

PARTICLE PHYSICS: THEMES AND CHALLENGES

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1 Introduction

We gather in Ho Chi Minh City with high expectations for the future of particle physics and high hopes for the future of science in Vietnam. We are in the midst of a revolution in our perceptions of nature, when the achievements of our science have brought our insights closer to everyday life than ever before. I will devote this lecture to seven themes that express the essence of our understanding—and our possibilities.

2 Elementarity

One of the pillars of our understanding is the identification of a set of fundamental constituents, the leptons

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix},$$

and the quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix},$$

which have no internal structure, no size, no form factors, and no excited states—so far as we know. The quarks are color triplets, so experience the strong interactions, whereas the leptons, color singlets, do not.

The charged leptons and the quarks are Dirac particles with gyromagnetic ratio $g = 2$ (+ the amount induced by interactions). The size of the fermions is smaller than the current limit of experimental resolution characterized by a radius $R \lesssim 10^{-17}$ cm.¹⁾ We don't yet know whether the neutrinos are massive or not. If neutrinos do have mass, they may be either Dirac or Majorana particles.²⁾

All the experimental evidence leads us to conclude that quarks and leptons are the fundamental (constituent) degrees of freedom at current energies. We regard them as elementary.

What if they were not? What if quarks and leptons were composite?

Approaching the compositeness scale from low energies, we would encounter new contact interactions that correspond to the exchange or rearrangement of constituents.³⁾ In quark-quark scattering, the conventional gluon exchange would be supplemented by a contact term of geometrical size and unknown Lorentz structure. In $\bar{p}p$ collisions, this new contribution would lead to an excess (over QCD) of hadron jets at large values of the transverse energy, where $\bar{q}q \rightarrow \bar{q}q$ is the dominant elementary reaction. In general, the angular distribution of the jets will differ from the standard QCD shape. If quarks and leptons have common constituents, a similar excess will be seen in dilepton production, from the elementary process $\bar{q}q \rightarrow \ell^+\ell^-$. At still higher energies, we expect to see the effects of excited q^* and ℓ^* states.⁴⁾ Finally, at energies well above the compositeness scale, quarks and leptons would begin to manifest form factors.

No experimental evidence except history suggests that quarks and leptons are composite. However, compositeness might explain the fermion mass spectrum, the existence of generations, and the relationship of quarks and leptons. No composite model has yet achieved these breakthroughs,⁵⁾ so the search for compositeness is a purely experimental exercise. The discovery of compositeness would alter our conception of matter in a fundamental way.

3 Symmetry

The other essential ingredient in the standard model is the notion that continuous local symmetries—gauge symmetries—determine the character of the fundamental interactions.

The simplest, and classic, example is the derivation of quantum electrodynamics from local phase invariance. The quantum mechanics of a free particle is invariant under global changes of phase of the wave function,

$$\psi(x) \rightarrow e^{i\theta} \psi(x). \quad (3.1)$$

This is the symmetry associated with charge conservation. Requiring a theory invariant under *local* changes of phase,

$$\psi(x) \rightarrow e^{i\theta(x)} \psi(x), \quad (3.2)$$

demands the introduction of a massless vector field, identified as the photon, and leads to a full theory of electrodynamics, QED.⁶⁾

The same general strategy can be applied to any continuous symmetry. That insight links the problem of building theories of the fundamental interactions to the search for the right symmetries to gauge. Let us review the electroweak theory as an example.

The crucial experimental clues for the construction of a gauge theory of the weak and electromagnetic interactions are the family pattern embodied in the left-handed weak-isospin doublets of leptons and quarks and the universal strength of the charged-current weak interactions. It is straightforward to construct the theory, which I will write down for one generation of leptons, idealizing the neutrinos as massless.

To incorporate $SU(2)_L$ weak-isospin symmetry, we define a left-handed doublet,

$$\mathbf{L} \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(1 - \gamma_5)\nu \\ \frac{1}{2}(1 - \gamma_5)e \end{pmatrix}, \quad (3.3)$$

and a right-handed singlet,

$$\mathbf{R} \equiv e_R = \frac{1}{2}(1 + \gamma_5)e. \quad (3.4)$$

To include electromagnetism, we define the weak hypercharge through $Q = I_3 + \frac{1}{2}Y$, so that $Y_L = -1$, $Y_R = -2$. The gauge group $SU(2)_L \otimes U(1)_Y$ allows two coupling constants, g for the $SU(2)_L$ gauge bosons $b_\mu^1, b_\mu^2, b_\mu^3$, and $\frac{1}{2}g'$ for the $U(1)$ gauge boson \mathcal{A}_μ . We may write the Lagrangian as

