Particle Physics: *The Standard Model*

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CERN Summer Lectures
17 – 27 July 2000

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SUMMARY: WHAT WE KNOW

→ FUNDAMENTAL CONSTITUENTS

- FEW IN NUMBER
- SIMPLE IN PROPERTIES

UNIFY QUARKS + LEPTONS ?

→ BASIC INTERACTIONS

- FEW IN NUMBER
- SIMPLE IN FORM

UNIFY INTERACTIONS ?

1

SUMMARY: WHAT WE KNOW

UNIFY PARTICLES + FORCES?

→ FUNDAMENTAL CONSTITUENTS

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UNIFY INTERACTIONS ?

UNIFICATION ≈ GENERALIZATION IS THE ESSENCE OF SCIENTIFIC EXPLANATION.

Examples:

- Electricity + magnetism + light (Maxwell)
- Atomic theory: relates thermodynamics + stat. mech. to Newtonian mechanics
- Chemistry + electricity (Faraday)
- Atomic structure + chemistry + QM

Key Points:

STRUCTURELESS CHARACTER OF THE QUARKS + LEPTONS

SUCCESS OF THE QUARK MODEL

hadron spectrum
quark-parton picture applied to
$\ell N \rightarrow \ell' +$ anything
$e^+e^- \rightarrow$ hadrons
$p\bar{p} \rightarrow$ jets

$SU(2)_L \otimes U(1)_Y$ ELECTROWEAK THEORY

(partial) unification
unitary + renormalizable (calculable)
$\rightarrow$ intermediate bosons $W^\pm$, Z^0
spurred discovery of neutral currents

NON-OBSERVATION OF FREE QUARKS

$\rightarrow$ notion of permanent confinement
"asymptotic freedom"
feeble int. at short distances
$\Rightarrow$ promise of unification

The World's Most Powerful Microscope

Fermilab's Tevatron Collider and Its Detectors

900-GeV protons: $c - 586$ km/h

1-TeV protons: $c - 475$ km/h

Improvement: 111 km/h!

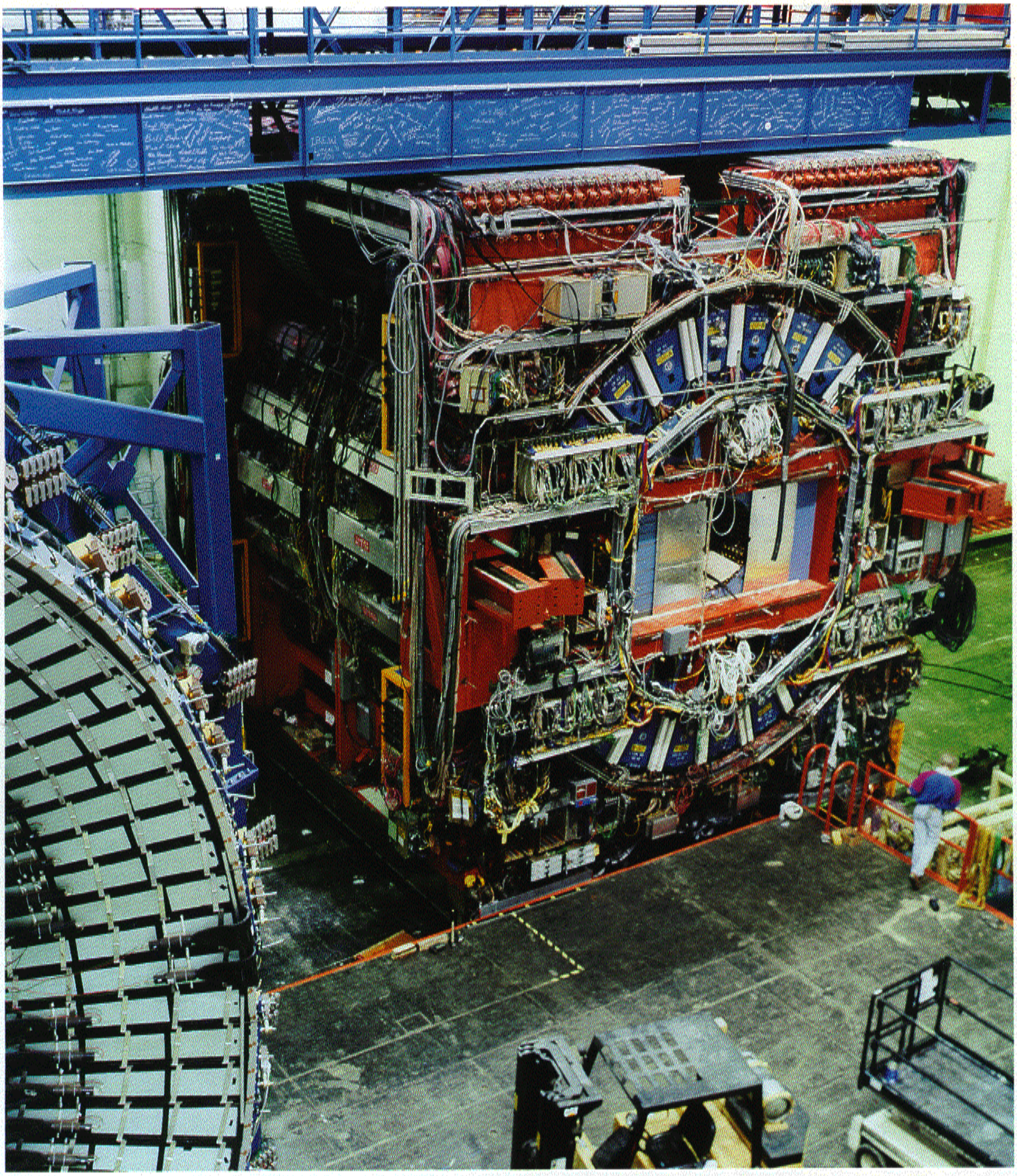
Protons pass my window 45 000 times per second

$\boxed{\text{Large Hadron Collider at CERN, 7-TeV protons: } c - 10 \text{ km/h}}$

Expect a 20× increase in luminosity in Tevatron Collider Run 2

PHYSICS TODAY

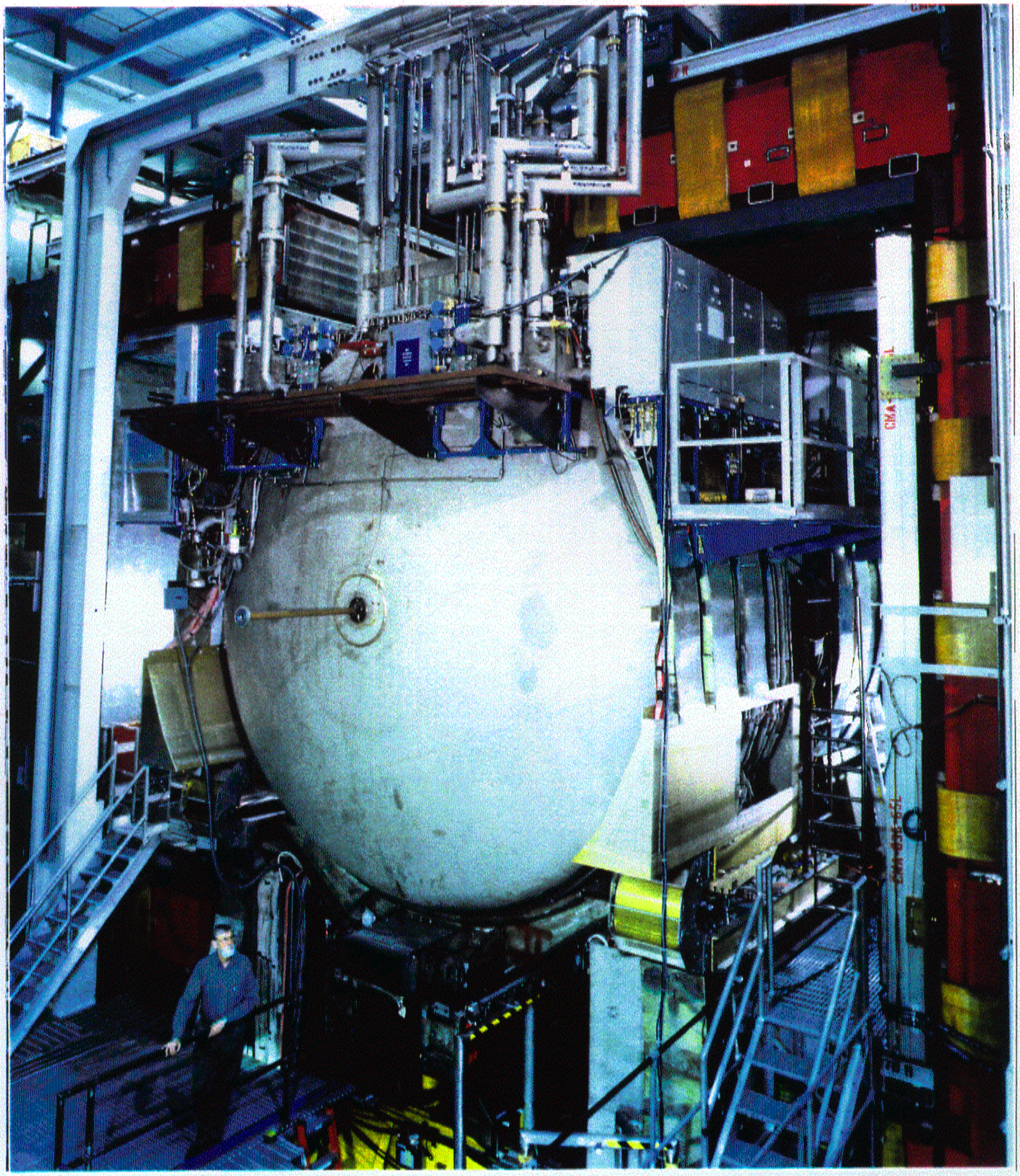
MAY 1997



THE REMARKABLE TOP QUARK

PHYSICS TODAY

MAY 1997



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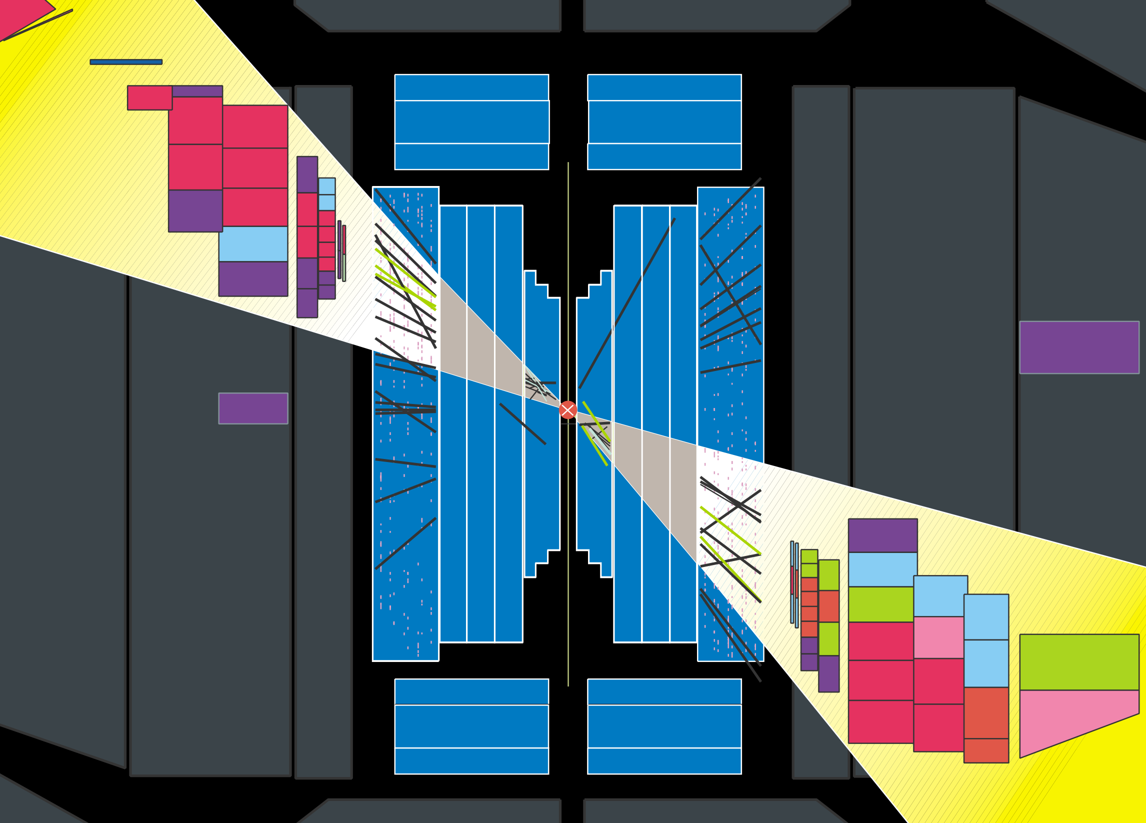
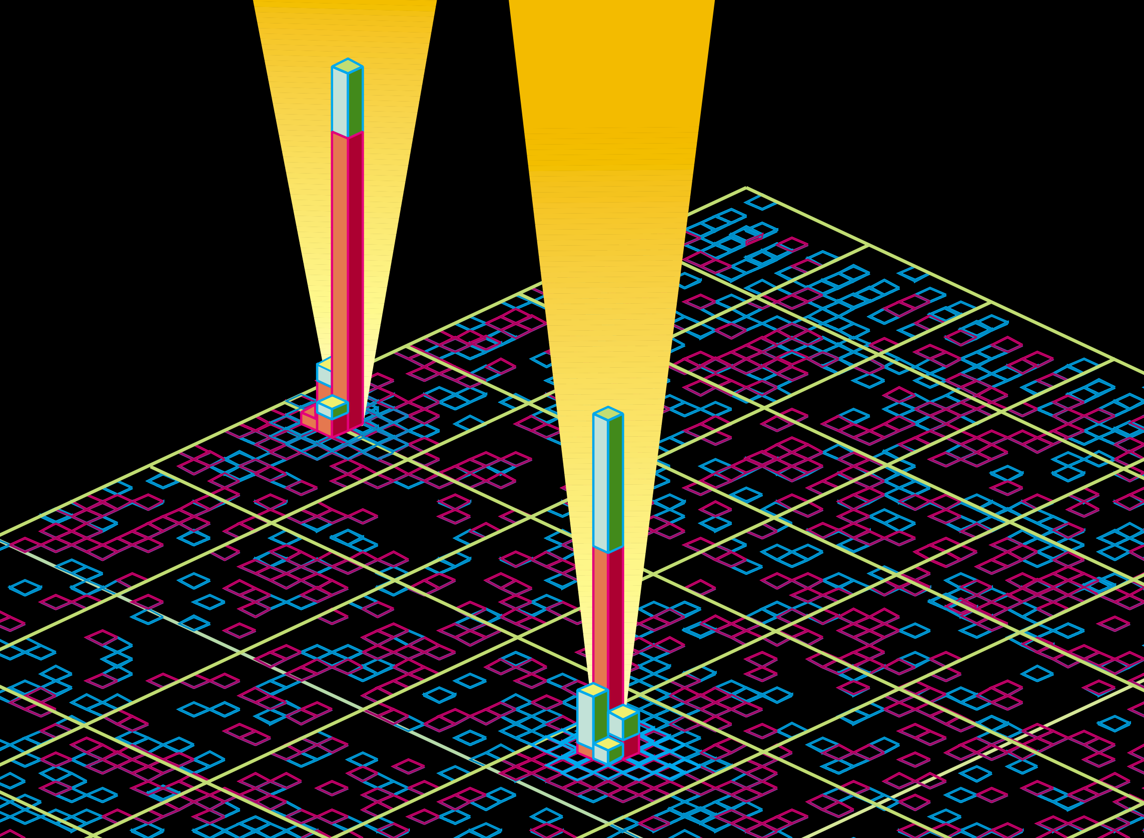




LEPTONS

SEEN AS FREE PARTICLES

WEAK • EM • GRAVITY

e^- $(0.510\ 999\ 07 \pm 0.000\ 000\ 15)\ \mathrm{MeV}/c^2$

$\tau > 4.3 \times 10^{23}\ \mathrm{y}$ (68% CL)

PRESUMED ABSOLUTELY STABLE

ν_e $\lesssim 10\text{–}15\ \mathrm{eV}/c^2$

μ^- $(105.658\ 389 \pm 0.000\ 034)\ \mathrm{MeV}/c^2$

$\tau = (2.197\ 03 \pm 0.000\ 04) \times 10^{-6}\ \mathrm{s}$

ν_μ $< \cancel{0.17}\ 0.19\ \mathrm{MeV}/c^2$ (90% CL)

τ^- $(1777.05^{+0.29}_{-0.26})\ \mathrm{MeV}/c^2$

$\tau = (290.2 \pm 1.2) \times 10^{-15}\ \mathrm{s}$

ν_τ $< 18.2\ \mathrm{MeV}/c^2$ (95% CL)

ALL SPIN-1/2, POINTLIKE ($\lesssim$ few $\times 10^{-17}$ cm)

FAMILY STRUCTURE

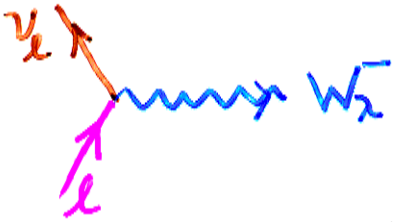
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

(ALMOST?) NO INTERACTIONS CROSS THESE FAMILY BOUNDARIES

"Charged current" $W^{\pm} \rightarrow \begin{pmatrix} \nu_\ell \\ \ell^- \end{pmatrix}_L$

WEAK INTERACTIONS TRANSFORM ONE LEPTON INTO ANOTHER, WITHIN A FAMILY

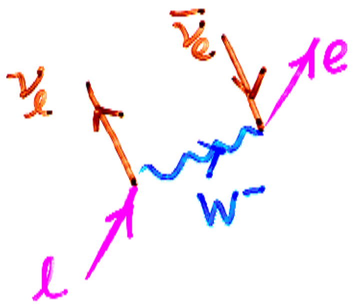
LEFT HANDED $\quad -i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{1/2} \bar{\nu}_\ell \gamma_\lambda (1-\gamma_5)\, \ell$



WEAK INTERACTIONS HAVE UNIVERSAL STRENGTH

TEST

COMPARE μ AND τ DECAY RATES



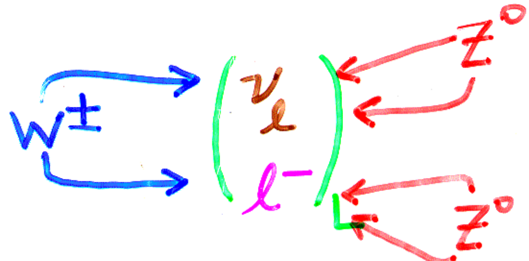
$$\Gamma(\ell^- \rightarrow \nu_\ell e^- \bar{\nu}_e) = \frac{G_F^2}{192\pi^3} m_\ell^5$$

COMMON FACTOR

$$G_F = 1.16639(2) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Cf. A. PICH, hep-ph/9701263

"Charged current"

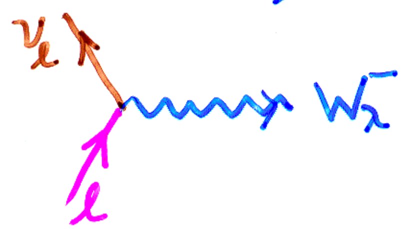


"Neutral Current"

WEAK INTERACTIONS TRANSFORM ONE LEPTON INTO ANOTHER, WITHIN A FAMILY

LEFT HANDED

$$-i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{1/2}\bar{\nu}_\ell\,\gamma_\lambda(1-\gamma_5)\,\ell$$



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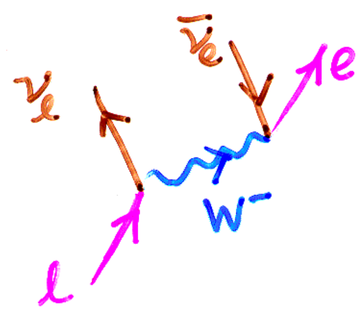

$$\Gamma(\ell^-\to\nu_\ell\, e^-\bar{\nu}_e)=\frac{G_F^2}{192\pi^3}m_\ell^5$$

COMMON FACTOR

$$G_F = 1.16639(2)\times10^{-5}\ \mathrm{GeV}^{-2}$$

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UNIVERSAL STRENGTH OF NC INTERACTIONS CONSISTENT WITH ALL EVIDENCE

NC AFFECTS BOTH LEFT-HANDED & RIGHT-HANDED ℓ^-

QUESTIONS

- WHY 3 FAMILIES OF LEPTONS?
- ARE THERE MORE?

LEP HAS LOOKED

$e^+e^- \to L^+L^-$ STABLE L^-: $M \gtrsim 84.2\ \mathrm{GeV}/c^2$ 95% CL

UNSTABLE $L^- \to \nu_L e^- \bar{\nu}_e$: $M \gtrsim 80.2\ \mathrm{GeV}/c^2$

$Z^0 \to \nu_L \bar{\nu}_L$ HARD TO OBSERVE

$\lambda_{int} \approx$ 50 MILLION KM OF WATER
(MEAN DISTANCE TO INTERACT)

OBSERVE BY NOT OBSERVING

$$N_\nu \equiv \underbrace{\Gamma(Z^0 \to \text{invisible})}_{} / \Gamma_{theory}(Z^0 \to \nu_i \bar{\nu}_i)$$

$$= \Gamma(Z^0 \to \text{all}) - \Gamma(Z^0 \to \text{visible})$$

this method

best determination

$$N_\nu = \cancel{3.07 \pm 0.12}$$

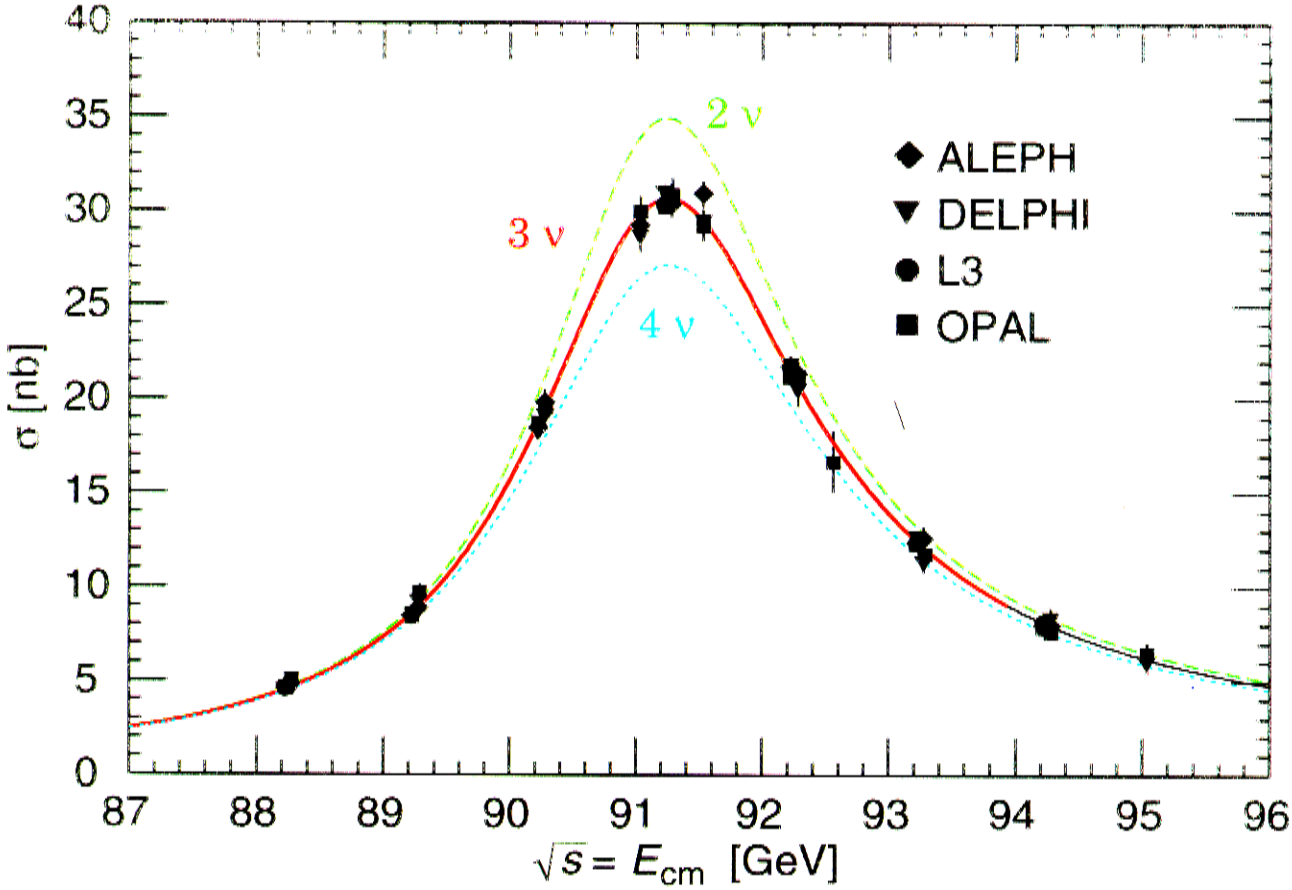
2.993 ± 0.011

2.994 ± 0.012

(G. Feldman + J. Steinberger, Sci. Am. 2/91)

$\Rightarrow$ ONLY 3 FAMILIES WITH MASSLESS (LIGHT) NEUTRINOS

Annihilation Cross Section Near M_Z

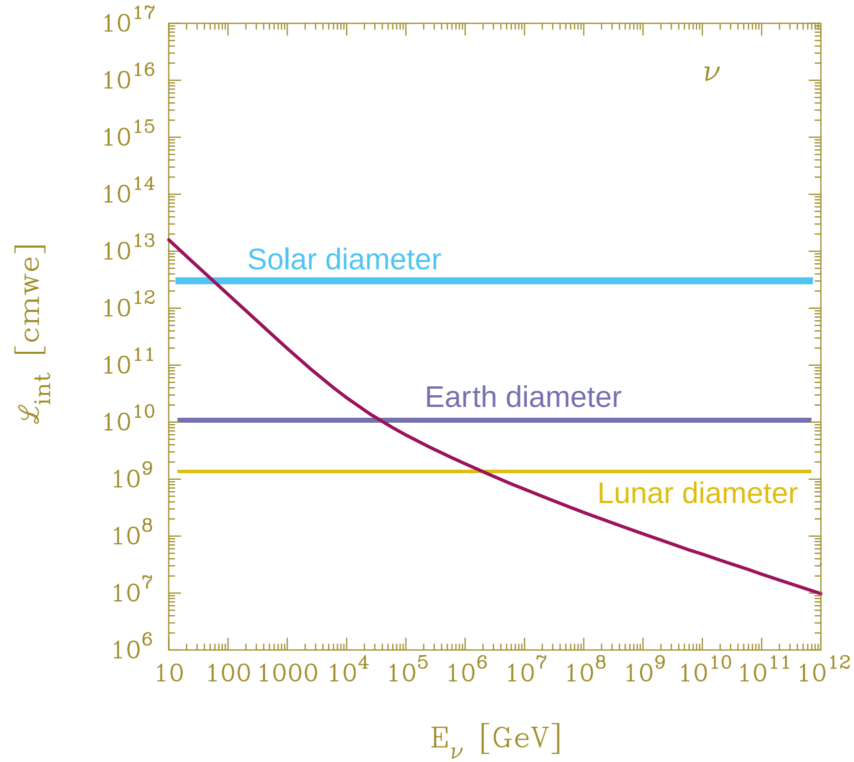


ν

Neutrinos are among the most abundant particles in the Universe

- Inside your body are more than 10 million (10^7) neutrinos left over from the Big Bang.
- Each second, some 10^{14} neutrinos made in the Sun pass through your body.
- Each second, about a thousand neutrinos made in Earth's atmosphere by cosmic rays pass through your body.
- Other neutrinos reach us from natural (radioactive decays of elements inside the Earth) and artificial (nuclear reactors) sources.

Neutrinos Traverse Vast Amounts of Material



Interaction Length of a 100-GeV ν = 25 million km H_2O ≈ 230 Earth diameters.

1 Earth diameter = 11 kilotonnes/ cm^2.
Atmosphere ≈ 10^3 cmwe vertical, ≈ 3.6×10^4 cmwe horizontal.

In Fermilab's neutrino beam, only 1 ν in 10^{11} will interact in your body.

Don't be neutrinos!

- IS LEPTON NUMBER CONSERVED ?

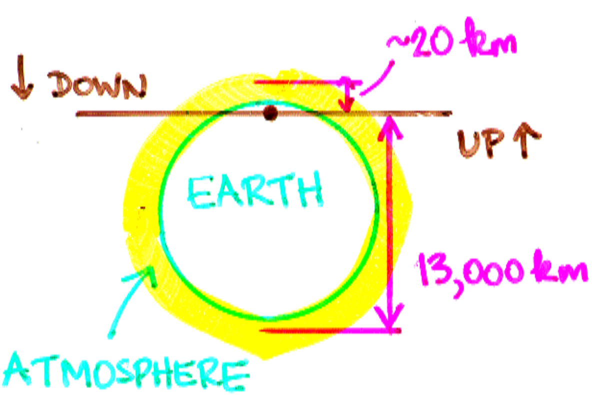
  $L-\bar{L}$?
  Separate e, μ, τ conservation ?

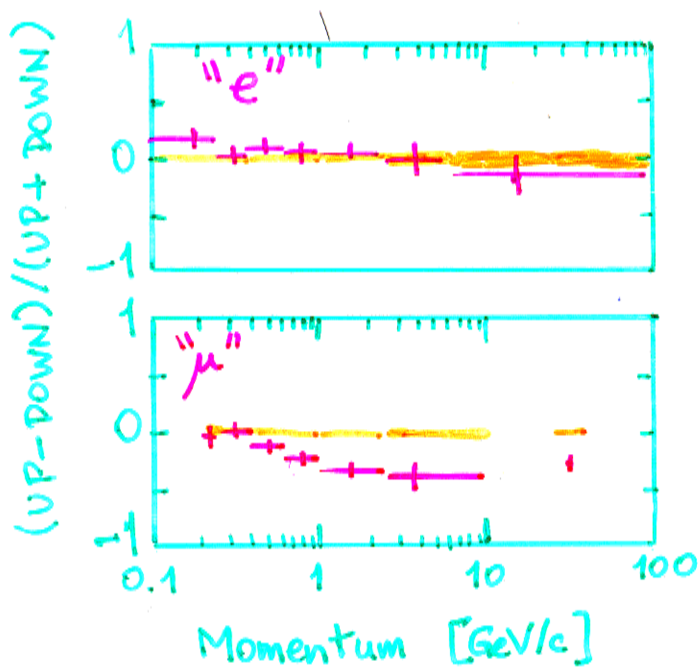
- ARE NEUTRINOS MASSLESS ?

- DO NEUTRINO FLAVORS MIX ?

  "Neutrino Oscillations"

Super-KamioKande





hep-ex/9807003

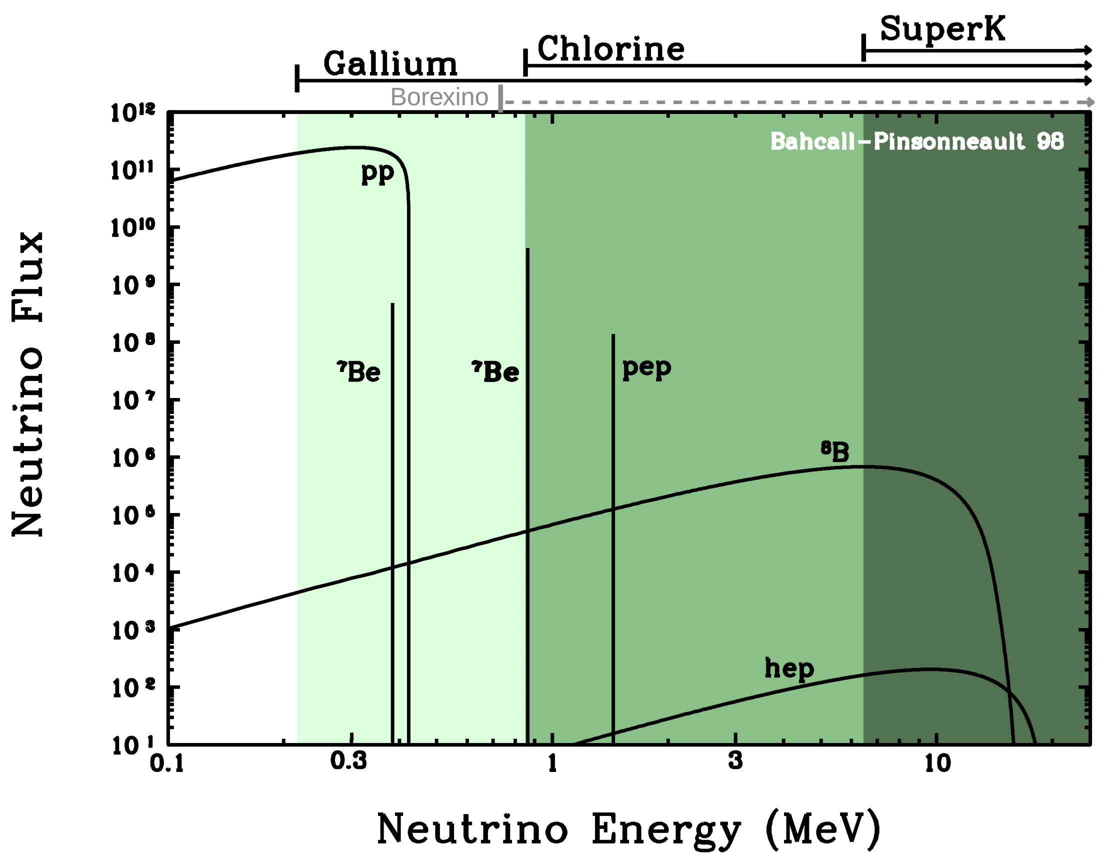
Suggests ν_e unchanged, but
$\nu_\mu \rightarrow \nu_\tau$ OR ν_x while propagating.

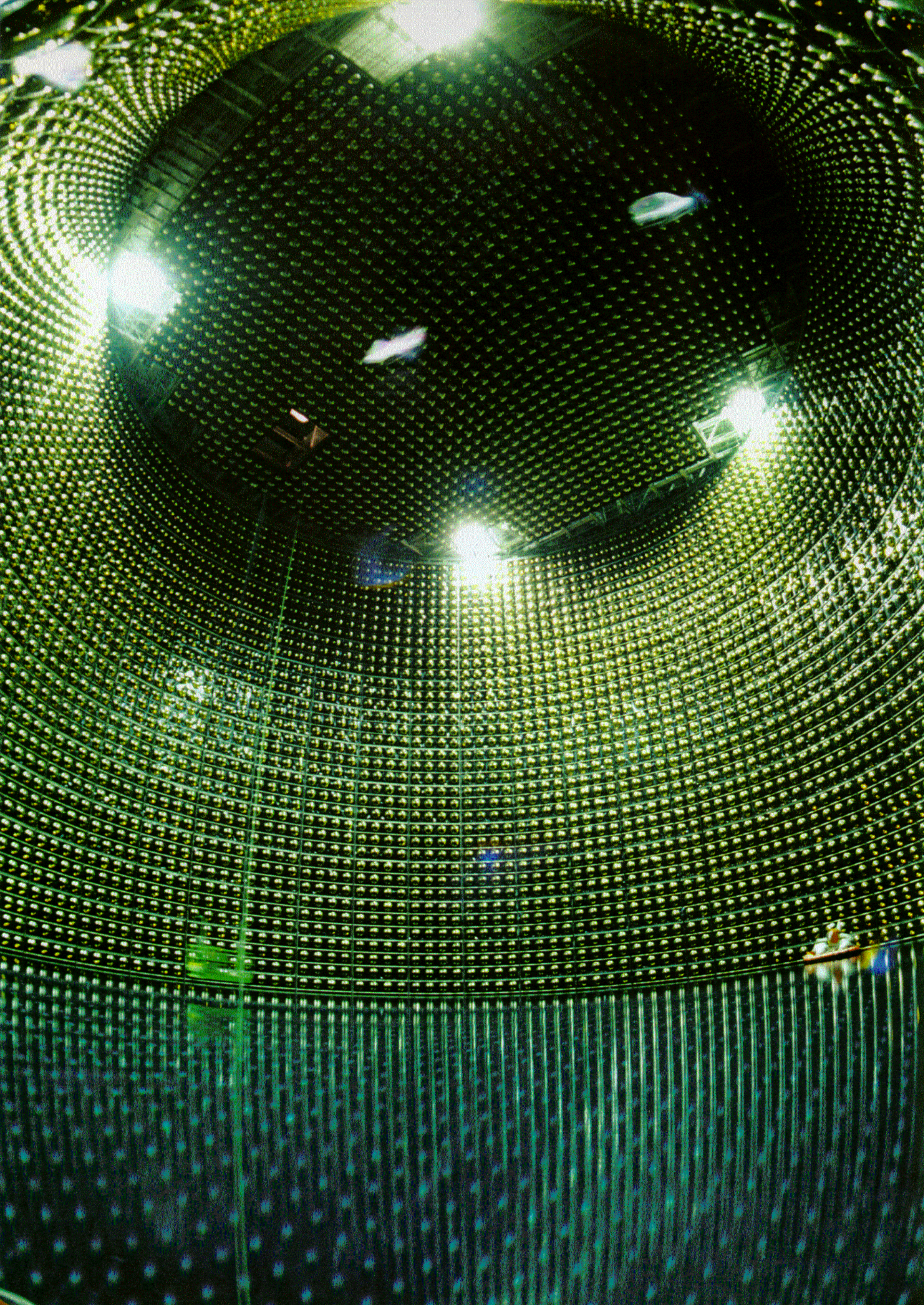
- ARE LEPTONS ELEMENTARY ?

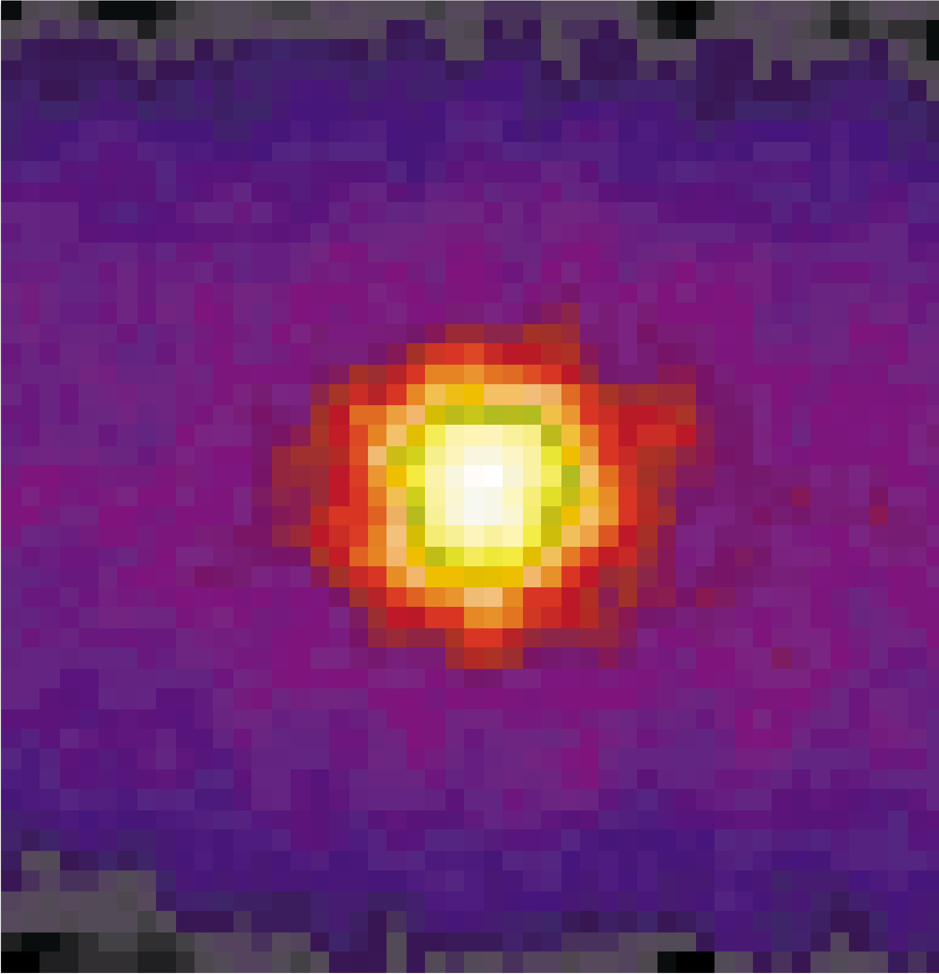
Detecting Neutrinos from the Sun

The nuclear burning that powers the Sun produces neutrinos as well as light and heat. Overall, . . .

