

$\boxed{\text{Lesson: QCD is not a good model for TC}}$

Keep in mind: In addressing problems of fermion mass, ETC is *much more ambitious* than global supersymmetry

Current ideas: K. Lane, "Two Lectures on TC," hep-ph/0202255

Review: Hill & Simmons, "Strong Dynamics & EWSB," hep-ph/0203079

Why is the Planck scale so large?

In particle physics terms, we have only tested Newtonian gravity to energy

$$E \approx 10^{-3}\ \text{eV}$$
$$\ll 1\ \text{TeV}$$
$$\lll M_{\text{Planck}} \qquad (10^{-31}\times\ \text{less!})$$
$$(1/M_{\text{Planck}} \approx 2 \times 10^{-35}\ \text{m})$$

What do we know about gravity at short distances?

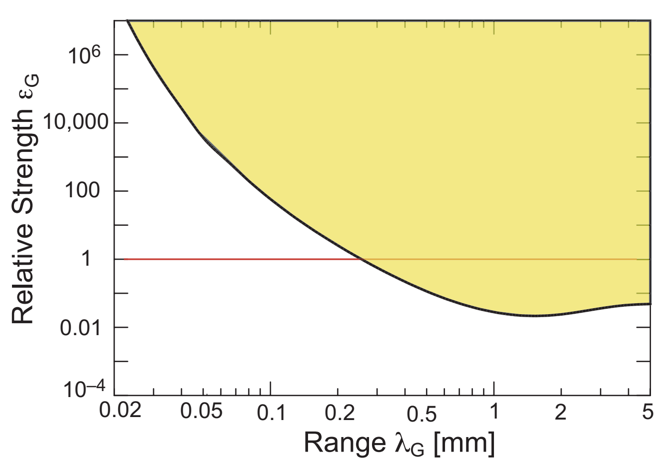
follows Newton's force law $(1/r^2)$ down to $r \lesssim 1$ mm

parameterize deviations:

$$V(r) = -\int dr_1 \int dr_2 \frac{G_{\text{N}}\rho(r_1)\rho(r_2)}{r_{12}}\left[1 + \varepsilon_{\text{G}} \exp(-r_{12}/\lambda_{\text{G}})\right]$$

Extra piece with gravitational strength ruled out down to $\lambda_{\text{G}} \approx 0.2$ mm

Limits on non-Newtonian force of gravity (Seattle)



Experiment leaves us free to consider modifications to Gravity even at (nearly) macroscopic distances

Suppose

at scale R

Gravity propagates in $4+n$ dimensions

Force law changes: Gauss's law $\Rightarrow$

$$G_{\mathrm{N}} \sim M_{\mathrm{Pl}}^{-2} \sim M^{\star\,-n-2} R^{-n}$$

$M^{\star}$: gravity's true scale

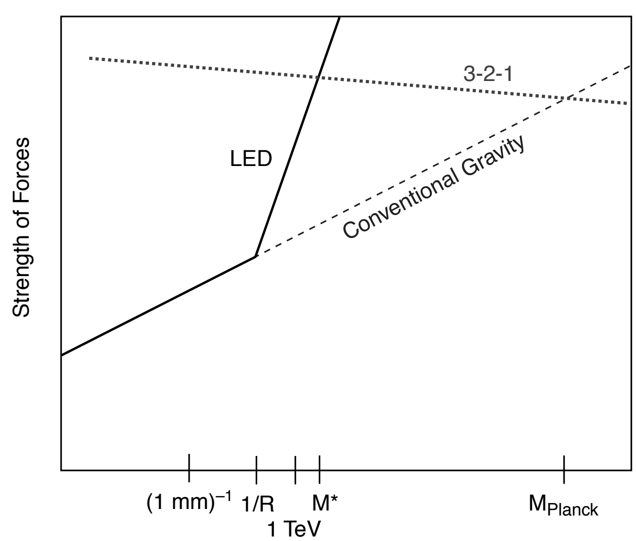
Example:

$$M^{\star} = 1 \text{ TeV}$$

$$\Rightarrow R \lesssim 10^{-3} \text{ m for } n = 2$$

Traditional: Use 4-d force law to extrapolate gravity to higher energies; $M_{\mathrm{P}} \sim$ scale where Gravity, SM forces are of comparable strength