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}}, \quad (3.5)$$

where $\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}^\ell F^{\ell\mu\nu} - \frac{1}{4}f_{\mu\nu}f^{\mu\nu}$, with $F_{\mu\nu}^\ell = \partial_\nu b_\mu^\ell - \partial_\mu b_\nu^\ell + g\varepsilon_{jkl}b_\mu^j b_\nu^k$ and $f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu$, and the matter term is

$$\mathcal{L}_{\text{leptons}} = \bar{\mathbf{R}}i\gamma^\mu \left(\partial_\mu + \frac{ig'}{2}\mathcal{A}_\mu Y \right) \mathbf{R} + \bar{\mathbf{L}}i\gamma^\mu \left(\partial_\mu + \frac{ig'}{2}\mathcal{A}_\mu Y + \frac{ig}{2}\vec{\tau} \cdot \vec{b}_\mu \right) \mathbf{L}. \quad (3.6)$$

Explicit mass terms for the gauge bosons or fermions are inconsistent with the gauge symmetry. Accordingly, this theory has a massless neutrino, a massless electron, and four massless electroweak gauge bosons. Nature has a massive electron, a massless neutrino, three massive gauge bosons, and but one massless electroweak gauge boson, the photon. The minimal solution to this mismatch is to hide the gauge symmetry by means of the Higgs mechanism. We introduce a complex weak-isospin doublet of scalar fields,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad (3.7)$$

with weak hypercharge $Y_\phi = +1$. Add to the Lagrangian a piece

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi), \quad (3.8)$$

where the gauge-covariant derivative is $\mathcal{D}_\mu = \partial_\mu + \frac{ig'}{2}\mathcal{A}_\mu Y + \frac{ig}{2}\vec{\tau} \cdot \vec{b}_\mu$, and the Higgs potential is $V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + |\lambda|(\phi^\dagger \phi)^2$. We are also free to add the interactions of the scalars with the fermions,

$$\mathcal{L}_{\text{Yukawa}} = -G_e \left[\bar{\mathbf{R}}(\phi^\dagger \mathbf{L}) + (\bar{\mathbf{L}}\phi) \mathbf{R} \right]. \quad (3.9)$$

If $\mu^2 < 0$, the vacuum state corresponds to a nonzero value of the scalar field, which we choose to be

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ (G_F \sqrt{8})^{-1/2} \end{pmatrix}. \quad (3.10)$$

(The last identification ensures that the theory reproduces the low-energy charged-current phenomenology.) The nonzero value of $\langle \phi \rangle_0$ hides (or breaks) the $SU(2)_L$ and $U(1)_Y$ symmetries, but preserves a residual invariance under $U(1)_{\text{EM}}$. The spectrum of the broken theory consists of a massless photon $A_\mu = \mathcal{A}_\mu \cos \theta_W + b_\mu^3 \sin \theta_W$, with coupling $gg'/\sqrt{g^2 + g'^2} \equiv e$; charged vector bosons $W_\mu^\pm = (b_\mu^1 \mp ib_\mu^2)/\sqrt{2}$, with $M_W^2 = \pi\alpha/G_F\sqrt{2}\sin^2 \theta_W$; a neutral intermediate boson $Z_\mu = b_\mu^3 \cos \theta_W - \mathcal{A}_\mu \sin \theta_W$, with $M_Z = M_W/\cos \theta_W$; a neutral Higgs scalar, with $M_H^2 = -2\mu^2 > 0$; and an electron with mass $m_e = G_e v/\sqrt{2}$. The predicted masses for the W^\pm and Z^0 are expressed in terms of the weak mixing parameter $\sin^2 \theta_W$, which is measured in neutral-current reactions. It is both a triumph and a frustration of the electroweak theory that spontaneous symmetry breaking plus Yukawa couplings generates fermion masses, for the Yukawa couplings are not calculable within the theory.

4 Consistency

The leptonic and hadronic charged weak currents are identical in form, characterized by the left-handed doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L, \quad (4.1)$$

etc. Renormalizability requires the absence of anomalies—quantum violations of classical symmetries or conservation laws. For the electroweak theory, the condition for anomaly freedom can be expressed in the requirement

$$\Delta Q = Q_{\text{R}} - Q_{\text{L}} = \sum_{\text{RH doublets}} Q - \sum_{\text{LH doublets}} Q = 0. \quad (4.2)$$

In an electroweak theory based on the lepton doublet $(\nu_e e)_L$, $\Delta Q^{(\text{leptons})} = -Q_{\text{L}} = +1 \neq 0$. To cancel the lepton anomaly, we could add right-handed fermions with appropriate charges, but no

right-handed charged-current interactions are known. More to the point, we can add a color triplet of left-handed quark doublets $(u\ d)_L$, for which $\Delta Q^{(\text{quarks})} = -3(\frac{2}{3} - \frac{1}{3}) = -1$, so that $\Delta Q = \Delta Q^{(\text{leptons})} + \Delta Q^{(\text{quarks})} = 0$.