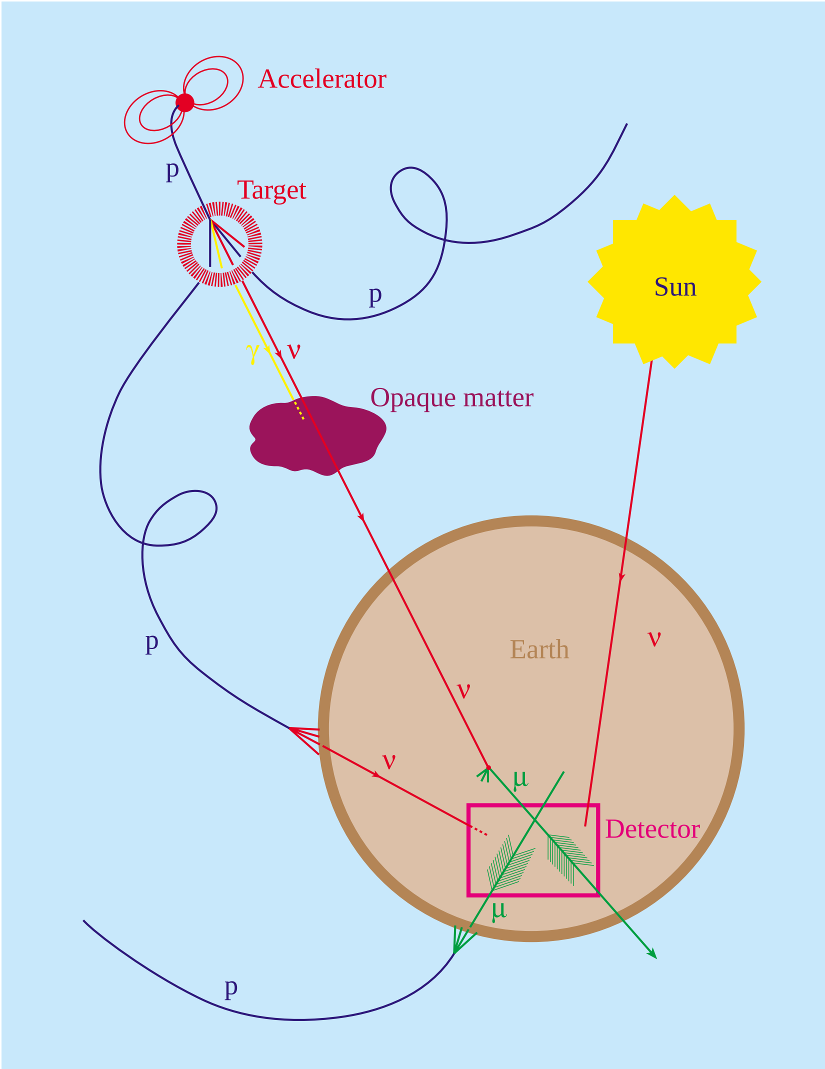
$$4p \longrightarrow\ ^4\mathrm{He} + 2e^+ + 2\nu_e + 25\ \mathrm{MeV}$$





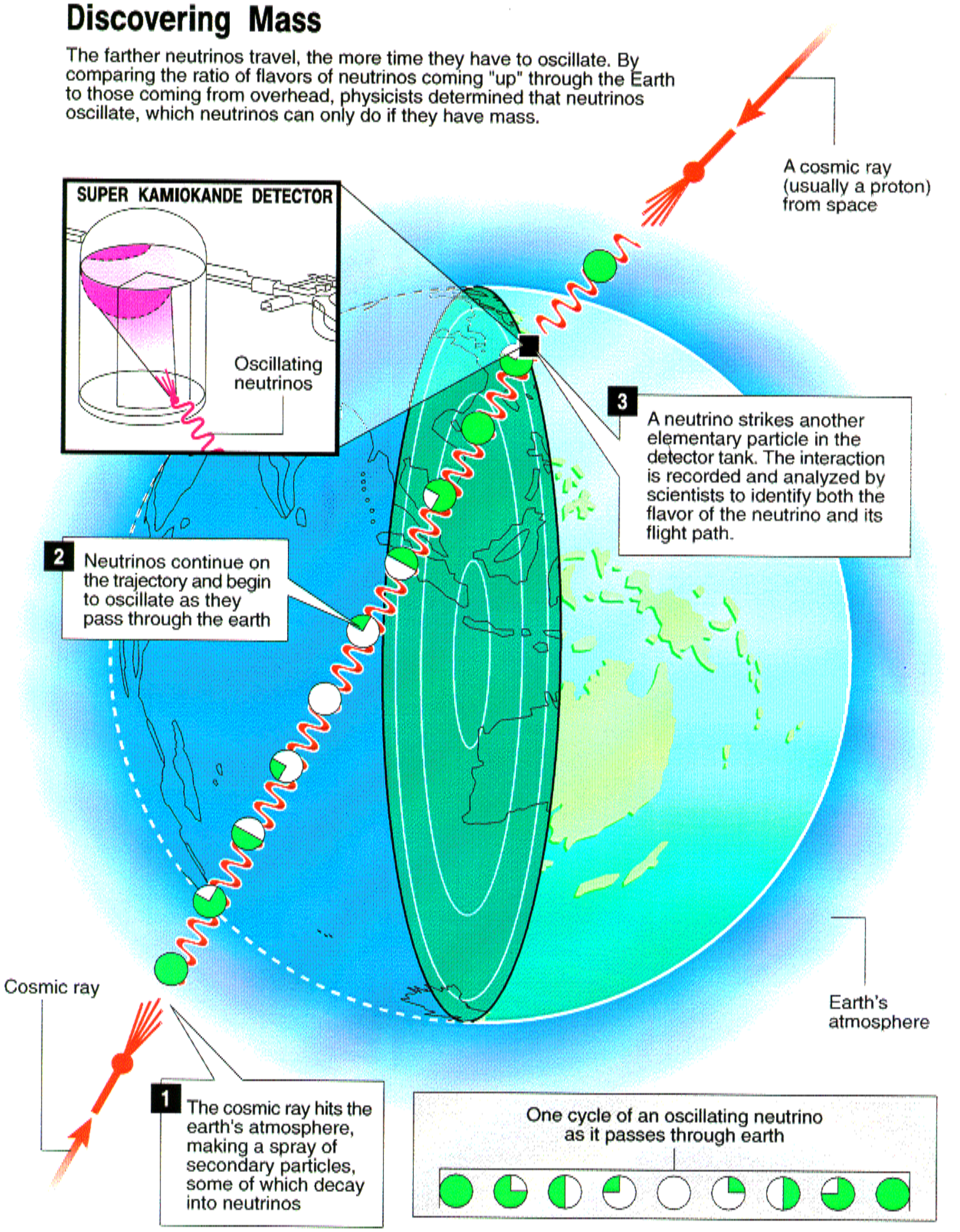


Cosmic Rays Produce Neutrinos in the Atmosphere



Discovering Mass

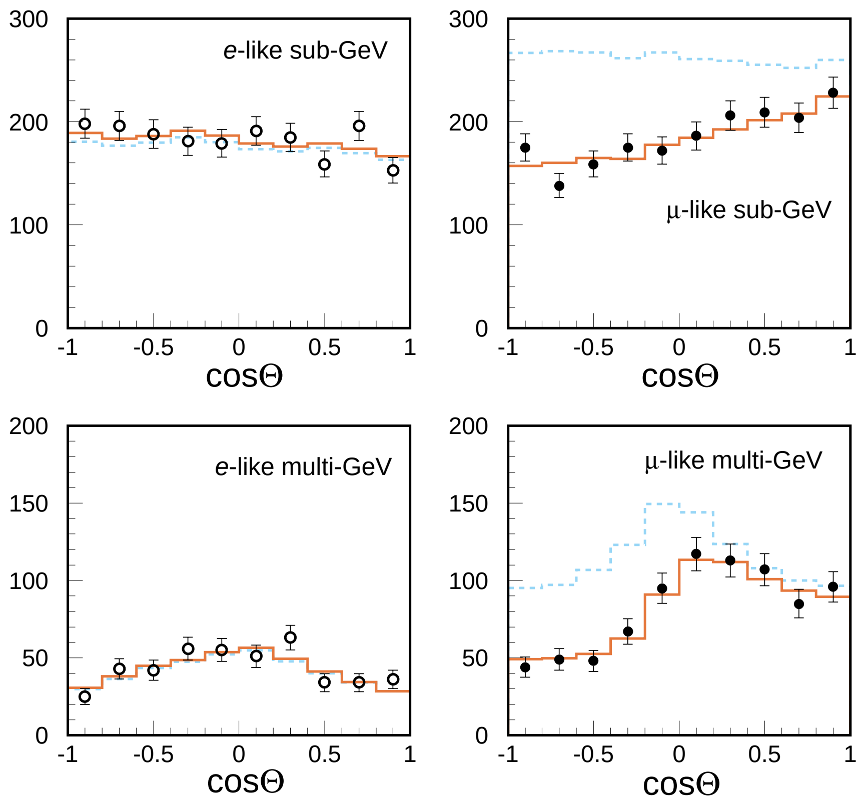
The farther neutrinos travel, the more time they have to oscillate. By comparing the ratio of flavors of neutrinos coming "up" through the Earth to those coming from overhead, physicists determined that neutrinos oscillate, which neutrinos can only do if they have mass.



University of Hawai'i media graphic

SuperK's Zenith-Angle Dependence

Downward ν ($\cos\theta = 1$) travel about 15 km.
Upward ν ($\cos\theta = -1$) travel up to 13 000 km.



Upward ν_μ, which travel longest path, are fewer than expected. Oscillations?

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Define the requirements for an experiment to measure the gyromagnetic ratio of the tau lepton, taking into account the τ lifetime and the anticipated result $g_\tau \approx 2$.

For background, become acquainted with the methods used to measure the magnetic anomalies of the electron [A. Rich and J. Wesley, *Rev. Mod. Phys.* **44**, 250 (1972); R. S. Van Dyck, Jr., P. B. Schwinberg, and H. G. Dehmelt, *Phys. Rev. Lett.* **38**, 93 (1977); R. S. Van Dyck, Jr., "Anomalous Magnetic Moment of Single Electrons and Positrons: Experiment," in *Quantum Electrodynamics,* edited by T. Kinoshita (World Scientific, Singapore, 1990), p. 322.] and muon [F. Combley, F. J. M. Farley, and E. Picasso, *Phys. Rep.* **68**, 93 (1981); F. J. M. Farley, and E. Picasso, "The Muon $g-2$ Experiments," in Kinoshita's *QED*, p. 479], and the magnetic dipole moments of the nucleons [N. F. Ramsey, *Molecular Beams*, Oxford University Press, Oxford, 1956] and unstable hyperons [L. Schachinger *et al.*, *Phys. Rev. Lett.* **41**, 1348 (1978); L. G. Pondrom, *Phys. Rep.* **122**, 57 (1985). For an interesting new technique, see D. Chen *et al.*, *Phys. Rev. Lett.* **69**, 3286 (1992), and V. V. Baublis, *et al.*, *Nucl. Inst. Meth.* **B90**, 150 (1994)]. Indirect determinations of $(g-2)_\tau$ are discussed by R. Escribano and E. Massó, "Improved Bounds on the Electromagnetic Dipole Moments of the Tau Lepton," (electronic archive: hep–ph/9609423); G. Köpp, D. Schaile, M. Spira, and P. Zerwas, *Z. Phys.* *C***65**, 545 (1995); M. A. Samuel and G. Li, *Int. J. Theor. Phys.* 33, 1471 (1994).

QUARKS

NOT OBSERVED AS FREE PARTICLES

STRONG · WEAK · EM · GRAVITY

Approximate family structure

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

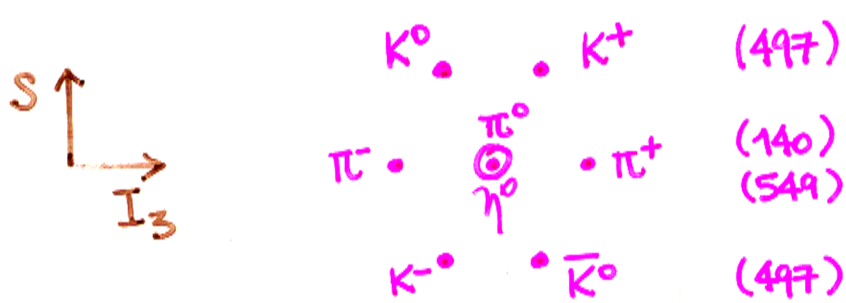
mass eigenstates $\neq$ flavor eigenstates

ALL SPIN-1/2, POINTLIKE ($r \lesssim \pi \times 10^{-17}$ cm)

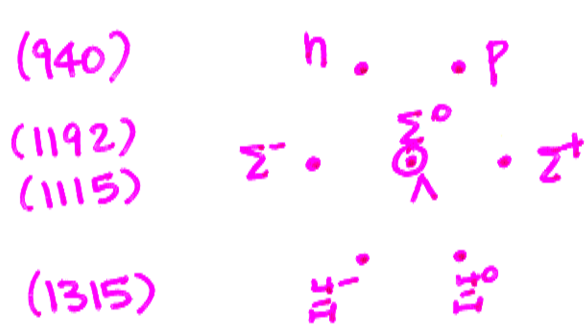
Smith, ARNPS 39, 73('89) $\boxed{\text{NO FREE QUARKS SEEN}}$

Hadron spectroscopy $\longrightarrow$ $SU(3)_{flavor}$

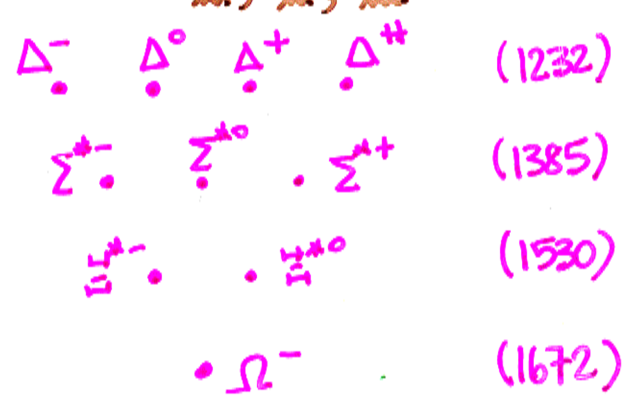
MESONS (spin 0) $\quad \underset{\sim}{1} \oplus \underset{\sim}{8}$



BARYONS $\quad \underset{\sim}{1}, \underset{\sim}{8}, \underset{\sim}{10}$



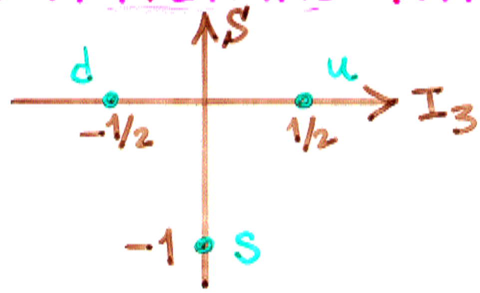
Spin-1/2



spin-3/2

Gell-Mann, Zweig

FUNDAMENTAL TRIPLET OF QUARKS



MESONS: $q\bar{q}$ $\quad \underset{\sim}{3} \otimes \underset{\sim}{3}^* = \underset{\sim}{1} \oplus \underset{\sim}{8}$

BARYONS: qqq $\quad \underset{\sim}{3} \otimes \underset{\sim}{3} \otimes \underset{\sim}{3} = \underset{\sim}{1} \oplus \underset{\sim}{8} \oplus \underset{\sim}{8} \oplus \underset{\sim}{10}$

- WHY THESE COMBINATIONS?

  Other possibilities?

  $qq\bar{q}\bar{q}$ $\quad 6q$ $\quad 4q\bar{q}$ $\quad \ldots$?

- WHAT DO FLAVORS MEAN?

QUARK PROPERTIES

Baryon number $\quad B(q) = 1/3$

Electric charge $\quad Q(u) = +2/3$

$Q(d) = Q(s) = -1/3$

Spin-1/2

TESTS

Charge

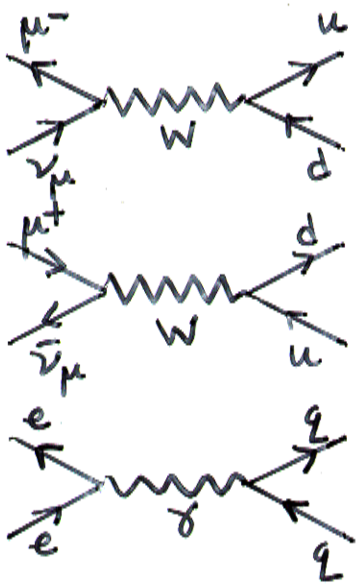
- Reproduces

uuu	Δ^{++}
uud	Δ^{+}
udd	Δ^{0}
ddd	Δ^{-}
sss	Ω^{-}

- Taking out overall coupling strengths,

$$\sigma(\ell T \to \ell' + X)$$

measures a "structure fcn" characteristic of the target



$\nu_\mu T \to \mu^- + X$ $\qquad$ $\nu_\mu d \to \mu^- u$

$F_2(\nu T)$ counts down quarks in T

$\bar\nu_\mu T \to \mu^+ + X$ $\qquad$ $\bar\nu_\mu u \to \mu^+ d$

$F_2(\bar\nu T)$ counts up quarks in T

$eT \to e + X$ $\qquad$ $eq \to eq$

$F_2(eT)$ counts up quarks $\times e_u^2$
$+$ down quarks $\times e_d^2$

ISOSCALAR TARGET (e.g. Deuterium)

$$\frac{F_2(eD)}{F_2(\nu D) + F_2(\bar\nu D)} = \frac{3e_u^2 + 3e_d^2}{3+3}$$

$$= \frac{1}{2}\cdot\frac{5}{9} = \frac{5}{18}$$

Observed values very close!

Spin

- Spectroscopy

$$q\bar{q} \rightarrow J^{PC} = \underbrace{0^{-+}, 1^{--}}_{L=0} \quad \underbrace{0^{++}, 1^{++}, 1^{+-}, 2^{++}}_{L=1} \ \cdots$$

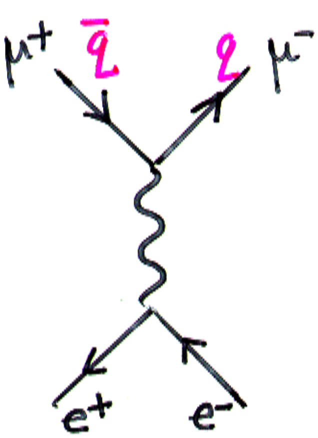
Nicest illustrations for heavy quarks ••

"IMPOSSIBLE" COMBINATIONS

0^{--}, 0^{+-}, 1^{-+} ... "exotic"

NOT ROUTINELY OBSERVED

See PRL 79, 1630 ('97) for recent claim of a 1^{-+} meson $\rightarrow \eta\pi^-$
$M \approx 1370$ MeV/c^2, $\Gamma \approx 385$ MeV

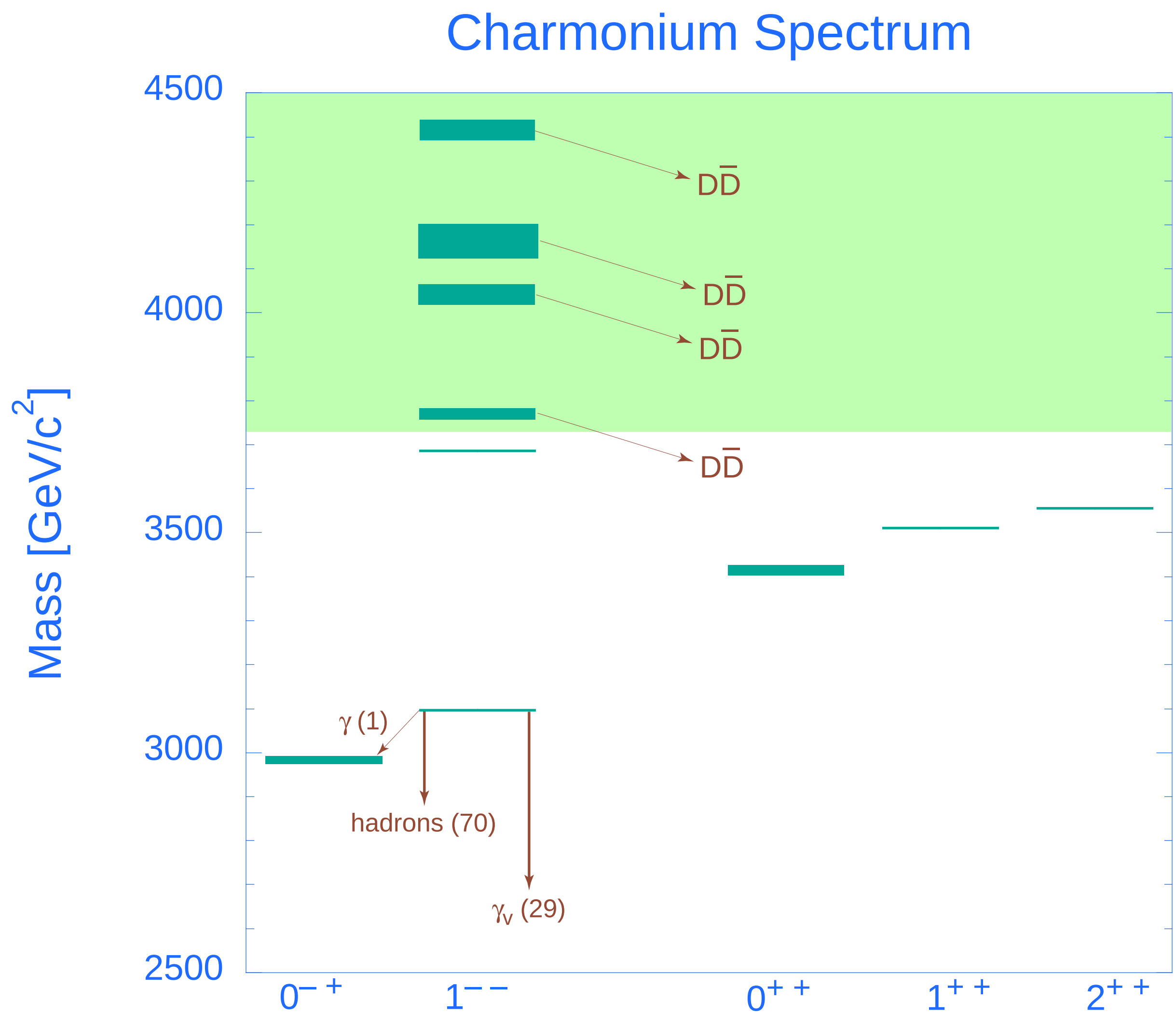


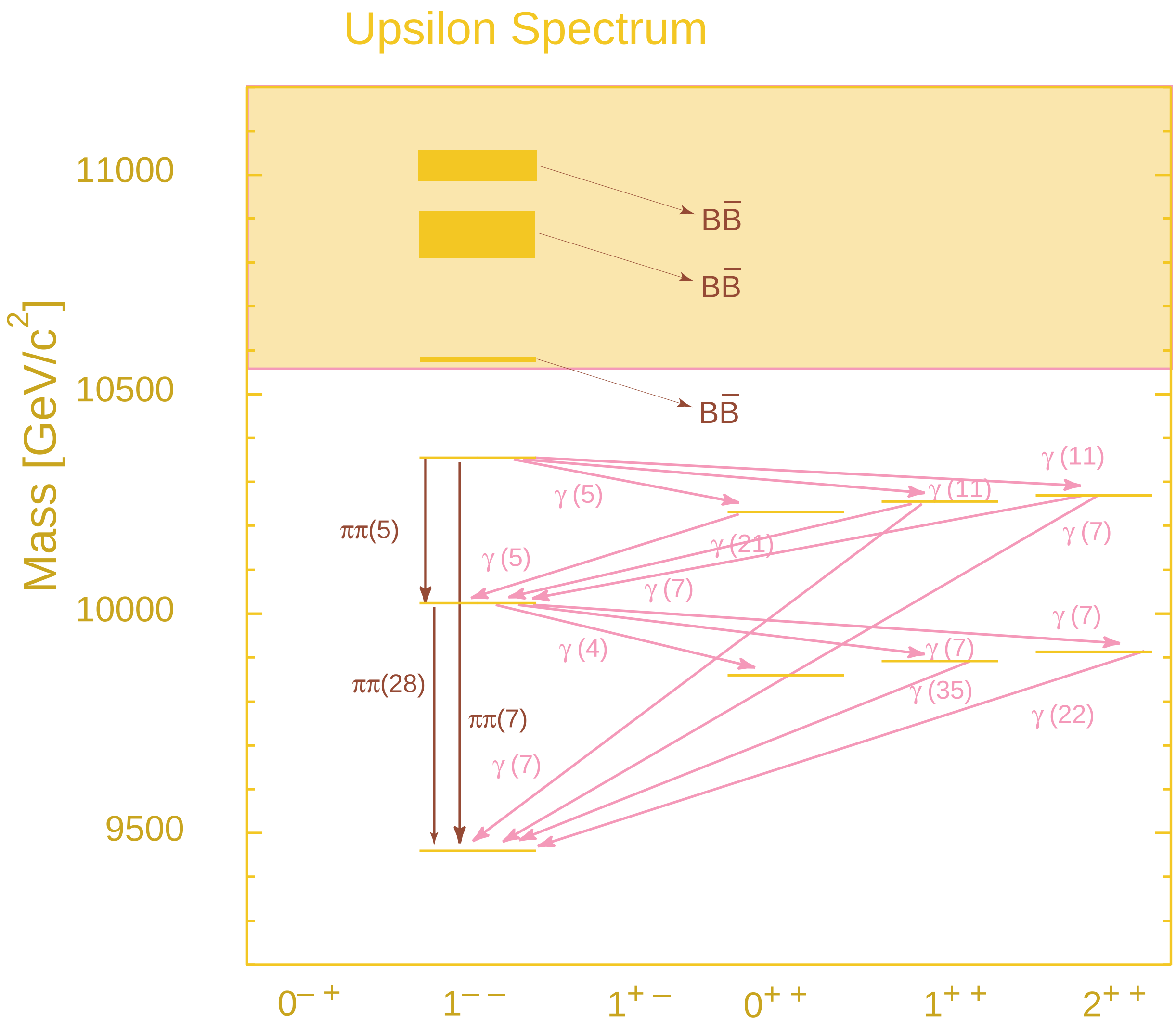
- Jet structure in $e^+e^- \rightarrow$ hadrons

$e^+e^- \rightarrow q\ \bar{q}$
$\quad\quad\quad\quad \bar{q} \rightarrow$ hadrons
$\quad\quad\quad\quad q \rightarrow$ hadrons

has same angular dist^{n} as

$e^+e^- \rightarrow \mu^+\mu^-$


Charmonium Spectrum
Mass [GeV/c^2]
4500
4000
3500
3000
2500
$D\bar{D}$
$D\bar{D}$
$D\bar{D}$
$D\bar{D}$
γ (1)
hadrons (70)
γ_v (29)
0^{-+}
1^{--}
0^{++}
1^{++}
2^{++}


Upsilon Spectrum
Mass [GeV/c^2]
11000
10500
10000
9500
$B\bar{B}$
$B\bar{B}$
$B\bar{B}$
ππ(5)
ππ(28)
ππ(7)
γ (5)
γ (5)
γ (4)
γ (7)
γ (21)
γ (7)
γ (11)
γ (11)
γ (7)
γ (7)
γ (7)
γ (35)
γ (22)
0^{-+}
1^{--}
1^{+-}
0^{++}
1^{++}
2^{++}

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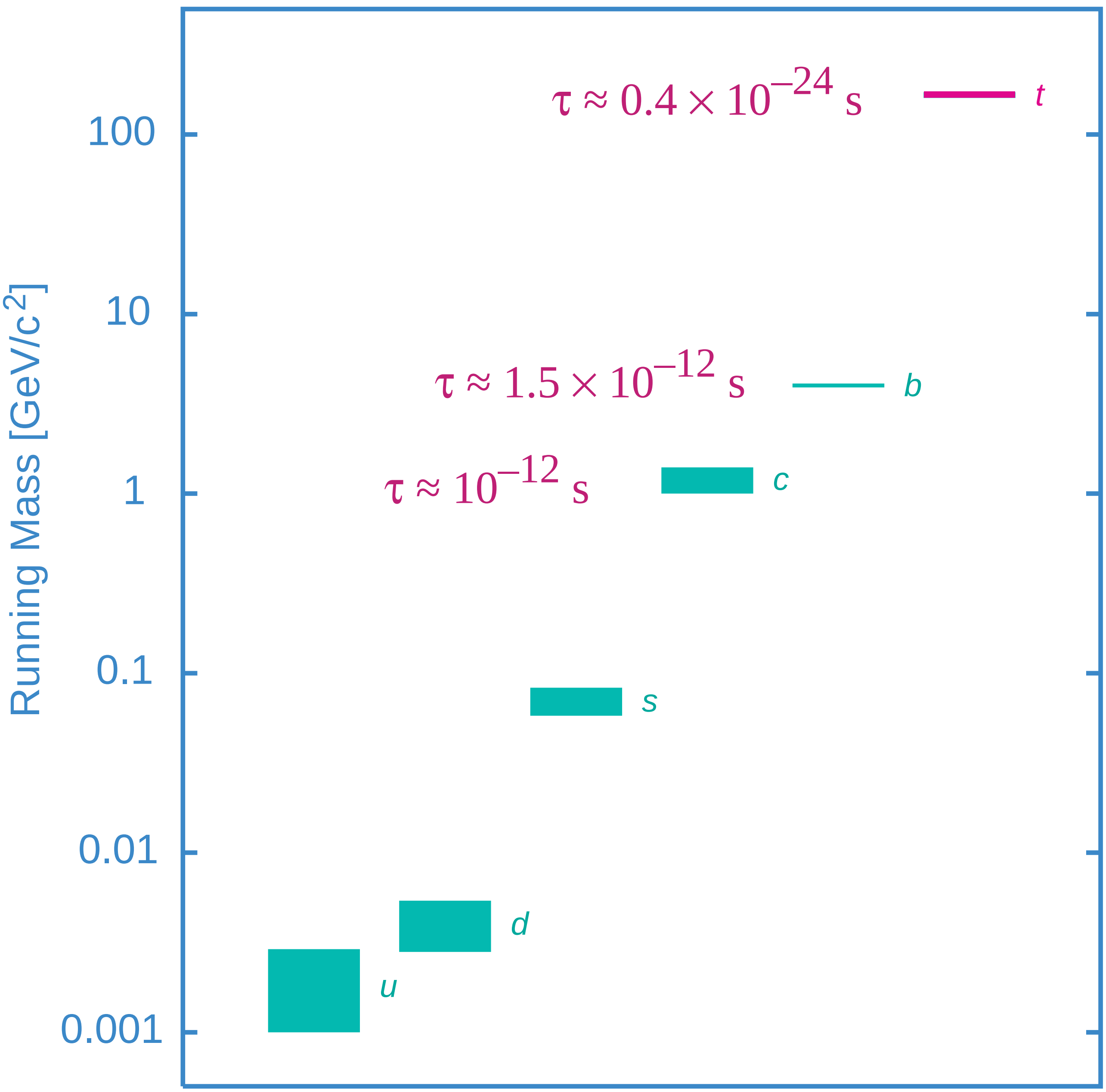
Consider bound states composed of a b-quark and a $\bar{b}$-antiquark. For $(b\bar{b})$ composites,

(a) Show that a bound state with orbital angular momentum L must have quantum numbers

$$C = (-1)^{L+S} \;\; P = (-1)^{L+1},$$

where S is the spin of the composite system.

(b) Allowing for both orbital and radial excitations, construct a schematic mass spectrum of $(b\bar{b})$ bound states. Label each state with its quantum numbers J^{PC}.


Running Mass [GeV/c2]
100
10
1
0.1
0.01
0.001
τ ≈ 0.4 × 10−24 s
t
τ ≈ 1.5 × 10−12 s
b
τ ≈ 10−12 s
c
s
d
u

THE COLOR OF QUARKS

SIMPLE QUARK MODEL CONFLICTS WITH PAULI'S EXCLUSION PRINCIPLE.

Δ^{++} WAVE FUNCTION

uuu WITH $\left.\begin{matrix} S = 3/2 \\ I = 3/2 \\ L = 0 \end{matrix}\right\}$ SYMMETRIC!

HIDDEN DEGREE OF FREEDOM?

"COLOR"

Suppose each quark FLAVOR comes in 3 distinct COLORS

$u_R \quad u_G \quad u_B$

$SU(3)_{color}$ SYMMETRY

- HADRONS ARE COLOR SINGLETS
- WAVE FCNS ANTISYMMETRIC IN COLOR, HENCE OVERALL

"EXPLAINS" WHY $q\bar{q}$, qqq ARE SPECIAL

$q\bar{q}$: $1 \oplus \cancel{8}$

qq : $\cancel{6} \oplus \cancel{3}^*$

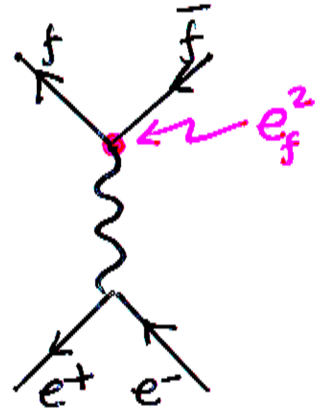
qqq : $1 \oplus \cancel{8} \oplus \cancel{8} \oplus \cancel{10}$

$4q$: $\not\supset 1$

$\vdots$

CONSEQUENCES / TESTS OF COLOR

$\rightarrow \quad R \equiv \dfrac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$



$$= 3 \sum_q e_q^2$$

$$= 3\left\{\left(\tfrac{2}{3}\right)^2 + \left(\tfrac{-1}{3}\right)^2 + \left(\tfrac{-1}{3}\right)^2\right\}$$

u d s

$$= 3 \cdot \tfrac{2}{3} = 2$$

(DATA)
$1\,\text{GeV} < \sqrt{s} < 4\,\text{GeV}$

HIGHER ENERGIES, INCLUDE

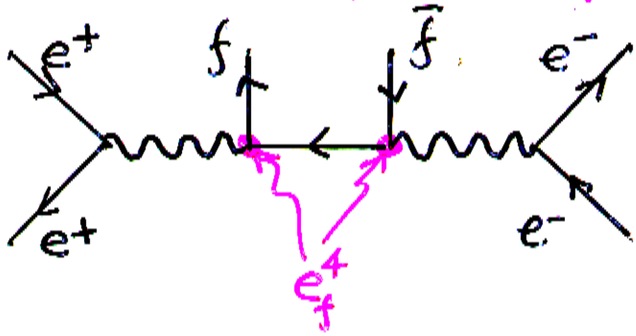
c: $3(2/3)^2 = 4/3$

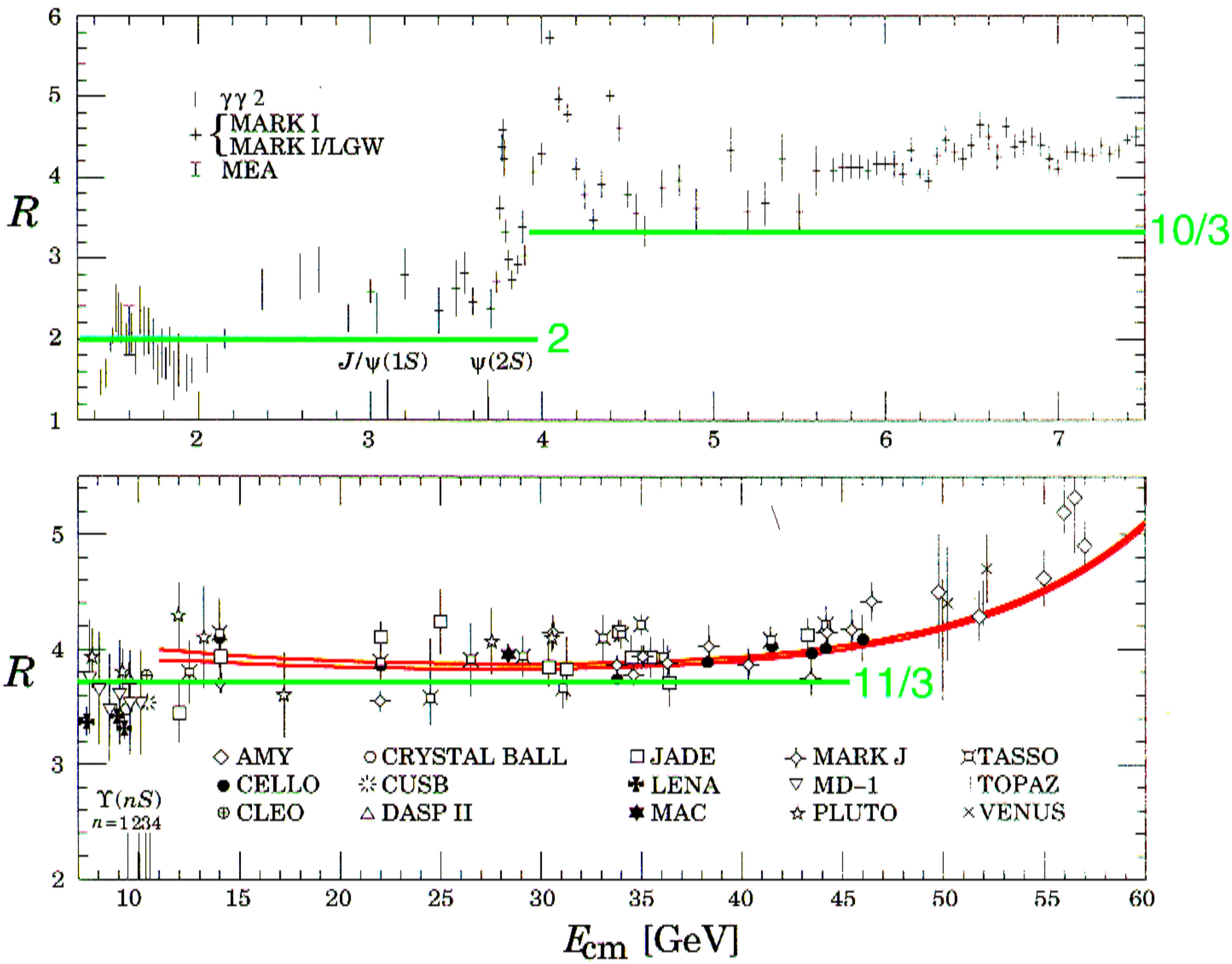
b: $3(-1/3)^2 = 1/3$

$\sum = 11/3$ ✓

$\rightarrow \quad R_{\gamma\gamma} \equiv \dfrac{\sigma(e^+e^- \rightarrow e^+e^- q\bar{q})}{\sigma(e^+e^- \rightarrow e^+e^-\mu^+\mu^-)} = 3 \sum_q e_q^4$

$$= 3\left\{\left(\tfrac{2}{3}\right)^4 + \left(\tfrac{-1}{3}\right)^4 + \left(\tfrac{-1}{3}\right)^4\right\} = \tfrac{2}{3} \checkmark$$





R
γγ 2
MARK I
MARK I/LGW
MEA
J/ψ(1S)
ψ(2S)
2
10/3
11/3
AMY
CELLO
CLEO
CRYSTAL BALL
CUSB
DASP II
JADE
LENA
MAC
MARK J
MD-1
PLUTO
TASSO
TOPAZ
VENUS
Υ(nS)
n=1234
E_{cm} [GeV]

2 THE IDEA OF GAUGE INVARIANCE

CLASSICAL ELECTRODYNAMICS

- Maxwell's eqn. for magnetic charge,

$$\nabla \cdot \underset{\sim}{B} = 0 \qquad \text{(no monopoles)}$$

is guaranteed by writing

$$\underset{\sim}{B} = \nabla \times \underset{\sim}{A} \qquad (\underset{\sim}{A}: \text{vector potential})$$

because $\nabla \cdot (\nabla \times \underset{\sim}{A}) = 0$.

Add any gradient to $\underset{\sim}{A}$:

$$\underset{\sim}{A} \to \underset{\sim}{A} + \nabla \Lambda$$

and the magnetic field is unchanged, because $\nabla \times (\nabla \Lambda) = 0$.

- Rewrite $\nabla \times \underset{\sim}{E} = -\partial \underset{\sim}{B} / \partial t$ as

$$\nabla \times \underbrace{(\underset{\sim}{E} + \partial \underset{\sim}{A} / \partial t)}_{\text{IDENTIFY AS } -\nabla V} = 0.$$

(V: scalar potential)

For $\underset{\sim}{E}$ to remain invariant under a shift in $\underset{\sim}{A}$, must change

$$V \to V - \partial \Lambda / \partial t$$

$$\underset{\sim}{E} = -\nabla V - \partial \underset{\sim}{A} / \partial t$$

$$\to -\nabla V + \cancel{\frac{\partial}{\partial t} \nabla \Lambda} - \partial \underset{\sim}{A} / \partial t - \cancel{\frac{\partial}{\partial t} \nabla \Lambda} \qquad \checkmark$$

COVARIANT NOTATION:

Field-strength tensor

$$F^{\mu\nu} \equiv \partial^\nu A^\mu - \partial^\mu A^\nu = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

$$= -F^{\nu\mu} \qquad \text{(WHERE } A^\mu = (V; \underset{\sim}{A})\text{)}$$

IS UNCHANGED BY THE "GAUGE TRANSFORM"

$$A^\mu \to A^\mu - \partial^\mu \Lambda$$

$$\boxed{\begin{array}{l}\text{MANY DIFFERENT 4-VECTOR}\\ \text{POTENTIALS DESCRIBE THE}\\ \text{SAME PHYSICAL CONSEQUENCES}\end{array}}$$

"GAUGE INVARIANCE"

- Remaining Maxwell equations

$$\nabla \cdot \underset{\sim}{E} = \rho$$

$$\nabla \times \underset{\sim}{B} = \underset{\sim}{J} + \frac{\partial \underset{\sim}{E}}{\partial t}$$

correspond to

$$\partial_\mu F^{\mu\nu} = -J^\nu = -(\rho; \underset{\sim}{J})$$

$$\hookrightarrow (-\nabla \cdot \underset{\sim}{E}\,;\ \partial \underset{\sim}{E}/\partial t - \nabla \times \underset{\sim}{B})$$

$$\partial_\nu J^\nu = -\partial_\nu \partial_\mu F^{\mu\nu} = 0$$

$$\boxed{\text{EM CURRENT IS CONSERVED}}$$

Now evaluate

$$\partial_\mu F^{\mu\nu} = -J^\nu$$

in terms of A^μ:

$$\Box A^\nu - \partial^\nu(\partial_\mu A^\mu) = J^\nu$$

"Lorentz gauge," $(\partial_\mu A^\mu = 0)$ & no sources:

$$\Box A^\nu = 0$$

$$\Box A^{\nu} = 0$$

Each component of A^{ν}, to be identified with the PHOTON FIELD, satisfies a (Klein-Gordon) wave eqn. for a massless field.

CONNECTION BETWEEN

Gauge invariance

Charge conservation

Massless vector fields

REMAINS TO BE UNDERSTOOD

PHASE INVARIANCE IN QUANTUM MECH.

NOW SUPPOSE WE KNEW QM,

BUT NOT E+M ...

"Derive" Maxwell's eqns. from a symmetry principle.