IF Gravity probes extra dimensions for $E \lesssim 1/R$, Gravity meets other forces at $E = M^{\star} \ll M_{\mathrm{P}}$

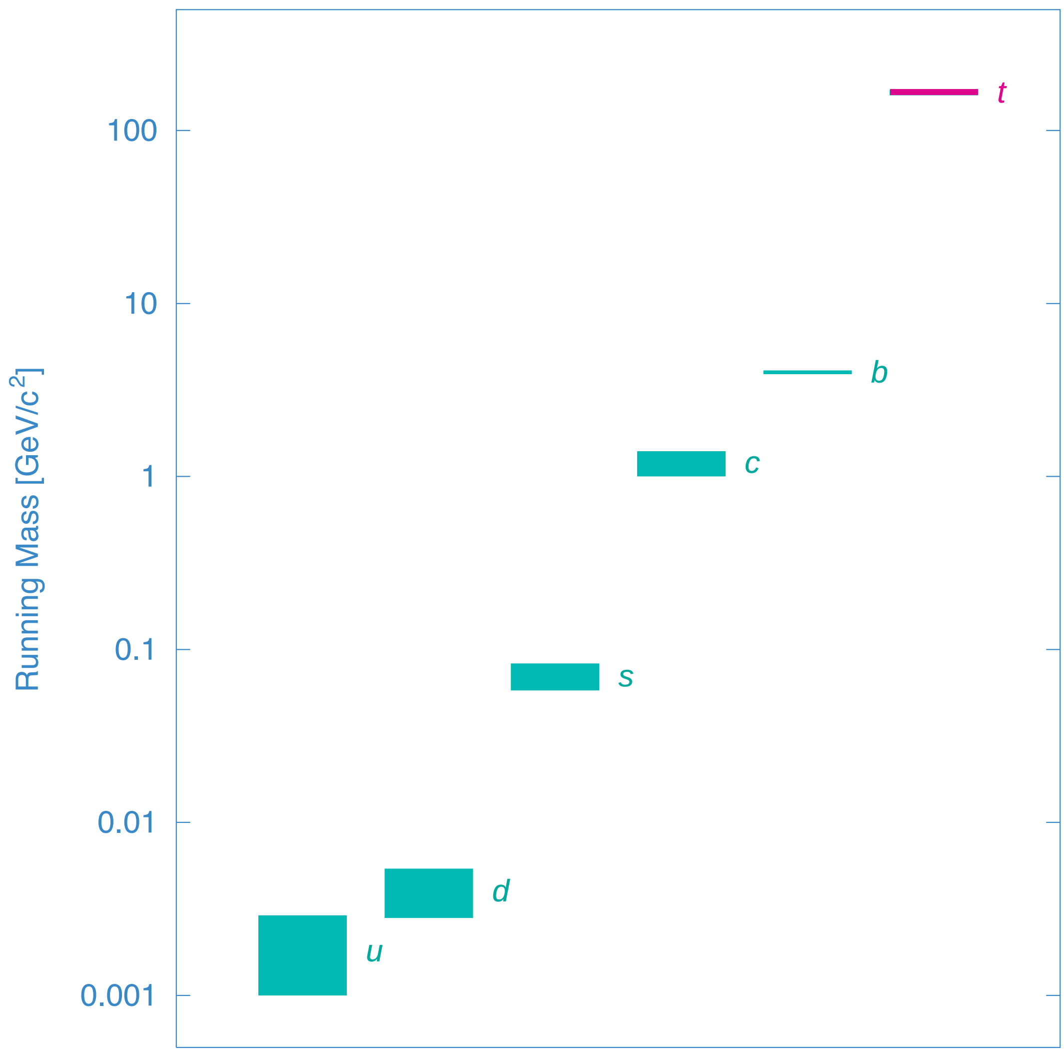


$\boxed{M_{\mathrm{P}} \text{ is a mirage (false extrapolation)!}}$

The problem of fermion masses

TC shows solving origin of EWSB will not necessarily give insight into fermion masses

Puzzling pattern of quark masses



$\overline{m}_q(m_q)$, u, d, s evaluated at 1 GeV

Underlying order on some other scale?