It is remarkable that a consistent theory of weak and electromagnetic interactions requires quarks as well as leptons. This suggests a deep connection between quarks and leptons that I take as an important clue toward a more complete theory.

5 Unity

Making connections is the essence of scientific progress. For monumental examples, think of the amalgamation of electricity and magnetism and light; of the recognition that heat is atoms in motion, which brought together thermodynamics and Newtonian mechanics; and of the realization that the chemical properties of substances are determined by the atomic and molecular structure of matter. Each of these unifications brought new understanding and illuminated phenomena beyond those that served as motivation.

What progress might we achieve by unifying the quarks and leptons, or the strong, weak, and electromagnetic interactions described by the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge theories, or both? The link between quarks and leptons implied by anomaly cancellation is reinforced by the following set of questions: Can we understand why (i) electric charge is quantized? (ii) $Q_p + Q_e = 0$? (iii) $Q_\nu - Q_e = Q_u - Q_d$? (iv) $Q_d = Q_e/3$? (v) $Q_\nu + Q_e + 3Q_u + 3Q_d = 0$?

What observations motivate a unified theory of the fundamental interactions? Beyond the similarities and links between quarks and leptons, we recognize that the $SU(2)_L \otimes U(1)_Y$ electroweak theory achieves only a partial unification of the weak and electromagnetic interactions, as evidenced by the fact that $\sin^2 \theta_W$ is a free parameter of the theory. Taken together, quantum chromodynamics and the electroweak theory have three distinct coupling parameters, $(\alpha_s, \alpha_{EM}, \sin^2 \theta_W)$ or, equivalently, $(\alpha_3, \alpha_2, \alpha_1)$. Might we reduce the number of independent couplings to two or one? As we shall review presently, the evolution of the gauge couplings suggests that coupling-constant unification might be possible.

The minimal example of a theory that unifies the quarks and leptons and the fundamental interactions is based on the gauge group $SU(5)$.⁷⁾ The gauge bosons of $SU(5)$ lie in the adjoint **24** representation. Decomposing these particles according to their $(SU(3)_c, SU(2)_L)_Y$ quantum numbers, we recognize

$$\begin{aligned} (\mathbf{8}, \mathbf{1})_0 & : \quad \text{gluons,} \\ (\mathbf{1}, \mathbf{3})_0 & : \quad W^+, W^-, W_3, \\ (\mathbf{1}, \mathbf{1})_0 & : \quad \mathcal{A}, \end{aligned} \tag{5.1}$$

the twelve gauge bosons of (unbroken) $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, plus twelve new force particles whose existence is implied by the unification:

$$\begin{aligned} (\mathbf{3}, \mathbf{2})_{-5/3} & : \quad X^{-4/3}, Y^{-1/3}, \\ (\mathbf{3}^*, \mathbf{2})_{5/3} & : \quad X^{4/3}, Y^{1/3}. \end{aligned} \tag{5.2}$$

These additional interactions mediate baryon- and lepton-number-violating processes. The fundamental fermions fit in the **5*** and **10** representations of $SU(5)$, with $\nu_e, e_L, d_L^c \in \mathbf{5}^*$ and $e_L^c, u_L, d_L, u_L^c \in \mathbf{10}$, where I have used charge-conjugate fields to represent the right-handed degrees of freedom.

It is a straightforward matter to compute the evolution of the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge couplings.⁸⁾ Writing $\alpha_i = g_i^2/4\pi$, we have to leading logarithmic approximation

$$1/\alpha_3(Q^2) = 1/\alpha_3(\mu^2) + b_3 \ln(Q^2/\mu^2), \tag{5.3}$$

where $4\pi b_3 = 11 - 2n_f/3 = 11 - 4n_g/3$, and $n_f(n_g)$ is the number of active flavors (generations) with mass $< \sqrt{Q^2}$;

$$1/\alpha_2(Q^2) = 1/\alpha_2(\mu^2) + b_2 \ln(Q^2/\mu^2), \tag{5.4}$$

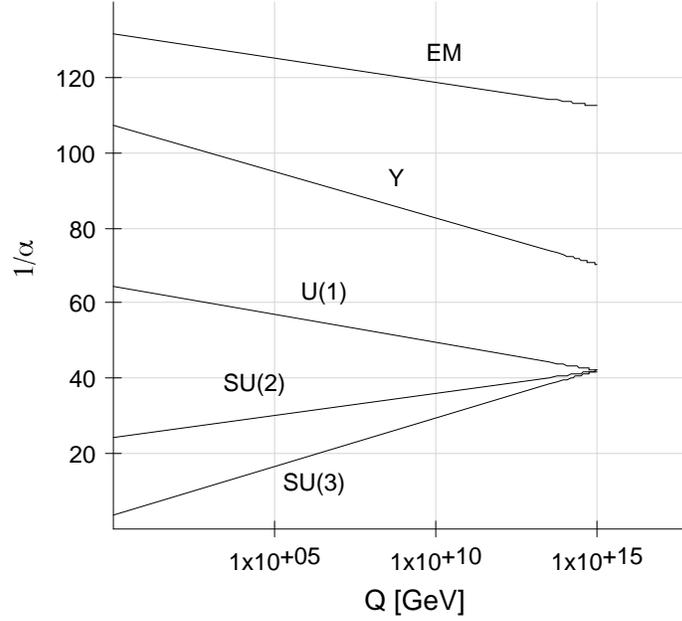


Figure 1: Evolution of running coupling constants in leading logarithmic approximation in the three-generation $SU(5)$ model.