QM STATE IS DESCRIBED BY A COMPLEX WAVE FUNCTION $\psi(x)$

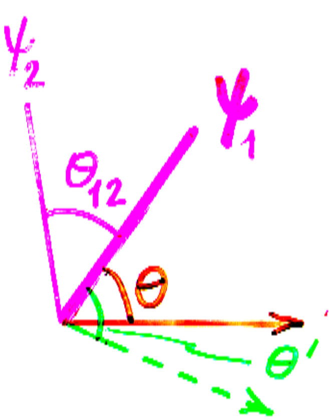
OBSERVABLES INVOLVE INNER PRODUCTS (EXPECTATION VALUES)

$$\langle \mathcal{O} \rangle = \int d^n \underset{\sim}{x} \; \psi^* \mathcal{O} \psi$$

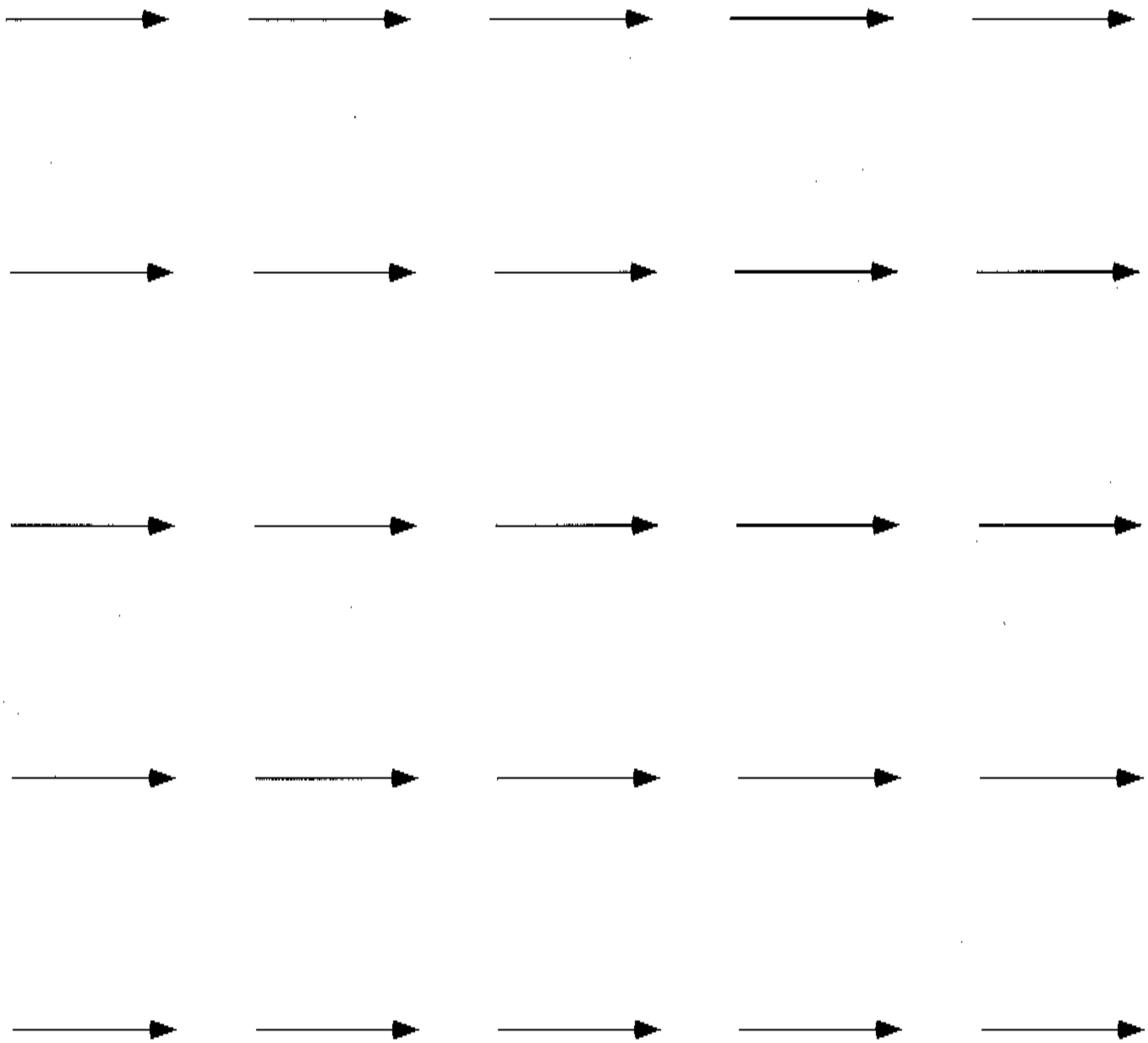
Hermitian op., $\mathcal{O}^\dagger = \mathcal{O}$

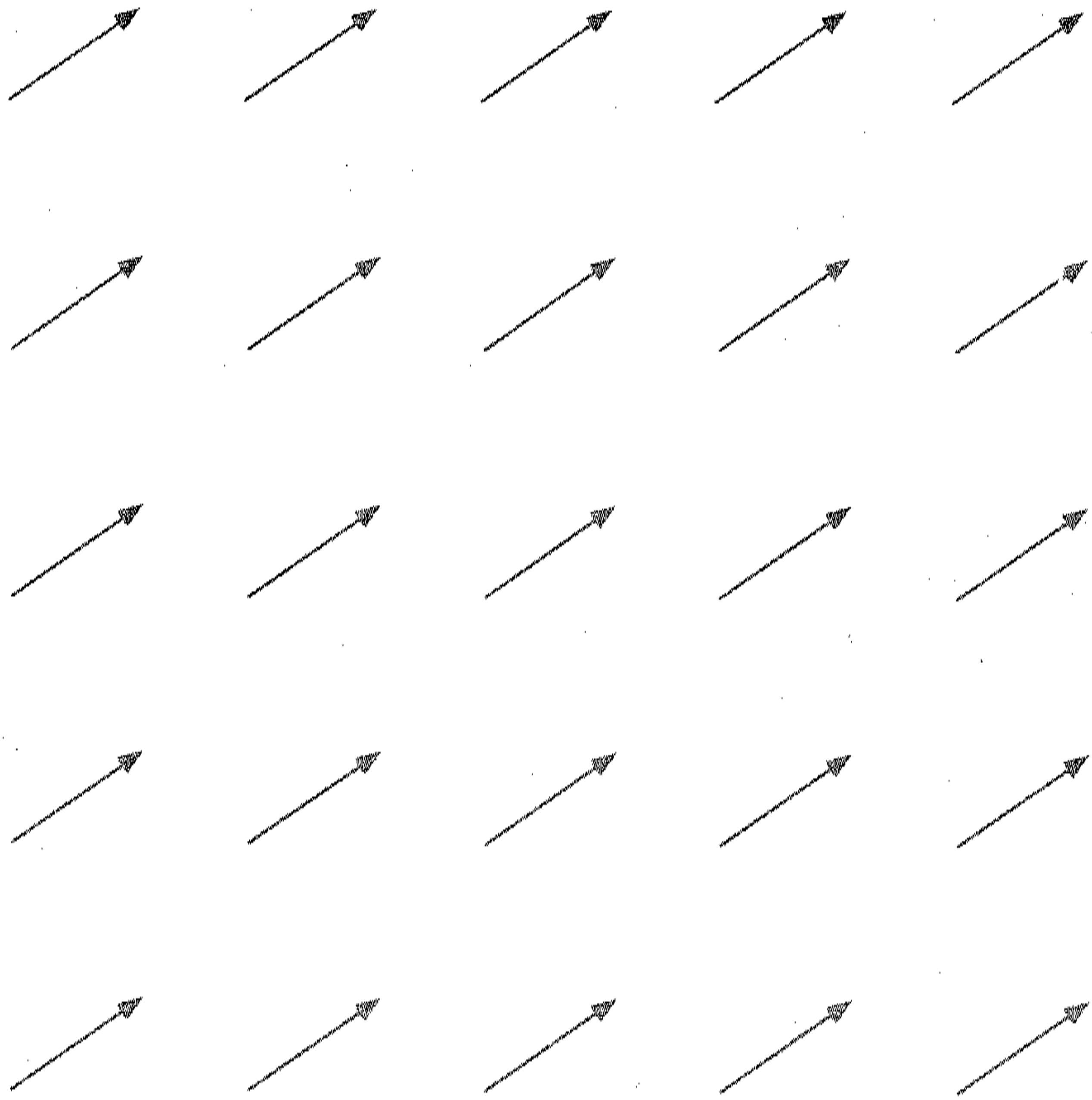
IF $\psi \to e^{i\theta}\psi$, THEN $\psi^* \to \psi^* e^{-i\theta}$

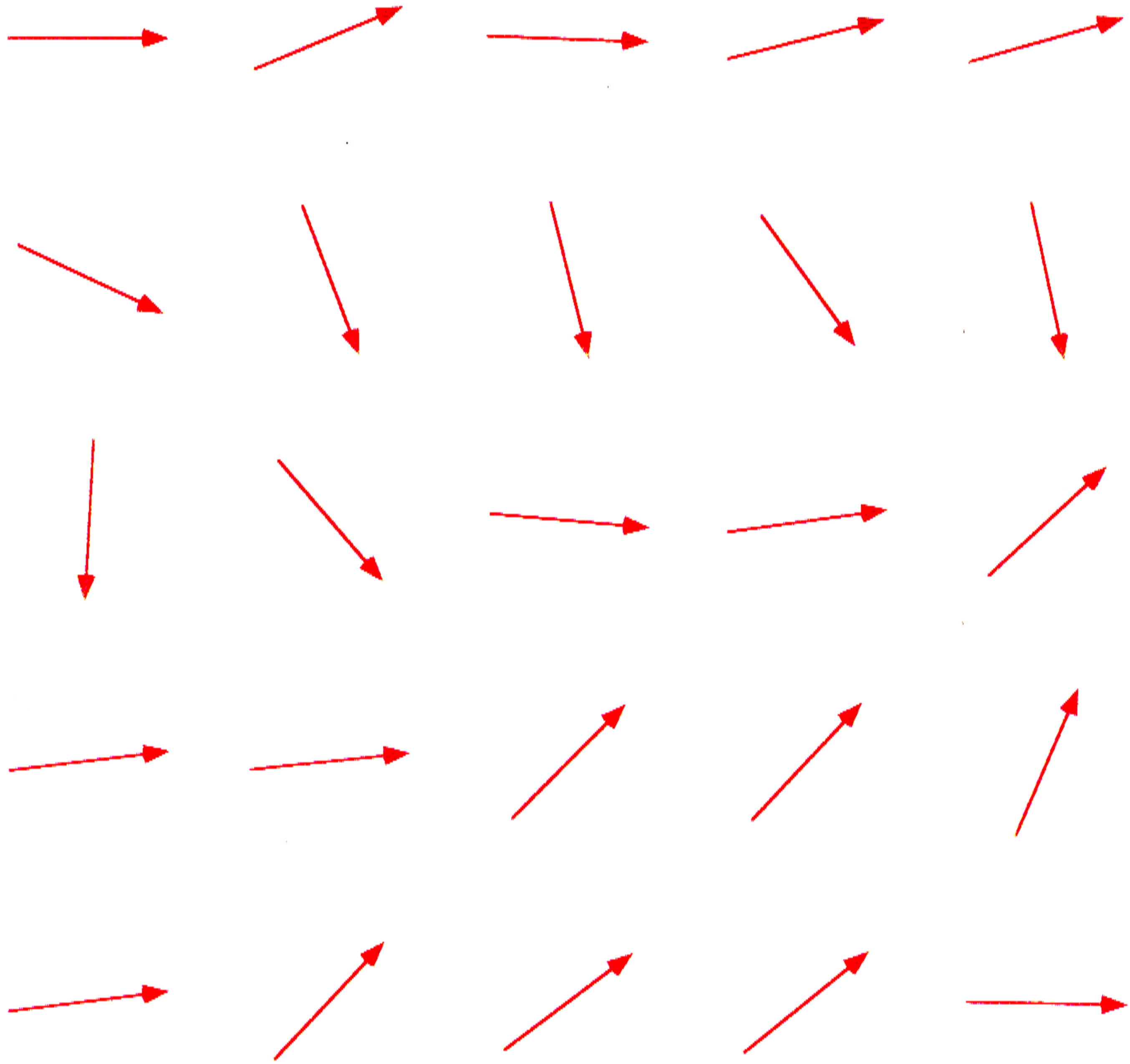
AND $\langle \mathcal{O} \rangle$ IS UNCHANGED, SO ...



THE ABSOLUTE PHASE IS CONVENTIONAL, CANNOT BE MEASURED. RELATIVE PHASES, MEASURED IN INTERFERENCE EXPERIMENTS, ARE UNAFFECTED BY A GLOBAL PHASE ROTATION







WHAT ABOUT A POSITION-DEPENDENT CHANGE OF PHASE?

$$\boxed{\psi \rightarrow e^{i\alpha(x)}\,\psi} \qquad (\star)$$

Here's the rub:

Schrödinger eqn. (eqn. of motion)

$$-\frac{\hbar^2}{2m}\nabla^2\psi = i\hbar\frac{\partial\psi}{\partial t}$$

involves derivatives.

So do some observables: E, p, ...

Under transformation $(\star)$

$$\partial_\mu\psi(x) \rightarrow \partial_\mu[e^{i\alpha(x)}\psi(x)] = e^{i\alpha(x)}[\partial_\mu\psi(x) + i\psi(x)(\partial_\mu\alpha(x))]$$

ADDITIONAL (4-VECTOR) TERM SPOILS LOCAL PHASE INVARIANCE

SCHRÖDINGER EQN FOR FREE PARTICLES IS NOT INVARIANT UNDER LOCAL PHASE ROTATIONS.

Can we fix it?

Modify eqns. of motion to accommodate local phase inv.?

Add a 4-vector to eat the offending 4-vector

REPLACE (everywhere!)

$$\partial_\mu \longrightarrow \mathcal{D}_\mu \equiv \partial_\mu + ie A_\mu$$

e: anticipates a theory of electromagnetism

How must A_μ transform?

REQUIRE THAT

$$\mathcal{D}_\mu \psi(x) \longrightarrow e^{i\alpha(x)} \mathcal{D}_\mu \psi(x) \qquad (**)$$

WHEN $\psi(x) \longrightarrow e^{i\alpha(x)} \psi(x)$

$$\begin{aligned}
(\partial_\mu + ieA'_\mu)(e^{i\alpha(x)}\psi(x)) &= \mathcal{D}'_\mu \psi' \\
&= e^{i\alpha(x)}[\partial_\mu + i(\partial_\mu \alpha) + ieA'_\mu]\psi \\
&= e^{i\alpha(x)}[\partial_\mu + ieA_\mu]\psi
\end{aligned}$$

TRUE IF

$$ieA'_\mu = ieA_\mu - i(\partial_\mu \alpha)$$

$$\boxed{A'_\mu = A_\mu - \frac{1}{e}(\partial_\mu \alpha)} \qquad \text{under } (*)$$

Since (**) is satisfied,

$$\psi^* \mathcal{D}_\mu \psi$$

is locally phase invariant

RECALL FROM ELEMENTARY QM

Momentum operator

$$p_\mu \sim i\hbar\,\partial_\mu$$

Replace $\partial_\mu \to \mathcal{D}_\mu = \partial_\mu + ieA_\mu$ momentum op. now

$$i\hbar\mathcal{D}_\mu = i\hbar\partial_\mu - eA_\mu$$
$$= p_\mu - eA_\mu$$

In classical electrodynamics, the momentum of a charged particle in an EM field.

$$\boxed{\text{FORM OF THE COUPLING } \mathcal{D}_\mu\psi \text{ BETWEEN MATTER AND THE EM FIELD IS PRESCRIBED (SUGGESTED) BY LOCAL PHASE INVARIANCE}}$$

The transformation law

$$A'_\mu = A_\mu - \frac{1}{e}(\partial_\mu\alpha)$$

has just the form of a gauge transformation in electrodynamics.

ORIGINS OF THIS STRATEGY

<u>H. Weyl (1921)</u> attempted to unify gravity and EM using a spacetime-dependent change of scale/calibration/gauge

Raum·Zeit·Materie

$$x_\mu \xrightarrow{dx_\mu}$$

Function	$f(x)$	$f(x+dx) = f(x) + \partial^\mu f(x)\cdot dx_\mu$
Scale	1	$1 + S^\mu(x)\,dx_\mu$
		$f(x) + \partial^\mu f(x)\cdot dx_\mu + f(x)\,S^\mu(x)\,dx_\mu + \cdots$

To first order in dx,

$$\Delta f = (\partial^\mu + S^\mu)\,f\cdot dx_\mu$$

Weyl: identify S^μ with EM 4-vector pot. (A^μ), incorporate EM into a geometrical theory.

<u>Eichinvarianz</u> $\longrightarrow$ étalon $\longrightarrow$ gage / gauge / jauge

"Gauge invariance"

Weyl's program FAILS. Need invariance under a change of <u>phase</u>, as we have seen.

STRATEGY EVOLVED, NAME REMAINED

ELECTRODYNAMICS

Potentials carry too much information.

Gauge invariance: Many different potentials produce same fields.

? DO FIELDS CONTAIN ALL THE NECESSARY PHYSICAL INFORMATION?

QM ANALYSIS $\Rightarrow$ NO!

Aharonov-Bohm Effect

NR free particle satisfies

$$-\frac{\hbar^2}{2m}\nabla^2\psi^0(\underset{\sim}{x},t) = i\hbar\frac{\partial\psi^0}{\partial t}(\underset{\sim}{x},t)$$

Impose a static vector potential $\underset{\sim}{A}(x)$:

$$\frac{(-i\hbar\nabla - eA)^2}{2m}\psi(\underset{\sim}{x},t) = i\hbar\frac{\partial\psi(\underset{\sim}{x},t)}{\partial t}$$

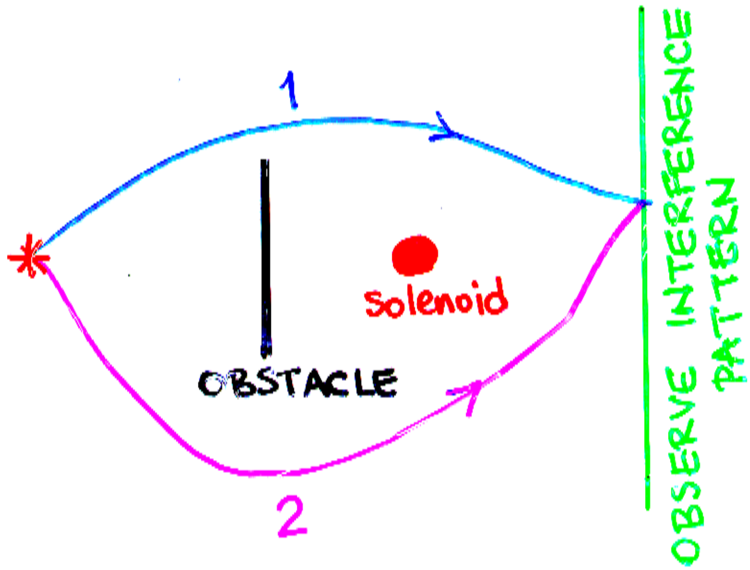
has solution

$$\psi(\underset{\sim}{x},t) = \psi^0(\underset{\sim}{x},t)\, e^{iS/\hbar}$$

where the classical action is

$$S = e\int d\underset{\sim}{x}\cdot\underset{\sim}{A}$$

Change in classical action $\Rightarrow$ phase shift



MAGNETIC FIELD = 0 ALONG PATHS

Vector potential $\underset{\sim}{A}$ cannot vanish everywhere:

$$\Phi = \int d\underset{\sim}{\sigma} \cdot \underset{\sim}{B} = \oint d\underset{\sim}{x} \cdot \underset{\sim}{A}$$

FIELD OFF: $\quad \psi^{0}(\underset{\sim}{x}, t) = \psi_1^{0}(\underset{\sim}{x}, t) + \psi_2^{0}(\underset{\sim}{x}, t)$

FIELD ON: $\quad \psi(\underset{\sim}{x}, t) = \psi_1^{0}(\underset{\sim}{x}, t)\, e^{iS_1/\hbar} + \psi_2^{0}(\underset{\sim}{x}, t)\, e^{iS_2/\hbar}$

$$S_i = e \int_{\text{path } i} d\underset{\sim}{x} \cdot \underset{\sim}{A}$$

INTERFERENCE EFFECTS (from $\psi^{*}\psi$) DEPEND ON

<u>Phase Difference</u> $(S_1 - S_2)/\hbar = (e/\hbar) \oint d\underset{\sim}{x} \cdot \underset{\sim}{A}$

Stokes' Thm: $(S_1 - S_2)/\hbar = (e/\hbar) \int d\underset{\sim}{\sigma} \cdot \underset{\sim}{B} = e\Phi/\hbar$

$\Phi \neq 0$: enclosed flux

Beams experience no EM forces, but the Vector Potential $\underset{\sim}{A}$ has a physical effect!

$\boxed{\text{See A. Tonomura, } \underline{\text{et al.}}\text{, PRL } \underline{56}\text{, 792 (1986).}}$

THE PATH-DEPENDENT ("NON-INTEGRABLE") PHASE

$$\exp\left[\frac{ie}{\hbar}\oint d\underset{\sim}{x}\cdot\underset{\sim}{A}\right] \longrightarrow \exp\left[-\frac{ie}{\hbar}\oint dx_\mu A^\mu\right]$$

CONTAINS

- MORE INFO THAN EM FIELDS
- LESS INFO THAN POTENTIALS

Note that the phase factor is gauge-invariant.

$\underset{\sim}{A} \to \underset{\sim}{A} + \nabla\Lambda$: Stokes

$$\rightarrow \int d\underset{\sim}{\sigma}\cdot\underbrace{(\nabla\times\nabla\Lambda)}_{=0}$$

COULD THE PHOTON HAVE A MASS?

Form of mass term

$$\mathcal{L}_\gamma = \frac{1}{2} m^2 A^\mu A_\mu$$

This violates local phase invariance, because

$$A^\mu A_\mu \rightarrow (A_\mu - \partial_\mu \alpha)(A^\mu - \partial^\mu \alpha) \neq A_\mu A^\mu$$

LOCAL PHASE INV. $\Rightarrow M_\gamma = 0$

<u>Best limits</u>

LAB: Test Gauss's Law

$M_\gamma < 10^{-14}$ eV PRL 26, 721 ('71)

LAB: Torsion balance / Cosmic $\underset{\sim}{A}$

$M_\gamma < 2 \times 10^{-16}$ eV PRL 80, 1826 ('98)

SPACECRAFT: Jupiter's $\underset{\sim}{B}$ (Pioneer X)

$M_\gamma < 6 \times 10^{-16}$ eV PRL 35, 1402 ('75)

UNIVERSE: Stability of Small Magellanic Cloud

$M_\gamma < 3 \times 10^{-27}$ eV (!)

Sov. Phys.-Usp. 19, 624 ('76)

LOCAL PHASE INVARIANCE → QED

DIRAC LAGRANGIAN

$$\mathcal{L}_{free} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

(leads to Dirac eqn

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0 \;)$$

Make the replacement

$$\partial_\mu \to \mathcal{D}_\mu = \partial_\mu + iqA_\mu(x)$$

Dirac Lagrangian becomes

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi = \mathcal{L}_{free} - qA_\mu \bar{\psi}\gamma^\mu\psi$$

$$= \mathcal{L}_{free} - J^\mu A_\mu$$

$J^\mu = q\bar{\psi}\gamma^\mu\psi$: CONSERVED EM CURRENT

DERIVED FROM GLOBAL PHASE INV.

Conservation Laws ← (Nöther's Theorem) → Continuous Symmetries

Local phase invariance prescribes form of interaction between matter + EM field.

Add kinetic (propagation) term for EM field:

$$\mathcal{L}_{QED} = \mathcal{L}_{free} - J^\mu A_\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

(locally phase invariant)

Particle Physics:
The Standard Model

Chris Quigg
Theoretical Physics Department
Fermi National Accelerator Laboratory
Chris.Quigg@cern.ch

CERN Summer Lectures
17 – 27 July 2000

3

3

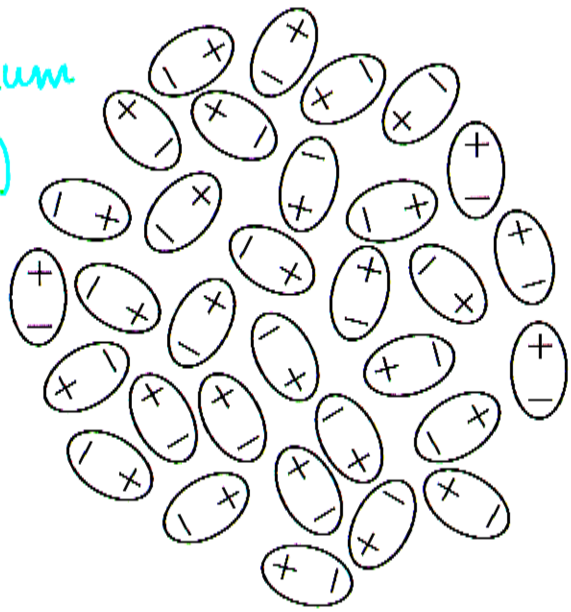
RUNNING COUPLING IN QED

Fine structure constant

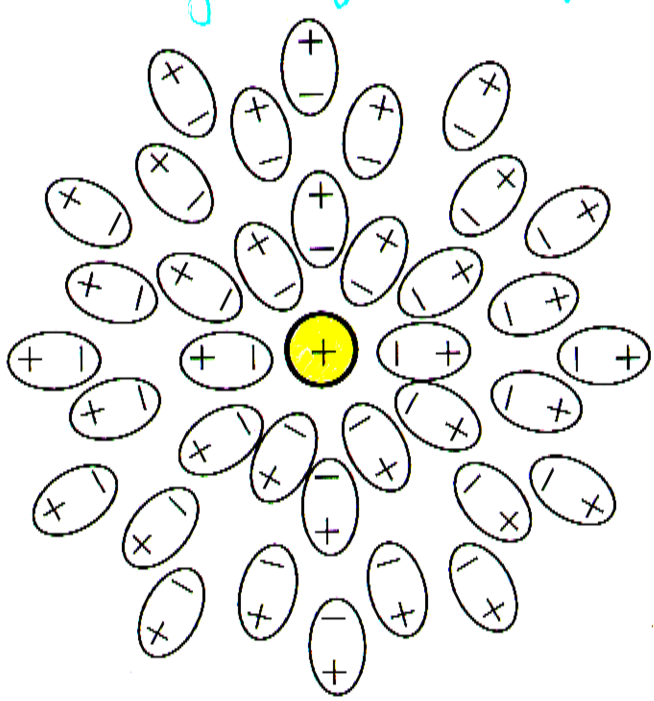
$\alpha = e^2/4\pi\hbar c = 1/137.035\ 989\ 5(61)$

Depends on distance / energy

Polarizable medium
(disordered dipoles)



Place a test charge $\oplus$ in the medium.
Attracts $\ominus$ charges of the dipoles



3

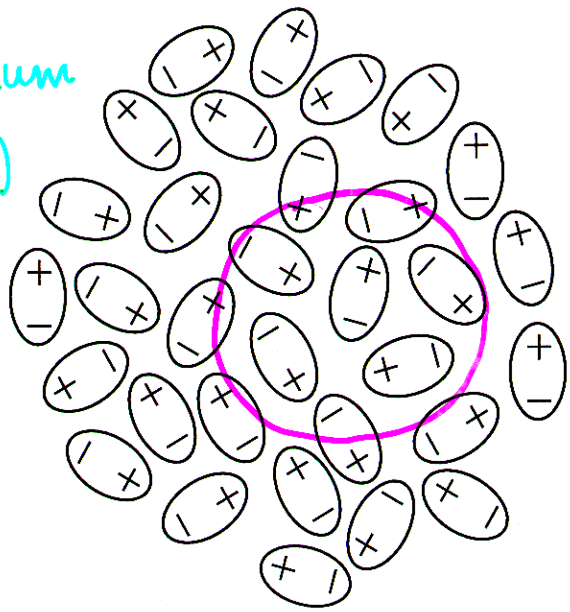
RUNNING COUPLING IN QED

Fine structure constant

$$\alpha = e^2/4\pi\hbar c = 1/137.035\,989\,5(61)$$

Depends on distance / energy

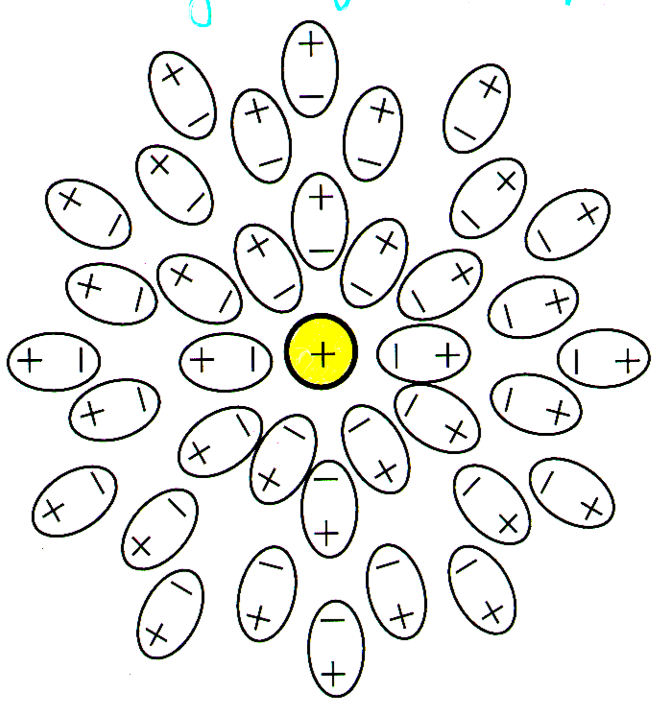
Polarizable medium (disordered dipoles)



Any finite volume contains zero net charge

Place a test charge ⊕ in the medium.

Attracts ⊖ charges of the dipoles



3

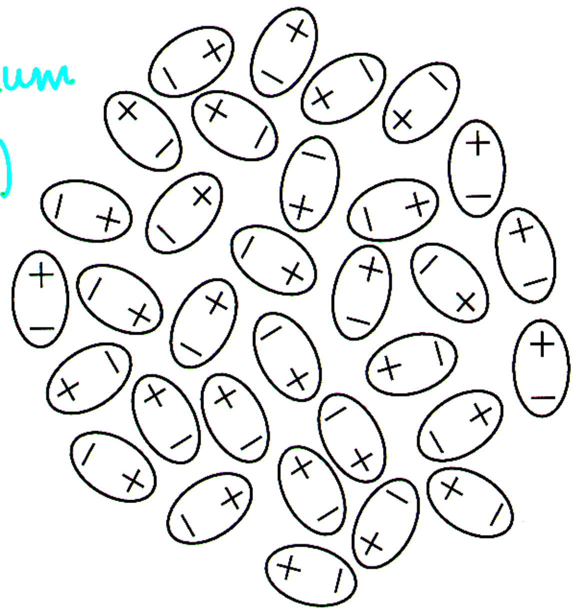
RUNNING COUPLING IN QED

Fine structure constant

$$\alpha = e^2/4\pi\hbar c = 1/137.035\,989\,5(61)$$

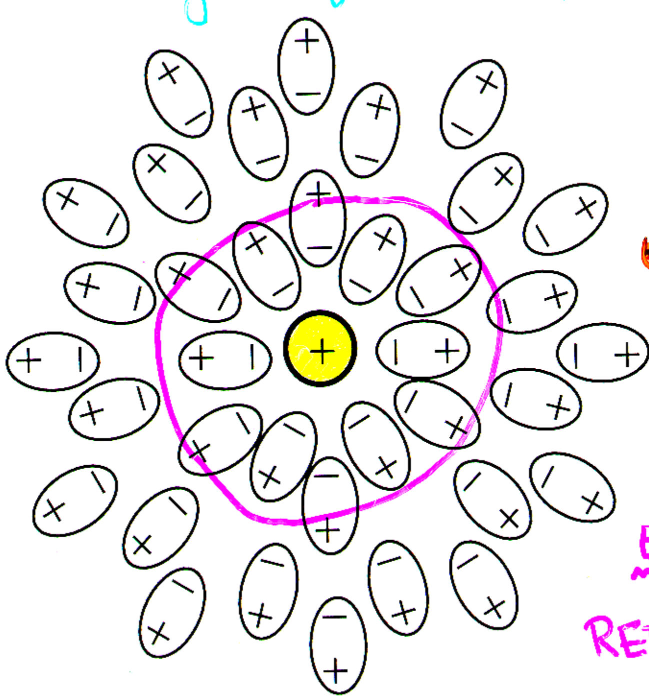
Depends on distance / energy

Polarizable medium
(disordered dipoles)



Place a test charge $+$ in the medium.
Attracts $-$ charges of the dipoles

Contains total charge
< test charge



Gauss Law:

$$Q_{eff} = \frac{Q_{test}}{\varepsilon} \leq Q_{test}$$

$\varepsilon \geq 1$: dielectric const.

$$\vec{E}_{med} = \vec{E}_{vac}/\varepsilon$$

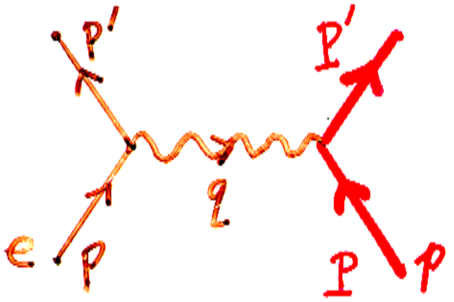
REDUCED BY SCREENING.

Recover full Q_{test} at very small distances $\Rightarrow$

EFFECTIVE CHARGE INCREASES AT SHORT DIST.

QED VACUUM AS A POLARIZABLE MEDIUM

Scatter electron from ∞ly massive proton



$$\mathcal{M} = -i e^2 \bar{u}(p') \gamma_\mu u(p) \frac{g^{\mu\nu}}{q^2} \bar{U}(P') \gamma_\nu U(P)$$

$$q = p - p'$$

$\lambda = \pm 1$: 2x helicity

Proton spinors for $M \gg E$

$$U_\lambda(P) = \sqrt{2M} \begin{pmatrix} \chi_\lambda \\ 0 \end{pmatrix}$$

$$\bar{U}_{\lambda'}(P') = \sqrt{2M} \left(\tilde{\chi}_{\lambda'}, 0 \right)$$

$$\bar{U}_{\lambda'}(P') \gamma_\nu U_\lambda(P) = 2M \tilde{\chi}_{\lambda'} \chi_\lambda \delta_{\nu 0}$$

$$= 2M \delta_{\lambda'\lambda} \delta_{\nu 0}$$

Electron spinors

$$u_\lambda(p) = \sqrt{E+m} \begin{pmatrix} \chi_\lambda \\ \frac{|\vec{p}|}{E+m} \chi_\lambda \end{pmatrix}$$

$$\gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

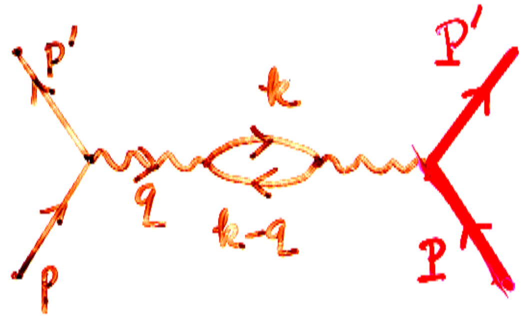
$$\underline{\gamma} = \begin{pmatrix} 0 & \underline{\sigma} \\ \underline{\sigma} & 0 \end{pmatrix}$$

Only γ_0 doesn't mix large + small components

$$\boxed{\mathcal{M} = -\frac{ie^2}{q^2} \bar{u}(p') \gamma_0 u(p) \cdot 2M \delta_{\lambda'\lambda}}$$

Coulomb interaction: interaction of lepton current with a purely timelike potential.

One-loop correction



$$\mathcal{M} = -\mathrm{Tr}\int \frac{d^4k}{(2\pi)^4}\Big[(+ie)\bar{u}(p')\gamma_\mu u(p)\cdot$$

$$\frac{-ig^{\mu\rho}}{q^2}(ie)\gamma_\rho \frac{i(\not{k}+m)}{k^2-m^2+i\varepsilon}(ie)\gamma_\sigma \frac{i(\not{k}-\not{q}+m)}{(k-q)^2-m^2+i\varepsilon}\cdot\frac{-ig^{\sigma\nu}}{q^2}\cdot$$

$$(-ie)\,\bar{U}(P')\gamma_\nu U(P)\Big]$$

⋮

FIVE PAGES OF CALCULATIONS

⋮

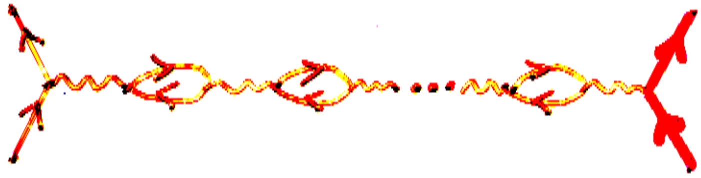
For $-q^2 \gg m^2$, replace $e^2 \equiv \alpha$ by the "running coupling constant"

$$\alpha_R(q^2) = \alpha_R(m^2)\left[1 + \frac{\alpha_R(m^2)}{3\pi}\log\left(\frac{-q^2}{m^2}\right) + \cdots\right]$$

Effective coupling increases at large q^2.

"vacuum polarization"

Sum higher orders in perturbation theory



$$\alpha_R(q^2) = \alpha_R(m^2)\Big[\underbrace{1 + \frac{\alpha_R(m^2)}{3\pi}\log\left(\frac{-q^2}{m^2}\right) + \left[\frac{\alpha_R(m^2)}{3\pi}\log\left(\frac{-q^2}{m^2}\right)\right]^2 + \dots}\Big]$$

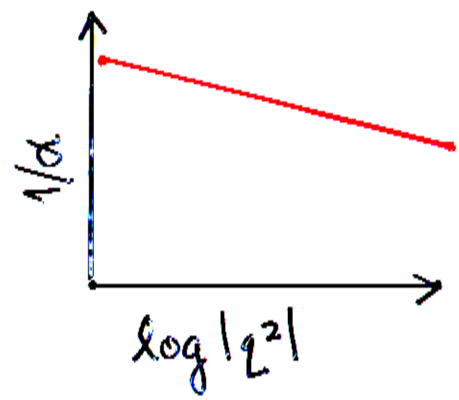
IDENTIFY $\sum_{n=0}^{\infty} x^n = 1/(1-x)$

$$\alpha_R(q^2) = \alpha_R(m^2)\Big/\left[1 - \frac{\alpha_R(m^2)}{3\pi}\log\left(\frac{-q^2}{m^2}\right)\right],$$

OR

$$\boxed{1/\alpha_R(q^2) = 1/\alpha_R(m^2) - \frac{1}{3\pi}\log\left(\frac{-q^2}{m^2}\right)}$$

Coupling constant increases at large q^2, short distances



Running α

Actual evaluation requires treating heavy leptons and quarks $(N_c e_q^2)$

(For light quarks, use $e^+e^- \to$ hadrons)

Typical result:

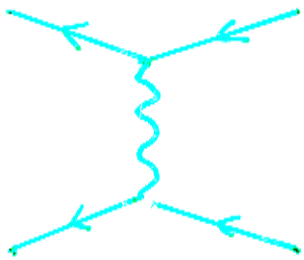
PL B356, 398 ('95)
ZP C67, 585 ('95)

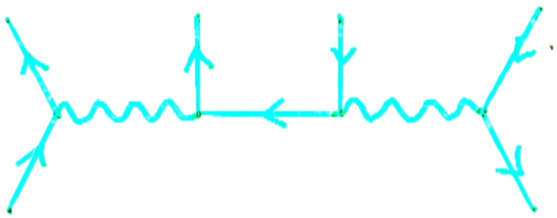
$$1/\alpha(M_Z^2) = 128.89 \pm 0.09$$

$\sim 6\%$ change from $1/\alpha(0)$

(Very important number for LEP analyses)

TOPAZ Collaboration at TRISTAN (KEK)

$e^+e^- \to \mu^+\mu^-$  measures $\alpha^2(58\text{ GeV})$

$e^+e^- \to e^+e^-\mu^+\mu^-$  measures $\alpha^4(0)$

$$1/\alpha(57.77\text{ GeV}) = 128.5 \pm 1.8 \pm 0.7$$

Significant change from

$$1/\alpha(0) = 137.036$$

I. Levine, et al. (TOPAZ Collaboration), PRL 78, 424 (1997)

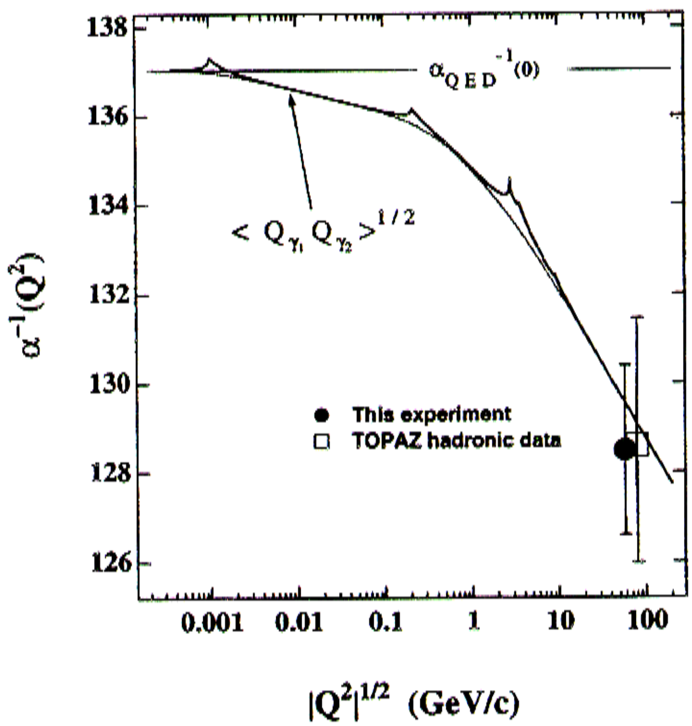


FIG. 2. The measured and theoretical electromagnetic coupling as a function of momentum transfer Q. The solid and dotted lines correspond to positive and negative Q^2 predictions, respectively. As we probe closer to the bare charge, its effective strength increases. $\langle Q_{\gamma_1} Q_{\gamma_2} \rangle^{1/2}$ denotes the square root of the median value for the product of the photon momentum transfers in the antitagged $e^+e^- \to e^+e^-\mu^+\mu^-$ sample. The hadronic data point has been shifted for display.

1998 CERN Summer Student Lecture Programme

Particle Physics: The Standard Model

Chris Quigg

* * * * *

If baryon number is absolutely conserved, the conservation law may be a consequence of a global phase symmetry like that of electromagnetism, with the electric charge replaced by baryon number.

(a) How would Newton's law of gravitation be modified if the baryonic phase symmetry were a *local* gauge invariance?

(b) In view of the close equality of inertial and gravitational masses imposed by the Eötvös experiment [P. G. Roll, R. Krotkov, and R. H. Dicke, *Ann. Phys. (NY)* **26**, 442 (1967)], what can be said about the strength of a hypothetical gauge interaction coupled to the baryon current? [Reference: T. D. Lee and C. N. Yang, *Phys. Rev.* **98**, 150 (1955).]

On the possibility of a massless gauge boson coupled to lepton number, see L. B. Okun, *Yad. Fiz.* **10**, 358 (1969) [English translation: *Sov. J. Nucl. Phys.* **10**, 206 (1969)]. For a critical reëxamination of the limits, see S. I. Blinnikov, A. D. Dolgov, L. B. Okun, and M. B. Voloshin, *Nucl. Phys.* **B458**, 52 (1996).

NON-ABELIAN GAUGE THEORIES

QED: Gauge Theory of local phase invariance based on the Abelian group $U(1)_{em}$.

Generalize to non-Abelian groups.

Yang+Mills, 1954

WHY?

$M_{proton} \simeq M_{neutron}$

+

Charge independence of nuclear forces.

ISOSPIN SYMMETRY OF THE STRONG (NUCLEAR) INTERACTIONS.

Isospin-invariant description for (free) equal-mass nucleons:

$$\mathcal{L}_0 = \bar{p}(i\gamma^\mu \partial_\mu - m)p + \bar{n}(i\gamma^\mu\partial_\mu - m)n$$

Composite spinor $\psi \equiv \begin{pmatrix} p \\ n \end{pmatrix}$

$$\boxed{\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi}$$

$\vec{\tau} = (\tau_1, \tau_2, \tau_3)$: Pauli matrices

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

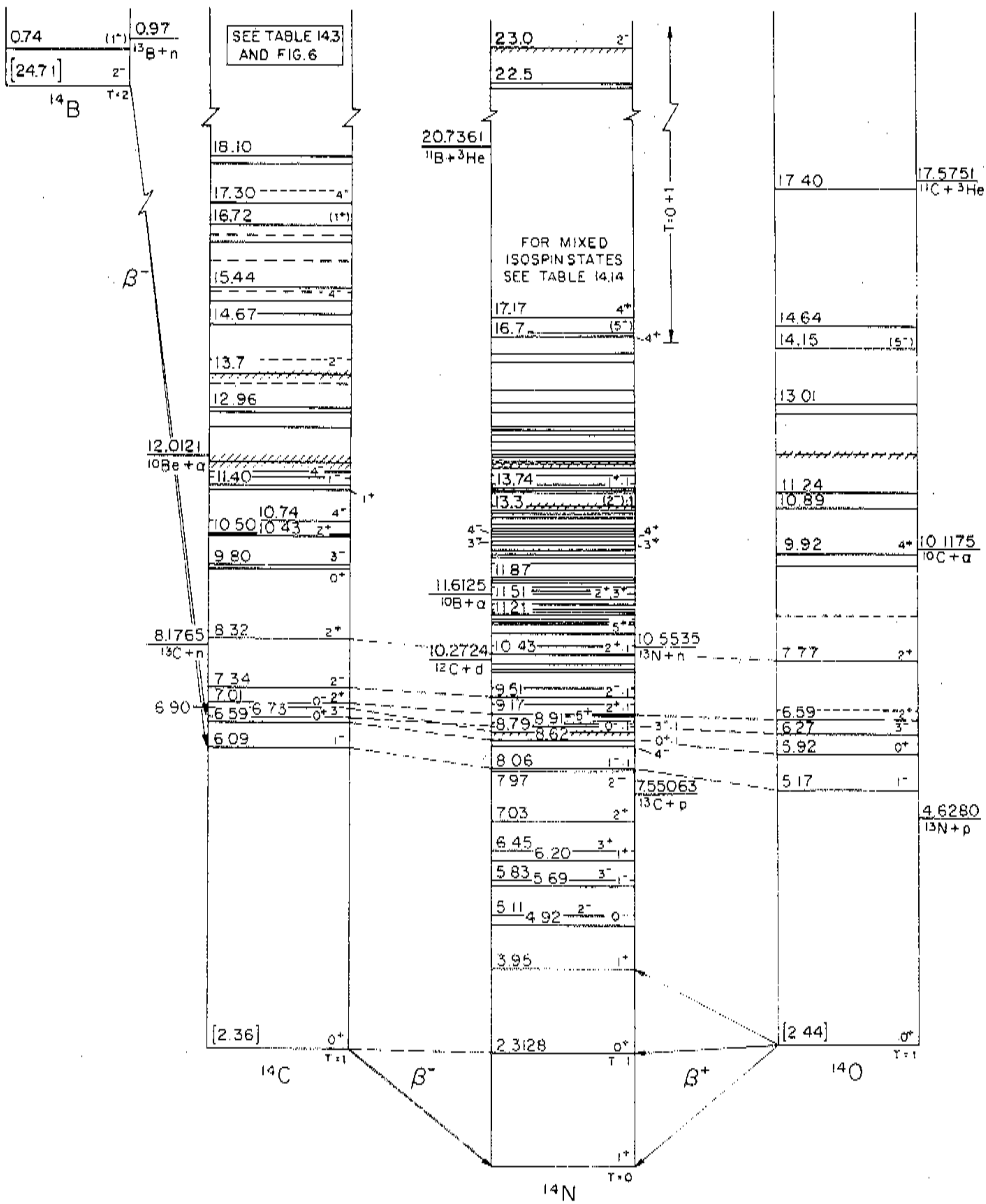
$\vec{\alpha}$: arbitrary 3-vector

Invariant under global isospin rot.,

$$\psi \to \exp\left[\frac{i\vec{\alpha}\cdot\vec{\tau}}{2}\right]\psi$$

with conserved isospin current

$$J^\mu = \bar{\psi}\gamma^\mu \frac{\vec{\tau}}{2}\psi$$



0.74
[24.71]
^{14}B
0.97
^{13}B+n
SEE TABLE 14.3 AND FIG. 6
18.10
17.30
16.72
15.44
14.67
13.7
12.96
12.0121
^{10}Be+α
11.40
10.50
10.74
10.43
9.80
8.1765
^{13}C+n
8.32
7.34
7.01
6.90
6.59
6.73
6.09
[2.36]
^{14}C
β$^-$
23.0
22.5
20.7361
^{11}B+^{3}He
FOR MIXED ISOSPIN STATES SEE TABLE 14.14
17.17
16.7
13.74
13.3
11.87
11.6125
^{10}B+α
11.51
11.21
10.2724
^{12}C+d
10.43
10.5535
^{13}N+n
9.51
9.17
8.79
8.91
8.62
8.06
7.97
7.55063
^{13}C+p
7.03
6.45
6.20
5.83
5.69
5.11
4.92
3.95
2.3128
^{14}N
β$^+$
17.40
17.5751
^{11}C+^{3}He
14.64
14.15
13.01
11.24
10.89
9.92
10.1175
^{10}C+α
7.77
6.59
6.27
5.92
5.17
4.6280
^{13}N+p
[2.44]
^{14}O

7 90

QUESTION:

Can we name the two nucleon states independently at every spacetime point?