Aside: varieties of neutrino mass

Chiral decomposition of Dirac spinor:

$$\psi = \tfrac{1}{2}(1-\gamma_5)\psi + \tfrac{1}{2}(1+\gamma_5)\psi \equiv \psi_{\mathrm{L}} + \psi_{\mathrm{R}}$$

Charge conjugate of RH field is LH:

$$\psi^c_{\mathrm{L}} \equiv (\psi^c)_{\mathrm{L}} = (\psi_{\mathrm{R}})^c$$

Possible forms for mass terms

Dirac connects LH, RH components of *same field*

$$\mathcal{L}_D = D(\bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}} + \bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}}) = D\bar{\psi}\psi$$

$\Longrightarrow$ mass eigenstate $\psi = \psi_{\mathrm{L}} + \psi_{\mathrm{R}}$

(invariant under global phase rotation $\nu \to e^{i\theta}\nu$, $\ell \to e^{i\theta}\ell$, so that lepton number is conserved)

Possible forms for mass terms (cont'd)

Majorana connects LH, RH components of *conjugate fields*

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{MA}} &= A(\bar{\psi}^c_{\mathrm{R}}\psi_{\mathrm{L}} + \bar{\psi}_{\mathrm{L}}\psi^c_{\mathrm{R}}) = A\bar{\chi}\chi \\
-\mathcal{L}_{\mathrm{MB}} &= B(\bar{\psi}^c_{\mathrm{L}}\psi_{\mathrm{R}} + \bar{\psi}_{\mathrm{R}}\psi^c_{\mathrm{L}}) = B\bar{\omega}\omega
\end{aligned}
$$

for which the mass eigenstates are

$$
\begin{aligned}
\chi &\equiv \psi_{\mathrm{L}} + \psi^c_{\mathrm{R}} = \chi^c = \psi_{\mathrm{L}} + (\psi_{\mathrm{L}})^c \\
\omega &\equiv \psi_{\mathrm{R}} + \psi^c_{\mathrm{L}} = \omega^c = \psi_{\mathrm{R}} + (\psi_{\mathrm{R}})^c
\end{aligned}
$$

$\mathcal{L}_{\mathrm{M}}$ violates lepton number by two units
$\Rightarrow$ Majorana ν can mediate $\beta\beta_{0\nu}$ decays

$$(Z, A) \to (Z+2, A) + e^- + e^-$$

Detecting $\beta\beta_{0\nu}$ would offer decisive evidence for the Majorana nature of ν

Specific framework: $SU(5)$, 2-step SSB

▷ $\mathbf{24}$ of scalars breaks

$$SU(5) \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

gives extremely large masses to leptoquark gauge bosons $X^{\pm 4/3}$ and $Y^{\pm 1/3}$

$\mathbf{24}$ does not occur in the $\bar{\text{L}}$R products

$$\begin{aligned} \mathbf{5}^* \otimes \mathbf{10} &= \mathbf{5} \oplus \mathbf{45} \\ \mathbf{10} \otimes \mathbf{10} &= \mathbf{5}^* \oplus \mathbf{45}^* \oplus \mathbf{50}^* \end{aligned}$$

that generate fermion masses, so quarks and leptons escape large tree-level masses

▷ $\mathbf{5}$ of scalars ($\supset$ SM Higgs) breaks

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_c \otimes U(1)_{\text{em}}$$

and endows fermions with mass; relates quark and lepton masses at M_U

$$\left.\begin{aligned} m_e &= m_d \\ m_\mu &= m_s \\ m_\tau &= m_b \end{aligned}\right\} \text{ at } M_U \text{ ; separate parameters } \left\{\begin{array}{c} m_u \\ m_c \\ m_t \end{array}\right.$$

Masses evolve from M_U to μ

$$\begin{aligned}
\ln\left[m_{u,c,t}(\mu)\right] &\approx \ln\left[m_{u,c,t}(M_U)\right] + \frac{12}{33-2n_f}\ln\left(\frac{\alpha_3(\mu)}{\alpha_U}\right) \\
&\quad + \frac{27}{88-8n_f}\ln\left(\frac{\alpha_2(\mu)}{\alpha_U}\right) - \frac{3}{10n_f}\ln\left(\frac{\alpha_1(\mu)}{\alpha_U}\right)
\end{aligned}$$

$$\begin{aligned}
\ln\left[m_{d,s,b}(\mu)\right] &\approx \ln\left[m_{d,s,b}(M_U)\right] + \frac{12}{33-2n_f}\ln\left(\frac{\alpha_3(\mu)}{\alpha_U}\right) \\
&\quad + \frac{27}{88-8n_f}\ln\left(\frac{\alpha_2(\mu)}{\alpha_U}\right) + \frac{3}{20n_f}\ln\left(\frac{\alpha_1(\mu)}{\alpha_U}\right)
\end{aligned}$$