where $4\pi b_2 = (22 - 4n_g - \frac{1}{2})/3$;

$$(5/3)(1/\alpha_1(Q^2)) = 1/\alpha_Y(Q^2) = 1/\alpha_Y(\mu^2) + b_Y \ln(Q^2/\mu^2), \quad (5.5)$$

where $4\pi b_Y = -20n_g/9$. We equate $\alpha_3, \alpha_2, \alpha_1$ at the unification scale M_U . To estimate the unification scale, we take

$$1/\alpha(M_Z^2) = 1/\alpha_Y(M_Z^2) + 1/\alpha_2(M_Z^2) = 129.08 \quad (5.6)$$

and

$$\alpha_3(M_Z^2) = 0.116, \quad (5.7)$$

whereupon

$$M_U \approx 10^{15} \text{ GeV}. \quad (5.8)$$

The characteristic evolution of the coupling constants is shown in Figure 1. Reality seems a little different.⁹⁾

Of special interest is the evolution of the weak mixing parameter

$$x_W = \sin^2 \theta_W = \alpha/\alpha_2. \quad (5.9)$$

The evolution equations for the gauge couplings yield

$$x_W(Q^2) = \frac{3}{8} - \alpha(Q^2) \frac{(3b_Y - 5b_2)}{8} \ln(Q^2/M_U^2) \quad (5.10)$$

$$= \frac{3}{8} + \frac{55\alpha(Q^2)}{48\pi} \ln(Q^2/M_U^2), \quad (5.11)$$

which is sketched in Figure 2. The $SU(5)$ prediction at the weak scale is

$$x_W(M_Z^2)|_{SU(5)} \approx 0.21, \quad (5.12)$$

which is both tantalizingly close to, and frustratingly far from, the LEP-SLD average value¹⁰⁾

$$x_W(M_Z^2)|_{\text{LEP+SLD}} = 0.23143 \pm 0.00028. \quad (5.13)$$

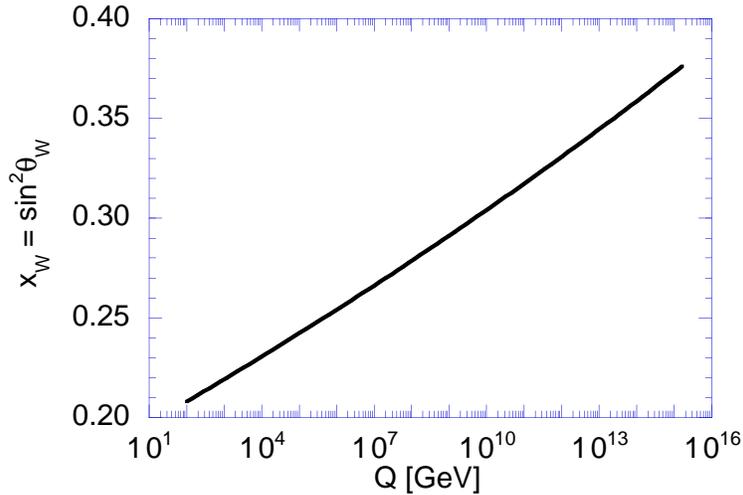


Figure 2: Evolution of the weak mixing parameter in the three-generation $SU(5)$ model.

Let us summarize the standing of the $SU(5)$ example of a unified theory. $SU(5)$ contains the standard-model gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ in a simple group, and thus reduces the number of independent couplings from three to one. By construction, the theory gives a correct description of the charged-current weak interactions. So far as the neutral-current interactions are concerned, it predicts a value of the weak mixing parameter that is close to, but not identical with, the observed value. $SU(5)$ naturally explains the quantization of electric charge. Since the electric-charge operator Q is a generator of $SU(5)$, the sum of electric charges over any representation must be zero. This means in particular that $Q(d^c) = (-1/3)Q(e)$. Proton decay is possible, through the action of the color-triplet gauge bosons X and Y .¹¹⁾ And of course, aspirations remain, even beyond a unified theory of the strong, weak, and electromagnetic interactions, for gravitation is omitted from the theory.