Can we turn the global $SU(2)_{isospin}$ invariance of the free-nucleon $\mathcal{L}$ into a local SU(2) invariance?

ANSWER:

Yes, but ...

as in QED, we must introduce a gauge field and interactions.

Require invariance of $\mathcal{L}$ under the local isospin rotation

$$\psi(x) \longrightarrow G(x)\,\psi(x)$$

with

2x2 in isospin

$$G(x) = \exp\left[\frac{i\vec{\tau}\cdot\vec{\alpha}(x)}{2}\right]$$

Find the usual problem:

$$\partial_\mu(G\psi) = G(\partial_\mu\psi) + (\partial_\mu G)\psi$$

Try the usual solution:

Introduce a gauge-covariant derivative

$\mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ in isospin

$$\mathcal{D}_\mu = \mathbb{I}\,\partial_\mu + ig\,B_\mu$$

with

$$B_\mu = \frac{1}{2}\vec{\tau}\cdot\vec{b}_\mu = \frac{1}{2}\tau^a b^a_\mu = \frac{1}{2}\begin{pmatrix} b^3_\mu & b^1_\mu - i b^2_\mu \\ b^1_\mu + i b^2_\mu & -b^3_\mu \end{pmatrix}$$

$$\vec{b}_\mu = (b^1_\mu, b^2_\mu, b^3_\mu)$$ isovector gauge fields

How must B_μ transform, in order that

$$\mathcal{D}_\mu \psi \longrightarrow \mathcal{D}'_\mu \psi' = G(\mathcal{D}_\mu \psi)$$

when $\psi \longrightarrow \psi' = G\psi$?

2 QUESTIONS

- Can we make the construction?
- Does the resulting theory have a physical application?

Find the transformation law

$$B'_\mu = G\left[B_\mu + (i/g)\,G^{-1}(\partial_\mu G)\right]G^{-1} \quad (*)$$

Observations:

(i) Abelian case of QED, where

$$G_{EM}(x) = e^{iq\alpha(x)} :$$

rule (*) gives

$$A'_\mu = G_{EM} A_\mu G_{EM}^{-1} + (i/q)(\partial_\mu G_{EM})\,G_{EM}^{-1}$$

$$= A_\mu + (i/q)(iq\,\partial_\mu \alpha) = A_\mu - \partial_\mu \alpha$$

Recover the familiar case!

(ii) Infinitesimal SU(2) transform

$$G(x) = 1 + \frac{i\vec{\alpha}(x)\cdot\vec{\tau}}{2} \qquad |\vec{\alpha}(x)| \ll 1$$

decode (*) in components:

$$\boxed{b'^{\,l}_\mu = b^l_\mu - \varepsilon_{jkl}\,\alpha^j\, b^k_\mu - \frac{1}{g}(\partial_\mu \alpha^l)}$$

gradient

isospin rotation
(absent in QED case)

Structure constants $\varepsilon_{jkl} \Rightarrow$ result depends on SU(2) group, not on the representation.

The Yang-Mills Lagrangian

So far:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi$$

$$= \mathcal{L}_0 - g\bar{\psi}\gamma^\mu B_\mu \psi = \mathcal{L}_0 - \underbrace{\frac{g}{2}\vec{b}_\mu \cdot \bar{\psi}\gamma^\mu \vec{\tau}\psi}_{}$$

gauge field interacting with the conserved (isospin) current

To complete the Y-M Lagrangian, must include a kinetic term that describes the propagation (+ interaction) of the gauge fields.

LOOK FOR

$$F_{\mu\nu} \equiv \frac{\vec{\tau}}{2} \cdot \vec{F}_{\mu\nu} = \frac{1}{2}\tau^a F^a_{\mu\nu}$$

TO CONSTRUCT

$$\mathcal{L}_{gauge} = -\frac{1}{4}\vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu} = -\frac{1}{2}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$$

3 PAGES OF CALCULATION $\longrightarrow$ SIMPLE RESULT

$$F_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu + ig[B_\nu, B_\mu]$$

$$= \frac{1}{ig}[\mathcal{D}_\nu, \mathcal{D}_\mu]$$

components:

$$F^\ell_{\mu\nu} = \partial_\nu b^\ell_\mu - \partial_\mu b^\ell_\nu + g\,\varepsilon_{jk\ell}\, b^j_\mu b^k_\nu$$

FULL Y-M LAGRANGIAN

$$\mathcal{L}_{YM} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - g\bar{\psi}\gamma^\mu B_\mu \psi - \frac{1}{2}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$$

$$\underbrace{g\bar{\psi}\gamma^\mu B_\mu \psi}_{= \; -\frac{g}{2}\vec{b}_\mu \cdot \bar{\psi}\gamma^\mu \vec{\tau}\psi} \qquad \underbrace{-\frac{1}{2}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})}_{= \; -\frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu}}$$

KEY DIFFERENCE FROM QED

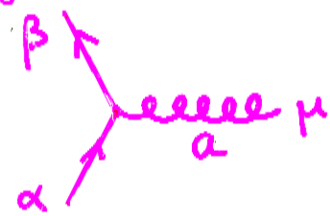
Existence of 3- and 4-gauge-boson interactions in YM.

Photons do not interact (directly) with photons in QED.

- Photons carry no charge
- YM gauge bosons carry generalized change (isospin)

Some implications for nuclear physics

Feynman rule



$$= -\frac{ig}{2}(\tau^a)_{\alpha\beta}\gamma^\mu$$

Suppose nuclear forces mediated by the isovector triplet of vector bosons. Interaction is

Repulsive for the I=1 NN state(s)

Attractive for the I=0 (deuteron) state

[Generally true that the SINGLET state is the maximally attractive channel.]

BUT, Nuclear forces are not mediated by massless vector particles, but by massive

$\pi, \rho, \omega, \sigma, \ldots$

YM Theory cannot serve the purpose for which it was conceived. (Good ideas never die!)

Particle Physics:
The Standard Model

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CERN Summer Lectures
17 – 27 July 2000

4

4

QCD – Theory of the Strong Interactions

QUARKS ÷ color triplets

LEPTONS ÷ color singlets

Suggests: COLOR ⇔ STRONG CHARGE

Formulate a color gauge theory based on $SU(3)_c$, family symmetry of

$$\psi = \begin{pmatrix} \psi_{red} \\ \psi_{green} \\ \psi_{blue} \end{pmatrix}$$

LAGRANGIAN

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi - \tfrac{1}{2}\mathrm{tr}(G_{\mu\nu}G^{\mu\nu})$$

Gauge-covariant derivative

$$\mathcal{D}_\mu = \partial_\mu + ig B_\mu$$

strong coupling constant

Gauge field $B_\mu = \frac{1}{2}\underset{\sim}{\lambda}\cdot\underset{\sim}{b}_\mu = \frac{1}{2}\lambda^l b^l_\mu$

3×3 matrix in color space

octet of colored GLUONS

λ's are SU(3) "Gell-Mann Matrices"

$$\mathrm{tr}(\lambda^\ell) = 0$$

$$\mathrm{tr}(\lambda^k \lambda^\ell) = 2\,\delta^{k\ell}$$

$$[\lambda^j, \lambda^k] = 2i\, f^{jk\ell}\, \lambda^\ell$$

antisymm. structure constants ($\varepsilon^{jk\ell}$ for SU(2))

$$\lambda_1 = \begin{array}{cc} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{array}{c} \bar{R} \\ \bar{B}, \\ \bar{G} \end{array} \\ \begin{array}{ccc} R & B & G \end{array} & \end{array} \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad (8.1.10)$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$

Gluon field-strength tensor

$$G_{\mu\nu} = \tfrac{1}{2}\, \underset{m}{G}_{\mu\nu} \cdot \underset{m}{\lambda} = \tfrac{1}{2}\, G^\ell_{\mu\nu}\, \lambda^\ell$$

$$= (ig)^{-1} [\mathcal{D}_\nu, \mathcal{D}_\mu]$$

$$= \partial_\nu B_\mu - \partial_\mu B_\nu + ig\,[B_\nu, B_\mu]$$

$$G^\ell_{\mu\nu} = \partial_\nu b^\ell_\mu - \partial_\mu b^\ell_\nu + g f^{jk\ell}\, b^j_\mu\, b^k_\nu$$

Quark-gluon interaction

$$\mathcal{L}_{int} = -\frac{g}{2}\, \bar{\psi} \gamma^\mu \lambda^\ell \psi\, b^\ell_\mu$$

YANG-MILLS $\quad -\frac{g}{2}\, \bar{\psi} \gamma^\mu \tau^a \psi\, b^a_\mu$

The effective (color) charge in QCD

In addition to vacuum polarization and screening as in QED, have the possibility of camouflage and antiscreening, because the gluon carries color.

GREEN QUARK AS TEST CHARGE

GLUON PROBE

GREEN-ANTIBLUE GLUON GOES ON A WALKABOUT, LEAVING BEHIND A BLUE QUARK, TO WHICH THE $R\bar{G}$ GLUON IS BLIND

The closer the gluon looks, the less of the test charge it sees, because the color charge has been dispersed in the gluon cloud.

VACUUM POLARIZATION / SCREENING

$\Rightarrow$ Effective charge is LARGER at short distances.

QED $$1/\alpha(q^2) = 1/\alpha(m^2) - \frac{1}{3\pi}\log(-q^2/m^2)$$

CAMOUFLAGE / ANTISCREENING

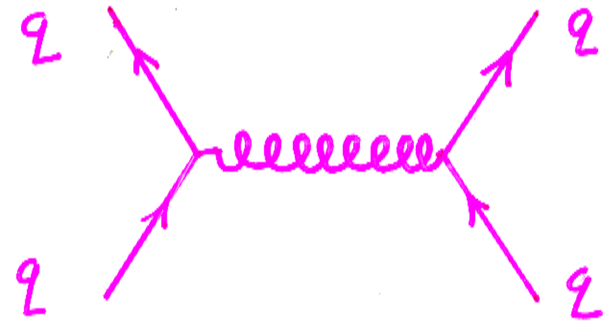
$\Rightarrow$ Effective charge is SMALLER at short distances

BOTH EFFECTS ARE PRESENT IN NON-ABELIAN GAUGE THEORIES.

WHICH PREVAILS?

ONE-LOOP CORRECTIONS TO $qq \to qq$

Lowest-order "TREE" diagram

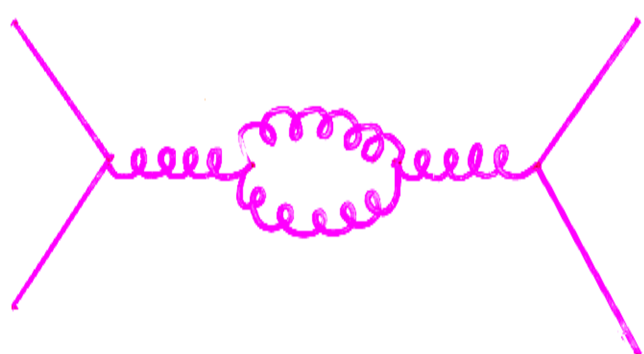


leads to the amplitude $\mathcal{M}_0$

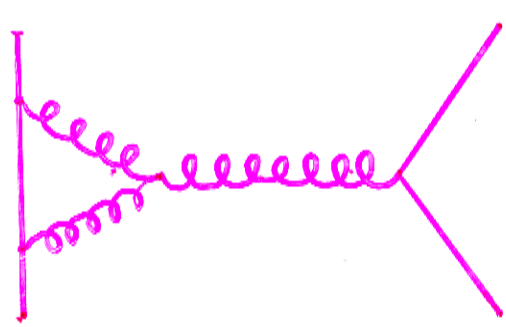
Include single loops, e.g.

Slightly symbolic / depends on gauge

QUARK LOOPS



BOSON LOOPS



VERTEX CORR

$qq \to qq$ AT ONE LOOP

$$\mathcal{M}_1 = \mathcal{M}_0 \left[1 + \frac{g^2}{16\pi^2} \log(-q^2/\mu^2) \left(\frac{2n_f}{3} - \frac{13N}{6} - \frac{2\cdot 3N}{4} \right) + \dots \right]$$

QUARK LOOPS

BOSON LOOPS

VERTEX CORR

non-Abelian terms

n_f: flavors

N: colors

DEFINE the strong coupling parameter

$$\alpha_s = g^2/4\pi$$

The **CHANGE** from $\mathcal{M}_0$ to $\mathcal{M}_1$ is equivalent to replacing α_s by

$$\alpha_s(q^2) = \alpha_s(\mu^2) \left[1 + \frac{\alpha_s(\mu^2)}{12\pi} \log(-q^2/\mu^2) \left(2n_f - 11N \right) + \mathcal{O}(\alpha_s^2) \right]$$

COMPARE QED:

$$\alpha(q^2) = \alpha(\mu^2) \left[1 + \frac{\alpha(\mu^2)}{3\pi} \log(-q^2/\mu^2) + \mathcal{O}(\alpha^2) \right]$$

$qq \to qq$ AT ONE LOOP

$$\mathcal{M}_1 = \mathcal{M}_0 \left[1 + \frac{g^2}{16\pi^2} \log(-q^2/\mu^2) \left(\frac{2n_f}{3} - \frac{13N}{6} - \frac{2 \cdot 3N}{4} \right) + \dots \right]$$

QUARK LOOPS

BOSON LOOPS

VERTEX CORR

non-Abelian terms

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N: colors

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COMPARE QED:

$$\alpha(q^2) = \alpha(\mu^2)\left[1 + \frac{\alpha(\mu^2)}{3\pi} \log(-q^2/\mu^2) + \mathcal{O}(\alpha^2) \right]$$

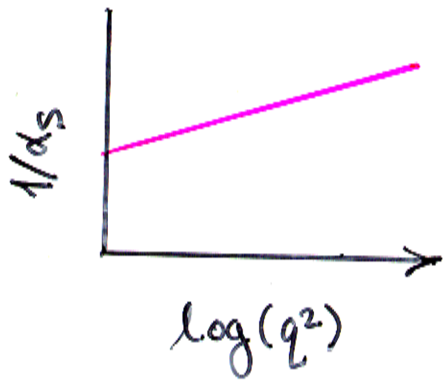
FERMION LOOPS: SCREENING

NON-ABELIAN TERMS: ANTISCREENING

RUNNING α_s

For QCD, with $N=3$, sum of most important terms gives

$$1/\alpha_s(q^2) = 1/\alpha_s(\mu^2) + \frac{(33-2n_f)}{12\pi} \log(-q^2/\mu^2)$$



"asymptotic freedom"

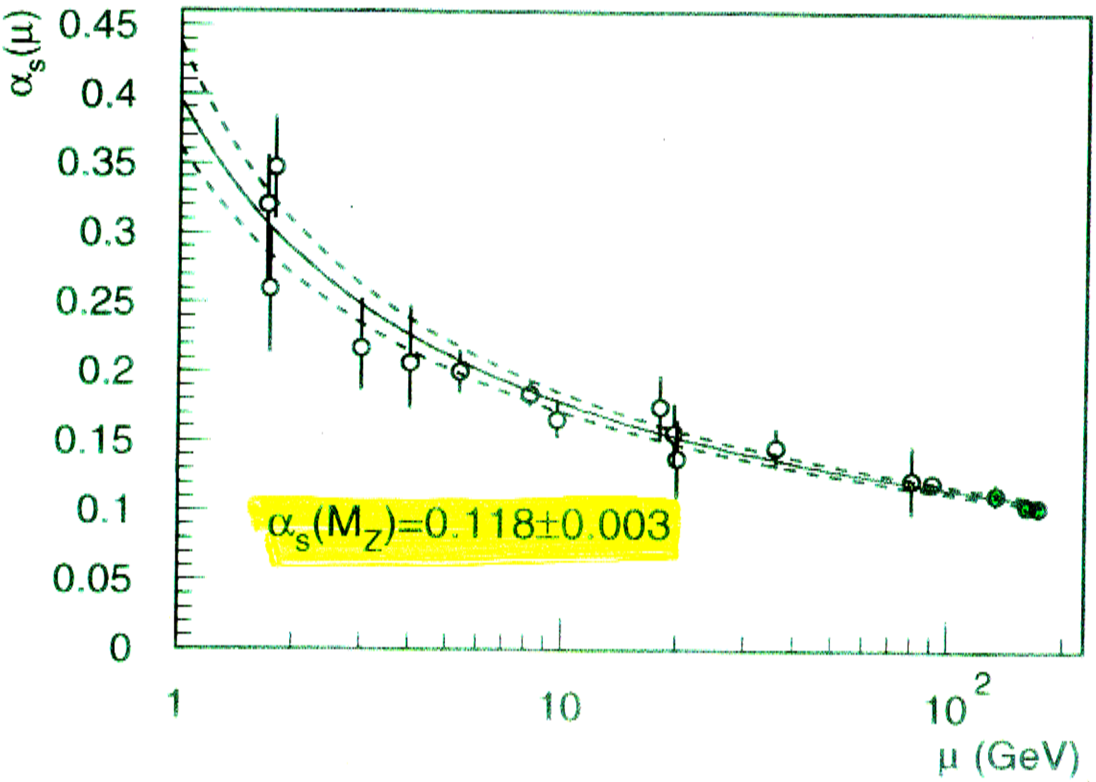
α_s diminishes at short distances / high q^2, provided

$n_f < 33/2 \rightarrow$ max. 16 flavors

Suggests a régime in which

$\alpha_s(q^2) \ll 1$,

where perturbation theory can be trusted for the strong interactions.

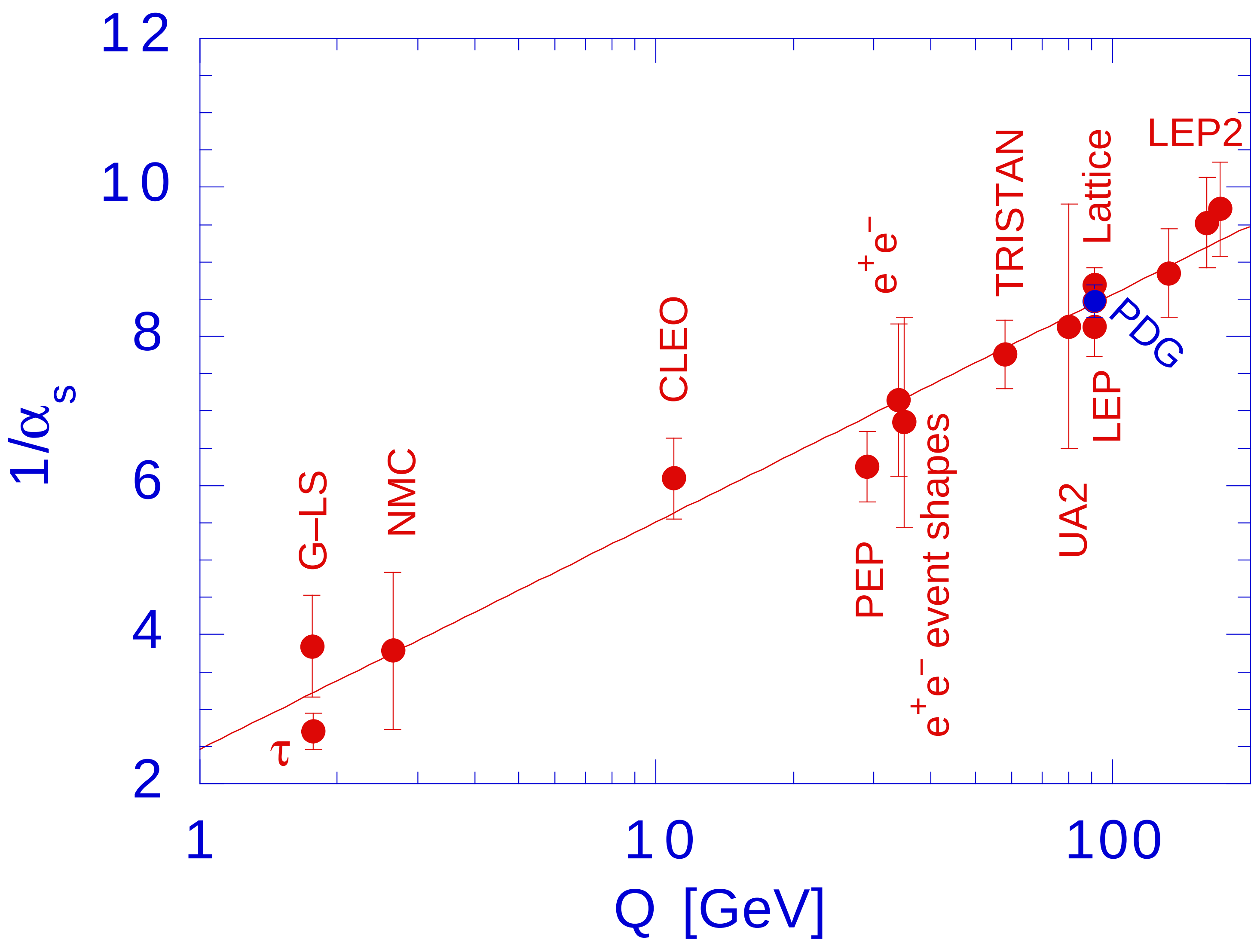


From S. Martí i García, "Review of α_s Measurements at LEP2," hep-ex/9704016

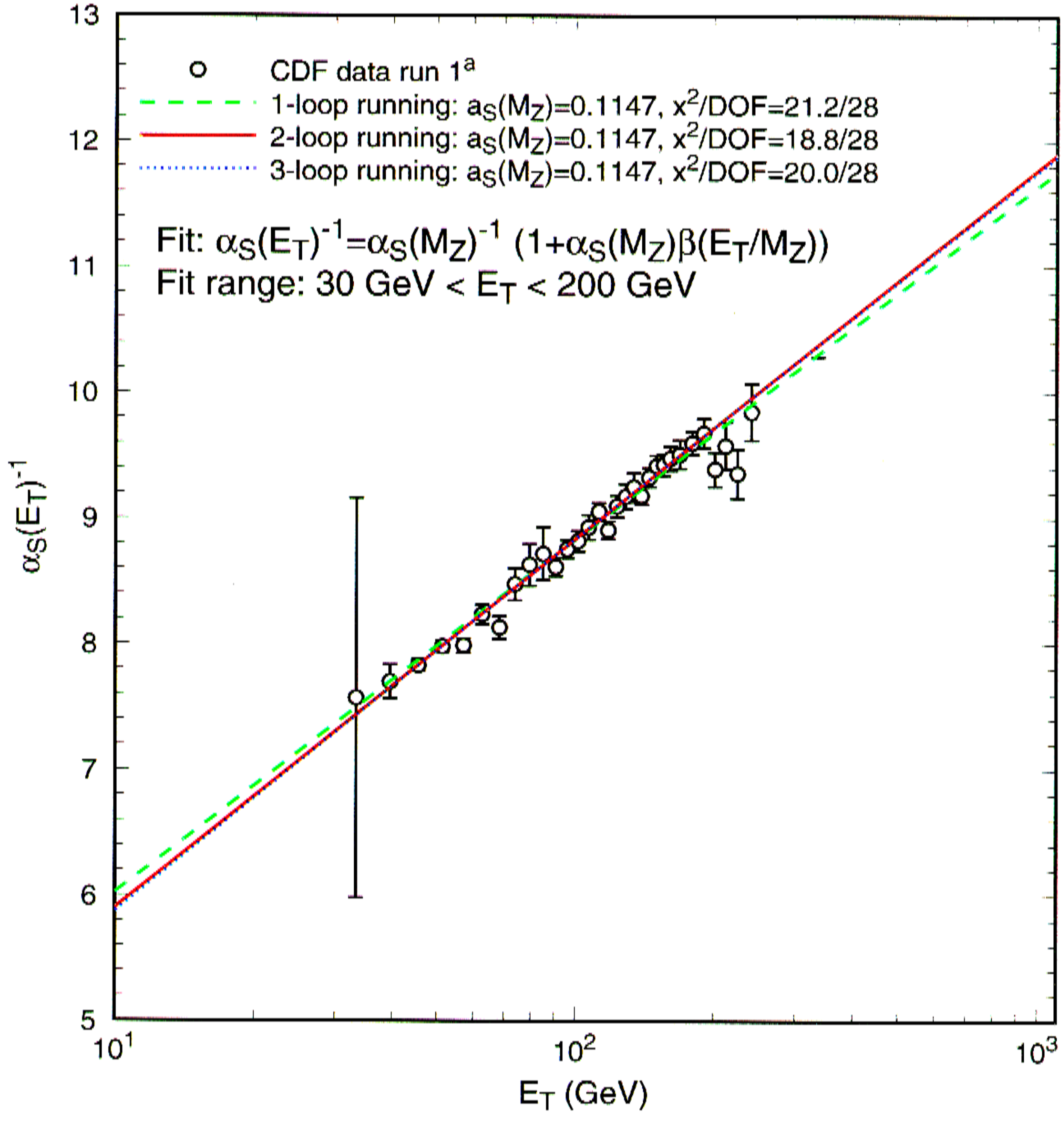
$\alpha_s(133\text{ GeV}) = 0.113 \pm 0.003 \pm 0.007$

$\alpha_s(161\text{ GeV}) = 0.105 \pm 0.003 \pm 0.006$

$\alpha_s(172\text{ GeV}) = 0.103 \pm 0.003 \pm 0.006$



12
10
8
6
4
2
$1/\alpha_s$
1
10
100
Q [GeV]
τ
G–LS
NMC
CLEO
PEP
e^+e^-
e^+e^- event shapes
TRISTAN
UA2
Lattice
LEP
PDG
LEP2



CDF data run 1[a]
1-loop running: $a_S(M_Z)$=0.1147, x^2/DOF=21.2/28
2-loop running: $a_S(M_Z)$=0.1147, x^2/DOF=18.8/28
3-loop running: $a_S(M_Z)$=0.1147, x^2/DOF=20.0/28
Fit: $\alpha_S(E_T)^{-1}=\alpha_S(M_Z)^{-1}\ (1+\alpha_S(M_Z)\beta(E_T/M_Z))$
Fit range: 30 GeV < E_T < 200 GeV
$\alpha_S(E_T)^{-1}$
E_T (GeV)

Where is the strong interaction strong?

$1/\alpha_s$ plot suggests

$\alpha_s \approx 1$ at $Q \sim 400$ MeV

... and where (perturbatively) weak?

already for $Q \approx 5$ GeV,

$\alpha_s \approx 0.2$,

a régime in which we can begin to feel comfortable about perturbation theory

The problem of hadron structure (proton mass, pion size, etc.) lives in the strong-coupling (non-perturbative régime).

Conventional to introduce QCD scale parameter Λ, defining (leading order)

$$\alpha_s(q^2) = 12\pi/(33-2n_f)\log(-q^2/\Lambda^2)$$

Some applications of Perturbative QCD

- QCD Corrections to parton-model formula for

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\to 3\sum_q e_q^2 \cdot \left\{1 + \underbrace{\frac{\alpha_s(Q^2)}{2\pi}} + O(\alpha_s^2)\ldots\right\}$$

FIG.

small (diminishing with Q^2), positive correction.

- Resolution of the parton model / permanent confinement puzzle.

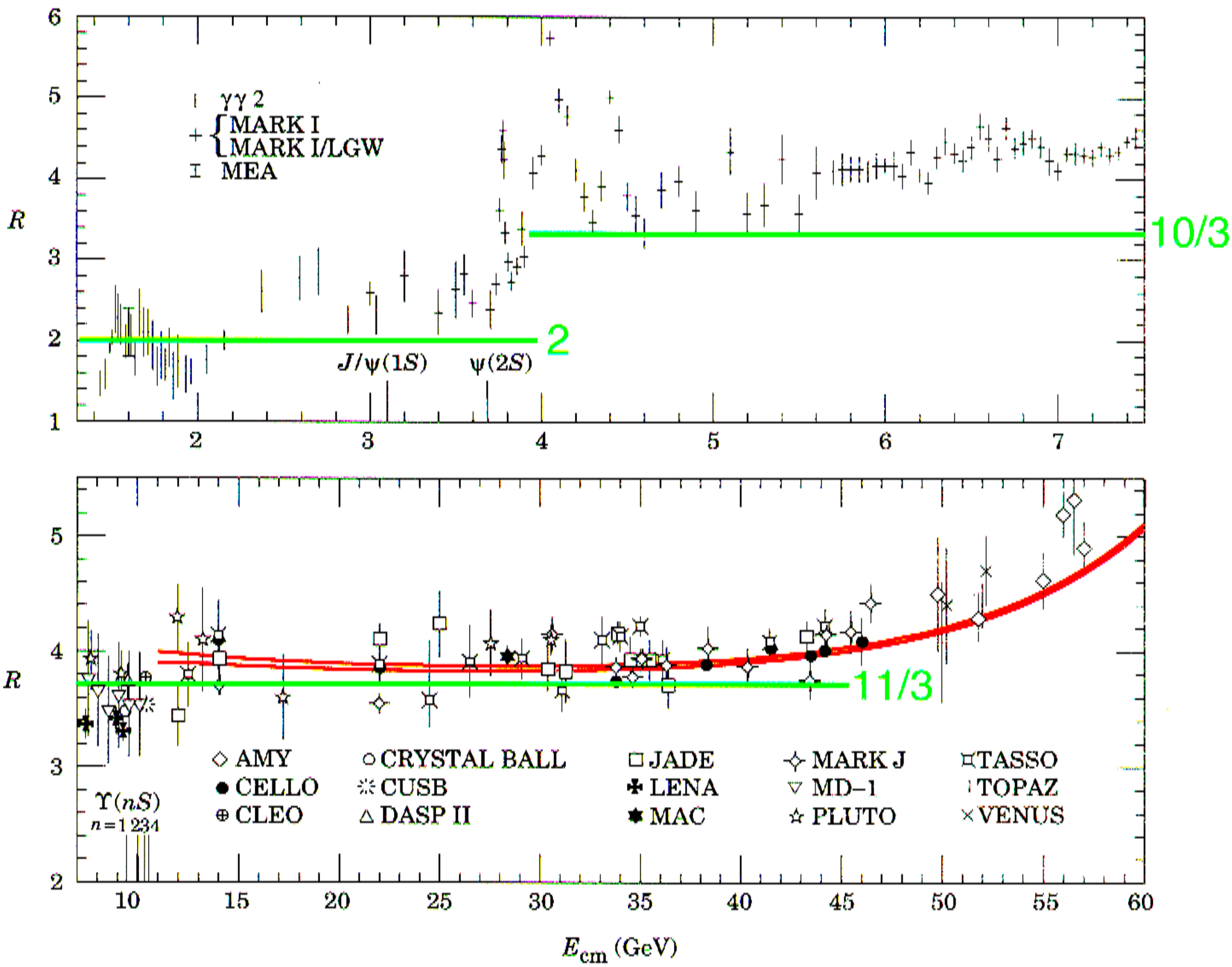
(GALLO LECTURES)

Predicts F_2 roughly independent of Q^2 and allows a controlled calculation of departures from exact scaling.

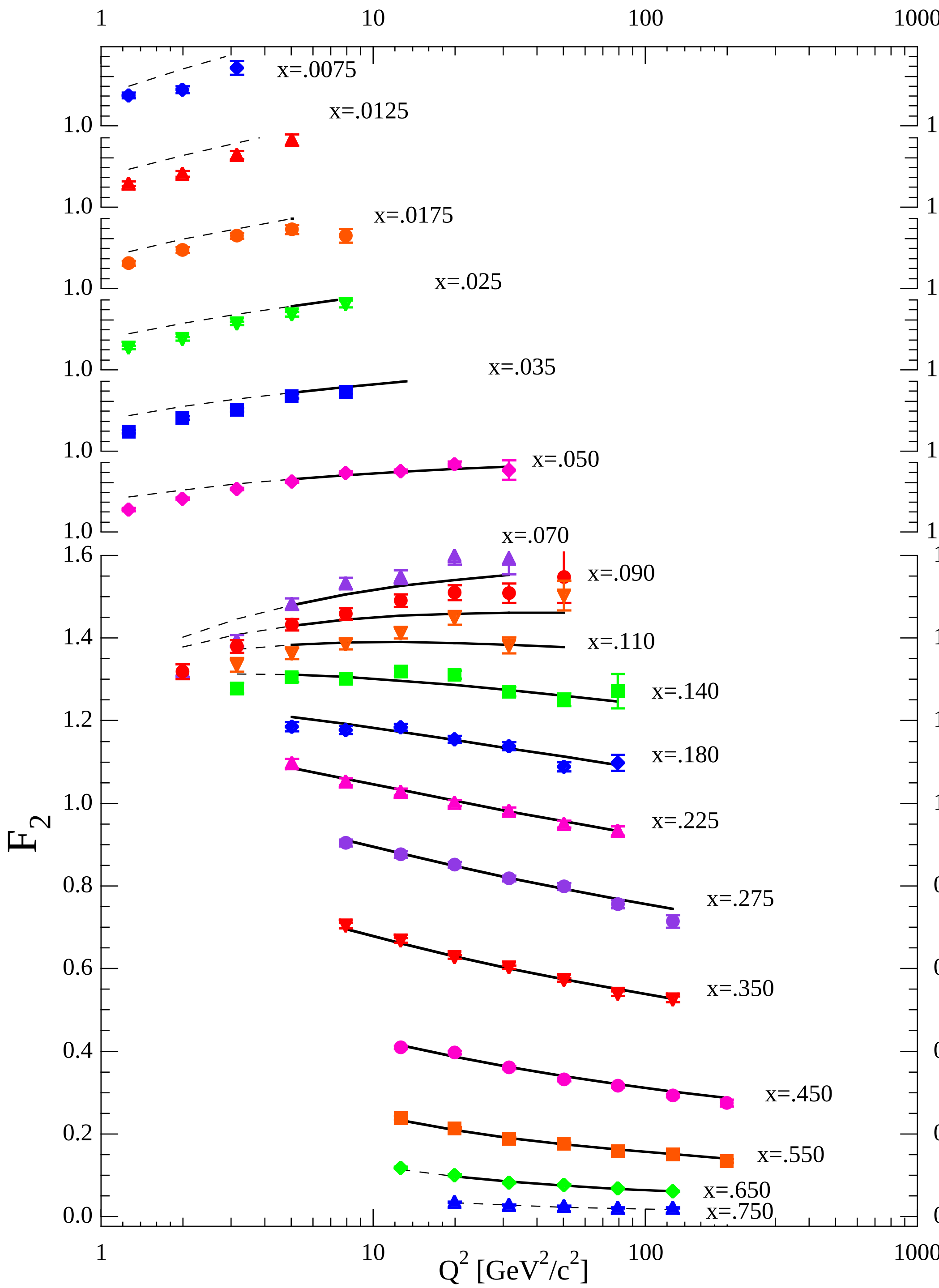
FIG.

Logarithmic variations

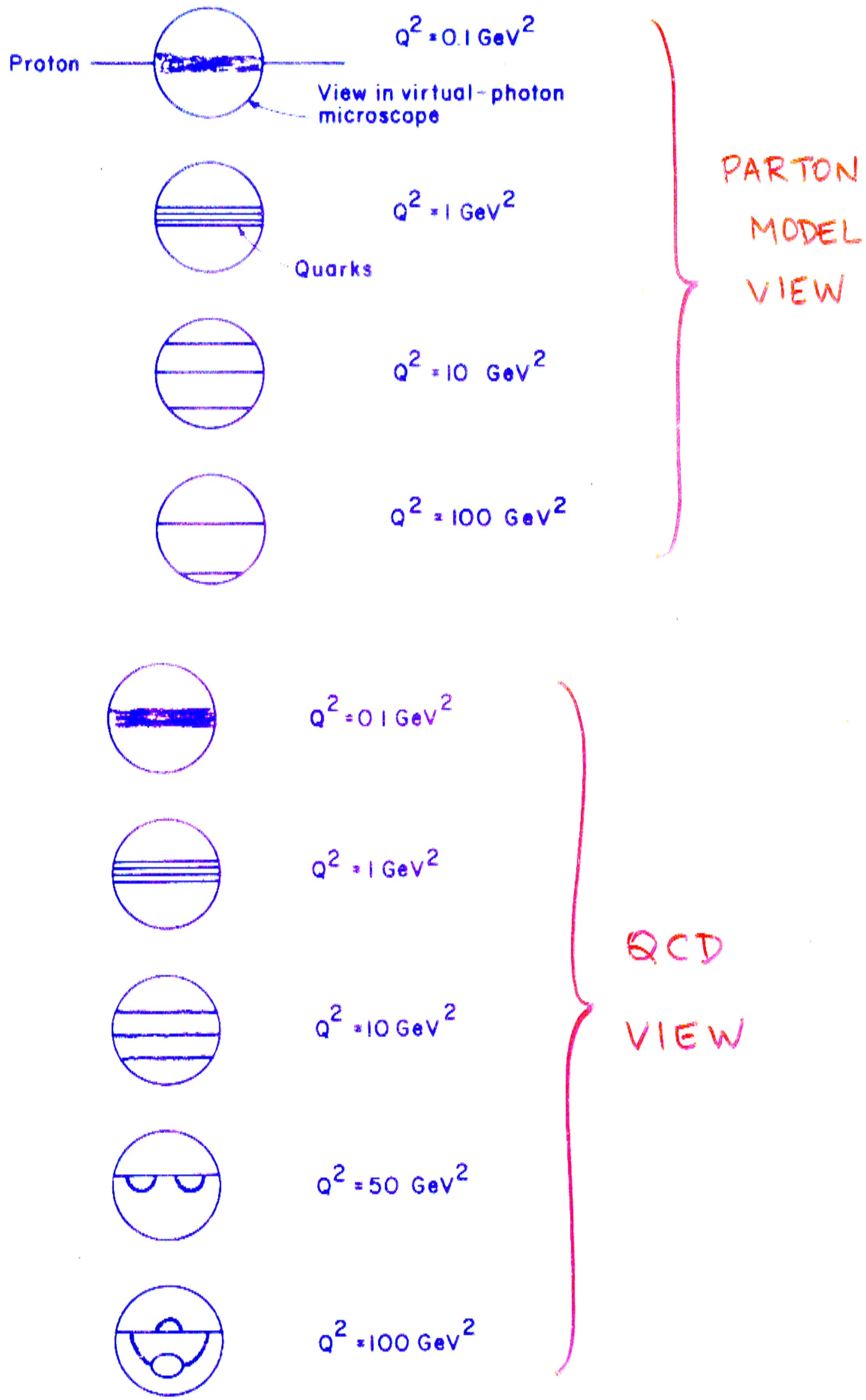
Non-asymptotically free theory predicts stronger (power-law) variations.



γγ 2
MARK I
MARK I/LGW
MEA
R
10/3
2
J/ψ(1S)
ψ(2S)
11/3
AMY
CELLO
CLEO
CRYSTAL BALL
CUSB
DASP II
JADE
LENA
MAC
MARK J
MD−1
PLUTO
TASSO
TOPAZ
VENUS
ϒ(nS)
n=1 2 3 4
E_cm (GeV)



1
10
100
1000
x=.0075
x=.0125
x=.0175
x=.025
x=.035
x=.050
x=.070
x=.090
x=.110
x=.140
x=.180
x=.225
x=.275
x=.350
x=.450
x=.550
x=.650
x=.750
1.0
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
F2
Q2 [GeV2/c2]



Proton
$Q^2 = 0.1\ GeV^2$
View in virtual-photon microscope
$Q^2 = 1\ GeV^2$
Quarks
$Q^2 = 10\ GeV^2$
$Q^2 = 100\ GeV^2$
PARTON MODEL VIEW
$Q^2 = 0.1\ GeV^2$
$Q^2 = 1\ GeV^2$
$Q^2 = 10\ GeV^2$
$Q^2 = 50\ GeV^2$
$Q^2 = 100\ GeV^2$
QCD VIEW

HIDDEN SYMMETRIES

Link between exact conservation laws and exact continuous (global) symmetries. Noether's Theorem (1918)

Some examples

energy	$\longleftrightarrow$	time translation
momentum	$\longleftrightarrow$	space translation
angular momentum	$\longleftrightarrow$	rotation invariance
electric charge	$\longleftrightarrow$	phase invariance

Two conditions:

- Invariance of the Lagrangian,

$$\delta \mathcal{L} = 0$$

- The unique physical vacuum is invariant under the symmetry transformations.

Then a classic analysis demonstrates the mass degeneracy of particle multiplets.

(Wigner's Group Theory)

Approximate (explicitly broken) sym.

isospin; flavor SU(3)

conservation of strangeness, ...

$$\mathcal{L} = \mathcal{L}_{\text{symmetric}} + \varepsilon\,\mathcal{L}_{\text{sym. breaking}}$$

Perturbation lifts degeneracy.

Splitting depends on ε,

vanishes as $\varepsilon \to 0$.

Examples:

- Traditional view of nuclear interaction arising from

$$\mathcal{L} = \mathcal{L}_{\text{strong}} + \mathcal{L}_{EM}(\alpha)$$

- Broken SU(3) as

$$\mathcal{L} = \underbrace{\mathcal{L}_{\text{strong}}}_{\text{respects SU(3)}} + \underbrace{\mathcal{L}_{\text{"medium strong"}}(m_s)}_{\text{singles out strangeness}}$$

USEFUL EXERCISE

Consider an *infinite ferromagnet*:

- CRYSTALLINE ARRAY OF SPINS (MAGNETIC DIPOLE MOMENTS) WITH NEAREST-NEIGHBOR INTERACTIONS

Interaction (EM!) is invariant under the group SO(3) of spatial rotations.

Disordered Phase ($T > T_{Curie}$):

Medium displays exact rotation symmetry (provided $\underset{\sim}{B} = 0$).

Thermal fluctuations are more important than dipole-dipole interactions $\Rightarrow$

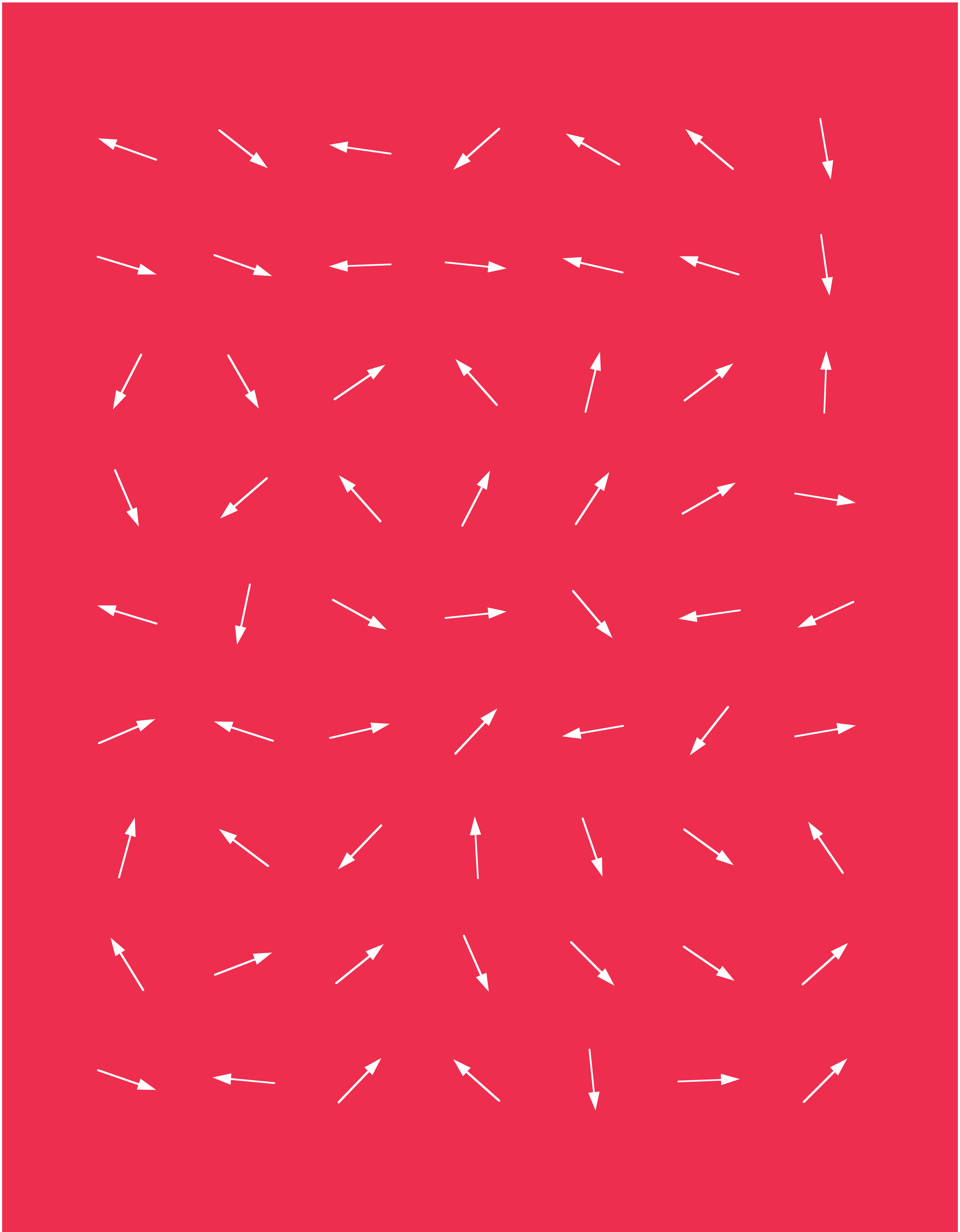
ORIENTATION OF SPINS IS RANDOM

$$\boxed{\text{SYMMETRY} \Leftrightarrow \text{DISORDER}}$$

Spontaneous magnetization $\langle \underset{\sim}{M} \rangle = 0$

No preferred direction in space

SO(3) invariance is MANIFEST



A perturbation can break the symmetry.