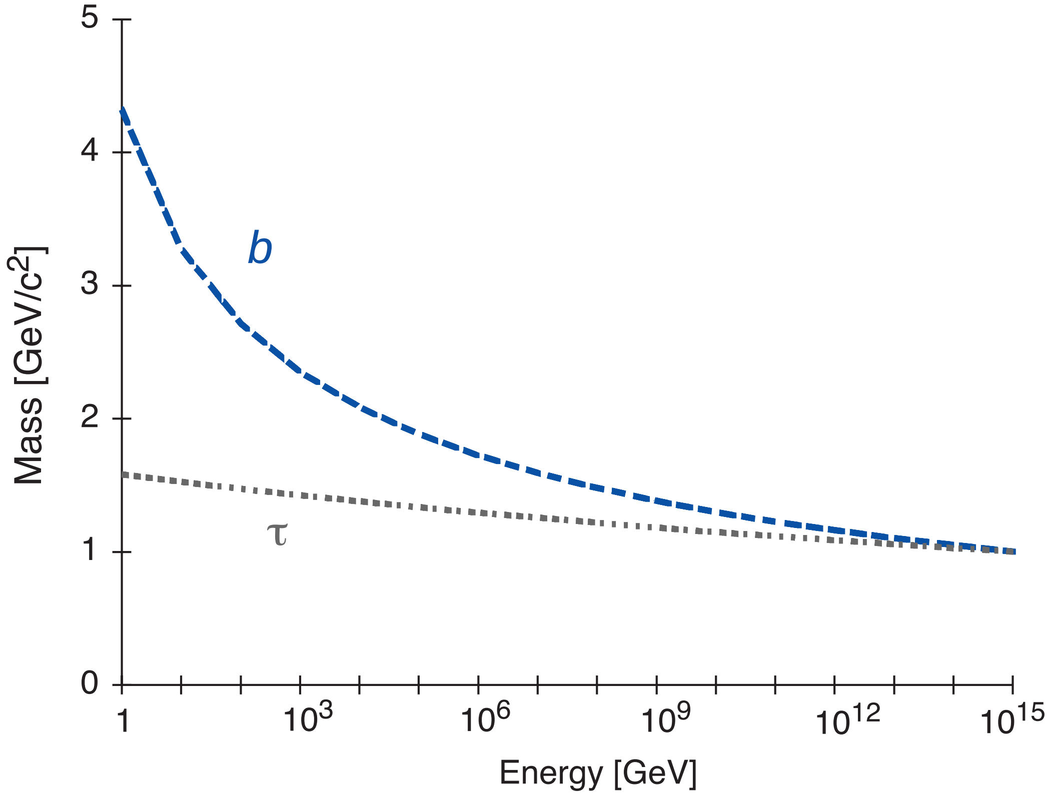
$$\begin{aligned}
\ln\left[m_{e,\mu,\tau}(\mu)\right] &\approx \ln\left[m_{e,\mu,\tau}(M_U)\right] \\
&\quad + \frac{27}{88-8n_f}\ln\left(\frac{\alpha_2(\mu)}{\alpha_U}\right) - \frac{27}{20n_f}\ln\left(\frac{\alpha_1(\mu)}{\alpha_U}\right)
\end{aligned}$$

n_f quark or lepton flavors (Higgs omitted)

Classic $SU(5)$ success: m_b and m_τ

$$\begin{aligned}
\ln\left[\frac{m_b(\mu)}{m_\tau(\mu)}\right] &\approx \cancel{\ln\left[\frac{m_b(M_U)}{m_\tau(M_U)}\right]} + \frac{12}{33-2n_f}\ln\left(\frac{\alpha_3(\mu)}{\alpha_U}\right) \\
&\quad - \frac{3}{2n_f}\ln\left(\frac{\alpha_1(\mu)}{\alpha_U}\right)
\end{aligned}$$

$n_f = 6$, $1/\alpha_U = 40$, $1/\alpha_s(\mu) = 5$, $1/\alpha_1(\mu) = 65$:



At low scale, $m_b = 2.91 m_\tau \approx 5.16$ GeV/c^2

...suggestive agreement with experiment

Perhaps all fermion masses arise on high scales, and show simple patterns there?