6 Identity

What makes a bottom quark a bottom quark, or an electron an electron? One of the great unsolved problems of the standard model is how to calculate fermion masses and mixing angles. In the electroweak theory, the Higgs mechanism produces fermion masses, as a result of spontaneous symmetry breaking. Recall that for a single generation of leptons, the Yukawa interaction is

$$\mathcal{L}_{\text{Yukawa}} = -G_e \left[\bar{\mathbf{R}}(\phi^\dagger \mathbf{L}) + (\bar{\mathbf{L}}\phi)\mathbf{R} \right], \tag{6.1}$$

where the left-handed lepton doublet is

$$\mathbf{L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \tag{6.2}$$

and $\mathbf{R} = e_R$ is the right-handed electron. The interaction (6.1) is the most general Lorentz scalar invariant under local $SU(3)_c \otimes U(1)_Y$ transformations. The electroweak theory offers no guidance about the value of the Yukawa coupling. For the electron, $G_e \approx 3 \times 10^{-6}$, while the analogous coupling for the top quark is $G_t \approx 1$.

For three generations of quarks and leptons, we can generalize (6.1) to

$$\mathcal{L}_{\text{Yukawa}} = \bar{u}_R^i U_{ij}(\bar{\phi}_u^\dagger \mathbf{Q}_j) + \bar{d}_R^i D_{ij}(\phi_d^\dagger \mathbf{Q}_j) + \bar{e}_R^i E_{ij}(\phi_d^\dagger \mathbf{L}_j), \tag{6.3}$$

where

$$\mathbf{Q}_j = \begin{pmatrix} u_j \\ d_j \end{pmatrix}_L, \quad \mathbf{L}_j = \begin{pmatrix} \nu_j \\ \ell_j \end{pmatrix}_L, \tag{6.4}$$

and U_{ij} , D_{ij} , and E_{ij} are complex 3×3 matrices. In the electroweak theory, $\phi_u = \phi_d = \phi$, with $\langle \phi \rangle_0$ given by (3.10), whereas in the minimal supersymmetric generalization, ϕ_u and ϕ_d are distinct. The ratio of their vacuum expectation values is parametrized as $\tan \beta = v_u/v_d$.

Unified theories imply relations among fermion masses. In the example of $SU(5)$, spontaneous symmetry breaking occurs in two steps. First, an adjoint **24** of scalars breaks

$$SU(5) \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y, \quad (6.5)$$

and gives very large masses to the $X^{\pm 4/3}$ and $Y^{\pm 1/3}$ gauge bosons. But because the **24** does not occur in the LR products $\mathbf{5}^* \otimes \mathbf{10} = \mathbf{5} \oplus \mathbf{45}$ and $\mathbf{10} \otimes \mathbf{10} = \mathbf{5}^* \oplus \mathbf{45}^* \oplus \mathbf{50}^*$, no fermion masses are generated at this stage. Electroweak symmetry is broken by the $SU(5)$ generalization of the usual Higgs mechanism, a **5** of scalars that contains the standard-model Higgs doublet:

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_{EM}. \quad (6.6)$$

This pattern of symmetry breaking leads to the relations

$$m_e = m_d, \quad m_\mu = m_s, \quad m_\tau = m_b, \quad (6.7)$$

at the unification scale. The masses of the charge-2/3 quarks are separate parameters m_u, m_c, m_t .

Like the values of coupling constants, the values of particle masses depend on the scale on which they are observed. In leading logarithmic approximation, the running masses of the up-like quarks evolve as

$$\begin{aligned} \ln m_{u,c,t}(\mu) &\approx \ln m_{u,c,t}(M_U) + \frac{12}{33 - 2n_f} \ln \left(\frac{\alpha_3(\mu)}{\alpha_U} \right) \\ &+ \frac{27}{88 - 8n_f} \ln \left(\frac{\alpha_2(\mu)}{\alpha_U} \right) - \frac{3}{10n_f} \ln \left(\frac{\alpha_1(\mu)}{\alpha_U} \right); \end{aligned} \quad (6.8)$$

the down-like quark masses run as

$$\begin{aligned} \ln m_{d,s,b}(\mu) &\approx \ln m_{d,s,b}(M_U) + \frac{12}{33 - 2n_f} \ln \left(\frac{\alpha_3(\mu)}{\alpha_U} \right) \\ &+ \frac{27}{88 - 8n_f} \ln \left(\frac{\alpha_2(\mu)}{\alpha_U} \right) + \frac{3}{20n_f} \ln \left(\frac{\alpha_1(\mu)}{\alpha_U} \right); \end{aligned} \quad (6.9)$$

and the masses of the charged leptons evolve as

$$\begin{aligned} \ln m_{e,\mu,\tau}(\mu) &\approx \ln m_{e,\mu,\tau}(M_U) \\ &+ \frac{27}{88 - 8n_f} \ln \left(\frac{\alpha_2(\mu)}{\alpha_U} \right) - \frac{27}{20n_f} \ln \left(\frac{\alpha_1(\mu)}{\alpha_U} \right). \end{aligned} \quad (6.10)$$