Static magnetic field $\underset{\sim}{B}$ biases the alignment of individual dipoles, and conceals the SO(3) invariance of the nearest-neighbor interaction

EXPLICIT BREAKING $SO(3) \longrightarrow SO(2)$

(ONE SYMMETRY AXIS REMAINS)

$$\langle \underset{\sim}{M}(\underset{\sim}{B}) \rangle \parallel \underset{\sim}{B}$$

FULL SO(3) SYMMETRY IS RESTORED AS $\underset{\sim}{B} \to 0$.

Ordered Phase ($T < T_c$):

Configurations of lowest energy have

$$\langle \underset{\sim}{M} \rangle \neq 0$$

Nearest-neighbor interaction favors PARALLEL ALIGNMENT OF SPINS

$$\boxed{\text{ORDER} \Longleftrightarrow \text{LESS SYMMETRY}}$$

Spontaneous magnetization

Ordered (Ferromagnetic) Phase

SO(3) invariance is spontaneously broken to SO(2)

SO(3) invariance of the EM interaction is HIDDEN because it is not manifest in the ground state.

Direction of the spontaneous magnetization is RANDOM: $\langle\langle \underset{\sim}{M} \rangle\rangle = 0$. Ground state is ∞ly degenerate. Properties don't depend on which one.

Particle Physics:
The Standard Model

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CERN Summer Lectures
17 – 27 July 2000

5

TOWARD THE ELECTROWEAK THEORY

GAUGE PRINCIPLE allows us to derive INTERACTIONS from SYMMETRIES.

Renormalizable, calculable theories

- vector gauge bosons
- "minimal coupling" to matter
- non-Abelian gauge bosons have self-interactions (possibility of asymptotic freedom)

Mathematically possible to construct a gauge theory for any simple or semi-simple gauge group.

$U(1)_{EM}$	QED
$SU(2)_{isospin}$	~~Yang-Mills~~
$SU(3)_{color}$	QCD

Are all the fundamental interactions

GAUGE INTERACTIONS?

Can we make a gauge theory of the weak interactions?

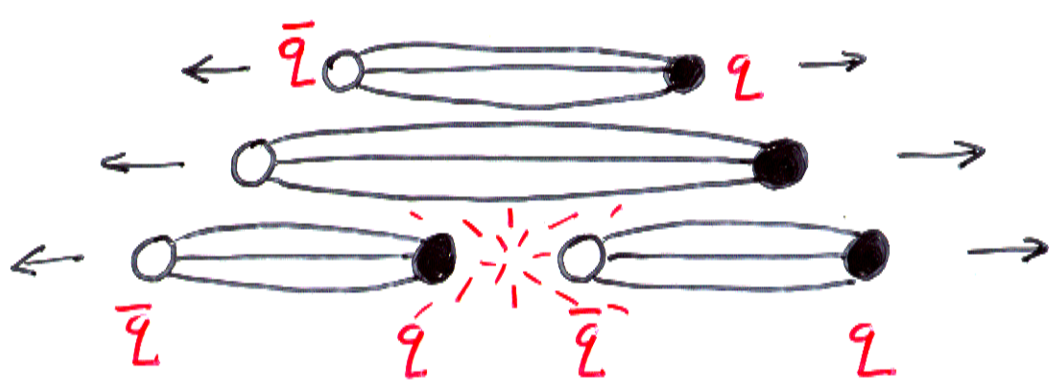
CHALLENGE: Gauge bosons are massless.

(Forces have ∞ range)

Range of weak interactions $\sim 10^{-15}$ cm

$\Rightarrow$ Mass of weak boson ~ 100 GeV/c^2

Short range ($\sim 10^{-13}$ cm) we experience for the strong interaction derives from its strength:



USEFUL ANALOGY FROM CONDENSED MATTER:

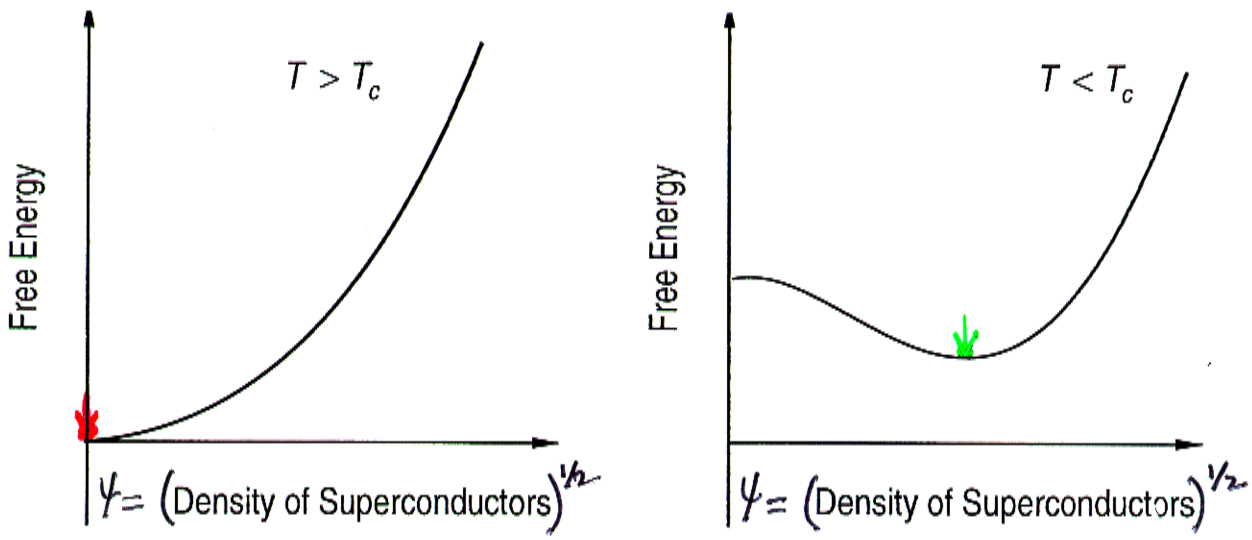
The Superconducting Phase Transition

SUPERCONDUCTOR:

- No electrical resistance
- Magnetic fields don't penetrate (Meissner effect)

Ginzburg–Landau description

Normal resistive carriers • Superconductors (No resistance)



$$G_{super}(\underset{\sim}{B}=0) = G_{normal}(\underset{\sim}{B}=0) + \alpha|\psi|^2 + \beta|\psi|^4$$

$T > T_c$: $\alpha > 0$, $\langle\psi\rangle_0 = 0$ — Resistive flow

$T < T_c$: $\alpha < 0$, $\langle\psi\rangle_0 \neq 0$ — Resistance-free flow

---

In a magnetic field $\underset{\sim}{B} \neq 0$, find that the PHOTON acquires a MASS within the superconductor.

A model for the weak bosons?

1998 CERN Summer Student Lecture Programme

Particle Physics: The Standard Model

Chris Quigg

* * * * *

The Ginzburg–Landau theory of superconductivity provides a phenomenological understanding of the Meissner effect: the observation that an external magnetic field does not penetrate the superconductor. Ginzburg and Landau introduce an "order parameter" ψ, such that $|\psi|^2$ is related to the density of superconducting electrons. In the absence of an impressed magnetic field, expand the free energy of the superconductor as

$$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha\,|\psi|^2 + \beta\,|\psi|^4\,,$$

where α and β are phenomenological parameters.

(a) Minimize $G_{\text{super}}(0)$ with respect to the order parameter and discuss the circumstances under which spontaneous symmetry breaking occurs. Compute $\langle|\psi|^2\rangle_0$, the value at which $G_{\text{super}}(0)$ is minimized.

(b) In the presence of an external magnetic field $\mathbf{B}$, a gauge-invariant expression for the free energy is

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{2} + \frac{1}{2m^*}\psi^*(-i\nabla - e^*\mathbf{A})^2\psi.$$

(The effective charge e^* turns out to be $2e$, because $|\psi|^2$ represents the density of Cooper pairs.) Derive the field equations that follow from minimizing $G_{\text{super}}(\mathbf{B})$ with respect to ψ and $\mathbf{A}$. Show that in the weak-field approximation ($\nabla\psi \approx 0$, $\psi \approx \langle\psi\rangle_0$) the photon acquires a mass within the superconductor. [Reference: V. L. Ginzburg and L. D. Landau, *Zh. Eksp. Teor. Fiz.* **20**, 1064 (1950); English translation: see *Men of Physics: Landau,* Vol. II, edited by D. ter Haar, Pergamon, New York, 1965. For further information, see §21.6 of Weinberg's *The Quantum Theory of Fields,* vol. 2, "Modern Applications" (Cambridge University Press, Cambridge, 1996).]

EW INTERACTIONS OF LEPTONS

Base our theory (in part) on the lepton family doublets and the $SU(2)_L$ symmetry we infer from them.

Left-handed weak-isospin doublet

$$L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

Right-handed weak-isospin singlet

$$R \equiv e_R = \frac{(1+\gamma_5)e}{2}$$

Assume no RH neutrino

Assign "weak hypercharge" Y so that

"Gell-Mann-Nishijima" $$Q = I_3 + \tfrac{1}{2}Y$$

$$\left.\begin{aligned} \nu_L:&\quad I_3 = +1/2 \qquad Q = 0 \\ e_L:&\quad I_3 = -1/2 \qquad Q = -1 \end{aligned}\right\} Y_L = -1$$

$$e_R: \quad I_3 = 0 \qquad Q = -1 \Rightarrow Y_R = -2$$

Choose as the gauge group

$SU(2)_L \otimes U(1)_Y$

GAUGE BOSONS:

$SU(2)_L \; \div \; b^1_\mu \quad b^2_\mu \quad b^3_\mu \qquad g$

$U(1)_Y \; \div \; a_\mu \qquad\qquad g'/2$

semi-simple (product) group

$\Rightarrow$ two independent coupling constants

MATTER TERM:

$$\mathcal{L}_{leptons} = \bar{L} i\gamma^\mu \left(\partial_\mu + \frac{ig}{2}\vec{b}_\mu \cdot \vec{\tau} + \frac{ig'}{2} a_\mu Y\right) L$$

$$+ \bar{R} i\gamma^\mu \left(\partial_\mu + \frac{ig'}{2} a_\mu Y\right) R$$

GAUGE-FIELD TERM:

$$\mathcal{L}_{gauge} = -\frac{1}{4} F^\ell_{\mu\nu} F^{\ell\mu\nu} - \frac{1}{4} f_{\mu\nu} f^{\mu\nu},$$

$$F^\ell_{\mu\nu} = \partial_\nu b^\ell_\mu - \partial_\mu b^\ell_\nu + g\,\varepsilon_{jk\ell}\, b^j_\mu b^k_\nu$$

$$f_{\mu\nu} = \partial_\nu a_\mu - \partial_\mu a_\nu$$

COMPLETE (UNBROKEN) LAGRANGIAN:

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{leptons}$$

NOT A REALISTIC THEORY!

(i) Cannot have a mass term for e, because symmetry treats the LH, RH fermions differently.

$$m\bar{\psi}\psi = m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$$

(ii) Theory has four massless EW gauge bosons. Nature has only one, the photon.

Example of SUPERCONDUCTIVITY showed us how SPONTANEOUS SYMMETRY BREAKING can endow GAUGE BOSONS with MASS.

Break $SU(2)_L \otimes U(1)_Y \longrightarrow U(1)_{EM}$

Will also see how FERMION MASS can arise from SSB.

Arrange that the VACUUM does not reflect the GAUGE SYMMETRY

Introduce an $SU(2)_L$ doublet of complex scalar fields

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \quad \text{with} \quad \left.\begin{matrix} I_3 = +\frac{1}{2} \\ I_3 = -\frac{1}{2} \end{matrix}\right\} Y_\varphi = +1$$

SCALAR LAGRANGIAN:

$$\mathcal{L}_{scalar} = (\mathcal{D}^\mu \varphi)^\dagger (\mathcal{D}_\mu \varphi) - V(\varphi^\dagger \varphi)$$

covariant derivative is

$$\mathcal{D}_\mu = \partial_\mu + \frac{ig}{2} \vec{b}_\mu \cdot \vec{\tau} + \frac{ig'}{2} a_\mu Y$$

potential is

$$V(\varphi^\dagger \varphi) = \mu^2 (\varphi^\dagger \varphi) + |\lambda| (\varphi^\dagger \varphi)^2$$

We may also add a

YUKAWA INTERACTION:

$$\mathcal{L}_{Yuk} = -G_e [\bar{R}(\varphi^\dagger L) + (\bar{L}\varphi)R]$$

new "Yukawa" coupling

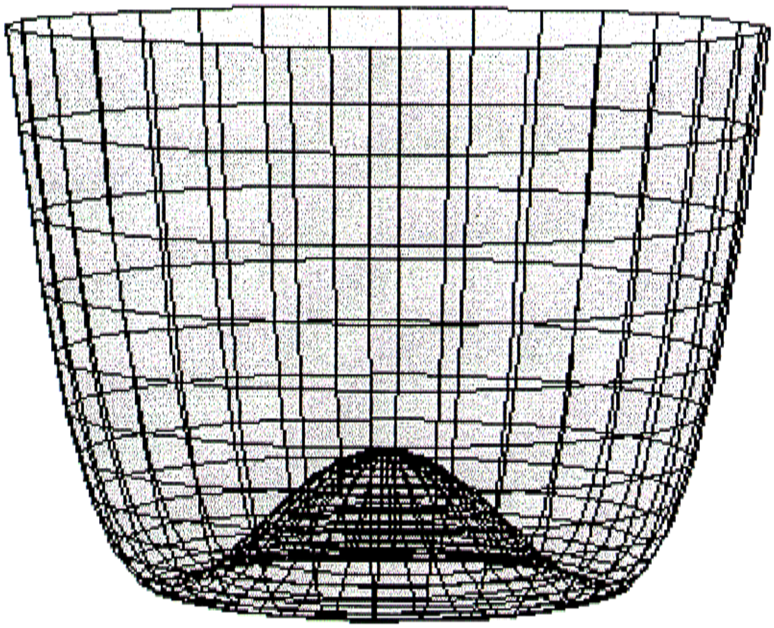
$SU(2)_L \otimes U(1)_Y$ invariant, Lorentz scalar

$$V(\varphi^\dagger\varphi) = \mu^2(\varphi^\dagger\varphi) + |\lambda|(\varphi^\dagger\varphi)^2$$

$\boxed{\text{CHOOSE} \quad \mu^2 < 0}$

MINIMUM OF POTENTIAL AT

$$|\varphi|^2 = -\mu^2/2|\lambda| \equiv v^2/2$$



Gauge symmetry is spontaneously broken.

Take $\langle\varphi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

breaks $SU(2)_L$ <u>and</u> $U(1)_Y$ ...

$$\tau_1\langle\varphi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0$$

$$\tau_2\langle\varphi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0$$

$$\tau_3\langle\varphi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0$$

$$Y\langle\varphi\rangle_0 = Y_\varphi\langle\varphi\rangle_0 = +1\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0$$

Addendum: Gauge symmetry and the vacuum state. (Refer to pp. 86-87)

SYMMETRY OPERATION G LEAVES THE VACUUM INVARIANT IF

$$G\langle\varphi\rangle_0 = \langle\varphi\rangle_0$$

We are interested in $SU(2)_L \otimes U(1)_Y$ gauge transforms

$$G = \exp\left(\frac{i\vec{\alpha}\cdot\vec{\tau}}{2}\right) \qquad \text{etc.}$$

which are built of infinitesimal transforms. Expand exponential:

$$G\langle\varphi\rangle_0 = \left(1 + \frac{i\vec{\alpha}\cdot\vec{\tau}}{2}\right)\langle\varphi\rangle_0$$

$$= \langle\varphi\rangle_0 \qquad \text{IFF} \qquad \vec{\tau}\langle\varphi\rangle_0 = 0$$

... but $U(1)_{EM}$ symmetry is respected by the vacuum state

$$Q\langle\varphi\rangle_0 = \frac{1}{2}(\tau_3+Y)\langle\varphi\rangle_0 = \begin{pmatrix} Y_\varphi+1 & 0 \\ 0 & Y_\varphi-1 \end{pmatrix}\langle\varphi\rangle_0$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

THE FOUR ORIGINAL GENERATORS ALL ARE BROKEN, BUT ELECTRIC CHARGE IS NOT.

- Have accomplished
  $$SU(2)_L \otimes U(1)_Y \longrightarrow U(1)_{EM}$$
- Expect the photon to remain massless
- Expect the remaining gauge bosons to acquire masses.

To analyze the outcome,

choose a gauge in which

$$\varphi = \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}$$

SCALAR LAGRANGIAN:

$$\mathcal{L}_{scalar} = \frac{1}{2}[(\partial_\mu \eta)(\partial^\mu \eta) + 2\mu^2 \eta^2]$$
$$+ \frac{(v+\eta)^2}{8}[g^2 |b_\mu^1 - i b_\mu^2|^2 + (g' a_\mu - g b_\mu^3)^2]$$

+ other interaction terms

To analyze the outcome,

choose a gauge in which

$$\varphi = \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}$$

SCALAR LAGRANGIAN:

$$\mathcal{L}_{scalar} = \frac{1}{2}\left[(\partial_\mu \eta)(\partial^\mu \eta) + 2\mu^2\eta^2\right]$$

$$+ \frac{(v+\eta)^2}{8}\left[\underbrace{g^2|b_\mu^1 - ib_\mu^2|^2}_{} + (g' a_\mu - g b_\mu^3)^2\right]$$

+ other interaction terms

- CHARGED GAUGE BOSONS

$$W^\pm = \frac{b_\mu^1 \mp i b_\mu^2}{\sqrt{2}}$$

$$\frac{g^2 v^2}{8}\left(|W_\mu^+|^2 + |W_\mu^-|^2\right)$$

VECTOR FIELD MASS TERM: $\frac{1}{2} m^2 A_\mu A^\mu$

$\Rightarrow$ $M_W = gv/2$

MASSIVE!

To analyze the outcome, choose a gauge in which

$$\varphi = \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}$$

SCALAR LAGRANGIAN:

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2}\left[(\partial_\mu \eta)(\partial^\mu \eta) + 2\mu^2\eta^2\right]$$

$$+ \frac{(v+\eta)^2}{8}\left[g^2|b_\mu^1 - i b_\mu^2|^2 + \underbrace{(g' a_\mu - g b_\mu^3)^2}\right]$$

+ other interaction terms

• NEUTRAL GAUGE BOSONS

Define orthonormal

$$Z_\mu \equiv \frac{-g' a_\mu + g b_\mu^3}{\sqrt{g^2+g'^2}}$$

ACQUIRES MASS

$$M_Z = \frac{\sqrt{g^2+g'^2}\,v}{2} \geq M_W$$

$$A_\mu \equiv \frac{g a_\mu + g' b_\mu^3}{\sqrt{g^2+g'^2}}$$

HAS NO MASS TERM

(HOPE TO IDENTIFY AS THE PHOTON)

To analyze the outcome,

choose a gauge in which

$$\varphi = \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}$$

SCALAR LAGRANGIAN:

$$\mathcal{L}_{scalar} = \frac{1}{2}\left[(\partial_\mu \eta)(\partial^\mu \eta) + 2\mu^2\eta^2\right] + \frac{(v+\eta)^2}{8}\left[g^2 |b^1_\mu - i b^2_\mu|^2 + (g' a_\mu - g b^3_\mu)^2\right]$$

$$+ \text{ other interaction terms}$$

- SCALAR FIELD η ACQUIRES

$$M_\eta^2 = -2\mu^2 > 0$$

THIS NEUTRAL SCALAR FIELD IS THE HIGGS BOSON OF THE ELECTROWEAK THEORY

We just determined that

$$M_W = gv/2$$

so we have

$$M_W^2 = 2v^2 G_F M_W^2/\sqrt{2}$$

$$\Rightarrow \quad v = (G_F\sqrt{2})^{-1/2} \approx 246 \text{ GeV}$$

so that

$$\langle\varphi\rangle_0 = \begin{pmatrix} 0 \\ (G_F\sqrt{8})^{-1/2} \end{pmatrix} \approx \begin{pmatrix} 0 \\ 174 \text{ GeV} \end{pmatrix}$$

"THE ELECTROWEAK SCALE"

DON'T (YET) HAVE A PREDICTION FOR M_W — NEED TO DETERMINE g.

HOW DO THE GAUGE BOSONS INTERACT?

Look back at

$$\mathcal{L}_{leptons} = \bar{L} i\gamma^\mu \left(\partial_\mu + \frac{ig}{2} \vec{b}_\mu \cdot \vec{\tau} + \frac{ig'}{2} a_\mu Y\right) L$$

$$+ \bar{R} i\gamma^\mu \left(\partial_\mu + \frac{ig'}{2} a_\mu Y\right) R$$

Recall $L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad R \equiv e_R$

HOW DO THE GAUGE BOSONS INTERACT?

Look back at

$$\mathcal{L}_{\text{leptons}} = \bar{L} i\gamma^\mu\left(\partial_\mu + \frac{ig}{2}\vec{b}_\mu\cdot\vec{\tau} + \frac{ig'}{2}\mathcal{A}_\mu Y\right)L$$

$$+\bar{R} i\gamma^\mu\left(\partial_\mu + \frac{ig'}{2}\mathcal{A}_\mu Y\right)R$$

Recall $L \equiv \begin{pmatrix}\nu\\ e\end{pmatrix}_L$ $\qquad R \equiv e_R$

• CHARGED-CURRENT INTERACTIONS

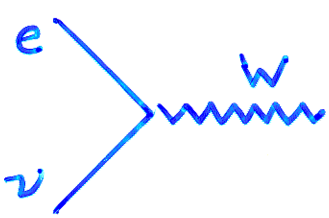
$$\mathcal{L}_{cc} = -\frac{g}{2}\bar{L}\gamma^\mu(\tau_1 b^1_\mu + \tau_2 b^2_\mu)L$$

with $b^1_\mu = \dfrac{W^+_\mu + W^-_\mu}{\sqrt{2}}$; $\quad b^2_\mu = \dfrac{W^-_\mu - W^+_\mu}{\sqrt{2}}$

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}}\left\{\bar{\nu}_L\gamma^\mu e_L W^+_\mu + \bar{e}_L\gamma^\mu \nu_L W^-_\mu\right\}$$

$$= -\frac{g}{2\sqrt{2}}\left\{\bar{\nu}\gamma^\mu(1-\gamma_5)e W^+_\mu + \bar{e}\gamma^\mu(1-\gamma_5)\nu W^-_\mu\right\}$$

FEYNMAN RULE:



$$= \frac{-ig}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$$

REPRODUCES RULE FROM LOW-ENERGY PHENOMENOLOGY

IF WE IDENTIFY $\quad \dfrac{g}{2\sqrt{2}} = \left(\dfrac{G_F M_W^2}{\sqrt{2}}\right)^{1/2}$

cf. p. 5 of notes

HOW DO THE GAUGE BOSONS INTERACT?

Look back at

$$\mathcal{L}_{leptons} = \bar{L} i\gamma^\mu \left(\partial_\mu + \frac{ig}{2}\vec{b}_\mu \cdot \vec{\tau} + \frac{ig'}{2} a_\mu Y\right) L$$

$$+ \bar{R} i\gamma^\mu \left(\partial_\mu + \frac{ig'}{2} a_\mu Y\right) R$$

Recall $L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad R \equiv e_R$

- NEUTRAL GAUGE-BOSON INT^{NS}

$$\mathcal{L}_0 = -\frac{g}{2}\bar{L}\gamma^\mu b^3_\mu \tau_3 L - \frac{g'}{2}(-1)\bar{L}\gamma^\mu a_\mu L$$

$$- \frac{g'}{2}(-2)\bar{R}\gamma^\mu a_\mu R$$

with $b^3_\mu = \dfrac{gZ_\mu + g'A_\mu}{\sqrt{g^2+g'^2}}$, $a_\mu = \dfrac{gA_\mu - g'Z_\mu}{\sqrt{g^2+g'^2}}$

$$\mathcal{L}_0 = -\frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_L \gamma^\mu \nu_L Z_\mu$$

$$+ \frac{Z_\mu}{\sqrt{g^2+g'^2}}\left[\bar{e}_L\gamma^\mu e_L \frac{(g^2-g'^2)}{2} - \bar{e}_R\gamma^\mu e_R g'^2\right]$$

$$+ \frac{gg'}{\sqrt{g^2+g'^2}} A_\mu \left[\bar{e}_L\gamma^\mu e_L + \bar{e}_R\gamma^\mu e_R\right]$$

HOW DO THE GAUGE BOSONS INTERACT?

Look back at

$$\mathcal{L}_{leptons} = \bar{L} i\gamma^\mu \left(\partial_\mu + \frac{ig}{2}\vec{b}_\mu \cdot \vec{\tau} + \frac{ig'}{2} a_\mu Y\right) L$$

$$+ \bar{R} i\gamma^\mu \left(\partial_\mu + \frac{ig'}{2} a_\mu Y\right) R$$

Recall $L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad R \equiv e_R$

• NEUTRAL GAUGE-BOSON INT$^{\underline{NS}}$

$$\mathcal{L}_0 = -\frac{g}{2}\bar{L}\gamma^\mu b_\mu^3 \tau_3 L - \frac{g'}{2}(-1)\bar{L}\gamma^\mu a_\mu L$$

$$- \frac{g'}{2}(-2)\bar{R}\gamma^\mu a_\mu R$$

with $b_\mu^3 = \dfrac{gZ_\mu + g' A_\mu}{\sqrt{g^2+g'^2}}$, $a_\mu = \dfrac{gA_\mu - g' Z_\mu}{\sqrt{g^2+g'^2}}$

$$\mathcal{L}_0 = -\frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_L \gamma^\mu \nu_L Z_\mu$$

$$+ \frac{Z_\mu}{\sqrt{g^2+g'^2}}\left[\bar{e}_L\gamma^\mu e_L \frac{(g^2-g'^2)}{2} - \bar{e}_R\gamma^\mu e_R g'^2\right]$$

$$+ \frac{gg'}{\sqrt{g^2+g'^2}} A_\mu \underbrace{\left[\bar{e}_L\gamma^\mu e_L + \bar{e}_R\gamma^\mu e_R\right]}_{\bar{e}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)e + \bar{e}\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)e = \bar{e}\gamma^\mu e}$$

$\equiv e$

vector interaction coupled to Q

HOW DO THE GAUGE BOSONS INTERACT?

Look back at

$$\mathcal{L}_{leptons} = \bar{L} i\gamma^\mu\left(\partial_\mu + \frac{ig}{2}\vec{b}_\mu\cdot\vec{\tau} + \frac{ig'}{2}a_\mu Y\right)L$$

$$+\bar{R} i\gamma^\mu\left(\partial_\mu + \frac{ig'}{2}a_\mu Y\right)R$$

Recall $L \equiv \begin{pmatrix}\nu \\ e\end{pmatrix}_L \qquad R \equiv e_R$

• NEUTRAL GAUGE-BOSON INT$^{\underline{NS}}$

$$\mathcal{L}_0 = -\frac{g}{2}\bar{L}\gamma^\mu b_\mu^3 \tau_3 L - \frac{g'}{2}(-1)\bar{L}\gamma^\mu a_\mu L$$

$$-\frac{g'}{2}(-2)\bar{R}\gamma^\mu a_\mu R$$

with $b_\mu^3 = \frac{gZ_\mu + g'A_\mu}{\sqrt{g^2+g'^2}}$, $a_\mu = \frac{gA_\mu - g'Z_\mu}{\sqrt{g^2+g'^2}}$

$$\mathcal{L}_0 = \boxed{-\frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_L\gamma^\mu\nu_L Z_\mu + \frac{Z_\mu}{\sqrt{g^2+g'^2}}\left[\bar{e}_L\gamma^\mu e_L\frac{(g^2-g'^2)}{2} - \bar{e}_R\gamma^\mu e_R g'^2\right]}$$

$$+\frac{gg'}{\sqrt{g^2+g'^2}}A_\mu\left[\bar{e}_L\gamma^\mu e_L + \bar{e}_R\gamma^\mu e_R\right]$$

"Weak Neutral Current" interactions — unknown before EW Theory — consequences of unification.

The Weak Mixing Angle

Define

$$\underset{U(1)_Y}{g'} = \underset{SU(2)_L}{g} \tan\theta_W$$

$$\sqrt{g^2+g'^2} = g/\cos\theta_W = g'/\sin\theta_W$$

We found

$$e \equiv \frac{gg'}{\sqrt{g^2+g'^2}}$$

so can write

$$g = e/\sin\theta_W \qquad g' = e/\cos\theta_W$$

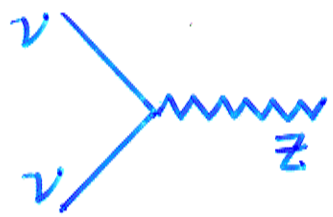
NOTE:

- 3 couplings e, g, g' are related in terms of θ_W
- The "weak" couplings g, g' are NOT SMALL compared to e

THE WEAKNESS OF THE WEAK INTERACTIONS IS DUE TO THE GREAT MASS OF THE GAUGE BOSONS

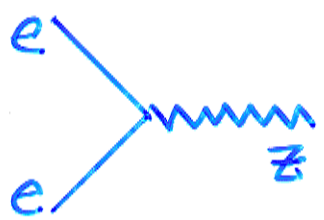
Feynman Rules for NC Interactions

$Z\nu\bar{\nu}$:



$$= \frac{-ig}{4\cos\theta_W}\gamma_\mu(1-\gamma_5)$$

$Ze\bar{e}$:



$$= \frac{-ig}{4\cos\theta_W}\gamma_\mu\left[R_e(1+\gamma_5)+L_e(1-\gamma_5)\right]$$

$$R_e = 2\sin^2\theta_W \qquad = -2Q_e\sin^2\theta_W$$

$$L_e = 2\sin^2\theta_W - 1 \qquad = 2I_3 - 2Q_e\sin^2\theta_W$$

TRY TO COMPUTE

$$\Gamma(Z\to\nu\bar{\nu}) = \frac{G_F M_Z^3}{12\pi\sqrt{2}}$$

$$\Gamma(Z\to e^+e^-) = \Gamma(Z\to\nu\bar{\nu})(L_e^2+R_e^2)$$

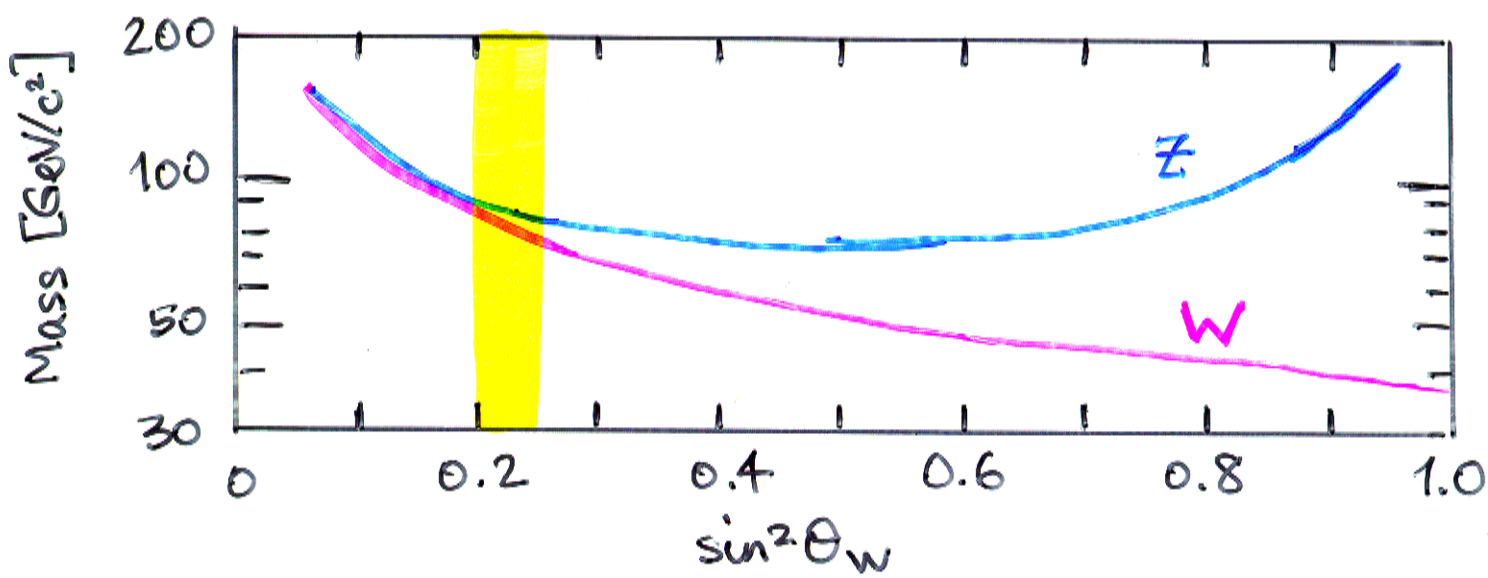
$$\Gamma(W\to e\nu) = \frac{G_F M_W^3}{6\pi\sqrt{2}}$$

GAUGE BOSON MASSES

We found

$$M_W^2 = g^2 v^2/4 = g^2/4G_F\sqrt{2}$$
$$= e^2/4G_F\sqrt{2}\,\sin^2\theta_W$$
$$= \pi\alpha/G_F\sqrt{2}\,\sin^2\theta_W$$
$$\approx \frac{(37.3\text{ GeV})^2}{\sin^2\theta_W}$$

$$M_Z^2 = (g^2+g'^2)v^2/4$$
$$= M_W^2/\cos^2\theta_W$$



Particle Physics:
The Standard Model

Chris Quigg
Theoretical Physics Department
Fermi National Accelerator Laboratory
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CERN Summer Lectures
17 – 27 July 2000

6

6

TESTS OF THE ELECTROWEAK THEORY

(Glashow, Salam, Weinberg)

Search for neutral-current interactions

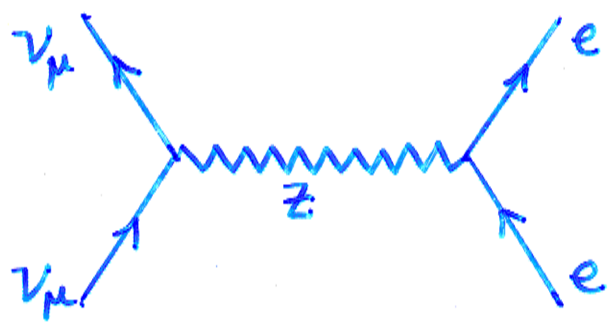
GARGAMELLE BUBBLE CHAMBER AT CERN

Look for interactions

$$\nu_\mu e \rightarrow \nu_\mu e$$

$$\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$$

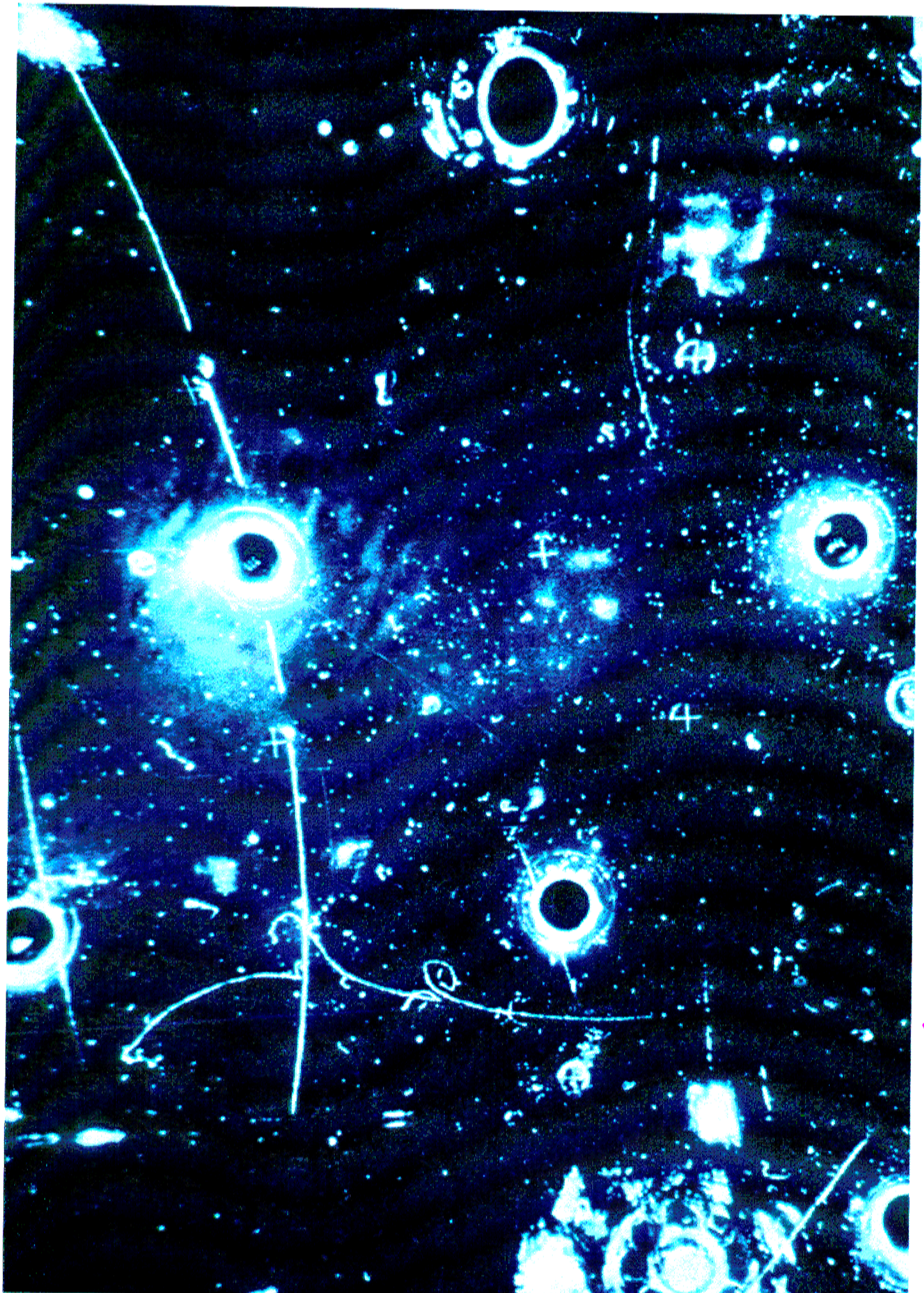
Cannot occur by cc interactions (different lepton families)



FIRST EVENT (Summer of '73)

$$\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$$

PL 46B, 121 ('73)



$\bar{\nu}_\mu$

http://preprints.cern.ch/archive/electronic/cern/PHO/photo-ex/60100.jpeg

FIRST $\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$

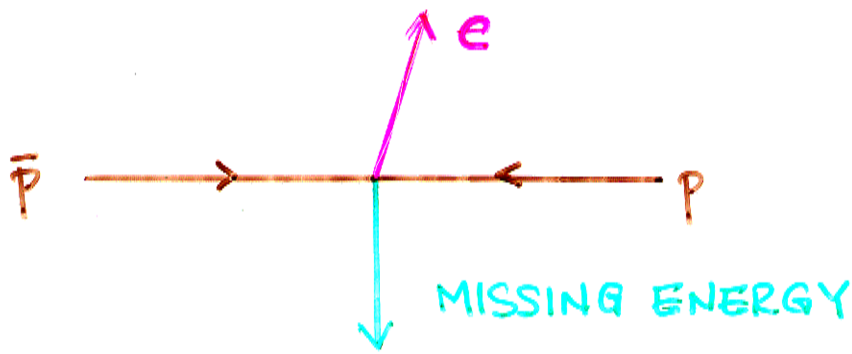
TO SEARCH FOR $W^{\pm}$, Z^{0} (at CERN)

SUPER PROTON SYNCHROTRON

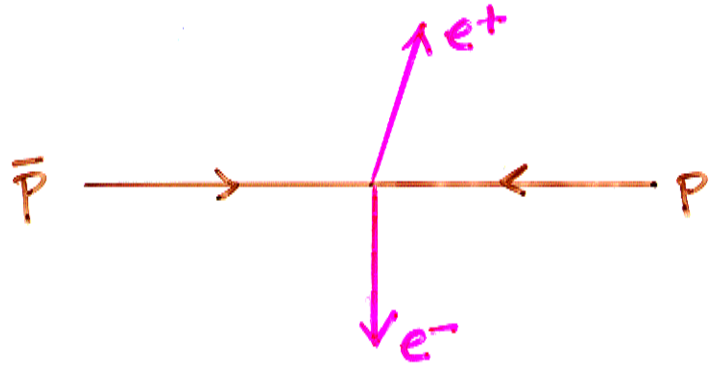
→ PROTON-ANTIPROTON COLLIDER "$Sp\bar{p}S$"

$E_{cm} = 540\ \text{GeV} \rightarrow 630\ \text{GeV}$

SIGNATURE OF $W \rightarrow e\nu$:



SIGNATURE OF $Z^{0} \rightarrow e^{+}e^{-}$



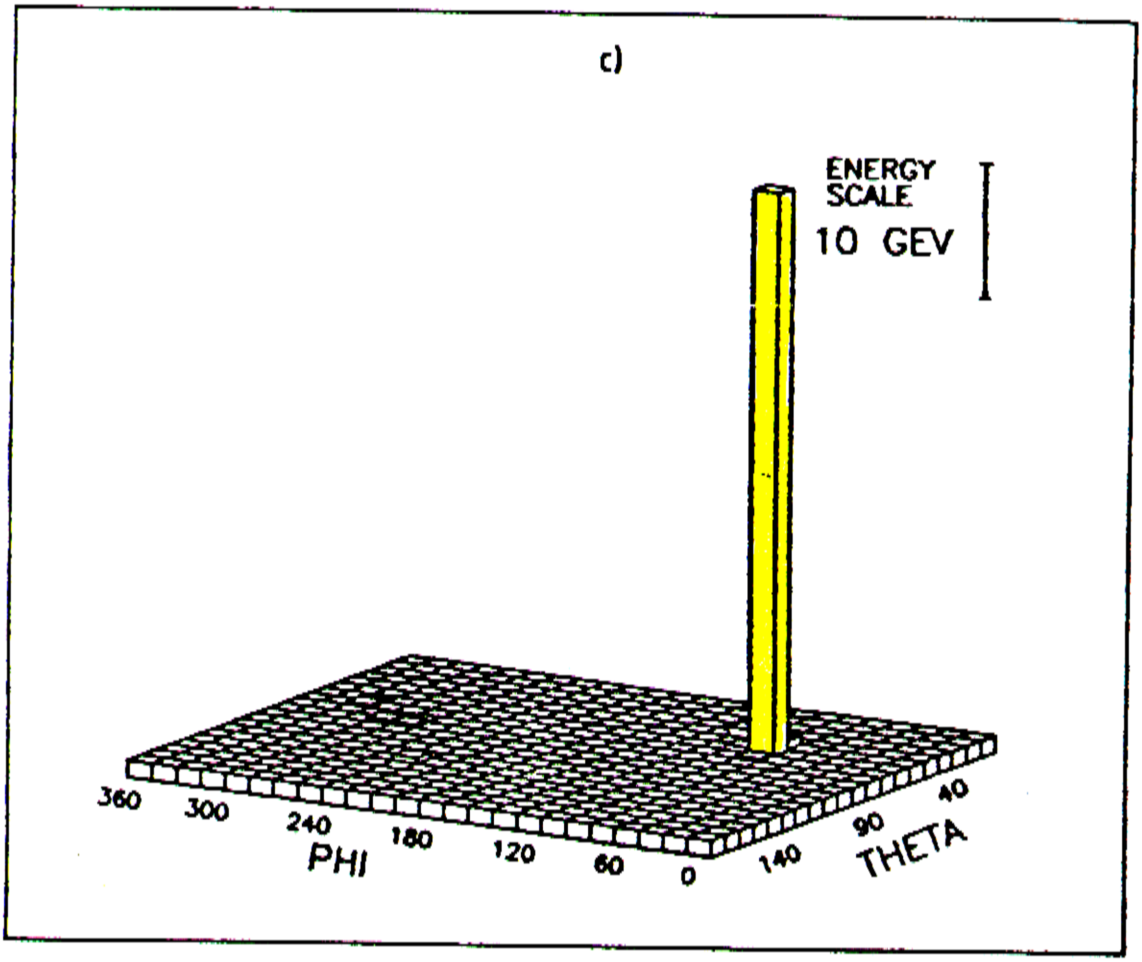
EXPERIMENTS UA1, UA2

1983: $W^{\pm}$, Z^{0} DISCOVERED

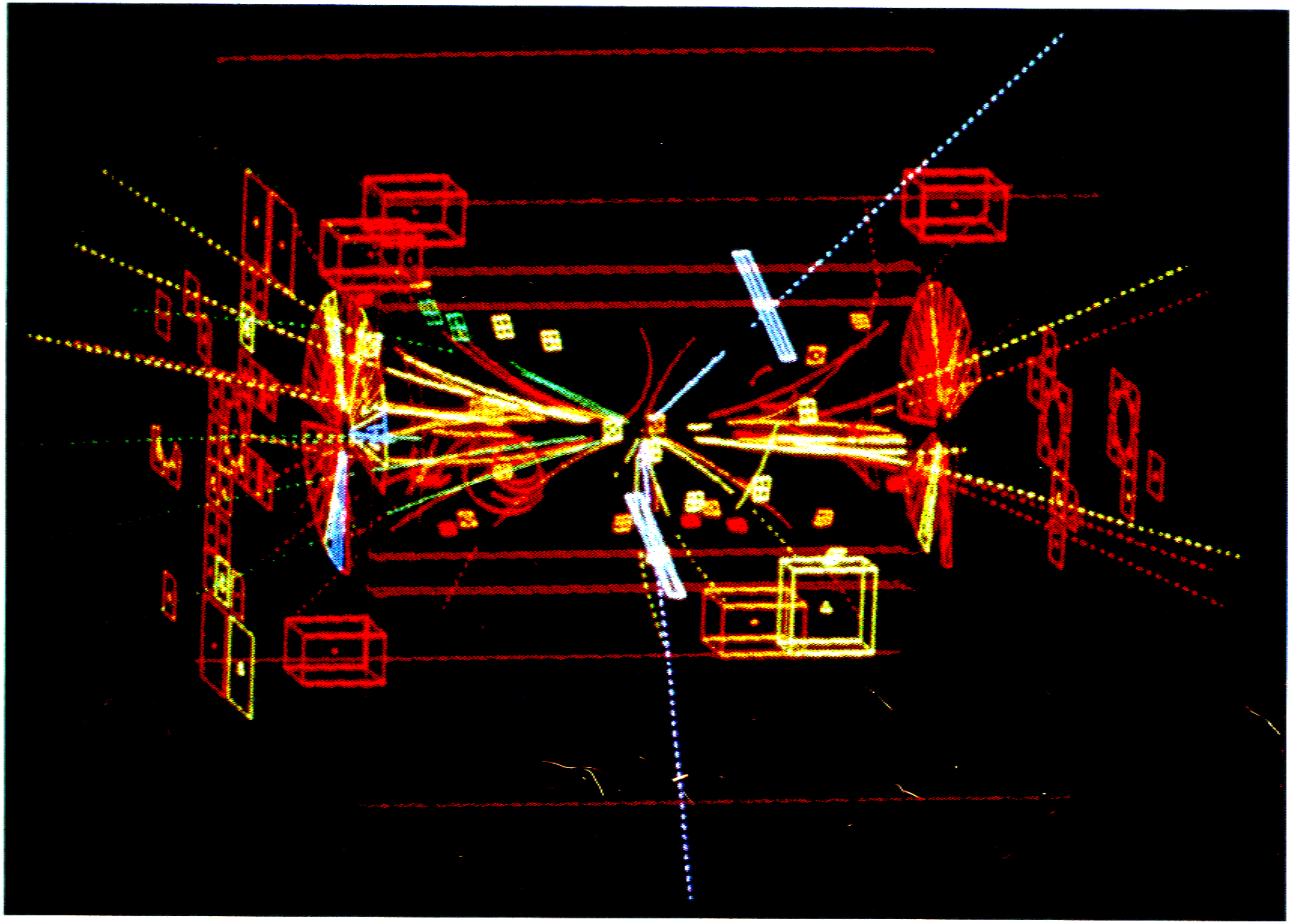
$W \to e\nu$ from UA2 at CERN

$$\bar{p}p \to W^{\pm} + \text{anything}$$
$$\hookrightarrow e^{\pm}\nu$$

$E_{cm} = 546$ GeV



ELECTRON + MISSING ENERGY



FIRST $Z^0 \rightarrow e^+e^-$ from UA1

$\bar{p}p \rightarrow Z^0 + \text{anything}$

$\hookrightarrow e^+e^-$

$Z^0 \rightarrow e^+e^-$ from UA1 at CERN

$\bar{p}p \rightarrow Z^0 + \text{anything}$
$\hookrightarrow e^+e^-$

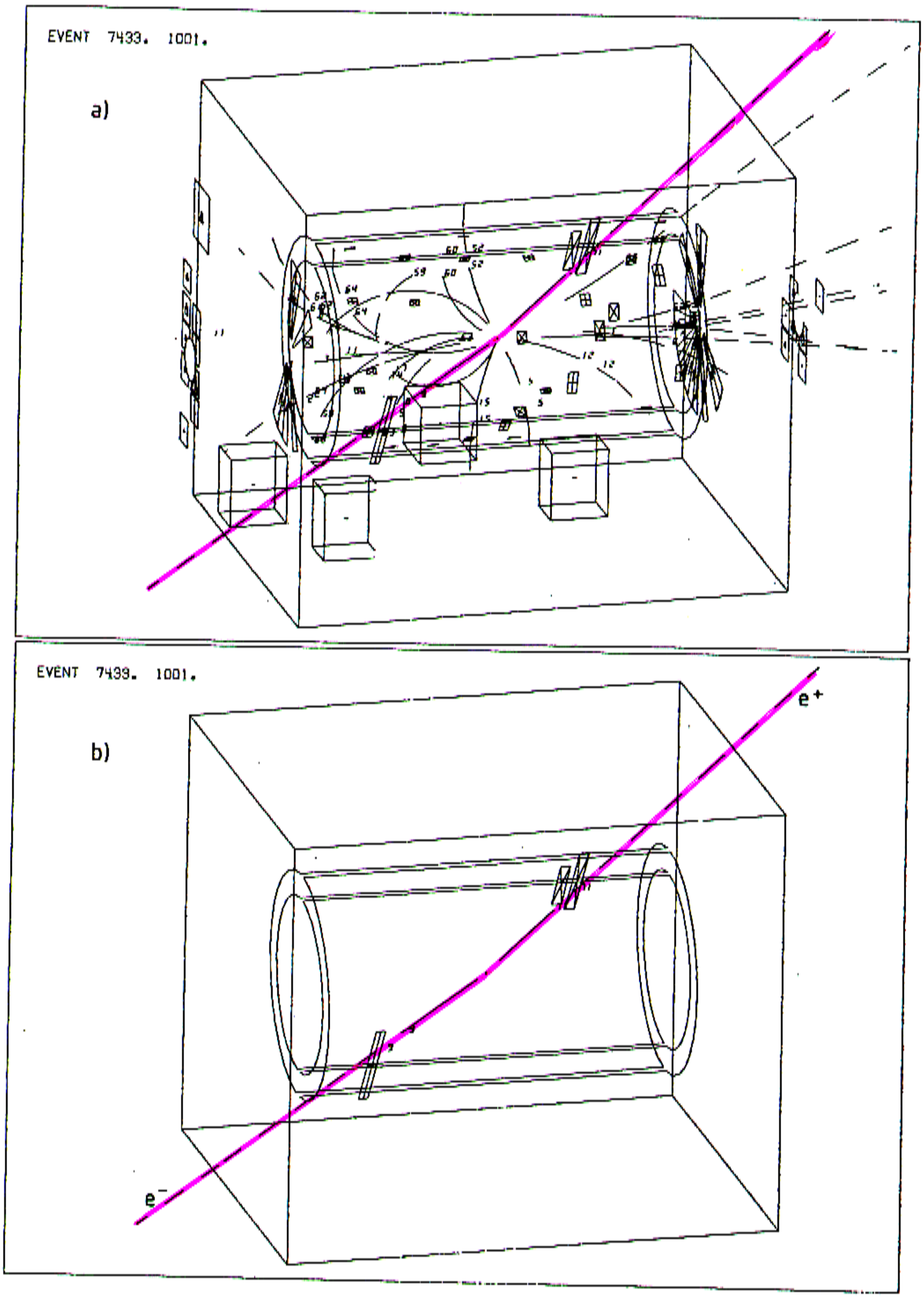


Fig. 2. (a) Event display. All reconstructed vertex associated tracks and all calorimeter hits are displayed. (b) The same, but thresholds are raised to $p_T > 2$ GeV/c for charged tracks and $E_T > 2$ GeV for calorimeter hits. We remark that only the electron pair survives these mild cuts.

Z° → e⁺e⁻ from UA1 at CERN

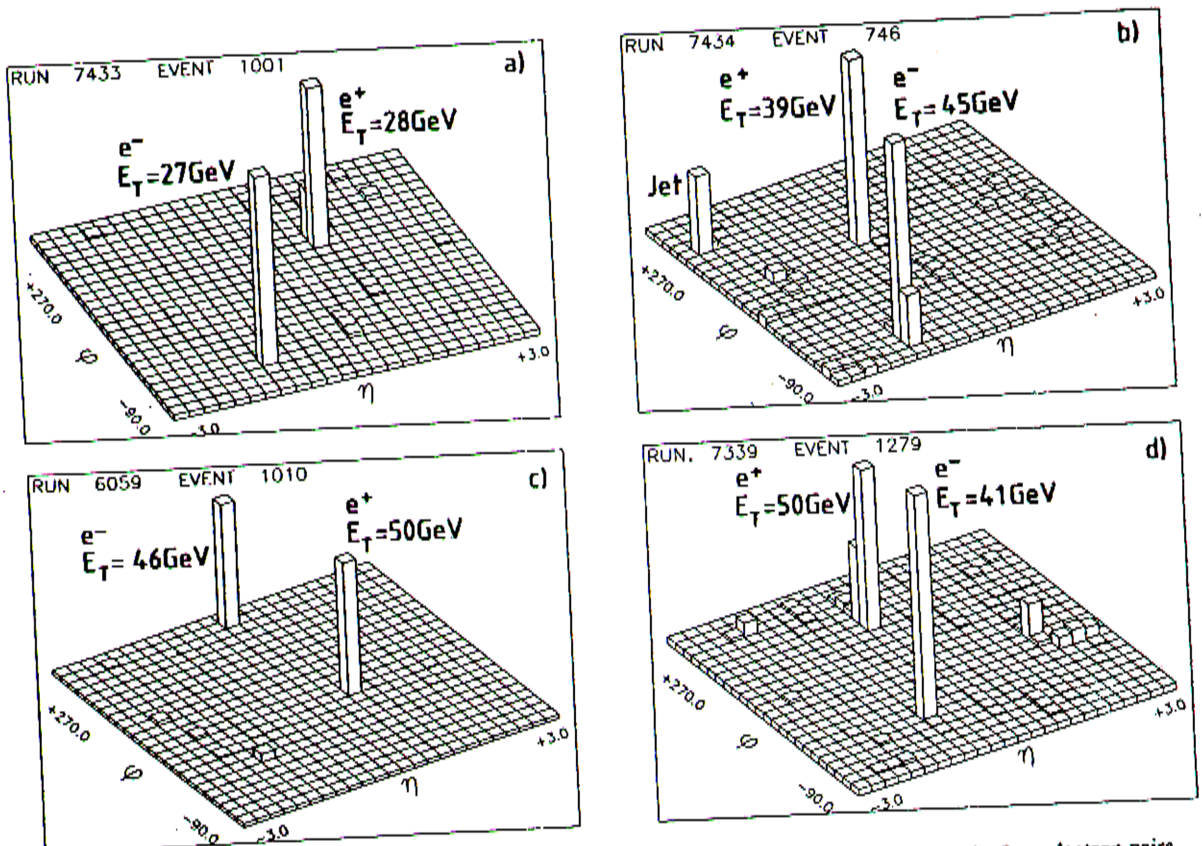


Fig. 3. Electromagnetic energy depositions at angles $>5°$ with respect to the beam direction for the four electron pairs.

ACCOUNTING FOR DEGREES OF FREEDOM

UNBROKEN THEORY

4 massless gauge bosons $\left.\begin{matrix} b_\mu^1 \\ b_\mu^2 \\ b_\mu^3 \\ a_\mu \end{matrix}\right\} \times 2 = 8$

Four scalar fields $\varphi^+, \varphi^0, \bar{\varphi}^0, \varphi^- = 4$

$$\overline{12}$$

AFTER SSB

3 massive gauge bosons $\left.\begin{matrix} W^+ \\ W^- \\ Z^0 \end{matrix}\right\} \times 3 = 9$

Massless photon A_μ $= 2$

Massive Higgs scalar η $= 1$

$$\overline{12}$$

THREE SCALAR D.O.F. HAVE BECOME THE LONGITUDINAL D.O.F. OF THE WEAK BOSONS

EXAMINE THE YUKAWA TERM

$$\mathcal{L}_{Yuk} = -G_e\left[\bar{R}(\varphi^{+}L) + (\bar{L}\varphi)R\right]$$

with $\varphi = \begin{pmatrix} 0 \\ \frac{v+\eta}{2} \end{pmatrix}$, we have

$$\mathcal{L}_{Yuk} = -\frac{G_e(v+\eta)}{\sqrt{2}}\left[\bar{e}_R e_L + \bar{e}_L e_R\right]$$

$$= -\underbrace{\frac{G_e v}{\sqrt{2}}}\,\bar{e}e \quad - \underbrace{\frac{G_e \eta}{\sqrt{2}}}\,\bar{e}e$$

Yukawa coupling between η and electrons

Electron mass term:

$$m_e = G_e v/\sqrt{2}$$

$$G_e \approx 3\times 10^{-6}$$

Electroweak theory provides the opportunity for fermions to acquire mass, but DOES NOT PREDICT the masses.

THE HIGGS BOSON

$$\mathcal{L}_{scalar} = \frac{1}{2}\left[(\partial_\mu \eta)(\partial^\mu \eta) + 2\mu^2\eta^2\right] + \ldots$$

NO SPECIFIC PREDICTION FOR M_H

$$M_H^2 = -2\mu^2 = 2|\lambda|v^2 > 0$$

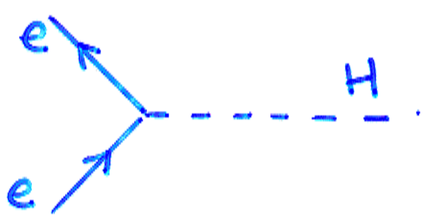
RECALL $V(\varphi^+\varphi) = \mu^2(\varphi^+\varphi) + |\lambda|(\varphi^+\varphi)^2;$

$$\langle\varphi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \qquad v = \sqrt{-\mu^2/|\lambda|}$$

(DON'T KNOW $|\lambda|$ or μ^2)

Yukawa interaction gives the

FEYNMAN RULE



$$-\frac{i m_e}{v} = -\frac{i G_e}{\sqrt{2}}$$

$$= -i m_e (G_F\sqrt{2})^{1/2} = \frac{-ig\, m_e}{2M_W}$$

(much smaller than the gauge coupling, for light fermions)

$$\boxed{H \to e^+e^-}$$

$$\mathcal{M} = -i m_e (G_F\sqrt{2})^{1/2}\ \bar{u}(p)\, v(q)$$

create e^- with ←   create e^+ with

$$p = \frac{M_H}{2}(1; \sin\theta, 0, \cos\theta) \qquad q = \frac{M_H}{2}(1; -\sin\theta, 0, -\cos\theta)$$

$$|\mathcal{M}|^2 = G_F\sqrt{2}\, m_e^2\ \mathrm{tr}(\not{q}\not{p})$$
$$= 4G_F\sqrt{2}\, m_e^2\ p\cdot q$$
$$= 2G_F\sqrt{2}\, m_e^2\, M_H^2$$

$$\frac{d\Gamma}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 M_H} = \frac{G_F m_e^2 M_H}{16\pi^2\sqrt{2}} \qquad \text{ISOTROPIC}$$

$$\Gamma = G_F m_e^2 M_H / 4\pi\sqrt{2}$$

GENERALIZE TO THREE GENERATIONS e, μ, τ

$$\Gamma(H \to \ell^+\ell^-) = G_F m_\ell^2 M_H / 4\pi\sqrt{2}$$

DOMINANT DECAY OF HIGGS BOSON (IF NOT TOO HEAVY) IS INTO THE HEAVIEST ACCESSIBLE PAIR OF FERMIONS. ($H \to W^+W^-$, Z^0Z^0 DOMINATE FOR A HEAVY HIGGS)

A HIGGS BOSON "MUST" EXIST

Study high-energy behavior of

$e^+e^- \to W_L^+ W_L^-$

LONGITUDINAL GAUGE BOSONS $\longleftrightarrow$ SCALARS

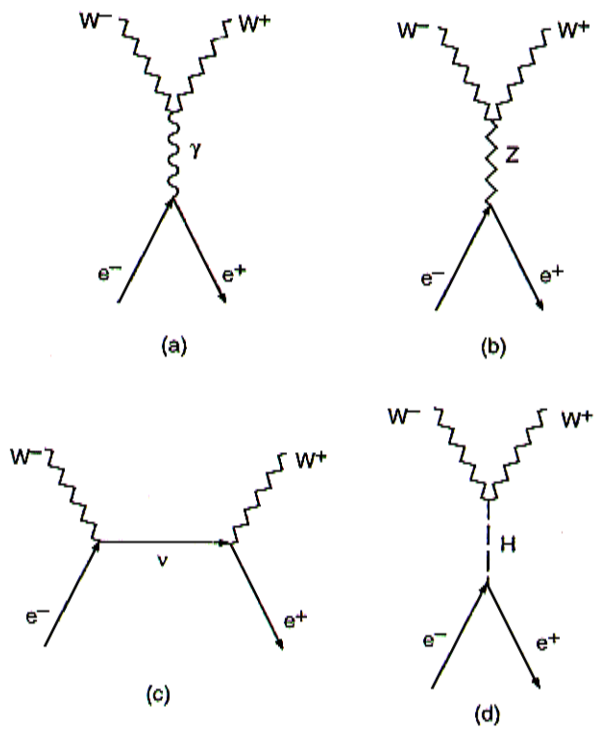


Figure 16: Lowest-order contributions to the $e^+e^- \to W^+W^-$ scattering amplitude.

J=1: $\mathcal{M}_a^{(1)}, \mathcal{M}_b^{(1)}, \mathcal{M}_c^{(1)}$ all $\propto s$
sum $\longrightarrow$ constant

Gauge-invariant couplings

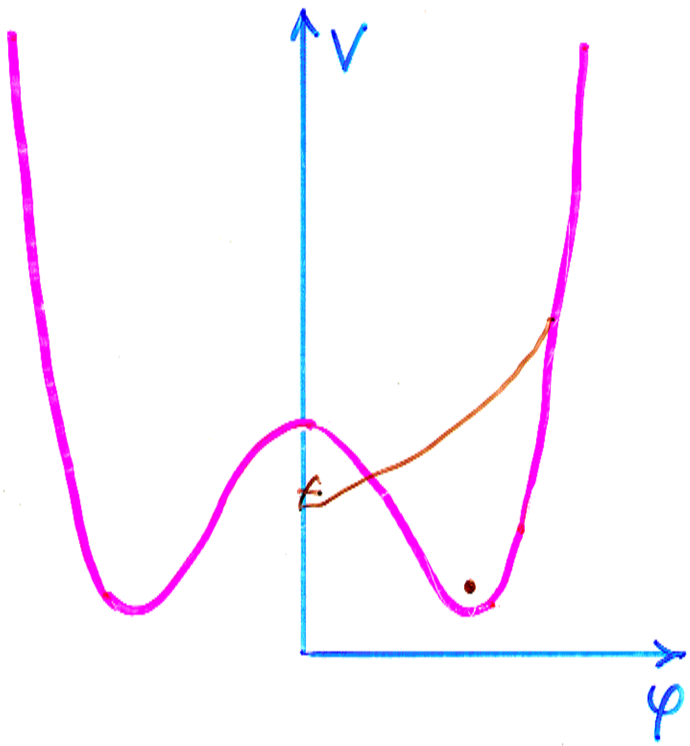
J=0: $\mathcal{M}_c^{(0)} \propto \sqrt{s}$ cancelled by $\mathcal{M}_d$

"wrong-helicity" electrons ($m_e \neq 0$)

$He\bar{e}$ coupling $\propto m_e$

Bounds on M_H

Within the electroweak theory, some consistency conditions restrict M_H.



- Compute quantum corrections to $V(\varphi^\dagger\varphi)$. Require that the minimum not be at $\varphi = 0$ (so that SSB still occurs).

$-4m_t^4 \cdots$

$$M_H^2 > \frac{3G_F\sqrt{2}}{16\pi^2}\left(2M_W^4 + M_Z^4 + \cdots\right)$$

A LOWER LIMIT

Thought Expt.

- COMPUTE AMPLITUDES FOR SCATTERING

$W_L^+W_L^-$ $\quad Z_L^0 Z_L^0$ $\quad HH$ $\quad HZ$

s-wave scattering amplitudes

$$\lim_{s \gg M_H^2} (a_0) \to \frac{-G_F M_H^2}{4\pi\sqrt{2}} \begin{pmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{pmatrix}$$

Thought Expt. • COMPUTE AMPLITUDES FOR SCATTERING

$W_L^+ W_L^- \quad Z_L^0 Z_L^0 \quad HH \quad HZ$

s-wave scattering amplitudes

$$\lim_{s \gg M_H^2} (a_0) \to -\frac{G_F M_H^2}{4\pi\sqrt{2}} \begin{pmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{pmatrix}$$

REQUIRE LARGEST EIGENVALUE RESPECT PARTIAL-WAVE UNITARITY

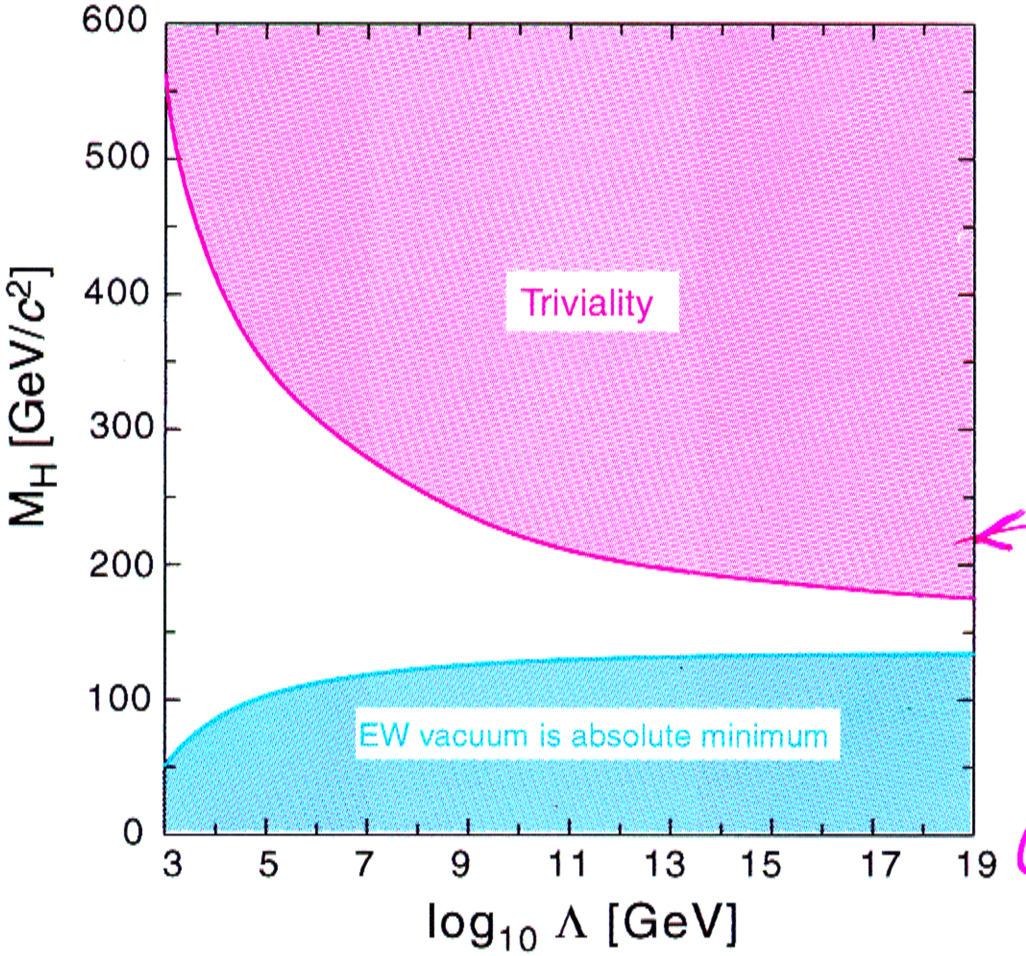
$$|a_0| \leq 1$$

$$\Downarrow \quad M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F}\right)^{1/2} = 1\ \mathrm{TeV}/c^2$$

IF TRUE, HIGGS BOSON WILL BE FOUND BELOW 1 TeV (at LEP2 or Tevatron or LHC)

IF FALSE, WEAK INTERACTIONS BECOME STRONG $\Rightarrow$ NEW PHENOMENA ON THE 1-TeV SCALE.

Generally, bounds depend on the energy up to which we assume the electroweak theory is the whole story.



require positive coefficient of $(\varphi^\dagger\varphi)^2$

ELECTROWEAK INTERACTIONS OF QUARKS

Completely parallel to the EW Theory of Leptons.

Based on the quark family doublets

Left-handed weak-isospin doublet

$$L_q \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L$$

2 RH weak-isospin singlets

$$R_u = u_R = \frac{(1+\gamma_5)}{2} u$$

$$R_d = d_R = \frac{(1+\gamma_5)}{2} d$$

Assign weak hypercharge Y

so that $Q = I_3 + \frac{1}{2} Y$

$$\left.\begin{array}{lll} u_L : & I_3 = +1/2 & Q = +2/3 \\ d_L : & I_3 = -1/2 & Q = -1/3 \end{array}\right\} Y_L = 1/3$$

u_R : $I_3 = 0$ $\quad Y = 2Q = 4/3$

d_R : $I_3 = 0$ $\quad Y = 2Q = -2/3$

Same form for the Lagrangian.

MATTER TERM:

$$\mathcal{L}_{quarks} = \bar{L}\, i\gamma^\mu\left(\partial_\mu + \frac{ig}{2}\vec{b}_\mu\cdot\vec{\tau} + \frac{ig'}{2}a_\mu Y\right)L$$
$$+ \sum_i \bar{R}_i\, i\gamma^\mu\left(\partial_\mu + \frac{ig'}{2}a_\mu Y\right)R$$

YUKAWA INTERACTION:

$$\mathcal{L}_{YuK} = -G_d\left[\bar{R}_d(\varphi^\dagger L_q) + (\bar{L}_q\varphi)R_d\right]$$

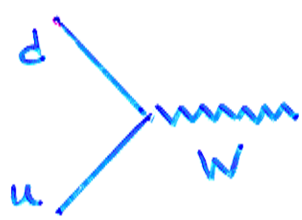
(JUST LIKE THE ELECTRON'S)

$$- G_u\left[\bar{R}_u(\bar{\varphi}^\dagger L_q) + (\bar{L}_q\bar{\varphi})R_u\right]$$

Spontaneous symmetry breaking proceeds as before.

FEYNMAN RULES

CHARGED CURRENT:



$$= \frac{-ig}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$$

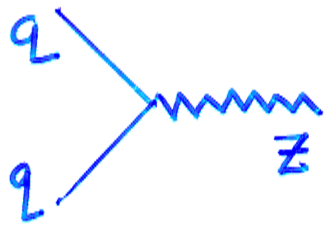
QUARK-LEPTON UNIVERSALITY—

same CC interactions

(same weak isospin assignments)

Feynman Rules for NC Interactions

$Zq\bar{q}$:



$$= \frac{-ig}{4\cos\theta_W}\gamma_\mu\left[R_q(1+\gamma_5)+L_q(1-\gamma_5)\right]$$

$$R_q = -2Q_q\sin^2\theta_W$$

$$L_q = 2I_3 - 2Q_q\sin^2\theta_W$$

just as for the leptons.

UNIVERSAL IN FORM, though not in value; depends on I_3, Y assignments.

The idealized one-generation model is very neat and attractive:

QUARK-LEPTON parallels are clear

QUARK-LEPTON UNIVERSALITY is automatic

BUT, It doesn't QUITE describe our world.

To get the correct charged current, REPLACE

$$L_q = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \text{BY} \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

$$d_\theta = d\cos\theta_c + s\sin\theta_c$$

$$\cos\theta_c = 0.9736 \pm 0.0010$$

"Cabibbo universality" accounts for relative strength of

$\pi \to \mu\nu, \quad K \to \mu\nu$

Now the neutral-current Lagrangian contains

$$\mathcal{L}_{Zq} = \frac{-g Z_\mu}{4\cos\theta_W}\Big\{\bar{u}\gamma^\mu[(1-\gamma_5)L_u + (1+\gamma_5)R_u]u$$

$$+\bar{d}\gamma^\mu[(1-\gamma_5)L_d + (1+\gamma_5)R_d]d \quad \cos^2\theta_c$$

$$+\bar{s}\gamma^\mu[(1-\gamma_5)L_d + (1+\gamma_5)R_d]s \quad \sin^2\theta_c$$

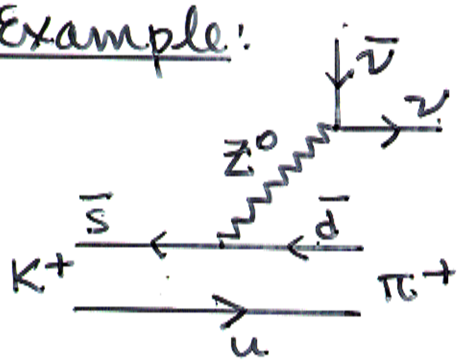
$$+\bar{d}\gamma^\mu[(1-\gamma_5)L_d + (1+\gamma_5)R_d]s \quad \sin\theta_c\cos\theta_c$$

$$+\bar{s}\gamma^\mu[(1-\gamma_5)L_d + (1+\gamma_5)R_d]d \quad \sin\theta_c\cos\theta_c$$

These have to be checked against NC observations. (Not right)

HISTORICALLY, these are the sign of TROUBLE: flavor- (strangeness-) changing NC

Example:



$$\frac{\Gamma(K^+\to\pi^+\nu\bar{\nu})}{\Gamma(K^+\to \text{all})} < \overbrace{\cancel{2.4\times10^{-9}}}^{(4.2^{+9.7}_{-3.5})\times10^{-10}}$$

not the level of first-order weak interactions!

Solution to flavor-changing neutral currents

(Glashow · Iliopoulos · Maiani)

EXPAND THE MODEL

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \qquad \begin{pmatrix}u\\ d_\theta\end{pmatrix}_L \quad \begin{pmatrix}c\\ s_\theta\end{pmatrix}_L$$

$$e_R,\ \mu_R \qquad\qquad u_R,\ d_R,\ s_R,\ c_R$$

where

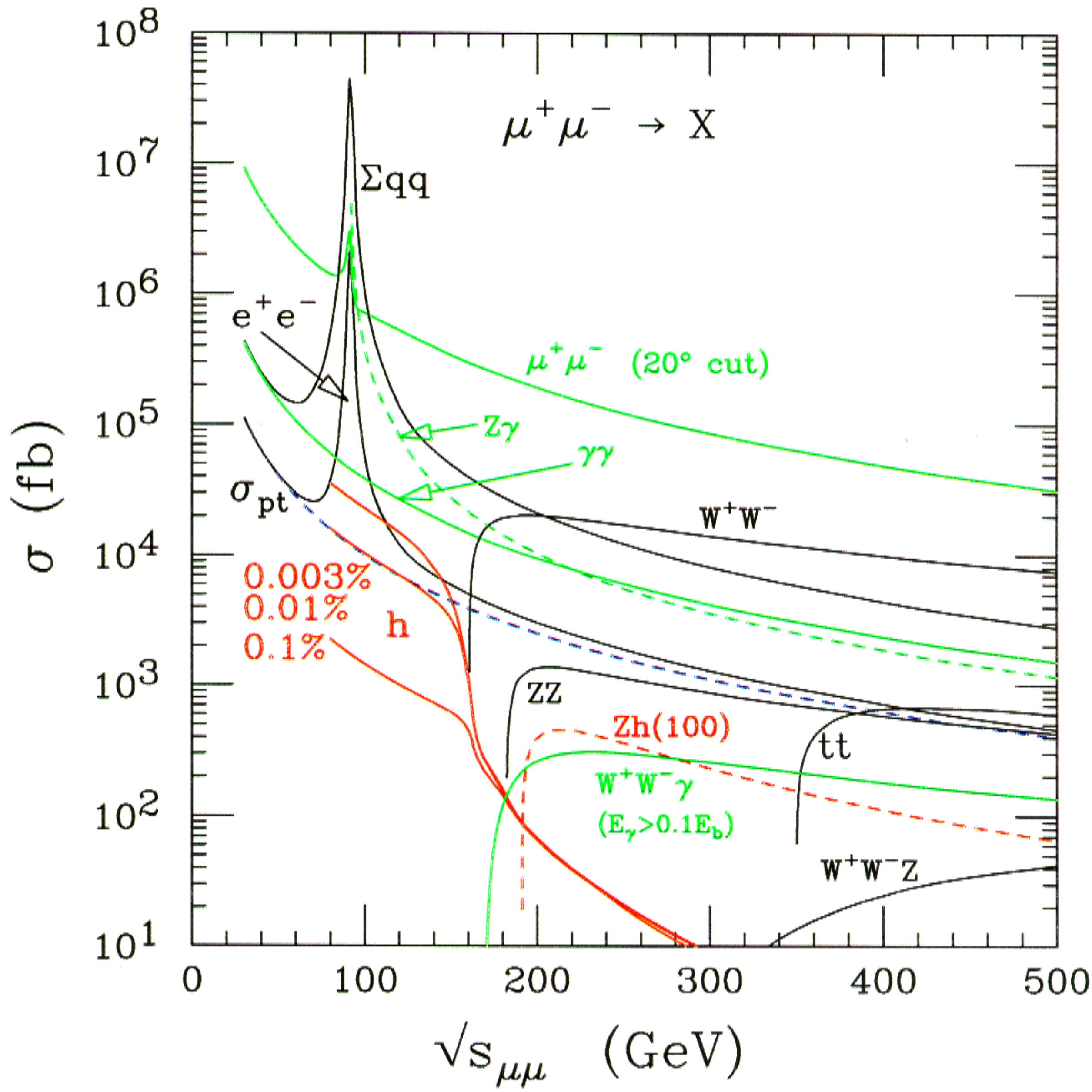
$$s_\theta = s\cdot\cos\theta_c - d\sin\theta_c$$

Requires a (then) new "charmed" quark

CROSS TERMS NOW CANCEL IN $\mathcal{L}_{Z\cdot q}$, LEAVING A FLAVOR DIAGONAL FEYNMAN RULE FOR NEUTRAL CURRENTS.

Generalizes to n quark doublets, provided quark mixing matrix is unitary.

(PREDICTION OF c QUARK)



$\mu^+\mu^- \to X$
Σqq
e^+e^-
$\mu^+\mu^-$ (20° cut)
$Z\gamma$
$\gamma\gamma$
σ_{pt}
W^+W^-
0.003%
0.01%
0.1%
h
ZZ
Zh(100)
tt
$W^+W^-\gamma$
(E_γ>0.1E_b)
W^+W^-Z
σ (fb)
$\sqrt{s}_{\mu\mu}$ (GeV)

SEARCHING FOR THE HIGGS BOSON

$e^+e^- \to H^0$: Cross section is small.

$$\sigma(e^+e^- \to H) = \frac{4\pi\,\Gamma(H\to e^+e^-)\,\Gamma(H\to \text{all})}{(s-M_H^2)^2 + M_H^2\,\Gamma^2(H\to\text{all})}$$

$$\left[\Gamma(H\to e^+e^-) = G_F\, m_e^2\, M_H / 4\pi\sqrt{2}\right]$$

$\mu^+\mu^- \to H^0$: Bigger by

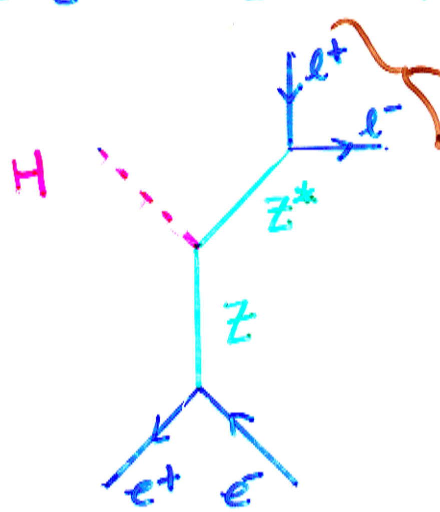
$$(m_\mu/m_e)^2 \approx 42{,}000$$

IS IT BIG ENOUGH?

IS A MUON COLLIDER FEASIBLE?

ON THE Z^0 RESONANCE:

$$e^+e^- \to Z^0 \to H Z^*$$
$$\hookrightarrow \ell^+\ell^-$$



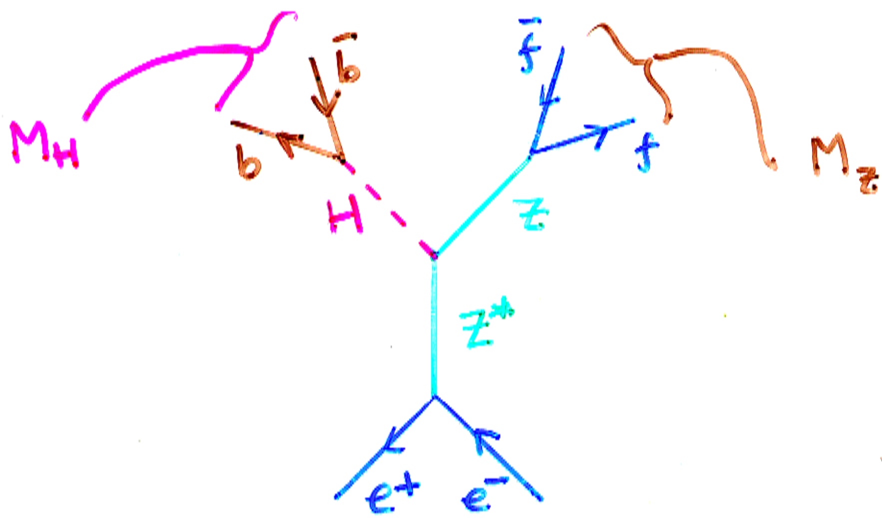
$< M_Z$

extensive searches at LEP

Currently most promising (LEP2)

ASSOCIATED PRODUCTION:

$$e^+e^- \to Z^* \to HZ$$



In the range of masses being explored

$$\Gamma(H\to b\bar{b})/\Gamma(H\to \text{all}) \approx 90\%$$

$$\Gamma(H\to \tau^+\tau^-)/\Gamma(H\to \text{all}) \approx 8\%$$

and we know that

$$\Gamma(Z\to q\bar{q})/\text{all} \approx 70\%$$

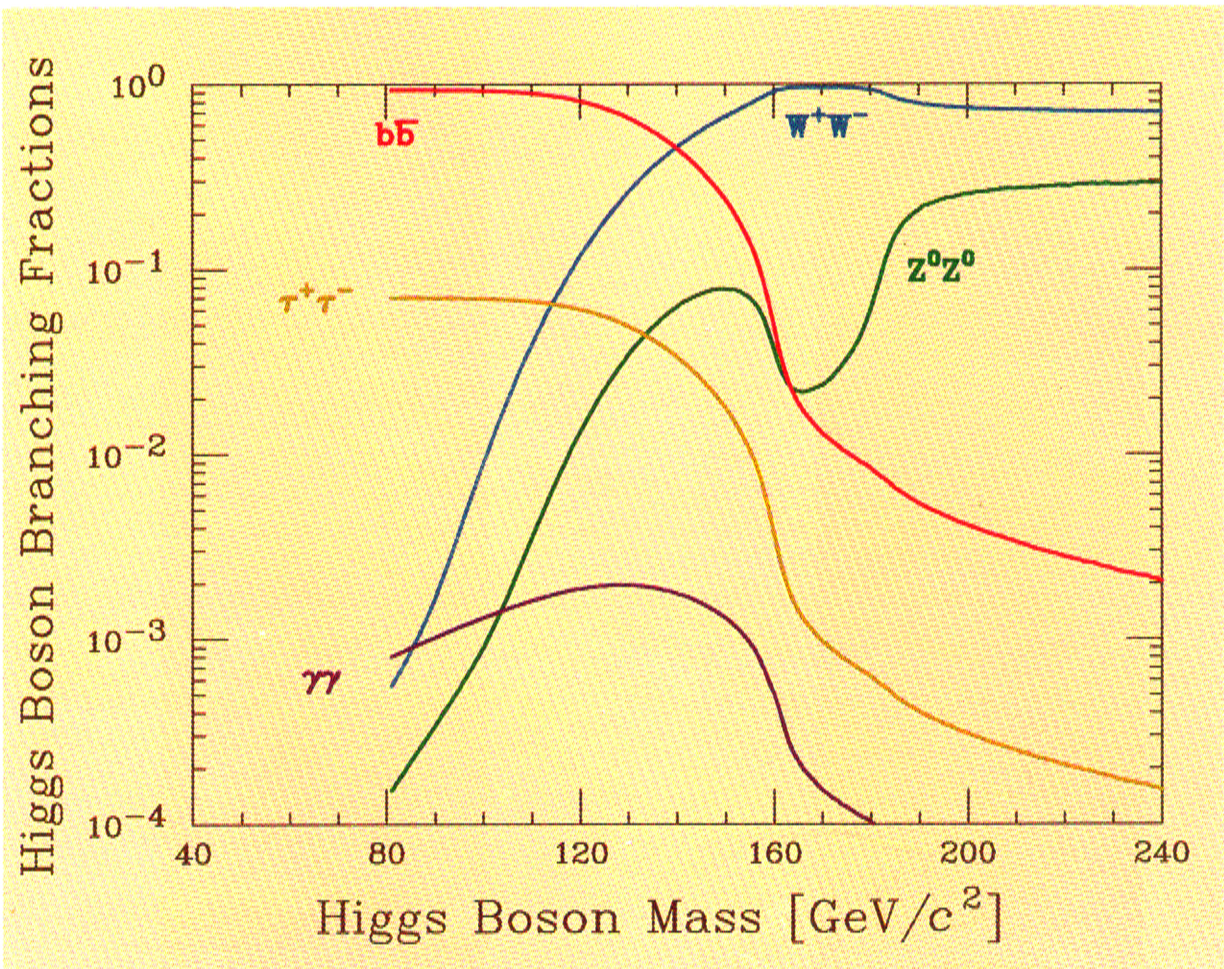
$$\Gamma(Z\to \nu\bar{\nu})/\text{all} \approx 20\%$$

$$\Gamma(Z\to e^+e^- \text{ OR } \mu^+\mu^- \text{ OR } \tau^+\tau^-)/\text{all} \approx 3\%$$

Current limits: $M_H \gtrsim 90$ GeV/c^2

(Janot)