not so pretty: $\underbrace{\frac{m_s}{m_d}}_{\approx 20} = \underbrace{\frac{m_\mu}{m_e}}_{\approx 200}$ $\mu \approx 1$ GeV

More elaborate SB scheme (scalar $\mathbf{45}$) can change

$$m_e/m_d \text{ at } M_U$$

Example: $m_s = \frac{1}{3}m_\mu, \quad m_d = 3m_e \quad$ at M_U

leads to low-energy predictions

$$\left.\begin{array}{c} m_s \approx \frac{4}{3}m_\mu \\ m_d \approx 12m_e \end{array}\right\} \text{ at } \mu \approx 1 \text{ GeV .}$$

$\boxed{\Longrightarrow \text{ a new strategy / a new industry}}$

- ▷ begin with supersymmetric $SU(5)$
  (advantages for $\sin^2\theta_W$, coupling constant unification, proton lifetime)
  or susy $SO(10)$ (to accommodate massive ν)
- ▷ find "textures" — simple patterns of Yukawa matrices that lead to successful predictions for masses and mixing angles
- ▷ interpret in terms of symmetry breaking patterns
- ▷ seek derivation / motivation for winning entry

(Reassuring that some schemes fail on m_t or $|V_{cb}|$)

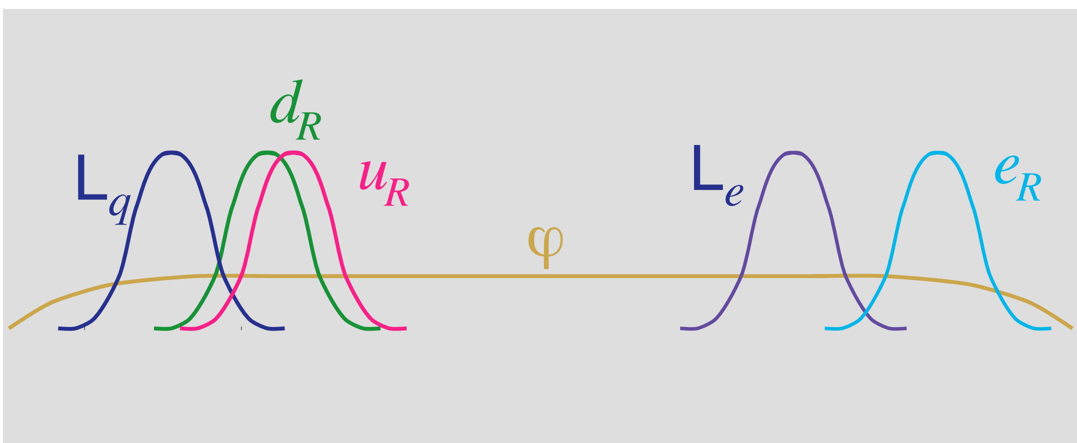
Other approaches to fermion mass

▷ Composite models

▷ Special role to top

▷ Other radiative mechanisms:
squark mass $\longrightarrow$ quark mass

▷ Topology of extra dimensions



In a decade or two, we can hope to . . .

Understand electroweak symmetry breaking

Observe the Higgs boson

Measure neutrino masses and mixings

Establish Majorana neutrinos ($\beta\beta_{0\nu}$)

Thoroughly explore CP violation in B decays

Exploit rare decays (K, D, . . .)

Observe neutron EDM, pursue electron EDM

Use top as a tool

Observe new phases of matter

Understand hadron structure quantitatively

Uncover QCD's full implications

Observe proton decay

Understand the baryon excess

Catalogue matter and energy of the universe

Measure dark energy equation of state

Search for new macroscopic forces

Determine GUT symmetry

Detect neutrinos from the universe

Learn how to quantize gravity

Learn why empty space is nearly weightless

Test the inflation hypothesis

Understand discrete symmetry violation

Resolve the hierarchy problem

Discover new gauge forces

Directly detect dark-matter particles

Explore extra spatial dimensions

Understand the origin of large-scale structure

Observe gravitational radiation

Solve the strong CP problem

Learn whether supersymmetry is TeV-scale

Seek TeV-scale dynamical symmetry breaking

Search for new strong dynamics

Explain the highest-energy cosmic rays

Formulate problem of identity

. . .

. . . and learn to ask the right questions