Accordingly, the masses of the b -quark and the τ -lepton evolve to different values at low energies:

$$\ln \left[\frac{m_b(\mu)}{m_\tau(\mu)} \right] \approx \frac{12}{33 - 2n_f} \ln \left(\frac{\alpha_3(\mu)}{\alpha_U} \right) + \frac{3}{2n_f} \ln \left(\frac{\alpha_1(\mu)}{\alpha_U} \right). \quad (6.11)$$

Choosing $n_f = 6$, $1/\alpha_U = 40$, $1/\alpha_3(\mu) = 5$, and $1/\alpha_1 = 65$, we compute (which is to say, predict)

$$m_b = 2.91m_\tau = 5.16 \text{ GeV}/c^2, \quad (6.12)$$

in good agreement with the facts. To make this simple estimate, I have neglected the change in evolution at top threshold. Higgs-boson contributions, omitted here, are important for the evolution of heavy-fermion masses. The top Yukawa coupling plays a crucial role in supersymmetric models.

Starting from the equalities $m_s(M_U) = m_\mu(M_U)$ and $m_d(M_U) = m_e(M_U)$, equations (6.9) and (6.10) lead to the prediction that at $\mu \approx 1$ GeV,

$$\frac{m_s}{m_d} = \frac{m_\mu}{m_e}. \quad (6.13)$$

This is less successful; empirically, the left-hand-side is about 20, and the right-hand-side about 200. A more elaborate scheme for breaking the electroweak symmetry—say, adding a **45** of scalars—can give rise to a different simple pattern of masses at the unification scale. The simple pattern

$$m_s(M_U) = \frac{1}{3}m_\mu(M_U), \quad m_d(M_U) = 3m_e(M_U) \quad (6.14)$$

leads to

$$m_s \approx \frac{4}{3}m_\mu, \quad m_d \approx 12m_e, \quad (6.15)$$

at $\mu = 1$ GeV.

The important point in these exercises is not that $SU(5)$ gives us an understanding of the pattern of fermion masses, but the more general lesson that a simple pattern at the unification scale can manifest itself in a complicated (irrational!) pattern at low energies. This insight has spawned a new strategy for making sense of the pattern of fermion masses—and a new industry for theorists.¹²⁾ Begin with a promising unified theory, like supersymmetric $SU(5)$, which has advantages over ordinary $SU(5)$ for $\sin^2 \theta_W$, coupling constant unification, and the proton lifetime, or supersymmetric $SO(10)$, which can accommodate massive neutrinos. Then find “textures,” simple patterns of Yukawa matrices that lead to successful predictions for masses and mixing angles. Interpret these in terms of patterns of electroweak symmetry breaking. Finally, seek a derivation of—or at least a motivating principle for—the winning entry. The proof that this program has predictive power is that some schemes fail for m_t or $|V_{cb}|$.

7 Opportunity

As successful as the electroweak theory is in describing experimental observations,¹³⁾ we do not need hints from experiment to know that the theory is incomplete.¹⁴⁾ We have only to look at the many parameters of the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge theories of the strong, weak, and electromagnetic interactions to see opportunities for a more predictive theory. The 6 quark masses, 3 charged-lepton masses, 4 quark-mixing parameters, 3 coupling constants, 2 parameters of the Higgs potential, and 1 strong (CP) phase make 19 parameters whose values are not explained by the standard model. Seventeen of these numbers lie in the domain of the electroweak theory. Next, we can inquire into the self-consistency and naturalness of the electroweak theory. The hierarchy, naturalness, and triviality problems indicate that the electroweak theory is not complete.

As an illustration of these shortcomings, let us ask why the electroweak scale is small. Note that we do have some understanding, from the evolution of coupling constants down from the unification scale, of why the strong interaction becomes strong at a scale of about 1 GeV.

The $SU(2)_L \otimes U(1)_Y$ electroweak theory does not explain how the scale of electroweak symmetry breaking is maintained in the presence of quantum corrections. The problem of the scalar sector can be summarized neatly as follows.¹⁵⁾ The Higgs potential is

$$V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + |\lambda|(\phi^\dagger \phi)^2. \quad (7.1)$$

With μ^2 chosen to be less than zero, the electroweak symmetry is spontaneously broken down to the $U(1)$ of electromagnetism, as the scalar field acquires a vacuum expectation value that is fixed by the low-energy phenomenology,

$$\langle \phi \rangle_0 = \left(\begin{array}{c} 0 \\ \sqrt{-\mu^2/2|\lambda|} \end{array} \right) \equiv \left(\begin{array}{c} 0 \\ v/\sqrt{2} \end{array} \right) = (G_F \sqrt{8})^{-1/2} \approx 175 \text{ GeV}. \quad (7.2)$$