Decay Probabilities



Future Prospects

- Tevatron Collider at Fermilab

  $\bar{p}p$ at 2.0 TeV

  can explore

  $$q\bar{q} \to H + W, \quad H + Z$$

  May be able to explore

  $$M_H \to 130 \text{ GeV}/c^2$$

  ? → 180 GeV/c^2

- Large Hadron Collider at CERN

  pp at 14 TeV

  can explore

  $$gg \to H$$

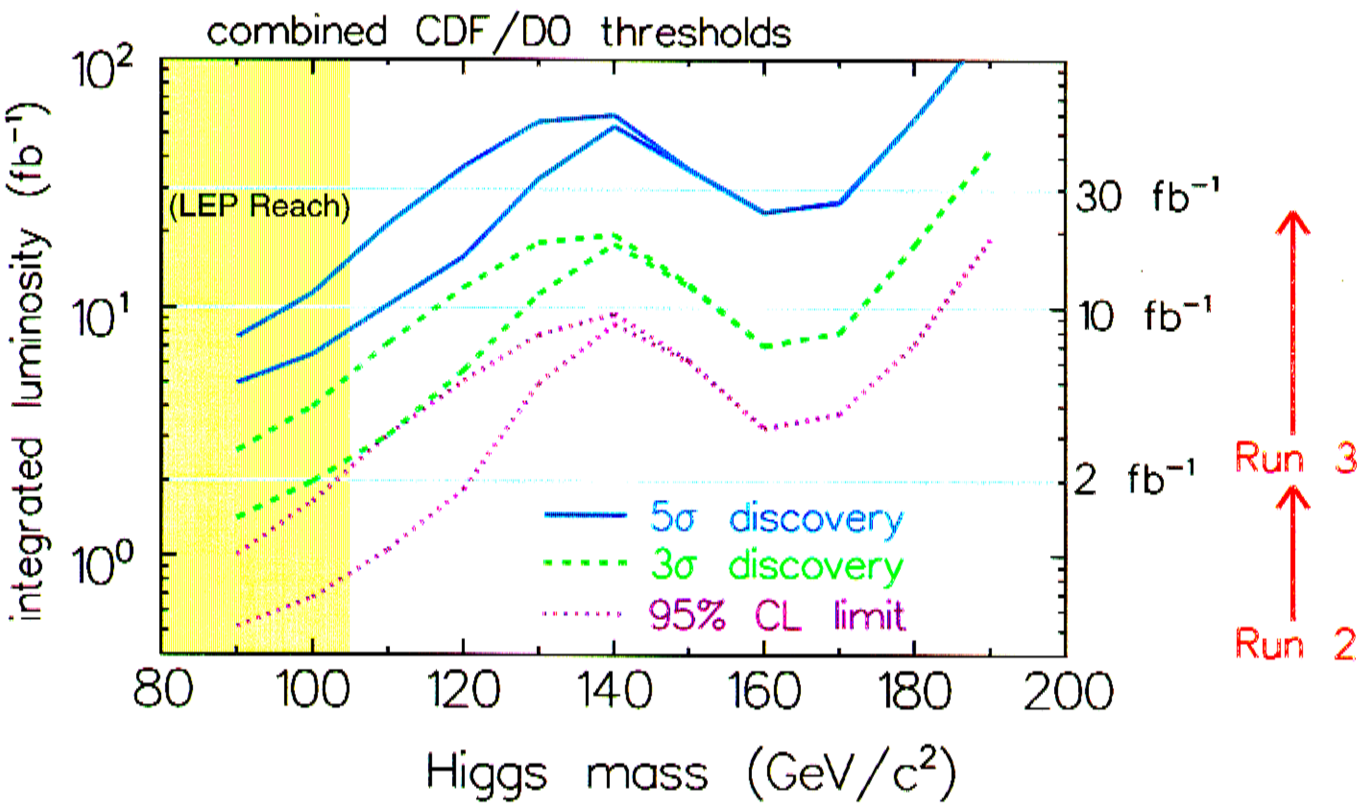
  $$W^+W^- \to H$$

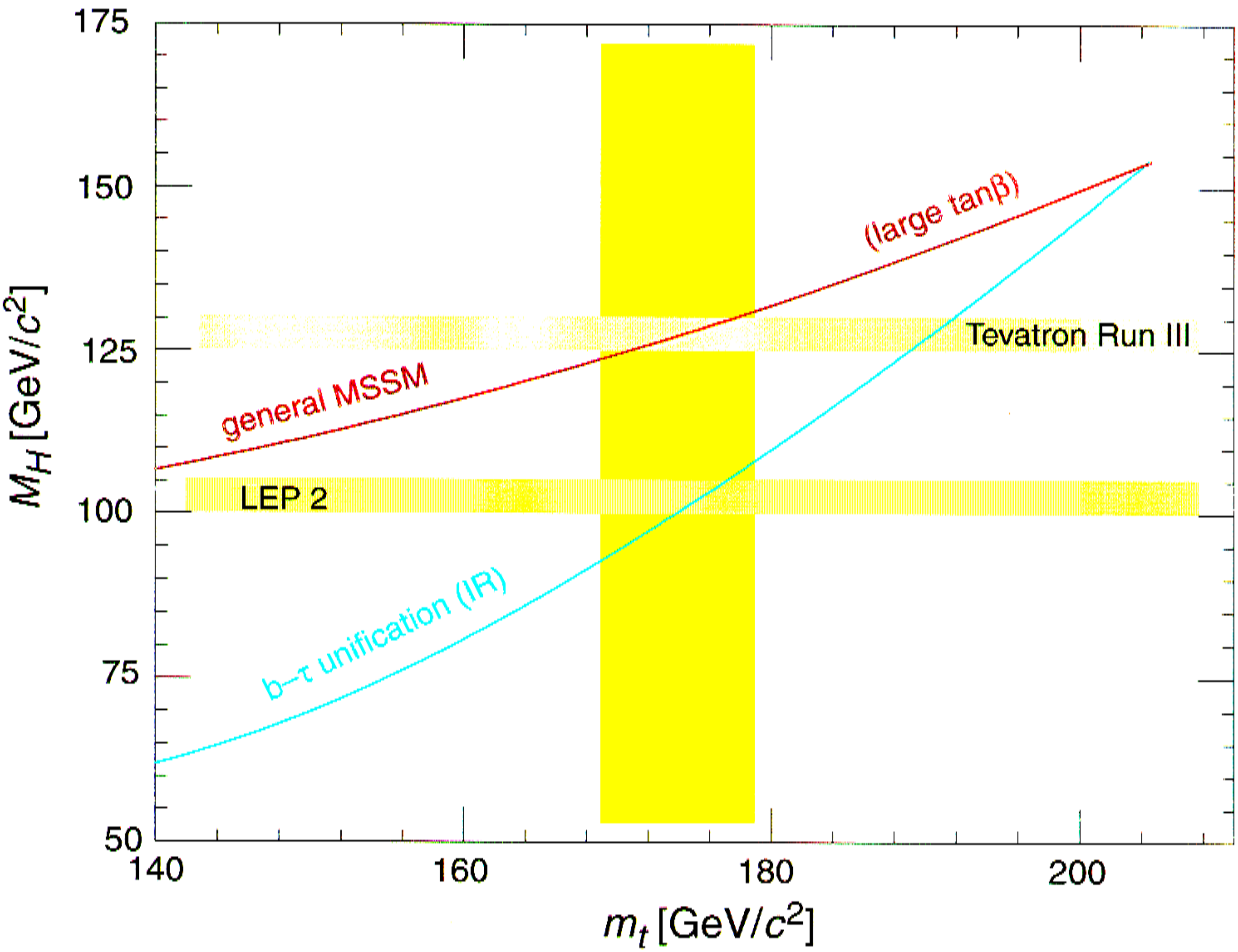
  $$q\bar{q} \to H + W, \quad H + Z$$

  $$\vdots$$

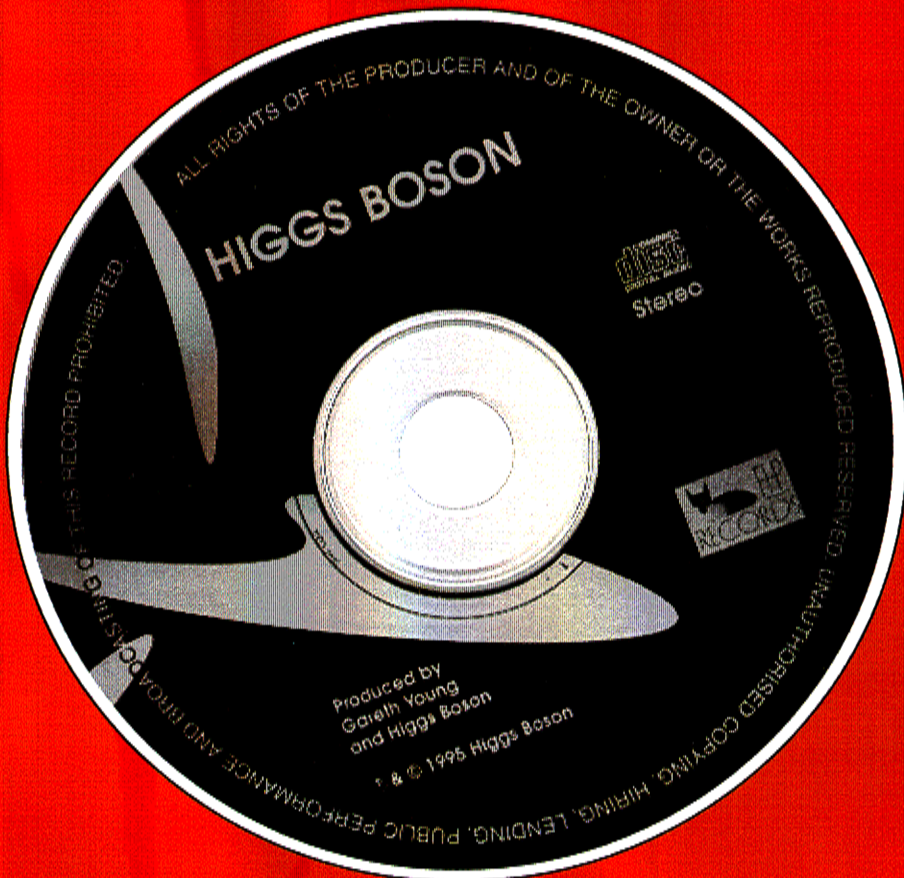
  through the interesting range of masses.

(Gianotti)


combined CDF/D0 thresholds
10^2
10^1
10^0
integrated luminosity (fb^{-1})
(LEP Reach)
30 fb^{-1}
10 fb^{-1}
2 fb^{-1}
Run 3
Run 2
5σ discovery
3σ discovery
95% CL limit
80
100
120
140
160
180
200
Higgs mass (GeV/c^2)



175
150
125
100
75
50
140
160
180
200
M_H [GeV/c^2]
m_t [GeV/c^2]
(large tanβ)
general MSSM
Tevatron Run III
LEP 2
b–τ unification (IR)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Cooled
Faith (Yourself)
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Particle Physics:
The Standard Model

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CERN Summer Lectures
17 – 27 July 2000

7

⑦ Another Look at Symmetries

SYMMETRY ⟷ INDISTINGUISHABLE
⟷ UNOBSERVABLE

UNOBSERVABLE	Transform.	Conserved
ABSOLUTE POSITION	$\vec{r} \to \vec{r} + \vec{\Delta}$	$\vec{p}$
" TIME	$t \to t + \delta$	E
" ORIENTATION	$\hat{r} \to \hat{r}'$	$\vec{L}$
" VELOCITY	$\vec{v} \to \vec{v}'$	
" RIGHT	$\vec{r} \to -\vec{r}$	$\mathbb{P}$
" FUTURE	$t \to -t$	$\mathbb{T}$
" CHARGE	$e \to -e$	$\mathbb{C}$
" PHASE		
⋮		

VIOLATION OF A SYMMETRY

→ OBSERVATION OF AN UNOBSERVABLE

... MEASURING A QUANTITY THAT WOULD VANISH IF THE SYMMETRY HELD ...

VARIETIES OF SYMMETRIES

- CONTINUOUS SPACETIME SYMMETRIES

  Poincaré invariance:

  translations +

  Lorentz transforms

  STARTING POINT FOR THEORIES

- PERMUTATION (IDENTICAL-PARTICLE) SYMMETRIES

  Spin + statistics:

  Bose-Einstein &

  Fermi-Dirac

  (DERIVED ... IN ≥ 3 SPACE DIM)

Recent development: ANYONS in 2+1 D

ACQUIRES PHASE $\begin{cases} 0 \text{ mod } 2\pi & \text{BOSON} \\ \pi \text{ mod } 2\pi & \text{FERMION} \\ \theta & \text{ANYON} \end{cases}$

Statistics interpolate

- INTERNAL SYMMETRIES

Global symmetries:

isospin, $SU(3)_{flavor}$

chiral symmetry

Gauge symmetries:

$SU(3)_{color}$

$SU(2)_L \otimes U(1)_Y$

$\vdots$

- DISCRETE SYMMETRIES

$\mathbb{P}$ - parity $\quad \vec{x} \to -\vec{x}$

$\mathbb{C}$ - charge conjugation

$\mathbb{T}$ - time reversal $\quad t \to -t$

We can prove the

$\mathbb{CPT}$ Theorem:

Lorentz Invariance

\+

Local field theory

\+

Spin / Statistics

under $\mathbb{CPT}$

PARTICLE $\longrightarrow$ ANTIPARTICLE

MOMENTUM FIXED

HELICITY REVERSED

all matrix elements

$\longrightarrow$ complex conjugates

CONSEQUENCES

Particle vs. Antiparticle

EQUAL MASSES •

OPPOSITE CHARGES &
FORM FACTORS →

EQUAL LIFETIMES ☆

CPT INVARIANCE

•	$(m_{W^+} - m_{W^-}) \,/\, m_{\text{average}}$	-0.002 ± 0.007
•	$(m_{e^+} - m_{e^-}) \,/\, m_{\text{average}}$	$<4 \times 10^{-8}$, CL = 90%
→	$\lvert q_{e^+} + q_{e^-} \rvert / e$	$<2 \times 10^{-18}$
→	$(g_{e^+} - g_{e^-}) \,/\, g_{\text{average}}$	$(-0.5 \pm 2.1) \times 10^{-12}$
☆	$(\tau_{\mu^+} - \tau_{\mu^-}) \,/\, \tau_{\text{average}}$	$(2 \pm 8) \times 10^{-5}$
→	$(g_{\mu^+} - g_{\mu^-}) \,/\, g_{\text{average}}$	$(-2.6 \pm 1.6) \times 10^{-8}$
•	$(m_{\pi^+} - m_{\pi^-}) \,/\, m_{\text{average}}$	$(2 \pm 5) \times 10^{-4}$
☆	$(\tau_{\pi^+} - \tau_{\pi^-}) \,/\, \tau_{\text{average}}$	$(6 \pm 7) \times 10^{-4}$
•	$(m_{K^+} - m_{K^-}) \,/\, m_{\text{average}}$	$(-0.6 \pm 1.8) \times 10^{-4}$
☆	$(\tau_{K^+} - \tau_{K^-}) \,/\, \tau_{\text{average}}$	$(0.11 \pm 0.09)\%$ $(S = 1.2)$
	$K^\pm \to \mu^\pm \nu_\mu$ rate difference/average	$(-0.5 \pm 0.4)\%$
	$K^\pm \to \pi^\pm \pi^0$ rate difference/average	[f] $(0.8 \pm 1.2)\%$
•	$\lvert m_{K^0} - m_{\overline{K}^0} \rvert \,/\, m_{\text{average}}$	[g] $<10^{-18}$
	phase difference $\phi_{00} - \phi_{+-}$	$(-0.1 \pm 0.8)^\circ$
	CPT-violation parameters in K^0 decay	
	real part of Δ	0.018 ± 0.020
	imaginary part of Δ	0.02 ± 0.04
	$(\lvert \frac{q_{\overline{p}}}{m_{\overline{p}}} \rvert - \frac{q_p}{m_p}) / \lvert \frac{q}{m} \rvert_{\text{average}}$	$(1.5 \pm 1.1) \times 10^{-9}$
→	$\lvert q_p + q_{\overline{p}} \rvert / e$	$<2 \times 10^{-5}$
→	$(\mu_p + \mu_{\overline{p}}) \,/\, \lvert \mu \rvert_{\text{average}}$	$(-2.6 \pm 2.9) \times 10^{-3}$
•	$(m_n - m_{\overline{n}}) \,/\, m_{\text{average}}$	$(9 \pm 5) \times 10^{-5}$
•	$(m_\Lambda - m_{\overline{\Lambda}}) \,/\, m_\Lambda$	$(-1.0 \pm 0.9) \times 10^{-5}$
☆	$(\tau_\Lambda - \tau_{\overline{\Lambda}}) \,/\, \tau_{\text{average}}$	0.04 ± 0.09
→	$(\mu_{\Sigma^+} + \mu_{\overline{\Sigma}^-}) \,/\, \lvert \mu \rvert_{\text{average}}$	0.014 ± 0.015
•	$(m_{\Xi^-} - m_{\overline{\Xi}^+}) \,/\, m_{\text{average}}$	$(1.1 \pm 2.7) \times 10^{-4}$
☆	$(\tau_{\Xi^-} - \tau_{\overline{\Xi}^+}) \,/\, \tau_{\text{average}}$	0.02 ± 0.18
•	$(m_{\Omega^-} - m_{\overline{\Omega}^+}) \,/\, m_{\text{average}}$	$(0 \pm 5) \times 10^{-4}$

NO THEORETICAL PRINCIPLE ENSURES THAT $\mathbb{C}, \mathbb{P}, \mathbb{T}$ ARE SEPARATELY RESPECTED

$\left.\begin{matrix} \mathbb{C} \\ \mathbb{P} \\ \mathbb{T} \end{matrix}\right\}$ ARE RESPECTED BY THE STRONG & EM INTERACTIONS

We built our theory of weak interactions to violate $\mathbb{P}$, maximally for CC interactions

$\gamma_\mu (1-\gamma_5)$: Vector − Axial vector

First experimental confirmation of $\mathbb{P}$-violation in weak decays:

detection of $\vec{J} \cdot \vec{p}_e$ correlation

C.S. Wu, et al. 1956

in β decay of polarized ^{60}Co

$\mathbb{P}: \vec{J} \longrightarrow \vec{J}$ nucleus spin

$\mathbb{P}: \vec{p}_e \longrightarrow -\vec{p}_e$ β momentum

Reflected in our decision to include only ν_L

$$\overset{\Rightarrow}{\longleftarrow} \; \nu_L \qquad \mathbb{P} \qquad \overset{\Rightarrow}{\longrightarrow} \; \cancel{\nu_R}$$

How do we know ν is LH?

- MEASURE μ^+ HELICITY IN

$$(\text{spin-0})\ \pi^+ \to \mu^+ \nu_\mu$$

$$\nu_\mu \xleftarrow{\Rightarrow} \pi^+ \xrightarrow{\Leftarrow} \mu^+$$

$$h(\nu_\mu) = h(\mu^+)$$

Bardon, PRL 7, 23('61)
Possoz, PL 70B, 265('77)

- MEASURE LONGITUDINAL POLARIZATION OF RECOIL NUCLEUS IN

$$\mu^- \ {}^{12}C \to {}^{12}B \ \nu_\mu$$

$J=0$ $\qquad$ $J=1$

INFER $h(\nu_\mu)$ BY ANG. MOM. CONS.

Roesch, AJP 50, 931('81)

- MEASURE LONGITUDINAL POLARIZATION OF RECOIL NUCLEUS IN

$$e^- \ {}^{152}Eu^m \to {}^{152}Sm^* \ \nu_e$$

$J=0$ $\qquad$ γ $\qquad$ $J=1$

INFER $h(\nu_e)$ FROM γ POL.

Goldhaber, PR 109, 1015('58)

NOTE: μ is forced to have "wrong helicity" in $\pi \to \mu\nu$

to give spin-0 for π

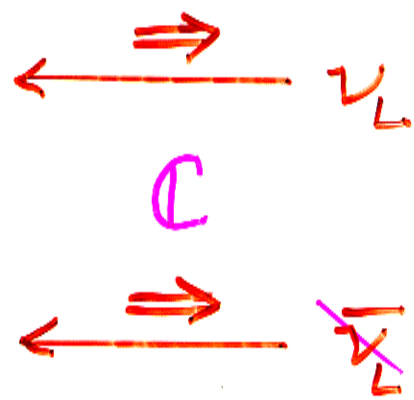
INHIBITS THE DECAY,

AND INHIBITS $\pi^+ \to e^+ \nu_e$ MORE:

$$\frac{\Gamma(\pi^+ \to e^+ \nu)}{\Gamma(\pi^+ \to \mu^+ \nu)} = 1.23 \times 10^{-4}$$

(amplitude $\propto m_{\ell^+}$)

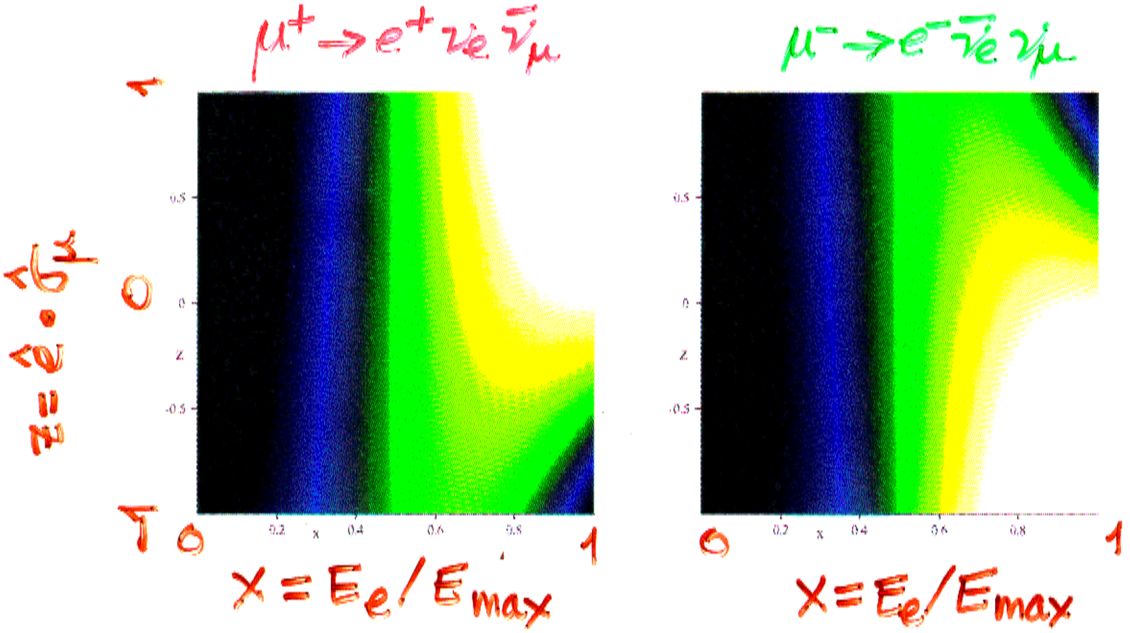
CHARGE-CONJUGATION INVARIANCE IS ALSO VIOLATED IN WEAK INT.



Example of Muon Decay

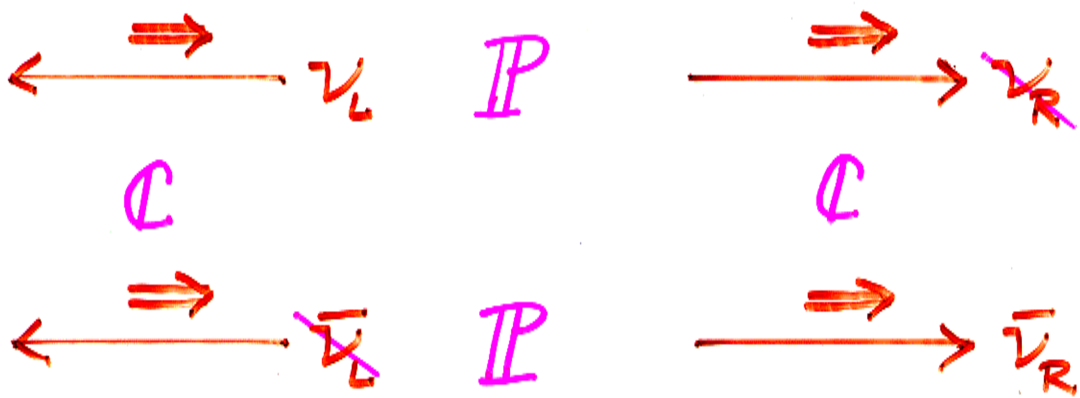
$$\frac{dN}{dx\,dz} = x^2(3-2x)\left[1 \pm z\,\frac{(2x-1)}{(3-2x)}\right]$$

e^+ follows $\hat{\sigma}_{\mu^+}$ $\qquad$ e^- avoids $\hat{\sigma}_{\mu^-}$



$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$
$\mu^- \to e^- \bar{\nu}_e \nu_\mu$
$z = \hat{e} \cdot \hat{\sigma}_\mu$
$x = E_e / E_{max}$
$x = E_e / E_{max}$

COULD THE WEAK INTERACTIONS RESPECT THE COMBINED OPERATION OF $\mathbb{CP}$ SYMMETRY?



ν_L and $\bar{\nu}_R$ exist in Nature

IN A $\mathbb{CPT}$-INVARIANT WORLD, $\mathbb{CP}$ SYMMETRY IS EQUIVALENT TO $\mathbb{T}$ SYMMETRY

The effect on transition amplitudes

initial and final momenta $\vec{p}_i, \vec{p}_f$

helicities λ_i, λ_f

$\mathbb{CP}$ relates

$$\mathcal{A}(i\{\vec{p}_i; \lambda_i\} \to f\{\vec{p}_f; \lambda_f\})$$

to

$$\mathcal{A}(\bar{i}\{-\vec{p}_i; -\lambda_i\}) \to \bar{f}\{-\vec{p}_f; -\lambda_f\})$$

$\mathbb{T}$ relates

$$\mathcal{A}(i\{\vec{p}_i; \lambda_i\} \to f\{\vec{p}_f; \lambda_f\})$$

to

$$\mathcal{A}(f\{-\vec{p}_f; \lambda_f\} \to i\{-\vec{p}_i; \lambda_i\})$$

$\mathbb{CPT}$ $\to$ $\mathcal{A}(\bar{f}\{p_f; -\lambda_f\} \to \bar{i}\{p_i; -\lambda_i\})$

WHY MIGHT WE SUSPECT $\cancel{CP}$?

(NON-HISTORICAL)

Can we understand why matter dominates over antimatter in the U?

DENSITY OF BARYONS IS

SMALL $n_B/n_\gamma \approx 4\times10^{-10}$

BUT IMPORTANT,

WHILE DENSITY OF ANTIBARYONS

IS NEGLIGIBLE

If the U began in a symmetric state of ZERO BARYON NUMBER, How did the present MATTER EXCESS evolve?

Sakharov's 3 conditions:

- Processes that violate BARYON #
- Departure from thermal eq$^{\underline{m}}$ while $\cancel{B}$
- $\cancel{CP}$

To illustrate, suppose that

$$X \text{ decays} \begin{cases} \nearrow B_1 \text{ with prob. } f \\ \searrow B_2 \quad '' \quad '' \ (1-f) \end{cases}$$

and its antiparticle

$$\bar{X} \text{ decays} \begin{cases} \nearrow \bar{B}_1 = -B_1 \text{ with } \bar{f} \\ \searrow \bar{B}_2 = -B_2 \quad '' \ (1-\bar{f}) \end{cases}$$

$\mathbb{CPT}$: $\Gamma(X \to \text{all}) = \Gamma(\bar{X} \to \text{all})$, so

STARTING FROM EQUAL POPULATIONS OF X AND $\bar{X}$, NET BARYON # IS

$$\Delta B = (f-\bar{f})B_1 + [(1-f)-(1-\bar{f})]B_2$$
$$= (f-\bar{f})(B_1 - B_2)$$

$\Delta B \neq 0$ REQUIRES $B_1 \neq B_2$ ($\cancel{B}$)

AND $f \neq \bar{f}$ ($\cancel{\mathbb{CP}}$)

Look for $\not{CP}$ in phenomena accessible to us...

... (may or may not have anything to do with $\not{CP}$ in B-violating processes in the Early Universe)

(Our current understanding of $\not{CP}$ does not instantly explain the observed excess of MATTER over ANTIMATTER.)

- Electric Dipole Moments of Particles

$$\vec{D} = \int d^3\vec{x} \; \vec{x}\, \rho(\vec{x}) \quad \propto \quad \vec{S}$$

(CAN ONLY SPECIFY ORIENTATION RELATIVE TO SPIN)

Under $\mathbb{P}$ $\quad \vec{D} \to -\vec{D} \qquad \vec{S} \to \vec{S}$

$\mathbb{T}$ $\quad \vec{D} \to \vec{D} \qquad \vec{S} \to -\vec{S}$

So $\vec{D} = 0$ unless $\cancel{\mathbb{P}}$ and $\cancel{\mathbb{T}}$

Nonzero EDM $\Rightarrow \cancel{\mathbb{T}}$ $(\cancel{\mathbb{CP}})$

VERY SENSITIVE EXPERIMENTS

- neutron
  first beams, now
  Grenoble, Petersburg — stored ultracold neutrons (5m/s)
  $^3He^{\uparrow}$
- electron
  Berkeley — atomic enhancement ^{205}Tl

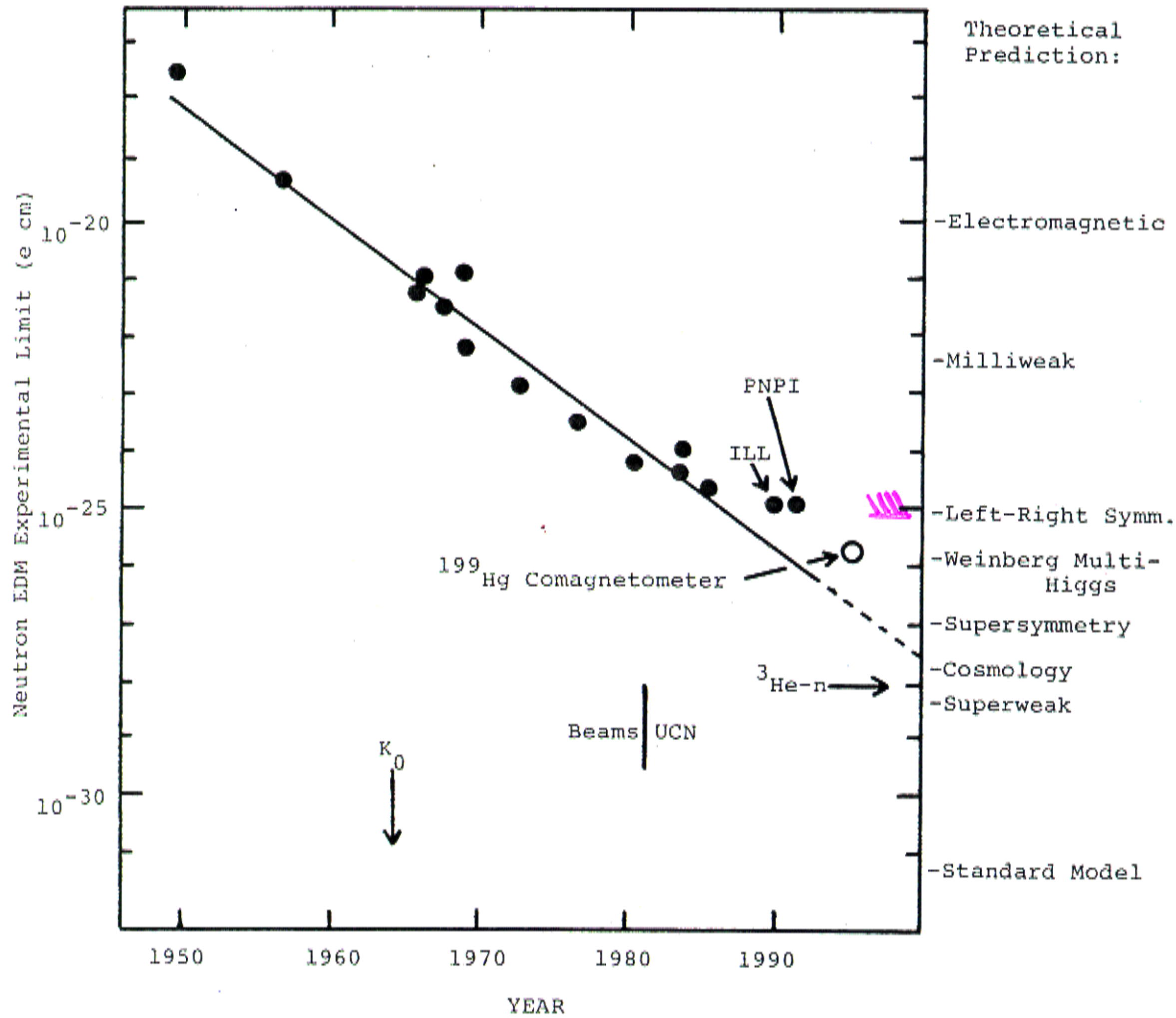
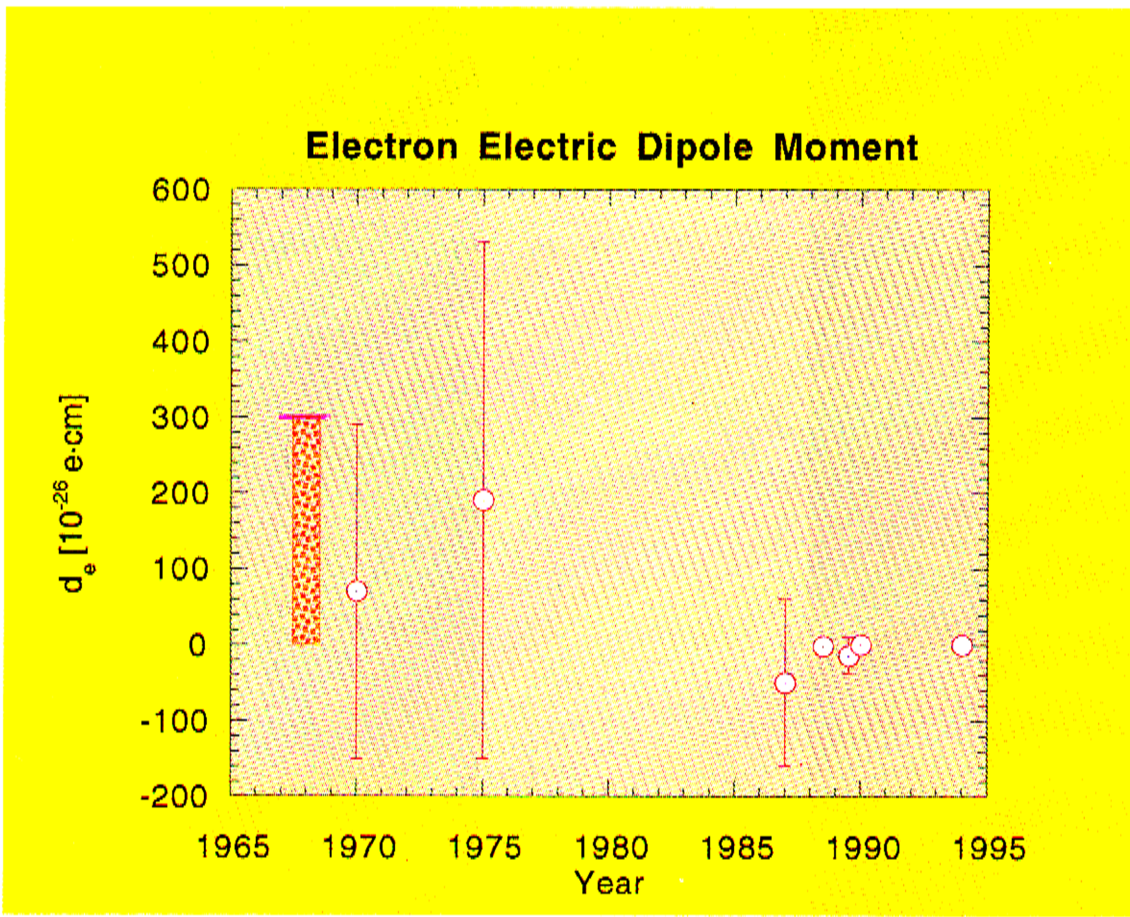


Fig. 2. The history of the experimental limit for the neutron EDM, along with some theoretical predictions. Since the observation of *CP* violation in K_0 decay (1964) the experimental limit for the EDM has been reducing at a constant rate. The first UCN EDM experimental results were reported in 1981–1982. The most recent limits, from the Petersburg Nuclear Physics Institute and from the Institut Laue-Langevin, are at the level of 10^{-25} *e* cm. For comparison, the sensitivity expected in the ^{199}Hg comagnetometer experiment is shown, along with that expected in our proposed ^{3}He–n comparison technique.

Current bound: $d_n < 0.97 \times 10^{-25}$ e·cm



Current bound: $(0.18 \pm 0.12 \pm 0.10) \times 10^{-26}$ e·cm

- Unequal particle/antiparticle transition rates

CP LEAR EXPERIMENT ('98)

PRODUCE K^0 OR $\bar{K}^0$
$(\bar{s}d)$ $(s\bar{d})$

IN THE REACTIONS

$$\bar{p}p \rightarrow \begin{array}{l} K^+\pi^-\bar{K}^0 \\ K^-\pi^+K^0 \end{array}$$

$K^\pm$ charge tags strangeness of $\overset{(-)}{K}{}^0$

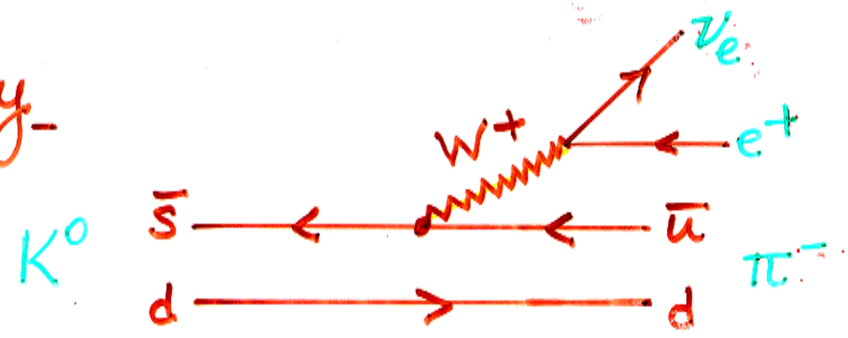
WEAK INTERACTIONS DON'T CONSERVE STRANGENESS, SO

K^0 AND $\bar{K}^0$ CAN MIX:

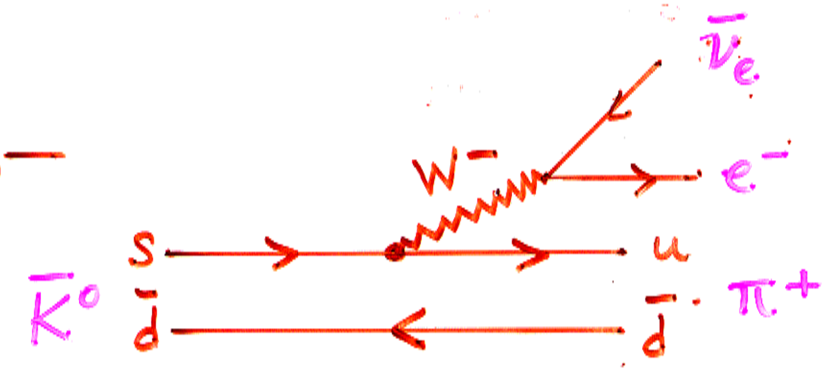
$$K^0 \leftrightarrow \bar{K}^0$$

Are the two rates equal?

K^0 decay



$\bar{K}^0$ decay



Measure

$$A \equiv \frac{\Gamma(\bar{K}^0_{t=0} \to e^+\pi^-\nu_{e\,t=\tau}) - \Gamma(K^0_{t=0} \to e^-\pi^+\bar{\nu}_{e\,t=\tau})}{+}$$

$$= (6.6 \pm 1.3_{stat} \pm 1.0_{syst}) \times 10^{-3}$$

Shows that

$$\Gamma(K^0 \to \bar{K}^0) \neq \Gamma(\bar{K}^0 \to K^0)$$

$\mathbb{T}$ VIOLATION!

CPLEAR A vs. τ

$$\frac{(K^0 \to \bar{K}^0) - (\bar{K}^0 \to K^0)}{+}$$

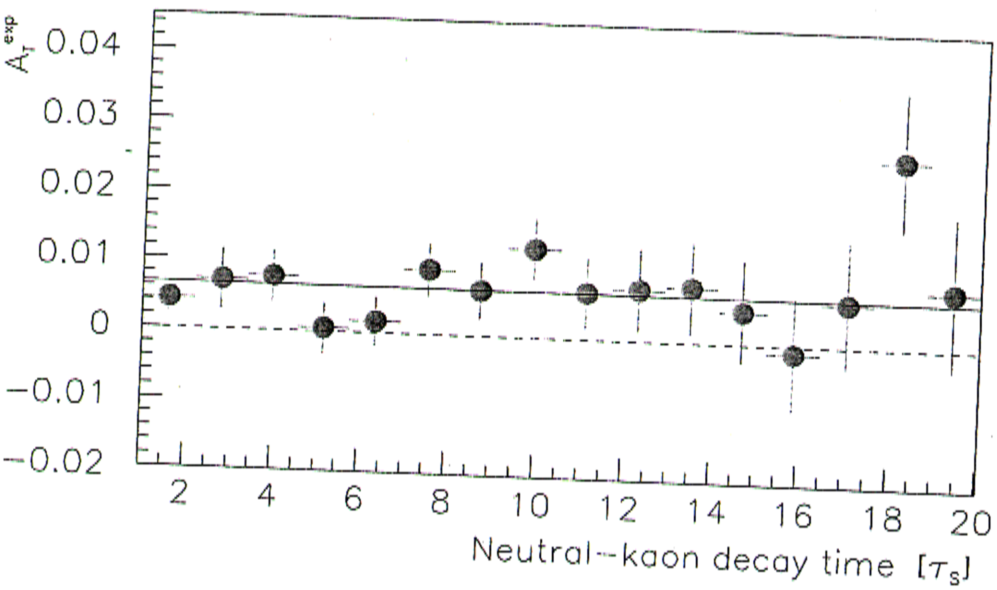


Figure 1: The asymmetry A_T^{exp} versus the neutral-kaon decay time (in units of τ_S). The solid line represents the fitted average $\langle A_T^{exp} \rangle$.