Three of the scalar degrees of freedom become the longitudinal components of the intermediate vector bosons W^+ , W^- , and Z^0 . The fourth emerges as a massive scalar particle, the Higgs boson, with mass given by

$$M_H^2 = 2|\lambda|v^2. \quad (7.3)$$

Beyond the classical approximation, scalar mass parameters receive quantum corrections from loops that contain particles of spins $J = 1, 1/2$, and 0 :

$$m^2(p^2) = m_o^2 + \underbrace{\text{---} \text{wavy} \text{---}}_{J=1} + \underbrace{\text{---} \text{loop} \text{---}}_{J=1/2} + \underbrace{\text{---} \text{circle} \text{---}}_{J=0} \quad (7.4)$$

The loop integrals are potentially divergent. Symbolically, we may summarize the content of (7.4) as

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \dots, \quad (7.5)$$

where Λ defines a reference scale at which the value of m^2 is known, g is the coupling constant of the theory, and the coefficient C is calculable in any particular theory. Instead of dealing with the relationship between observables and parameters of the Lagrangian, we choose to describe the variation of an observable with the momentum scale. In order for the mass shifts induced by radiative corrections to remain under control (*i.e.*, not to greatly exceed the value measured on the laboratory scale), either Λ must be small, so the range of integration is not enormous, or new physics must intervene to cut off the integral.

If the fundamental interactions are described by an $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry, *i.e.*, by quantum chromodynamics and the electroweak theory, then the natural reference scale is the Planck mass,

$$\Lambda \sim M_{\text{Planck}} \approx 10^{19} \text{ GeV}. \quad (7.6)$$

In a unified theory of the strong, weak, and electromagnetic interactions, the natural scale is the unification scale,

$$\Lambda \sim M_U \approx 10^{15}\text{-}10^{16} \text{ GeV}. \quad (7.7)$$

Both estimates are very large compared to the scale of electroweak symmetry breaking. We are therefore assured that new physics must intervene at an energy of approximately 1 TeV, in order that the shifts in m not be much larger than $v/\sqrt{2}$.

Only a few distinct scenarios for controlling the contribution of the integral in (7.5) can be envisaged. The supersymmetric solution¹⁶⁾ is especially elegant. Exploiting the fact that fermion loops contribute with an overall minus sign (because of Fermi statistics), supersymmetry balances the contributions of fermion and boson loops. In the limit of unbroken supersymmetry, in which the masses of bosons are degenerate with those of their fermion counterparts, the cancellation is exact:

$$\sum_{\substack{i=\text{fermions} \\ +\text{bosons}}} C_i \int dk^2 = 0. \quad (7.8)$$

If the supersymmetry is broken (as it must be in our world), the contribution of the integrals may still be acceptably small if the fermion-boson mass splittings ΔM are not too large. The condition that $g^2\Delta M^2$ be “small enough” leads to the requirement that superpartner masses be less than about 1 TeV/ c^2 .

A second solution to the problem of the enormous range of integration in (7.5) is offered by theories of dynamical symmetry breaking such as technicolor.¹⁷⁾ In technicolor models, the Higgs boson is composite, and new physics arises on the scale of its binding, $\Lambda_{TC} \simeq O(1 \text{ TeV})$. Thus the effective range of integration is cut off, and mass shifts are under control.

A third possibility is that the gauge sector becomes strongly interacting.¹⁸⁾ This would give rise to WW resonances, multiple production of gauge bosons, and other new phenomena at energies of 1 TeV or so. It is likely that a scalar bound state—a quasi-Higgs boson—would emerge with a mass less than about $1 \text{ TeV}/c^2$.

We cannot avoid the conclusion that some new physics must occur on the 1-TeV scale. This is the principal sharp motivation for multi-TeV hadron colliders—for the LHC. We seek to complete our understanding of electroweak symmetry breaking. A thorough investigation of the 1-TeV scale promises to solve the problem of gauge-boson masses and give us insight, if not a complete solution, into the origin of fermion masses. The large step in energy and sensitivity will also test the underpinnings of the standard model by allowing us to search for new forces, for composite quarks and leptons, and for new forms of matter.

8 Relevance

Physics is possible because we can analyze phenomena on one energy scale without understanding all energy scales. In other words, we need not understand everything before we can begin to answer something. In quantum field theory, it is frequently possible to identify the relevant degrees of freedom on some energy scale, and to formulate effective field theories that make sense in a restricted domain. Decoupling theorems codify the statement that degrees of freedom that come into play on a high scale do not matter on a low scale.

But the fact that we can formulate a consistent description of low-energy phenomena without understanding everything that happens all the way up to very high energies must not blind us to the additional insights that information from higher energies, or shorter distances, can bring. Early in this century, our scientific ancestors learned that to explain why a table is solid, or why a metal gleams, we must explore the atomic and molecular structure of matter at a billionth of human dimensions, where the laws of quantum mechanics take over from the customs of daily life. The recent discovery of the top quark in experiments at a billionth of the atomic scale inspires us to reconsider how the microworld influences our surroundings.

It is popular to say that top quarks were created in great numbers in the early moments after the big bang some fifteen billion years ago, disintegrated in a fraction of a second, and vanished from the scene until my colleagues learned to create them in the Tevatron at Fermilab. That would be reason enough to be interested in top: to learn how it helped sow the seeds for the primordial universe that has evolved into the world of diversity and change we live in. But it is not the whole story; it invests the top quark with a remoteness that hides its real importance—and understates the immediacy of particle physics. The real wonder is that here and now, every minute of every day, top affects the world around us. I would like to close by giving one striking example of top's influence on the everyday.¹⁹⁾

Consider a unified theory of the strong, weak, and electromagnetic interactions—three-generation $SU(5)$, say—in which all coupling constants take on a common value, α_U , at some high energy, M_U . If we adopt the point of view that the value of the coupling constant is fixed at the unification scale, then the value of the QCD scale parameter Λ_{QCD} depends on the mass of the top quark. If we evolve the $SU(3)_c$ coupling, α_s , down from the unification scale in the spirit of Georgi, Quinn, and Weinberg,⁸⁾ then the leading-logarithmic behavior is given by

$$1/\alpha_s(Q) = 1/\alpha_U + \frac{21}{6\pi} \ln(Q/M_U) \quad , \quad (8.1)$$

for $M_U > Q > m_t$. In the interval between m_t and m_b , the slope $(33 - 2n_f)/6\pi$ (where n_f is the number of active quark flavors) steepens to $23/6\pi$, and then increases by another $2/6\pi$ at every quark threshold. At the boundary $Q = Q_n$ between effective field theories with $n - 1$ and n active flavors, the coupling constants $\alpha_s^{(n-1)}(Q_n)$ and $\alpha_s^{(n)}(Q_n)$ must match. This behavior is shown by the solid line in Figure 3.

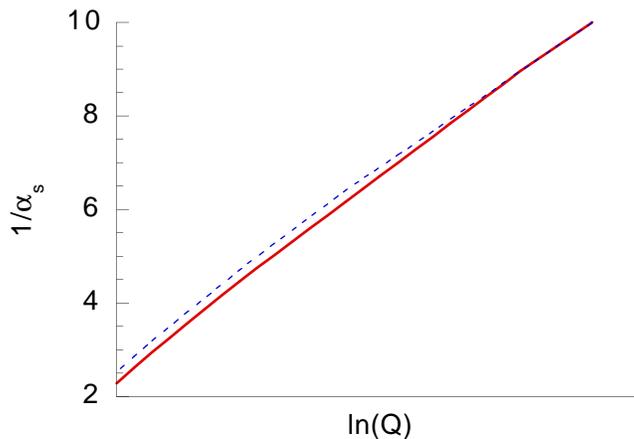


Figure 3: Two evolutions of the strong coupling constant.

To discover the dependence of Λ_{QCD} upon the top-quark mass, we use the one-loop evolution equation to calculate $\alpha_s(m_t)$ starting from low energies and from the unification scale, and match:

$$1/\alpha_U + \frac{21}{6\pi} \ln(m_t/M_U) = 1/\alpha_s(m_c) - \frac{25}{6\pi} \ln(m_c/m_b) - \frac{23}{6\pi} \ln(m_b/m_t) . \quad (8.2)$$

Identifying

$$1/\alpha_s(m_c) \equiv \frac{27}{6\pi} \ln(m_c/\Lambda_{\text{QCD}}) , \quad (8.3)$$

we find that

$$\Lambda_{\text{QCD}} = e^{-6\pi/27\alpha_U} \left(\frac{M_U}{1 \text{ GeV}} \right)^{21/27} \left(\frac{m_t m_b m_c}{1 \text{ GeV}^3} \right)^{2/27} \text{ GeV} . \quad (8.4)$$

The scale parameter Λ_{QCD} is the only dimensionful parameter in QCD; it determines the scale of the confinement energy that is the dominant contribution to the proton mass. We conclude that, in a simple unified theory,

$$M_{\text{proton}} \propto m_t^{2/27} . \quad (8.5)$$

The dotted line in Figure 3 shows how the evolution of $1/\alpha_s$ changes if the top-quark mass is reduced. We see from Equations (8.4) and (8.5) that a factor-of-ten decrease in the top-quark mass would result in a 20% decrease in the proton mass. We can't fully understand the origin of one of the most important parameters in the everyday world—the mass of the proton—without knowing the properties of the top quark.

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Footnotes and References

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19. I owe these insights to discussions with Bob Cahn. For additional examples of the links between particle physics and the quotidian, see R. N. Cahn, “The Eighteen Parameters of the Standard Model in Your Everyday Life,” to appear as a Colloquium in *Reviews of Modern Physics*.