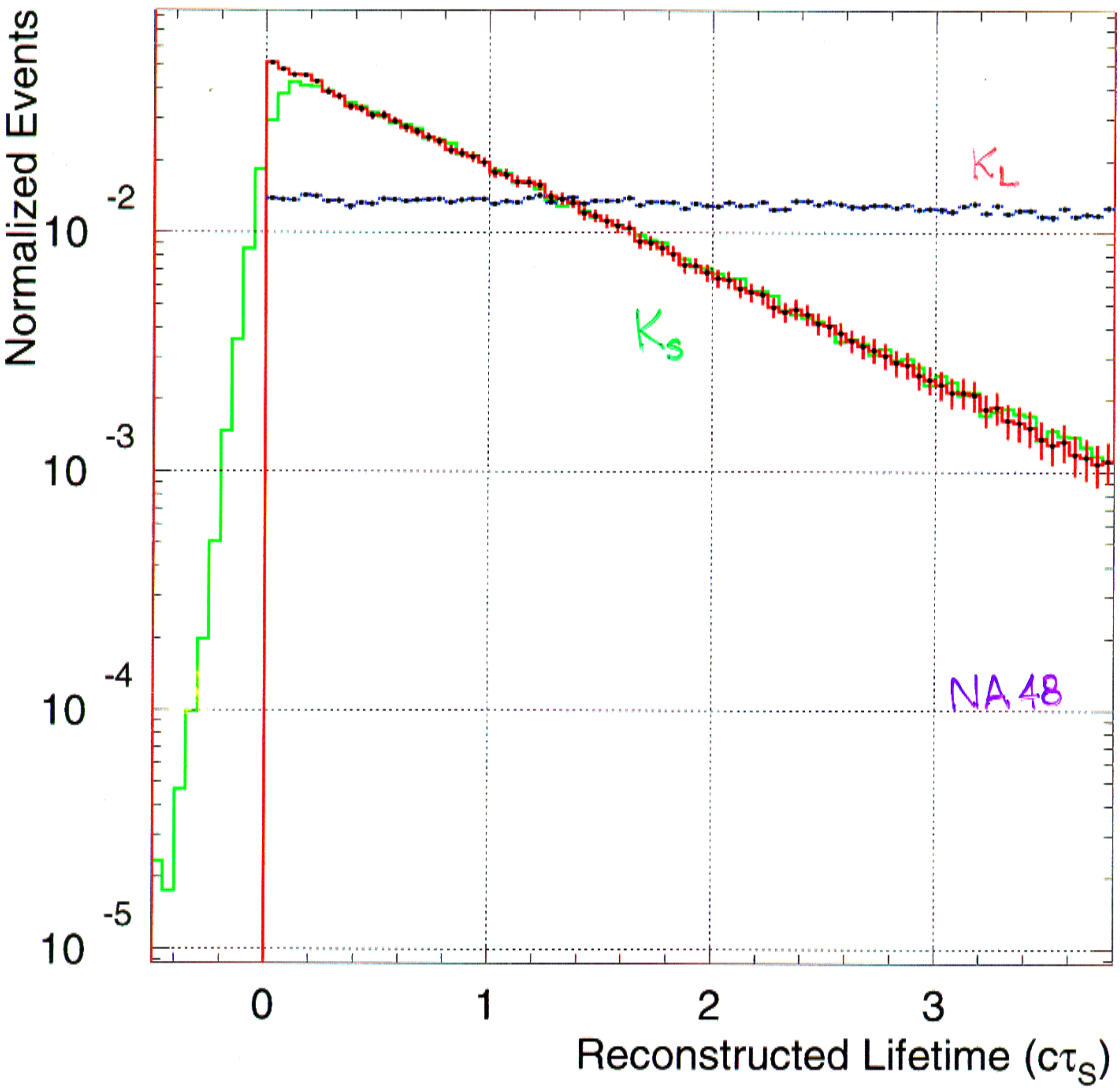
BECAUSE $K^0 \longleftrightarrow \bar{K}^0$, PHYSICAL EIGENSTATES ARE

$|K_S\rangle = (|K^0\rangle + |\bar{K}^0\rangle)/\sqrt{2}$ $\quad$ $CP\,|K_S\rangle = +|K_S\rangle$

$|K_L\rangle = (|K^0\rangle - |\bar{K}^0\rangle)/\sqrt{2}$ $\quad$ $CP\,|K_L\rangle = -|K_L\rangle$

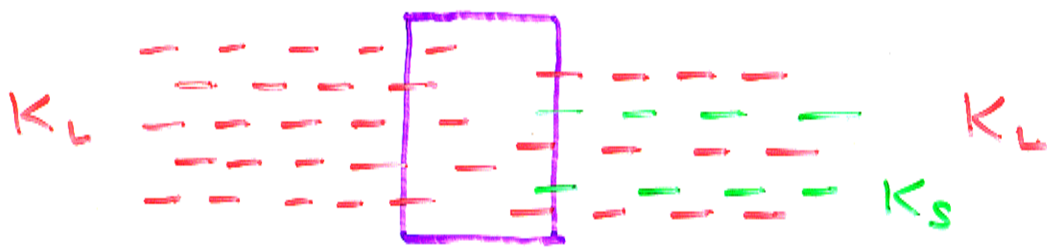
if CP $\begin{cases} K_S \to \pi\pi & \tau_S \simeq 0.9\times10^{-10}\,s & c\tau_S = 2.67\,cm \\ K_L \not\to \pi\pi & \tau_L \simeq 5.2\times10^{-8}\,s & c\tau_L \simeq 15.5\,m \\ \quad \to \pi\pi\pi & & \end{cases}$

$M_L - M_S \approx 3.5\ \mu eV$



TWO IMPORTANT PHENOMENA

- Regeneration
  - PRODUCE K^0 (equal mix of K_L, K_S)
  - WAIT $\gtrsim 10\,\tau_S$. BEAM IS PURE K_L
    $\Rightarrow$ equal mix of K^0, $\bar{K}^0$
  - PASS THROUGH A (THIN) OBSTACLE



$\bar{K}^0$ IS ABSORBED MORE THAN K^0

OUTPUT IS A MIXTURE OF

K_L, K_S "REGENERATED"

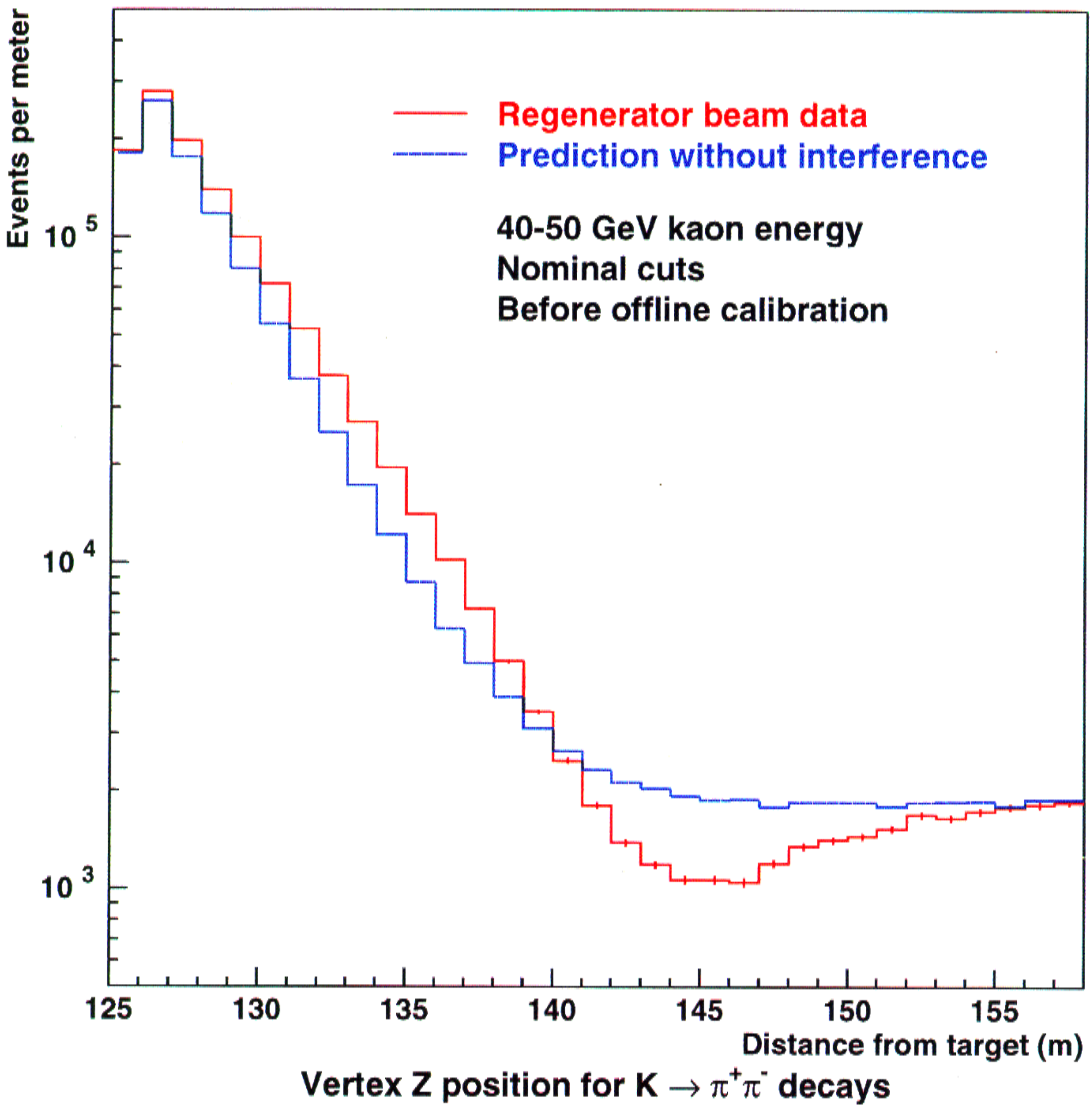
- • $\cancel{CP}$ DECAY $K_L \to \pi\pi$ (~ 0.3%)

  SMALL EFFECT, UNLIKE $\cancel{C}$ OR $\cancel{P}$

  $\Rightarrow$ $K_L - K_S$ INTERFERENCE

KTeV at Fermilab

First look at E832 1996 run: kaon interference



Within the EW theory we have developed, the gauge interactions respect CP invariance. The only place CP violation can arise is in the mixing of the quark mass eigenstates to form the flavor (W.I.) eigenstates. The matrix $\mathbf{V}$ in the charged current

$$(\bar{u}, \bar{c}, \bar{t}, \ldots)\, \gamma_\mu (1-\gamma_5)\, \mathbf{V} \begin{pmatrix} d \\ s \\ b \\ \vdots \end{pmatrix}$$

must contain complex elements.

≥ 3 families, and

$m_u \neq m_c \neq m_t \cdots \qquad m_d \neq m_s \neq m_b \cdots$

KOBAYASHI · MASKAWA

n×n quark mixing matrix has

$n(n-1)/2$ quark mixing ∡s

$(n-1)(n-2)/2$ complex phases

IS THE QUARK MIXING MATRIX THE (ONLY) ORIGIN OF ~~CP~~ ?

- Direct and indirect ~~CP~~ for neutral Kaons: NA48/KTeV/KLOE...
- ~~CP~~ violation in hyperon decay: Hyper-CP
- ~~CP~~ in B decays: Babar/Belle/CDF/DØ/HERA-B ...

1998 CERN Summer Student Lecture Programme

Particle Physics: The Standard Model

Chris Quigg

* * * * *

Outline a "three-neutrino experiment" to establish that a neutral, penetrating beam of ν_τ materializes into τ upon interacting in matter. [For background, look at the first two-neutrino experiment, J. Danby, *et al.*, *Phys. Rev. Lett.* **9**, 36 (1962). See also the Nobel Lectures of "Murder, Inc.," (as they called themselves), Mel Schwartz, Jack Steinberger, and Leon M. Lederman, reprinted in *Rev. Mod. Phys.* **61**, 527, 533, 547 (1989).] What would provide a copious source of ν_τ? What energy would be advantageous for the detection of the produced τ? What characteristics would be required of the detector? What are the important backgrounds, and how would you handle them? Some information about a three-neutrino experiment in analysis at Fermilab can be found at `http://fn872.fnal.gov`.

Particle Physics:
The Standard Model

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CERN Summer Lectures
17 – 27 July 2000

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UNIFIED THEORIES

First 7 lectures: Things that are true

Lecture 8: Things I would like to be true

TWO THEMES

(1) The idea of unification

... of the fundamental forces

... of the basic constituents

«GENERALIZATION»

Will argue from our knowledge of the standard model for the inevitability – subject to experimental check – of a UNIFIED THEORY of

quarks + leptons

strong, weak, EM interactions

(2) The human scale is not privileged for understanding Nature, and may even be disadvantaged.

NOT A SUDDEN REALIZATION

1920s Quantum Mechanics:

to understand

- why a rock is solid

OR

- why a metal gleams

we must understand its structure on a scale $10^9 \times$ SMALLER than the human scale, and we must understand the rules that prevail there.

WHAT IS NEW: Our comfort at cruising between different scales of momentum and distance. (Renormalization group)

Provocative formulation

The discovery that the human scale is not preferred is as important as the discoveries that

- our human location is not privileged (Copernicus, Galileo, Newton)

and

- there is no preferred inertial frame (Einstein)

and will prove to be as influential.

It may well be that

certain scales are preferred for understanding particular globally important aspects of Nature

- why (how it came to be that) $\alpha \approx 1/137$ and $\alpha_s(5\text{ GeV}) \approx 1/5$
- why quark and lepton masses have the seemingly unintelligible pattern they do.

WHY UNIFY?

- Quarks and leptons are structureless spin-½ particles. (How) are they related?

- EW universality:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

- Anomaly cancellation requires quarks and leptons.

- Can the three distinct coupling parameters $(\alpha_s, \alpha_{EM}, \sin^2\theta_W)$ be reduced to 2 or 1?

- We have seen that α_{EM} increases with Q^2, while α_s decreases. Is there a unification point where all couplings become equal?

... why unify?

- Why is charge quantized?

$$Q(p) + Q(e) = 0$$

$$Q(\nu) - Q(e) = Q(u) - Q(d)$$

$$Q(d) = \frac{1}{3} Q(e)$$

$$Q(\nu) + Q(e) + 3Q(u) + 3Q(d) = 0$$

THESE QUESTIONS LEAD US TOWARD

- A QUARK-LEPTON CONNECTION
- A MORE COMPLETE ELECTROWEAK UNIFICATION,

  (simple) $G \supset SU(2)_L \otimes U(1)_Y$
- "GRAND" UNIFICATION OF THE STRONG, WEAK, AND EM INTERACTIONS

$$G \supset SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

⋮

TOWARD A UNIFIED THEORY

Seek a unifying symmetry group

$$G \supset SU(3) \otimes SU(2) \otimes U(1)$$

that contains the known int$^{\underline{ns}}$.

SMALLEST CANDIDATE IS SU(5)

As usual, gauge bosons are contained in the adjoint (24-dimensional) representation.

Label SU(5) states by their

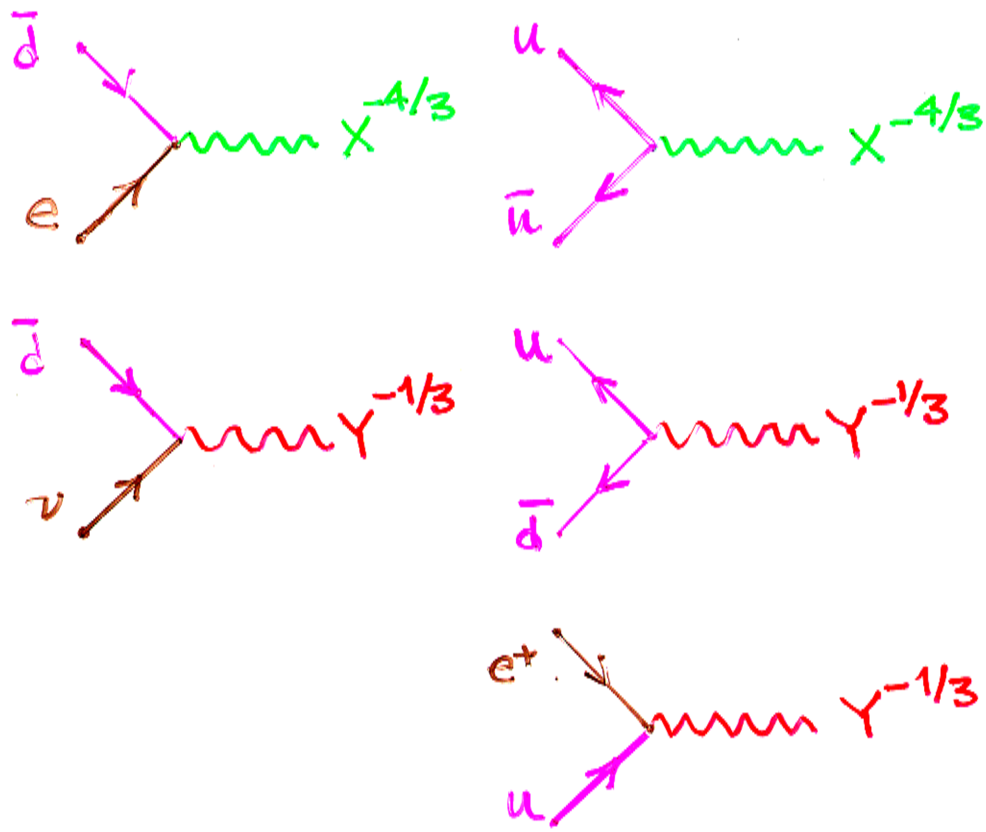
$(\underset{\sim}{SU(3)}, \underset{\sim}{SU(2)})_Y$ content

$$\underset{\sim}{24}\left\{\begin{array}{ll} (\underset{\sim}{8}, \underset{\sim}{1})_0 & \text{gluons} \\ (\underset{\sim}{3}, \underset{\sim}{2})_{-5/3} & \rule{0pt}{0pt} \\ (\underset{\sim}{3}^*, \underset{\sim}{2}^*)_{5/3} & \left.\right\} \text{12 new gauge bosons (consequence of unification)} \\ (\underset{\sim}{1}, \underset{\sim}{3})_0 & b_1, b_2, b_3 \;\; \Big\} \;\; W^+ \; W^- \\ (\underset{\sim}{1}, \underset{\sim}{1})_0 & a \;\; \Big\} \;\; Z^0 \; A \end{array}\right.$$

Recall that the price (or reward!) of EW unification was a new interaction, weak neutral current

Generally true that new interactions are required to complete the unifying group.

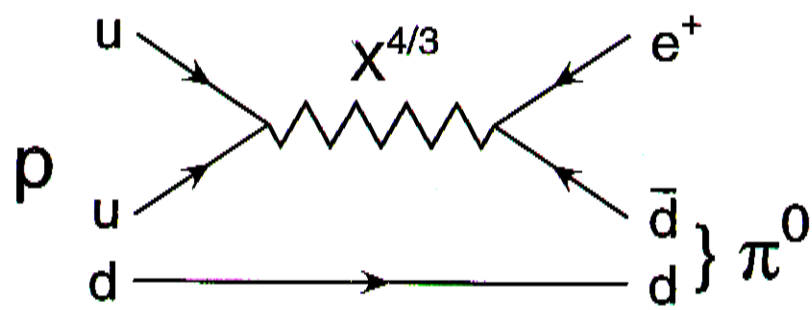
NEW GAUGE BOSONS ARE "LEPTOQUARKS" MEDIATE THE TRANSITIONS



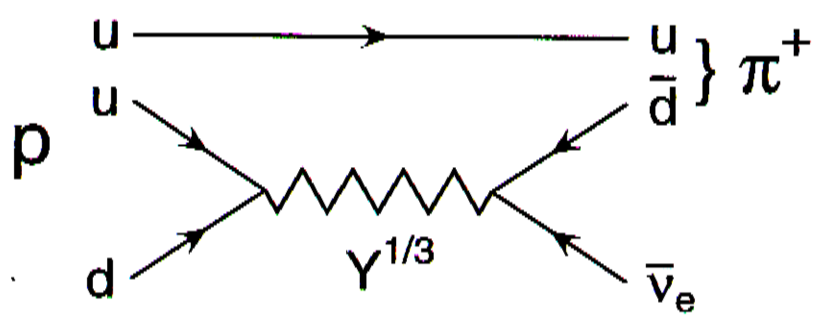
THESE NEW INTERACTIONS CAN MEDIATE

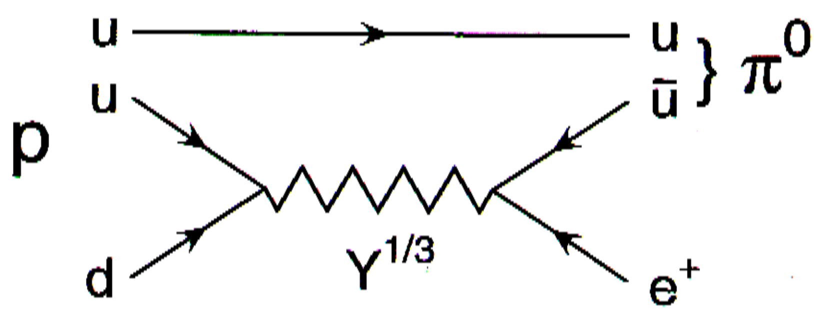
PROTON DECAY

X, Y MUST ACQUIRE VERY LARGE MASSES, OR PROTON DECAYS TOO FAST



expt: $\tau(p \to e^+\pi^0) > 5.5 \times 10^{32}$ y





Processes that change LEPTON NUMBER or BARYON NUMBER are essential to understand the MATTER EXCESS in the Universe

INTERACTIONS IN SU(5)

SU(5) IS A SIMPLE GROUP

⇒ A SINGLE COUPLING CONSTANT, g_5

The SU(5) unified theory prescribes the relative normalization of

g (now call g_2) $SU(2)_L$

g' $U(1)_Y$

$$\boxed{g'^2 = \frac{3}{5} g_2^2}$$

From our definition,

$$g' = g_2 \tan\theta_W$$

we see that $\tan^2\theta_W = \frac{3}{5}$,

so that

$$\sin^2\theta_W = \frac{3}{8}$$

in unbroken SU(5)

Coupling Constant Unification

Consider the evolution of the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ couplings.

STRONG:

$$1/\alpha_3(Q^2) = 1/\alpha_3(\mu^2) + b_3 \log(Q^2/\mu^2)$$

$$4\pi b_3 = 11 - 4n_g/3 \qquad b_3 = 7/4\pi$$

$SU(2)_L$:

$$1/\alpha_2(Q^2) = 1/\alpha_2(\mu^2) + b_2 \log(Q^2/\mu^2)$$

$$4\pi b_2 = \frac{22-4n_g}{3} - \frac{n_{Higgs}}{6} \qquad b_2 = \frac{19}{24\pi}$$

$U(1)_Y$:

$$1/\alpha_Y(Q^2) = (5/3)\cdot 1/\alpha_1(Q^2)$$
$$= 1/\alpha_Y(\mu^2) + (5/3)\cdot b_1 \log(Q^2/\mu^2)$$

$$1/\alpha_1(Q^2) = 1/\alpha_1(\mu^2) + b_1 \log(Q^2/\mu^2)$$

$$4\pi b_1 = -\frac{4n_g}{3} - \frac{n_H}{10} \qquad b_1 = -\frac{41}{40\pi}$$

Running couplings ...

SUPPOSE THAT THE COUPLINGS ARE UNIFIED AT SOME HIGH SCALE u:

$$1/\alpha_1(u^2) = 1/\alpha_2(u^2) = 1/\alpha_3(u^2) \equiv 1/\alpha_U$$

ESTIMATE u AND FIND $1/\alpha_U$.

Since we know

$$1/\alpha(M_Z^2) = 128.89 \pm 0.09$$

and

$$1/\alpha_3(M_Z^2) = 8.75 \pm 0.22$$

we can extrapolate

$$1/\alpha(Q^2) \equiv 1/\alpha_Y(Q^2) + 1/\alpha_2(Q^2)$$

$$= (8/3)\cdot 1/\alpha_u + (b_Y + b_2)\log\left(\frac{Q^2}{u^2}\right)$$

and $(8/3)\cdot 1/\alpha_3(Q^2)$

$$\Rightarrow \quad u \simeq 1.2 \times 10^{15} \text{ GeV}$$

$$1/\alpha_u \simeq 42.1$$

Now, we can predict $\sin^2\theta_W$

$$x_W \equiv \sin^2\theta_W = \alpha/\alpha_2$$

$$= \frac{1/\alpha_2}{1/\alpha_Y + 1/\alpha_2}$$

Compute

$$x_W(Q^2) = \frac{3}{8} - \underbrace{\frac{5}{8}(b_1 - b_2)}_{109/96\pi}\alpha(Q^2)\log\left(\frac{Q^2}{u^2}\right)$$

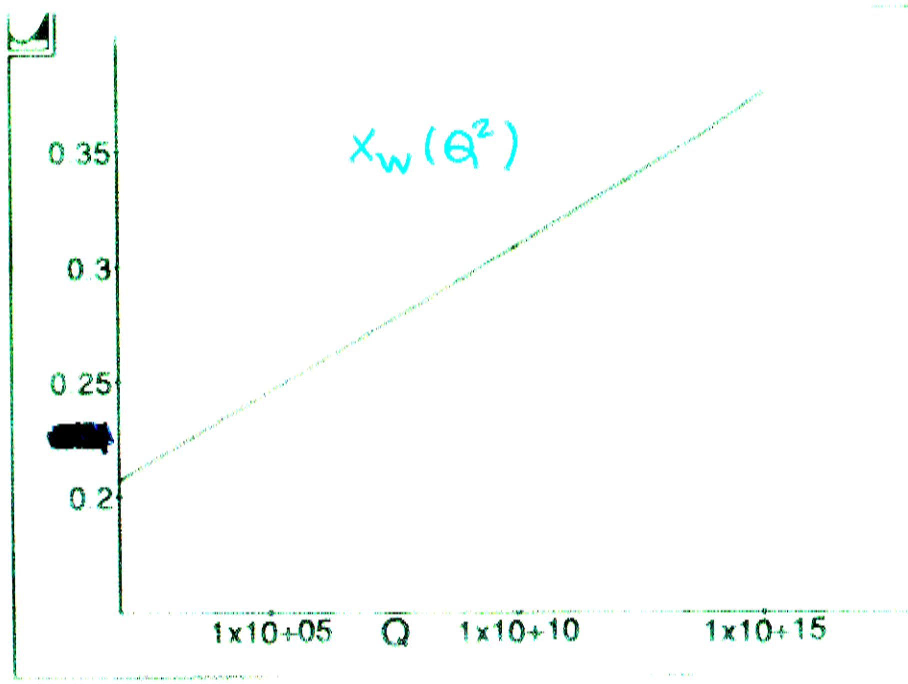
$$x_W(M_Z^2) = 0.207$$

"close to" BUT FAR FROM MEASURED

$$x_W = 0.231$$

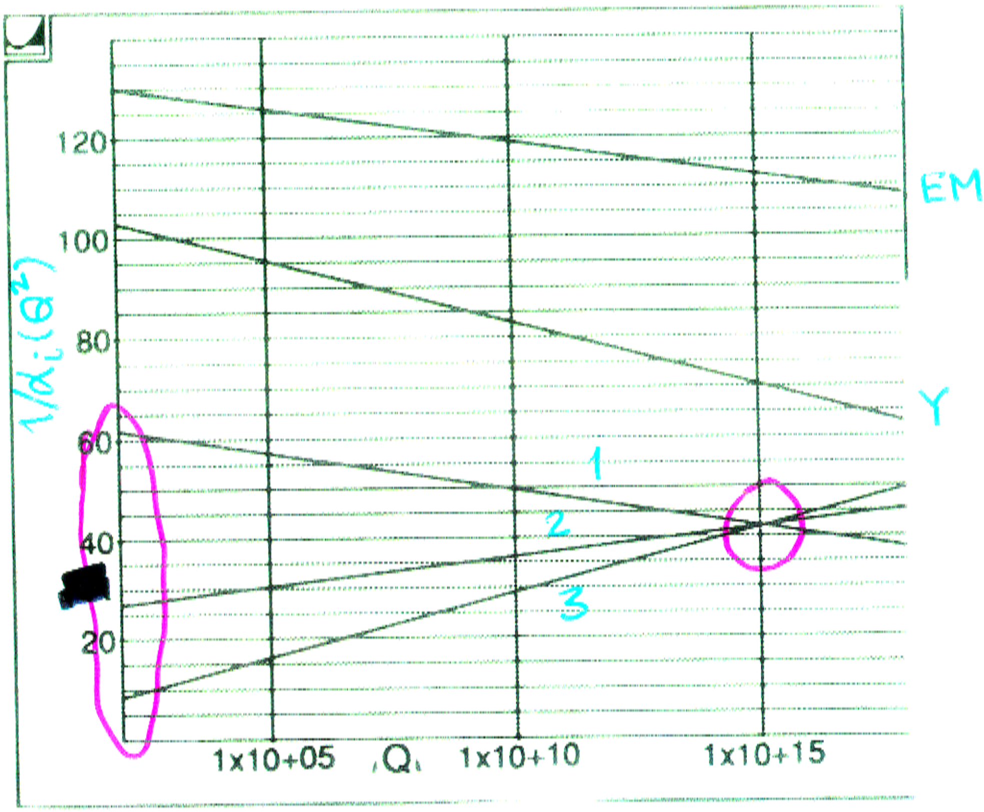
a remarkable prediction, but not quite a quantitative triumph.

SU(5)



195th

IDEALIZED SU(5)



Couplings in idealized SU(5)

COMPUTE

$$1/\alpha_2(M_Z^2) = x_W(M_Z^2) \cdot 1/\alpha(M_Z^2)$$
$$= 26.89$$

$$1/\alpha_Y(M_Z^2) = 103.00 = 1/\alpha - 1/\alpha_2$$

$$1/\alpha_1(M_Z^2) = 0.6 \cdot 1/\alpha_Y(M_Z^2)$$
$$= 61.80$$

FIG

Couplings in idealized SU(5)

COMPUTE

$$1/\alpha_2(M_Z^2) = x_W(M_Z^2)\cdot 1/\alpha(M_Z^2)$$
$$= 26.89 \quad (30.06)$$

$$1/\alpha_Y(M_Z^2) = \underset{(99.83)}{103.00} = 1/\alpha - 1/\alpha_2$$

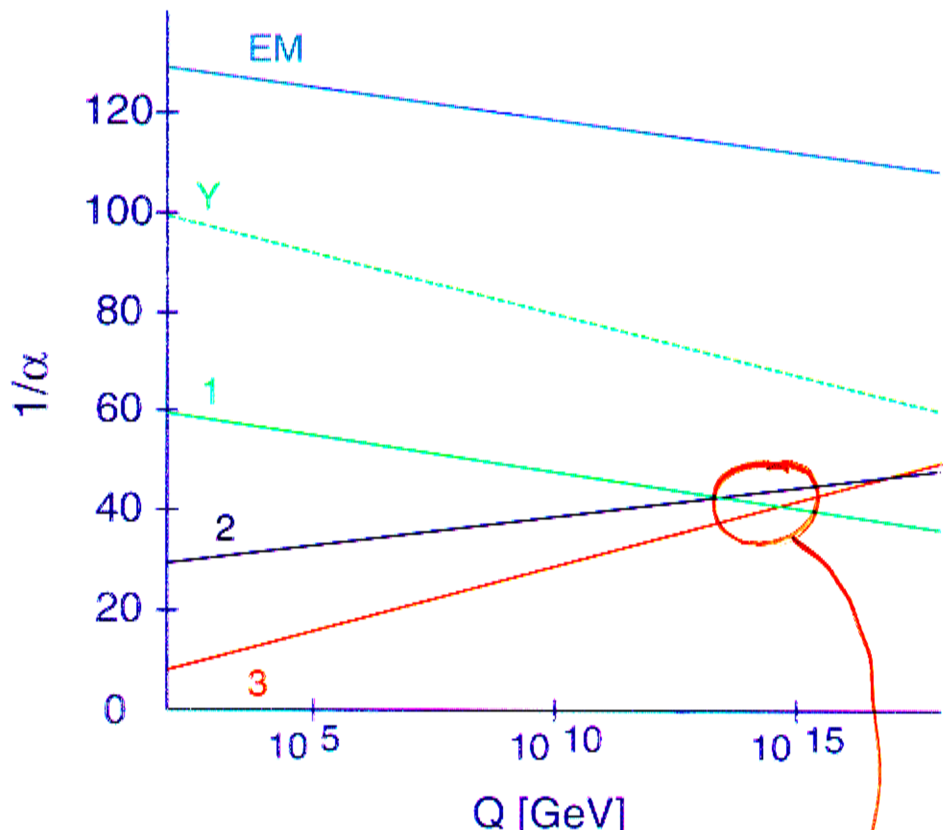
$$1/\alpha_1(M_Z^2) = 0.6\cdot 1/\alpha_Y(M_Z^2)$$
$$= 61.80 \quad (59.90)$$

FIG

USING THE MEASURED $x_W = 0.231$,
FIND (EXPERIMENT)

Close, but not perfect, agreement.

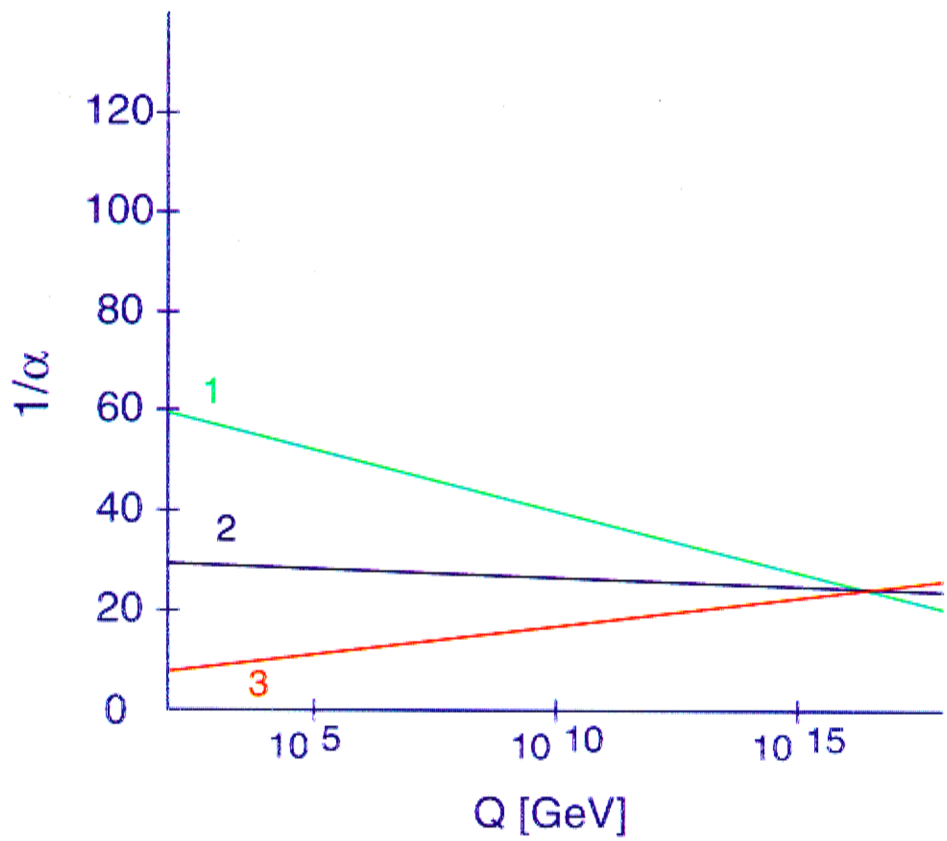
EVOLVE USING THE MEASURED COUPLINGS: SU(5) + experimental inputs



$\alpha_1^{-1}, \alpha_2^{-1}, \alpha_3^{-1}$ do not meet in a single point

Equivalent to the observation that the SU(5) prediction for $x_w(M_Z^2)$ misses the mark.

In supersymmetric SU(5) ...



FERMION MASSES IN SU(5)

Still no specific predictions for individual masses.

Depending on pattern of SYMMETRY BREAKING, expect relations among fermion masses at the unification scale.

ONE SIMPLE PATTERN:

$\underset{\sim}{24}$ of scalars breaks

$$SU(5) \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

and gives (very large) masses to $X^{\pm 4/3}, Y^{\pm 1/3}$

Then $\underset{\sim}{5}$ of scalars breaks

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_{EM}$$

and endows fermions with mass.

PREDICTION:

$$\left.\begin{aligned} m_e &= m_d \\ m_\mu &= m_s \\ m_\tau &= m_b \end{aligned}\right\} \text{at the unification scale}$$

m_u, m_c, m_t are separate parameters

RUNNING MASSES

$$\ln[m_{u,c,t}(\mu)] \simeq \ln[m_{u,c,t}(u)] + \frac{12}{33-2n_f}\ln\left(\frac{\alpha_3(\mu)}{\alpha_u}\right) + \frac{27}{88-8n_f}\ln\left(\frac{\alpha_2(\mu)}{\alpha_u}\right) - \frac{3}{10n_f}\ln\left(\frac{\alpha_1(\mu)}{\alpha_u}\right)$$

$$\ln[m_{d,s,b}(\mu)] = \ln[m_{d,s,b}(u)] + \frac{12}{33-2n_f}\ln\left(\frac{\alpha_3(\mu)}{\alpha_u}\right) + \frac{27}{88-8n_f}\ln\left(\frac{\alpha_2(\mu)}{\alpha_u}\right) + \frac{3}{20n_f}\ln\left(\frac{\alpha_1(\mu)}{\alpha_u}\right)$$

$$\ln[m_{e,\mu,\tau}(\mu)] = \ln[m_{e,\mu,\tau}(u)] + \frac{27}{88-8n_f}\ln\left(\frac{\alpha_2(\mu)}{\alpha_u}\right) - \frac{27}{20n_f}\ln\left(\frac{\alpha_1(\mu)}{\alpha_u}\right)$$

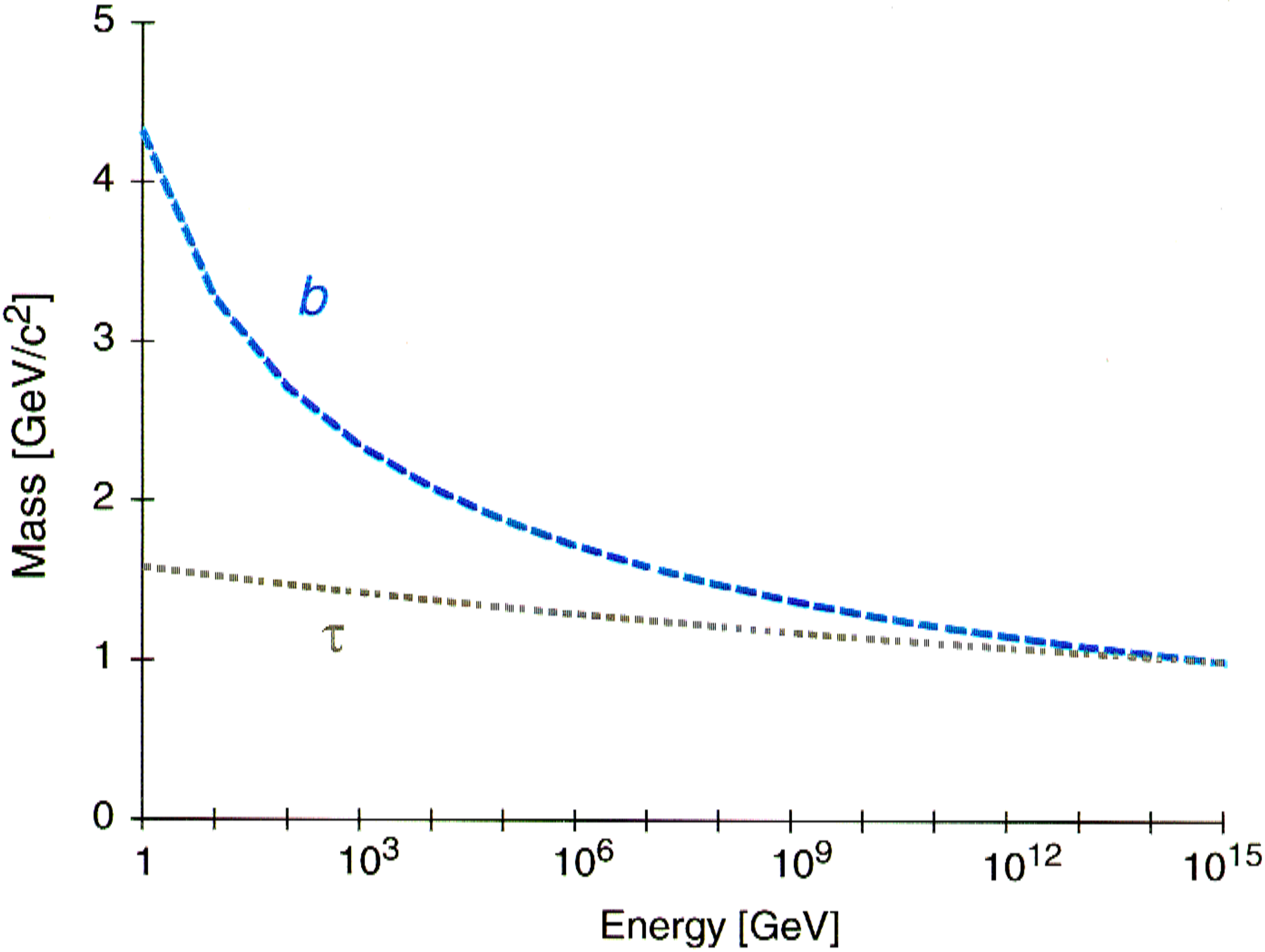
Quark masses run visibly
Lepton masses run little

$$\ln\left[\frac{m_b(\mu)}{m_\tau(\mu)}\right] \approx \cancel{\ln\left[\frac{m_b(u)}{m_\tau(u)}\right]} + \frac{12}{33-2n_f}\ln\left(\frac{\alpha_3(\mu)}{\alpha_u}\right) + \frac{3}{2n_f}\ln\left(\frac{\alpha_1(\mu)}{\alpha_u}\right)$$

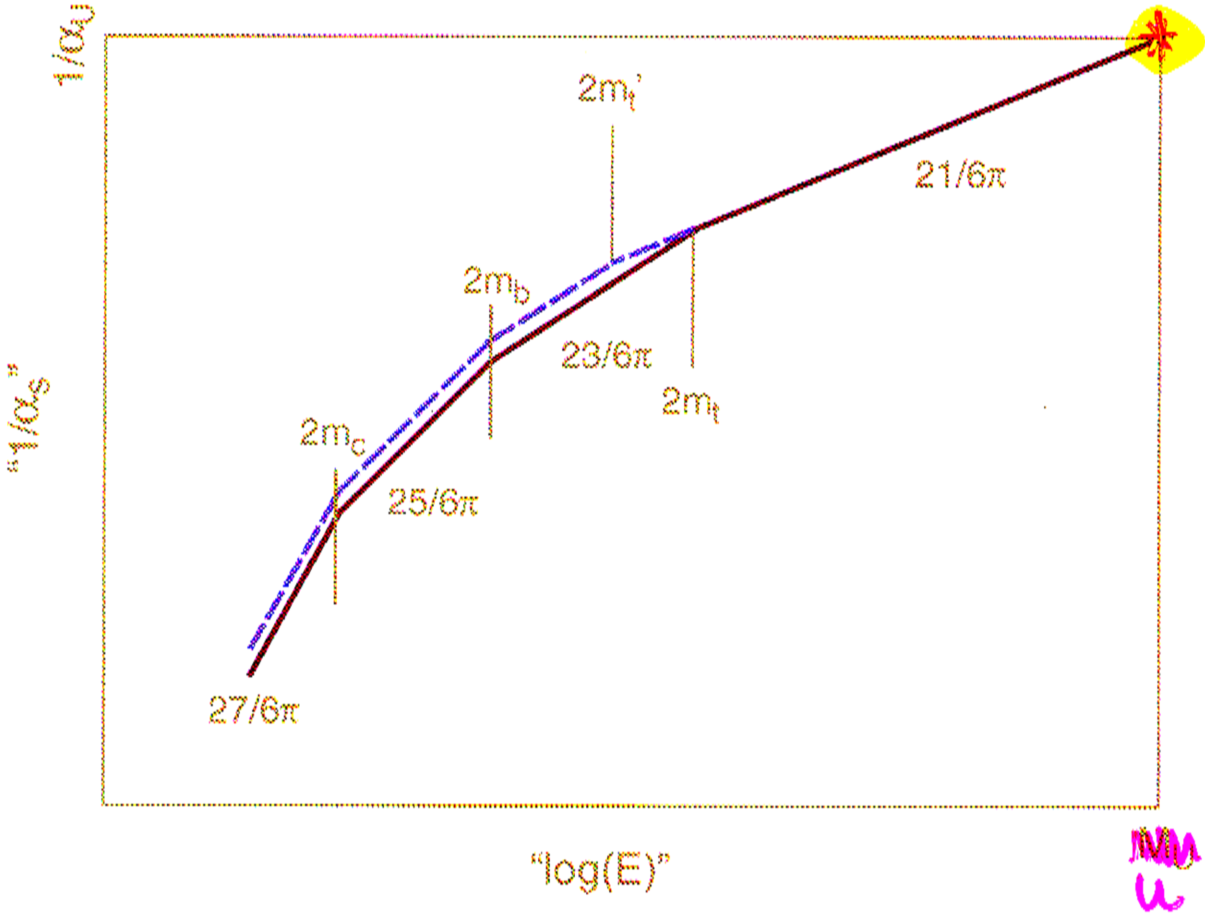
with $n_f = 6$, $1/\alpha_u = 40$, $1/\alpha_3(\mu) = 5$, $1/\alpha_1(\mu) = 65$

COMPUTE (PREDICT!)

$$m_b = 2.91\, m_\tau \approx 5.2 \ \mathrm{GeV}/c^2$$


5
4
3
2
1
0
Mass [GeV/c²]
b
τ
1
10³
10⁶
10⁹
10¹²
10¹⁵
Energy [GeV]

m_t INFLUENCES α_s



SMALLER $m_t \longleftrightarrow$ SMALLER α_s

How does Λ_{QCD} depend upon m_t?

Calculate $\alpha_s(2m_t)$ evolving up from low energies and down from the unification scale, and match:

$$1/\alpha_U + \frac{21}{6\pi}\ln(2m_t/M_U) = 1/\alpha_s(2m_c) - \frac{25}{6\pi}\ln(m_c/m_b) - \frac{23}{6\pi}\ln(m_b/m_t)$$

Identifying

$$1/\alpha_s(2m_c) \equiv \frac{27}{6\pi}\ln(2m_c/\Lambda_{\text{QCD}}) \; ,$$

we find that

$$\Lambda_{\text{QCD}} = e^{-6\pi/27\alpha_U}\left(\frac{\overset{u}{\cancel{M_U}}}{1\ \text{GeV}}\right)^{21/27}\left(\frac{2m_t\cdot 2m_b\cdot 2m_c}{1\ \text{GeV}^3}\right)^{2/27}\ \text{GeV} \; .$$

We have learned from (lattice) QCD that

$$M_{\text{proton}} \approx C\Lambda_{\text{QCD}}$$

$$\boxed{\frac{M_{\text{proton}}}{1\ \text{GeV}} \propto \left(\frac{m_t}{1\ \text{GeV}}\right)^{2/27}}$$

All this is but a dream.

Still, examine it by a few experiments.

Nothing is too wonderful to be true,
if it be consistent with the laws of nature
and in such things as these,
Experiment is the best test of such
consistency.

Michael Faraday
Research notes, 19th March 1849